TWO CONTRIBUTIONS OF JAPANESE LESSON STUDY FOR THE MATHEMATICS TEACHER EDUCATION: THE EFFECTIVE TERMINOLOGY FOR DESIGNING LESSONS AND AS A DRIVING FORCE TO PROMOTE SUSTAINABLE STUDY GROUPS

Duas contribuições da Lesson Study japonesa para a formação de professores de matemática: a terminologia eficaz para planejamento de aulas e como força promotora de grupos de estudo sustentáveis

Masami Isoda

Raimundo Olfos

Soledad Estrella

Yuriko Y. Baldin

Abstract

Along the history of 150 years, Japanese Lesson Study has brought contributions to the mathematics education of contemporary times, two of them presented in this paper as the education of schoolteachers to understand *the effective lesson planning* aligned to a school curriculum, and *the meaning of a sustainable study group* to implement Lesson Study outside Japan. The investigation considered the Lesson Study principles, in which the establishment of a common terminology for the mathematics lesson planning is found as essential to sustain the implementation of Study Groups. The paper presents a structure of curriculum design with the perspective of the development of mathematical thinking, illustrated with an example of the terminology for the multiplication in primary school textbook, and the organizational route that enabled the establishment of a sustainable Study Groups in Lesson Study in Chile. The paper presents reflections on the possibilities to work Lesson Study outside Japan.

Keywords: Lesson Study; Effective lesson planning; Mathematical thinking; Lesson Study Group; Terminology and curriculum design.

Resumo

Após evoluir por 150 anos, a Lesson Study Japonesa está trazendo contribuições para a educação matemática dos tempos contemporâneos. Duas delas são apresentadas neste artigo: a formação de professores para entender *o planejamento eficaz das aulas*, alinhadas a um currículo escolar, e *o significado de um grupo de estudo sustentável* para implementar a Lesson Study fora do Japão. A investigação considerou os princípios da Lesson Study em que o estabelecimento de uma terminologia comum para o planejamento da aula de matemática é considerado essencial para sustentar a implementação de Grupos de Estudos. O artigo apresenta um esquema de desenho curricular na perspectiva de desenvolvimento do pensamento matemático, com exemplo da terminologia para a multiplicação no ensino básico e um percurso organizacional na criação de um Grupo de Estudos sustentável em Lesson Study no Chile. O artigo apresenta reflexões sobre as possibilidades de trabalhar Lesson Study fora do Japão.

Palavras-chave: Lesson Study; Planejamento eficaz de aulas; Pensamento matemático; Grupo de Estudos de Lesson Study; Terminologia e desenho curricular.

Introduction

The attention of mathematics education researchers, especially of those focused on the investigation of Teacher Education issues, has increasingly been driven to the efficacy of the Japanese Lesson Study on the professional development courses and on the students´ mathematics learning activities in the classrooms highlighted after the publication Stigler and Hiebert (1999) that followed the International Mathematics and Science Study (TIMSS) in 1995. The core principle

of Japanese Lesson Study (LS) has been, since its origin in 19th century, the effective learning of the students through the classroom activities, especially with the problem solving (ISODA, 2015a), described by Stigler and Hiebert (1999) as the Problem Solving Approach that constitutes today the well-known theory of Japanese teaching approach, one aspect of this theory was discussed in the Panel 1 of the International Seminar of Lesson Study in Teaching of Mathematics (SILSEM), in May 2021.

Expanding the ideas presented in the Panel, this paper aims to focus on two crucial contributions of the Lesson Study (LS) to the Mathematics Teacher Education, completing with more details the abstracts in the Proceedings of the SILSEM. For this purpose, it is necessary to mention the historical context of the beginning of the now called Lesson Study in Japan.

The Lesson Study in Japan started in 1873 with the establishing of the Education code and the Normal School for the preparation of teachers to a new school system that demanded innovated discourse management in students´ centered learning of the curriculum content (ISODA et al., 2007). In Brazil and many other countries, LS in mathematics is often regarded only as a methodological process of "investigating" a lesson, whereas the Lesson Study in Japan has always been a *research activity* of the teachers with the support of *the educational system* aiming at the development of teacher to focus on the learning of students in the classroom lessons (SHIMIZU; CHINO, 2015). Therefore, it is necessary to understand the challenges coming out from the cultural differences in educational contexts that would promote a significant implementation of Lesson Study projects taking the best of its core principles. Among many dimensions of LS that can be considered as the legacy from its evolution in time, this paper highlights two: -the importance of a defined terminology for the concepts and procedures of the basic mathematics teaching and the development of a curriculum; - the dimension of characterizing an organizational structure of school system in local contexts that allows an effective implementation of LS in cultures outside Japan.

The section 2 of this paper elaborates the concept of a *common terminology* that underpins the design of an effective school mathematics curriculum, at the level of primary education. The terminology supports the lesson planning throughout the disciplinary content, along with the competency of the teacher/instructor to realize the lesson plan as well as to communicate the ideas precisely. The education of teachers who use a terminology with exact meanings is crucial for the development of school textbooks and professional improvement (ISODA, 2015a; ISODA; OLFOS, 2021). The section 2 retrieves historically the meaning of the research activity done by teachers to ground the objective of the terminology in theorizing the teacher learning to teach. An example of the importance of the terminology for the adequacy of the school mathematics textbooks is illustrated by the comparison between two textbooks in Chile.

The section 3 is motivated by the exemplary implementation of a collaborative project of the implementation of LS in Chile, where the political structure of the educational system as well as the co-acting of an international collaboration network play important roles for the success of lasting and effective Lesson Study Groups (LSG). It is noteworthy that the process of the implementation of LSG in Chile and the activities in Chilean school classrooms, attends what Shimizu and Chino (2015) have already indicated as the three co´s that characterize the Japanese LS developed as a functional network: *cooperation, collaboration and co-acting*. The Asian Pacific Economic Cooperation (APEC) - Lesson Study project, led by the University of Tsukuba and Professor Masami Isoda together with APEC Countries leading representatives, is grounded by the principle of three co´s. Moreover, through the progress made in the implementation of LS project in those countries, the schematic theory of systematization of a basic terminology to

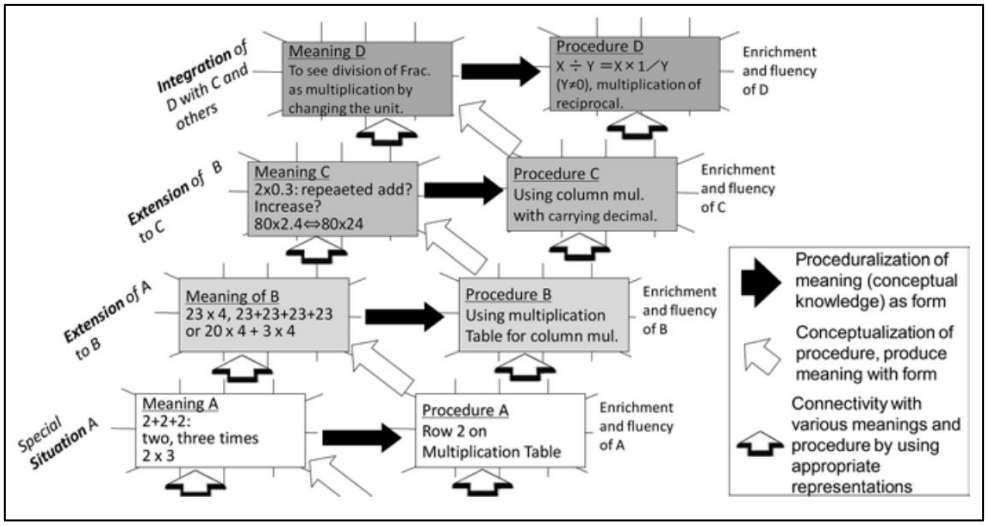
the concept of a school mathematics curriculum has emerged, enabling the indication of a profile for the professional development of pre and in service teachers. An outcome of this project grounded by this theory in the Southeast Asian Ministers of Education Organization (SEAMEO) is the standards for mathematics teacher education. This paper refers also to this aspect of APEC LS project as contribution to the theme of the Panel 1 of SILSEM. Moreover, the section 2 explains the theory of designing lessons to guide the interweaving of the levels of a productive lesson through the curriculum.

Terminology for Designing Lesson

The Elementary School at the University of Tsukuba, Japan, has originated Lesson Study since 1873 for the innovation of school education through the annual national lesson study meeting and several teacher training programs. Japanese original mathematics, Wasan, was established, during the 17th century, based on Chinese mathematics. It had established the tradition to pose the question each other for learning mathematics and to use mathematics textbooks for ordinary people that included several questions and cartoons to promote learning. On these bases, Japanese Problem Solving Approach has established what is currently well-known worldwide since the TIMSS-VIDEO (http://www.timssvideo.com/japan-mathematics-lessons) as well as the Dialectic Approach as in the pages 11-14 of (ISODA;OLFOS, 2021). For Lesson Study in Japanese mathematics education, the product is not only limited to these approaches which develop students who learn mathematics by and for themselves, called nowadays student agency (ORGANIZAÇÃO PARA A COOPERAÇÃO E DESENVOLVIMENTO ECONÔMICO, 2019), but also theories of mathematics education with the terminology for designing curriculum, textbook, and planning the lesson. Therefore, a key principle to design mathematics curriculum has been *extension and integration* since 1968 as reconsideration of the policy to design textbooks on mathematization (1943) and the policy to promote mathematical activity through the reflection (1947) for New Math Era. It was established earlier than the reinvention principle by Freudenthal, in 1973.

The principle of *extension and integration* was set for promoting the development of mathematical thinking. For example, in Japanese problem-solving approach, every teacher should plan their lesson based on what students already learned by setting unknown problem, which will be considerable if students use the already learned. Current mathematics educators may call it Zone of Proximal Development by Vygotsky. On the other hand, Japanese explain it by the sequence for designing classes, unit plan, yearly plan, textbooks across the grades and curricula. Indeed, this definition stimulate teachers to consider each class based on task(problem)-sequence, under the curriculum and textbook, for knowing what content of learning is actually new for students, who already learned something and when the new content will be used for learning further content.

Figure 1- The diagram of extension and integration of the process of multiplication in the task sequence: conceptual and procedural knowledge.



Source: Isoda and Olfos (2021, p. 8).

For designing this sequence, Japanese mathematics educators produced two types of terminologies. First type is a terminology to distinguish conceptual differences in the mathematical content of learning. In mathematics education, the learning trajectory commonly used for *a learning unit* is explained by the terms in such terminology. However, Japanese teachers do more than this, they also use the terminology to explain a long-term sequence going beyond the grades. In both cases, while explaining the trajectory, we must distinguish the conceptual difference in each part. Figure 1 shows the extension and integration process of multiplication, from single digit multiplication to fraction: every meaning and procedure are distinguished by terminology. Thus, first type of terminology explains the conceptual change processes.

Another terminology relates to developing the *mathematical thinking* under the task sequence on the extension and integration principle. This second type of terminology promotes the way to overcome inconsistency on the conceptual change processes. On Problem Solving Approach, it enhances firstly the analogical reasoning (or abduction) to approach an unknown problem or an extended problem by using what students already learned, and to promote deductive reasoning as well as inductive reasoning. To develop student agency, we must embed opportunities into the task-sequence for students to think mathematically, recursively. In discussing the theme of mathematical thinking as such, it had been categorized, in 1960´s, by several technical terms for explaining mathematical ideas, mathematical ways of thinking in general, mathematical values and attitude for mathematical mindset. The most well-known work of categorization was done by Shigeo Katagiri, which was translated into English (ISODA; KATAGIRI, 2012). As the leader of his lesson study groups, Katagiri edited more than 130 books for lesson study with his group of teachers who designed the classes to develop *mathematical thinking*. Each book includes 20 lessons and around 2,600 lesson study experiences are embedded in. However, the results of a lesson study are unusual to appear in a book, although if 5% appeared in the books by his efforts, we could estimate that his theory was demonstrated in more than 50,000 cases. In his theory, the category of mathematical thinking functions to design the appearance of *mathematical thinking on planned task sequence in classrooms* that enables students to recognize it as the entity (object) to be appreciated for the development of the mindset to think like it. His group of study demonstrated that students could think

mathematically in their classroom under an appropriate task-sequence to develop it.

Other mathematics educators also promoted lesson studies for problem solving approach. In 1989, Masami Isoda established his own lesson study group to demonstrate his theory of mathematics education for his dialectic approach, having published 10 books on this subject among his 57 books. The diagram of the developing sequence throughout a curricular topic shown in Figure 1 was originated in 1991 with his Sapporo lesson study group, which results were published in Japanese (https://s-opac.sap.hokkyodai.ac.jp).

Developing terminologies for local teachers: first international challenges and the case of Chile

All the works mentioned in the previous section were done in Japan. Since the 80´s, Japanese efforts began to be known outside Japan after the high achievement on the International Association for the Evaluation of Educational Achievement (IEA), especially in 1964 and 1981, which demonstrated Japanese high-quality education, and several international cooperation projects have been established since the 80´s. Existence of specific terminologies for teaching and learning was known to the researchers who visited Japan at that time. They were astonished by meeting ordinary teachers, not researchers, who use such terminologies, but they did not consider the adaptation to their countries because Lesson Study was not part of the major role of researchers who were training PhD students. Exceptional case is *the Open-ended Approach,* which was originally published in Japanese in 1977, and translated into English (BECKER; SHIMADA, 1997). At that time, to establish the national curriculum standards was a priority in USA and England. In the 90´s, the Japan International Cooperation Agency (JICA) started the first bilateral project for mathematics education in Philippines, officially, and Japanese mathematics educators began to transfer these terminologies. However, the adaptations of the terminologies to the local context constituted also challenges, because the country curriculum and textbooks were far from their experience in Japan. Around the Millennium, JICA project in Honduras began to adapt Japanese textbooks and managed the teacher training projects. International collaborations projects for textbooks have been expanded. Under these situations, APEC Lesson Study project (APEC-LS) started in 2006, and JICA Chile project was run at the same time. In 2000´s, the issues of 21st century curriculum pushed international curriculum synchronization, and every country developed their own curriculum through learning and comparing the curricula. Indeed, Isoda and Olfos compared seven countries' curricula for the multiplication, the first comparison been in the 00´s (ISODA; OLFOS, 2009) and the second comparison in 10´s (ISODA; OLFOS, 2021). In their comparison, the second one was more synchronized than the first comparison, even with differences. It is a result of the efforts by curriculum specialists and mathematics educators in each country who refereed other countries curriculum and the textbooks for their own reform activity. On this context of synchronization, a regional movement also started, notably the Southeast Asia- Basic Education Standards: Common Core Regional Learning Standards (SEA-BES CCRLS) in Mathematics, as the mathematics framework for Southeast Asia 11 countries. These standards included *the mathematical idea*, *thinking*, *value and attitude* (MANGAO; AHMAD; ISODA, 2017; GAN; ISODA; TEH, 2021) into the theory of designing curricula which constitutes a renovation of Katagiri's framework. Nowadays, we are coming into the era to develop, revise and share the terminologies for teachers who design curriculum, textbook, and lesson plans to be worked in the local theories of mathematics lesson study, in their language, educational system, and culture.

Under this context, the Center for Research on International Cooperation in Educational Development (CRICED) of the University of Tsukuba, Japan, collaborated with 7 major institutions in 7 countries and have been developing the online program for preservice and in-service teachers (https://www.criced.tsukuba.ac.jp/seameo_

online-program.html). From Chile, Raimundo Olfos and Soledad Estrella contributed having established a lesson study group as association, published three books for lesson study in Spanish and developed national textbooks with their own adaptations. The next section explains the *terminology* for introductory stage of multiplication for early years of the elementary education through a comparison of textbooks, between traditional edition and new edition, both accepted as national textbooks in Chile.

Terminology for conceptualization of multiplication in Chilean curriculum and texts

Typical Chilean teachers try to teach procedure, though there are also teachers trying to make mathematics sense for students. Nevertheless, to develop student agency, teachers need to establish task sequences for students aiming at the sense making (McCALLUM, 2018). Textbook should be a necessary tool to provide the task sequences. The newest National Curriculum in Chile (MINISTÉRIO DA EDUCAÇÃO DO CHILE, 2012) does not explain a specific definition for multiplication like in Japan, but it provides the following mathematics learning objective, Nr 11 for grade 2 students:

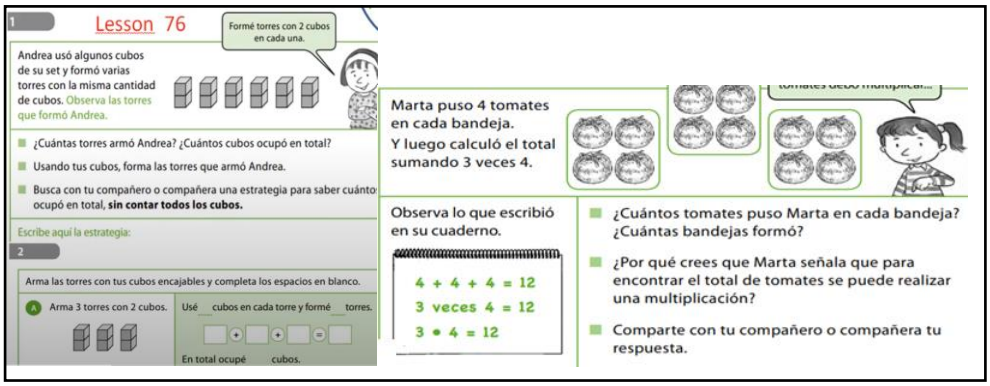
> Student shows **understanding** of multiplication: using concrete and pictorial representations; showing multiplication as an addition of **equal addends;** using distributivity as a strategy to build **tables** of 2, of 5 and of 10, solving problems that involve tables of 2, of 5 and of 10.
>
> (MINISTÉRIO DA EDUCAÇÃO DO CHILE, 2012, p. 232)

Here, addition of equal addends implicates accumulation. In grade 4, the term *factor* is currently introduced replacing the terms *multiplicand* and *multiplier* included in previous versions of the national curriculum. In the following, the lessons in two official Chilean textbooks will be explained with the use of terminology of multiplicand and multiplier in the case of introducing the multiplication for second grade students. First textbook, called here Textbook A, is traditional one and the second textbook, Textbook B, is an adaptation of a Gakko Tosho editorial version, which was managed by Masami Isoda and Soledad Estrella for Spanish translation and curricular adaptation (ISOLDA, 2021) during 2020 (OLFOS; ISODA; ESTRELLA, 2021).

In the Textbook A, different meanings of multiplication are introduced without explicit relations to the students´ knowledge: first one is accumulation, and second one is multiplication rows based on the property of commutativity represented by array/area diagram.

The question is to count how many towers of two blocks we have, as in the Figure 2: the task starts counting by two. The process refers to an accumulation and then transit to represent it as a multiplication. The solution to another situation of counting three baskets with four oranges in each basket is registered as 3 times 4. And the *dot* is used to represent the term *times* ($3 \bullet 4$). If we explain this notation by the terminology, the first number (3 *times*) is a multiplier and the second number (4) is the multiplicand.

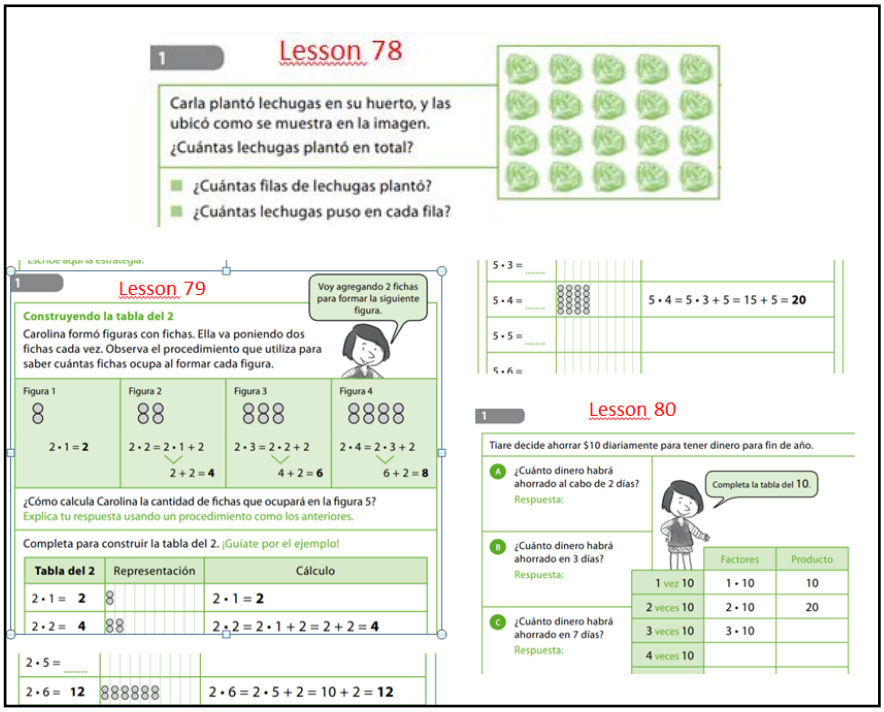
Figure 2- Introduction of multiplication (Textbook A)



Source: Ministério da Educação do Chile (2013, p. 7 ~ 14).

In the next Figure 3, after the commutativity been discussed, the meaning of multiplication is changed as $2 \times 4 = 2 \times 3 + 2$, instead of $2 \times 4 = 4 + 4$ which was learned as an accumulation in previous lessons. Then, $2 \times 4 = 2 \times 3 + 2 = (2 \times 2 + 2) + 2 = ((2 \times 1 + 2) + 2) + 2 = 2 + 2 + 2 + 2$. This implicates that 2 is multiplicand and '4 times' is multiplier. For the table with row 2, 2×4 is discussed as '2, 4 times' instead of '2 times 4'. It can be perceived that it is an inconsistency, however it is not said that there is a contradiction because it was learned *after the commutativity*. We observe that we changed the • sign in the book to the × sign in this text for the sake of familiarity and coherence with the rest of the text.

Figure 3- Introducing rows for multiplication (Textbook A)



Source: Ministério da Educação do Chile (2013, p. 7 ~14).

The second text, called here Textbook B, which corresponds to an adaptation of a publication from the Gakko Tosho editorial, considers the following approach to the conceptualization of multiplication. To avoid the inconsistency

of Textbook A, the order of rows is changed as in Figure 4.

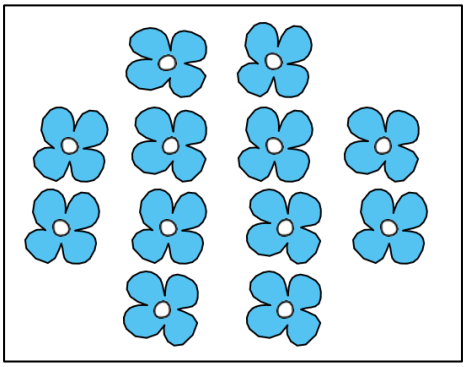
Figure 4 - Introduction of row 2 for the multiplication, in Textbook B

Source: Isoda (2020, p. 35).

Here 4 × 2 = 2 +2 +2 +2, however if we compare each multiplication in row 2, it can be seen as 4 × 2 = 3 × 2 +2. It shows the property of adding the multiplicand 2 to next expression in row 2, consecutively. In other words, the product is increased by the multiplicand on each row, consecutively. On this row 2 notion, this property is explained by accumulation. In Japanese original textbook, 4 × 2 is represented by 2 × 4 which means "2, four times" in Japanese grammar. With Japanese grammar, the inconsistency like in Textbook A does not happen. On the other hand, if we directly translate it in Chile with Spanish grammar, it appeared inconsistent as we have seen in Textbook A. Thus, the change of the order of row 2 like in the Figure 4 keeps the same consistency as the Japanese.

Another feature of Textbook B is the introduction of the meaning of multiplication as following: multiplication is firstly introduced to distinguish the situation explained with words "number of groups" (multiplier) and "size of groups" (multiplicand). So, the multiplication is introduced as (multiplier) × (multiplicand). Next, the accumulation is used to find answer to the problems in *concrete situation* to interpret the use of 'times' in the context. After doing the activities and apprehending the full understanding of the situations in which the concepts are clear, the meanings and the multiplication were introduced in systematic way. Especially, explaining the situation of a multiplication with the use of the terms "number of groups" (multiplier) and "size of groups" (multiplicand) supports the visualization of a situation as case of multiplication, even when the situation does not appear directly as a case for multiplication, like in the following Figure 5.

Figure 5- If we move top two to bottom, it is 4 × 3



Source: Isoda (2020, p. 34).

By establishing such consistency that can be well explained by terminologies, the Textbook B, an adapted edition of Gakko Tosho, has been selected by more than 70% of Chilean schools.

Lesson Study in the teaching of mathematics and statistics in Chile: more than a decade of advances and findings

In Chile, since more than a decade ago, the Group for the Lesson Study of the Institute of Mathematics of the Pontifical Catholic University of Valparaiso (IMA-GEC), co-designs, implements, develops and theorizes on Lesson Study in communities of teachers of the educational system, and disseminates its research through publications and books, and is dedicated to teacher training programs. A professional development course for in-service teachers, carried out in 2006 and 2007, was related to the Chile-Japan Program until 2010, and financed by the Center for Improvement, Experimentation and Pedagogical Research (CPEIP) dependent on the Ministry of Education of

Chile (MINEDUC); in particular, aspects of Mathematics Education and the Lesson Study were offered, and the teachers began to work collaboratively on lessons following the methodology of Lesson Study.

Currently, with the support of the Ministry of Education of Chile (MINEDUC), massive online courses are being carried out to update primary school teachers that promote the problem-solving approach under the Lesson Study methodology (e.g., Program Sumo Primero 2019-2021). Particularly in the Institute of Mathematics-IMA, the methodology of Lesson Study is applied in various undergraduate courses and explicitly in its master's degree in Mathematics Education. In addition, the MINEDUC designed, during 2020 and 2021, a set of mathematics textbook for first to sixth grades, based on Japanese books and, at the same time, on the results from the Japanese experiences of Lesson Study with the researchers of the Institute of Mathematics, all of them under the direct advice of Professor Masami Isoda, from the University of Tsukuba, Japan.

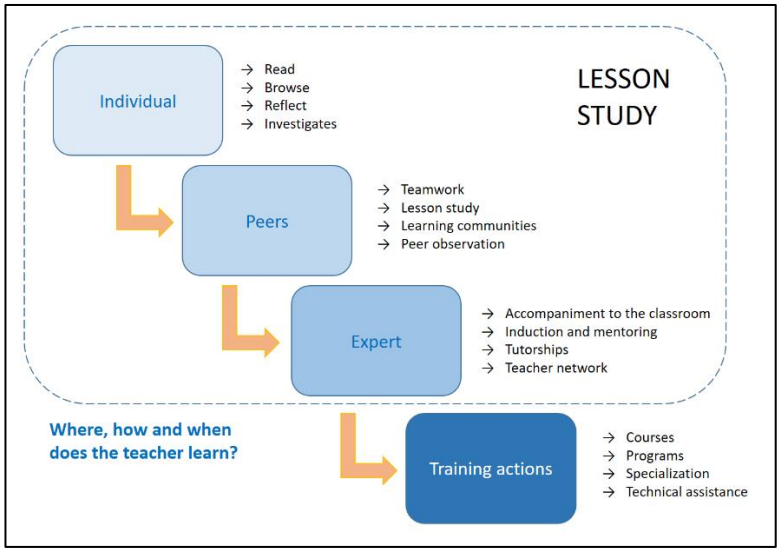
In the future, the members of the Lesson Study Group plan to continue focused on the Lesson Study as the object of research, to generate evidence of its impact on the professional development of teachers, that is, on the achievement of student learning and, more generally, on the country's mathematics education, so that this methodology will be part of the initial and continuous teacher training courses, and its use will be disseminated throughout the country and, as far as possible, in Latin America.

National Teacher Professional Development System

In Chile, in 2016 a Law created the Teacher Professional Development System, which is one of the pillars of the current Chilean Educational Reform. In this system, transformations are established to solve and intervene in matters of teaching professionalism, the needs to support their performance and their assessment. The challenge of CPEIP is that each teacher has a formal or informal learning space, individually or among peers, inside or outside the establishment, within the framework of a progression in professional development and their teaching career.

The CPEIP recognizes methodologies that can favor teacher training and its values, as well as the dissemination of teamwork, learning communities, peer observation, accompaniment in the classroom, which, in our experience, represent essential aspects of the Lesson Study, as illustrated in Figure 6 (the dotted border frame was positioned by the authors).

Figure 6- Methodologies that contribute to the improvement of the teaching process in Chile



Source: (MINEDUC-CPEIP, 2018, in Spanish).

Next, what we understand by Lesson Study is specified, the advances and findings that have been followed by the PUCV academics are discussed, who have been able to implement the methodology in training teachers at primary school and especially the mathematics teachers who teach in high schools. At the regional level, the Lesson Study has been disseminated through face-to-face open lessons, and lately at the national level, in northern and southern regions, through open online lessons.
(https://estudiodeclases.cl/videos/;https://estadisticatemprana.cl/publicaciones/#revista).

Lesson Study from the Institute of Mathematics in Valparaíso

Lesson Study has been recognized as a collaborative activity between teachers and researchers in processes in which they study one or more lessons to improve teaching and progress in their professional development. In many cases Lesson Study has entered the schools through the leading teachers themselves, establishing there a learning community that learns from experience and shares best practices when studying a lesson focused on discipline-specific content.

In 2008, the Lesson Study Group of the Institute of Mathematics of the Pontifical Catholic University of Valparaiso (GEC-IMA) was founded, and gradually the academics Arturo Mena-Lorca and Raimundo Olfos promoted the incorporation of this methodology in the Mathematics Pedagogy programs, Magister in Mathematics Education, and in research for the Doctorate in Mathematics Education. Likewise, some academics, for example, Raimundo Olfos and Sergio Morales, steadily incorporated this methodology in the training teachers of primary education at the PUCV School of Pedagogy, and in 2021, with the cooperation of Soledad Estrella, massive online courses are being carried out to update primary school teachers and to promote the problem-solving approach under the Lesson Study.

The efforts of GEC-IMA have focused on disseminating this methodology of teacher professional development, trying to be faithful to the principles of the Japanese Lesson Study, understood as: (1) the teachers who participate in the Lesson Study seek to develop experience and to learn something new, and also to improve a specific lesson; (2) includes significant time spent to study or to create educational materials; (3) it takes place over several weeks, and not in few hours; and (4) professionals with greater expertise and specialized knowledge contribute during the lesson planning process and the discussion after its implementation (ELLIOTT, 2019). As an essential feature, GEC-IMA encourages teachers to discuss essential concepts that students need to learn, to respond to current study plans, and to consider current knowledge of students.

GEC-IMA carries out annually and permanently National Conferences and Regional Conferences, with public lessons and courses, seeking to share good teaching practices of mathematics generated under the Lesson Study, and promote the formation of Communities of Lesson Study as a strategy of Continuing Teacher Professional Development (PONTIFICIA UNIVERSIDAD CATÓLICA DE VALPARAÍSO).

During the years 2020 and 2021, members of GEC IMA worked on the adaptation of mathematics school texts from the prestigious Japanese publisher Gakko Tosho, from grades 1 to 4 (for example, ISODA; ESTRELLA, 2020; ESTRELLA; ISODA, 2020). These textbooks, delivered by MINEDUC, provide teachers with ideas about how to help and support their students to learn mathematics in a meaningful way, to learn to think and to learn mathematics by themselves, in line with the Lesson Study.
(ISOLDA, 2021).

These features are consonant with the principle of three *co´s* that characterizes a sustainable Lesson Study Groups.

Some research findings

Academics from the PUCV Institute of Mathematics, and in particular

some GEC- IMA participants, direct or participate in government-funded projects and/or programs that use Lesson Study as research and/or development strategy. The themes of such research deal with the teaching of statistics, mathematics, or STEM activities for education in pre-school, primary and secondary school, together with educational proposals in initial teacher training and continuing training.

Currently, GEC encourages the formation of Lesson Study groups in schools and high schools, through four modalities, all with the support of GEC IMA: 1- formation of a Lesson Study group within the same educational establishment; 2- formation of a Lesson Study group with teachers from different educational establishments; 3- GEC IMA invites different professors to form a Lesson Study group with format 1 and /or 2 on the university premises and with continuous support for 2 months; and 4- creation of Lesson Study groups, national or international, that are formed to design and implement a lesson in synchronous online mode. This last experience has been developed in 2018 and 2019, with GEC IMA and countries such as Brazil and the US, implementing and improving STEM-type lessons, and at the same time integrating cultures and languages.

The courses and investigations are developed under the problem-solving approach, since it is appreciated that this approach incorporates how to help children to learn, to solve a closed problem with a single solution or an open problem with several solutions (e.g.,, BALDIN et al., 2018). Isoda and Nakamura (2010) outlined five steps of this approach: pose a problem; estimate solutions (plan and predict the solution); solve it independently; explain to others and compare the different approaches to the solution (or the different solutions); and later, integrate the proposals, argue, and explain the learned new knowledge.

Different theories and approaches of mathematics education and statistics education have allowed us to analyze the design and implementation of lessons generated by the Lesson Study, in numerous articles and book chapters (e.g., ESTRELLA; MENA- LORCA; OLFOS, 2018). In general, and through qualitative and interpretative analyzes, the dynamics of the Lesson Study improves mutual support and shared reflection (e.g., OLFOS; ESTRELLA; MORALES, 2015). Estrella et al. (2020) investigate the design of tasks with high cognitive demand and the ability of teachers to maintain such cognitive demands during the implementation of the lessons; Vidal-Szabó et al. (2020) contribute to workspace theory by characterizing a statistics lesson related to exploratory data analysis in which students reason and communicate their ideas with data-based evidence; in (ISODA et al., 2021), six researchers from Japan, Chile and Brazil implemented the Lesson Study between two countries, and studied the digital competence of a teacher involved in the implementation of a cross-border lesson for classrooms in Brazil and Chile. This year 2021, Isoda and Olfos publish their fourth book that incorporated different researchers, commemorating an earlier edition published in 2011, “Teaching of multiplication: from the study of Japanese lessons to the Iberoamerican proposals” in which the prominent professor Ubiratan D'Ambrosio collaborated. (ISODA; OLFOS, 2021)

Final considerations on the Chilean experience of implementation of Lesson Study

This year 2021, GEC IMA celebrated 13 years of continuous work carrying out its mission, that is, to contribute to the professional development of teachers in learning communities, to curriculum development, to the enrichment of mathematical experiences in the classroom and to the improvement of the quality and equity of education through the Lesson Study as a form of action research. GEC IMA is projected as a research, innovation and development center with national and international scope, made up of professors, researchers and future professors who contribute to the development of the curriculum, to the professionalization of teachers through the establishment of networks of teacher professional development communities that

they investigate, from their schools, how to enhance the mathematical knowledge, skills and attitudes that students need to adapt to the world around them.

All the actions carried out by the GEC-IMA Lesson Study group seek what is proposed by Elliott (2019), that is, to transform the organization of the work of teachers in the classrooms, creating more school space for them to spend investigating their lessons. Having groups of teachers who spend a couple of hours a week in Lesson Study is an alternative for professional development that far exceeds other strategies for continuing teacher training, not only because it contemplates more comprehensive and lasting professional learning, but also for the horizontality of the methodology and the strategy that generates greater reflection, security, leadership and teaching commitment (ESTRELLA et al., 2018).

Finally, GEC IMA as a pole in Latin America, longs for a more expanded and shared conception of the Lesson Study, with its listening approach based on mutual learning and respect. The study of a lesson is also based on the fruitful assumption of continuous self-improvement, which allows it to enter the school dynamics of many teachers and cultures of different countries. We recognize in this methodology of teacher professional development the real feasibility of the transformation of the classroom, the school and the system school.

Concluding remarks

After more than two decades of the projects to internationalize the concepts and the benefits of Lesson Study in the cultures outside the original Japanese setting, we learn that the Japanese Lesson Study in Mathematics carries itself a long history of pedagogical trajectory that must be taken into consideration by other cultures to understand and to implement the key features of this methodology of professional development of schoolteachers. This paper brings two important dimensions for the process of adequation of the Lesson Study principles in this regard. The establishment of correct terminology for mathematics content across the curriculum and the textbooks that must be commonly shared and understood by distinct cultures is pointed out in the section 2. This aspect is essential in the formation of mathematics teachers capable to distinguish the conceptual and proceduralization evolutions of the curricular concepts across the grades program, especially in the elementary education. The second dimension brought in this paper refers to the complex challenge of the implementation of Lesson Study projects in cultures outside Japan, elaborated through the experience of a successful case of Chile that can shed light to the possibilities of expanding as a model to other cultures alike, in Latin America. The case of Chile with its advances and findings indicates in their grounding principles those pointed by Shimizu and Chino (2015) as three co´s: cooperation, collaboration (including at the international level) and co-acting (through the micro local context of Chilean school teachers, researchers, governmental support) present in the section 3, through the elaboration of different forms of connection among the agents of the educational system.

As final words, the role of the Lesson Study that contributes to the improvement of the Quality Mathematics Education comprehends more than the two dimensions discussed in this paper. The Lesson Study contributes to the knowledge about the Mathematical Thinking, Teacher Education, Curriculum Development, Design of Teaching and Learning Materials, to mention a few. The journey to implement and to diffuse the Lesson Study is a complex enterprise that will bring all the educators as protagonists of the education system in each cultural environment.

References

BALDIN, Y.Y; ISODA, M.; OLFOS, R.; ESTRELLA, S. A STEM cross-border lesson on energy for primary education under APEC lesson study Project. *In:* HSIEH, F. J. (Ed.). **Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education**, Vol 1, Taipei, Taiwan: EARCOME, pp 236-247, 2018. ISBN 978-986-05-5783-1.

BECKER, J. P.; SHIMADA, S. **The Open-Ended Approach:** A new proposal for Teaching Mathematics. Reston: NCTM, 1997. ISBN: 0-87353-430-1.

ELLIOT, J. Quality criteria for lesson and learning studies as forms of action research. **International Journal for Lesson and Learning Studies**, 2019.

ESTRELLA, S.; ISODA, M. **Suma Primero: manual del docente, 4° básico**. Valparaíso: Ediciones Universitarias de Valparaíso, 2020. ISBN 978-956-17-0865-5.

ESTRELLA, S.; MENA-LORCA, A.; OLFOS, R. Lesson Study in Chile: A very promising but still uncertain path. In: **Mathematics Lesson Study Around the World,** p. 105-122. Cham: Springer, 2018.

ESTRELLA, S.; ZAKARYAN, D.; OLFOS, R.; ESPINOZA, G. How teachers learn to maintain the cognitive demand of tasks through Lesson Study. **Journal of Mathematics Teacher Education**, v. 23, p. 293-310, 2020.

GAN, T. H.; ISODA, M.; TEH, K. H. **Mathematics Challenges for Classroom Practice at Lower Secondary Level**. Penang, Malaysia: SEAMEO-RECSAM, 2021. Disponível em:

http://www.recsam.edu.my/tag/2021_recsamtsukuba_mathbook.pdf

ISODA, M. Dialectic on the Problem Solving Approach : Illustrating Hermeneutics as the Ground Theory for Lesson Study in Mathematics Education**.** *In:* CHO, S. J. (Ed.) **Selected Regular Lectures from the 12th International Congress on the Mathematics Education**. Swaziland: Springer, p. 355-381, 2015a.

ISODA, M. The Science of Lesson Study in the Problem Solving Approach. *In:* INPRASITHA, M. *et al.* (Eds.). **Lesson Study:** Challenges in Mathematics Education. Singapore: World Scientific. p. 81-108, 2015b. Disponível em: https://www.worldscientific.com/doi/10.1142/9789812835420_0006.

ISODA, M. **Suma Primero 1º Básico. Texto del Estudiante Tomo 2.** Japan/Chile: University of Tsukuba/MINEDUC, 2020. Disponível em: https://www.curriculumnacional.cl/docentes/Educacion-General/Matematica/Matematica-2-basico/227534:Sumo-Primero-2-Basico-Texto-del-Estudiante-Tomo-2

ISODA, M.; ESTRELLA, S. **Suma Primero:** libro del estudiante, 1° básico. Valparaíso: Ediciones Universitarias de Valparaíso, 2020. ISBN 978-956-17-0880-8.

ISODA, M.; ESTRELLA, S.; ZAKARYAN, D.; BALDIN, Y.Y.; OLFOS, R.; ARAYA, R. Digital competence of a teacher involved in the implementation of a cross-border lesson for classrooms in Brazil and Chile. **International Journal for Lesson and Learning Studies**, Vol. ahead-of-print No. ahead-of-print, 2021. Disponível em: https://doi.org/10.1108/IJLLS-05-2021-0045.

ISODA, M.; KATAGIRI, S. **Mathematical thinking:** How to develop it in the classroom. Singapore: World Scientific, 2012. Disponível em: https://www.worldscientific.com/worldscibooks/10.1142/8163

ISODA, M.; NAKAMURA, T. Mathematics Education Theories for Lesson Study: Ptroblem Solving Approach and the Curriculum through Extension and Integration. **Journal of Japan Society of Mathematical Education**, v. 92, p. 5, 2010.

ISODA, M.; OLFOS, R. **Enseñanza de la multiplicación:** Desde el estudio de clases japonés a las propuestas iberoamericanas. Valparaíso: Ediciones Universitarias de Valparaíso, 2009.

ISODA, M.; OLFOS, R. **Teaching Multiplication with Lesson Study:** Japanese and Ibero-American Theories for International Mathematics Education. Cham: Springer Nature, 2021.

ISODA, M.; STEPHENS, M.; OHARA, Y.; MIYAKAWA, T. (Eds). **Japanese Lesson Study in Mathematics:** its Impact, Diversiy and Potential for Educational Improvement**.** Singapore: World Scientific, 2007.

MANGAO, D. D.; AHMAD, N. J.; ISODA, M. **SEAMEO Basic Education Standards (SEA-BES):** Common Core Regional Learning Standards (CCRLS) in Mathematics and Science. Penang, Malaysia: SEAMEO RECSAM, 2017. Disponível em:

http://www.recsam.edu.my/sub_SEA-BES/images/docs/CCRLSReport.pdf

MCCALLUM, W. G. **Making sense of mathematics and making mathematics make sense.** In: **Proceedings of ICMI Study 24 School Mathematics Curriculum Reforms:**

challenges, changes, and opportunities. Tsukuba: University of Tsukuba. p. 1-8, 2018.

MINISTÉRIO DA EDUCAÇÃO DO CHILE .MINEDUC-CPEIP. **Plan local de formación:** un instrumento para potenciar el desarrollo profesional. 2018.

https://www.cpeip.cl/wp-content/uploads/2020/01/Infografia-Plan-Local-v5.pdf

MINISTÉRIO DA EDUCAÇÃO DO CHILE. **Las Bases Curriculares para la Educación Básica 2012**. 2012. Disponível em: https://www.curriculumnacional.cl/portal/Tipo/Asociados-a-la-Base-Curricular/Partes-de-los-Programas/14598:Las-Bases-Curriculares-para-la-Educacion-Basica-2012.

ORGANIZAÇÃO PARA A COOPERAÇÃO E DESENVOLVIMENTO ECONÔMICO – OECD. Student Agency for 2030, 2019. Disponível em: https://www.oecd.org/education/2030-project/teaching-and-learning/learning/student-agency.

OLFOS, R.; ESTRELLA, S.; MORALES, S. Clase pública de un estudio de clases de estadística: Una instancia de cambio de creencias en los profesores. **Revista Electrónica Educare**, v. 19, n. 3, p. 1-17, 2015.

OLFOS, R.; ISODA, M.; ESTRELLA, S. Más de una década de Estudio de Clases en Chile: hallazgos y avances. **Revista Paradigma** (Edición Cuadragésimo Aniversario: 1980-2020), v. l. XLI, p. 190-221, 2020.

OLFOS, R.; ISODA, M.; ESTRELLA, S. **Chile:** Country´s presentation. 2021. Disponível em: https://www.youtube.com/watch?v=34GkW9gWfzl

PONTIFICIA UNIVERSIDAD CATÓLICA DE VALPARAÍSO. **Jornada Estudio de clases**. Valparaíso, Comunicaciones IMA PUCV. Disponível em: https://www.youtube.com/watch?v=apaqdO8-kBs

SHIMIZU, S.; CHINO, K. History of Lesson Study to develop good practices in Japan. *In:* INPRASITHA, M. *et al.* (Eds.). **Lesson Study:** Challenges in Mathematics Education. Singapore: World Scientific. p. 123-140, 2015. Disponível em:

https://www.worldscientific.com/doi/abs/10.1142/9789812835420_0008

STIGLER, J. W.; HIEBERT, J. **The teaching gap**. New York: Free Press, 1999.

VIDAL-SZABÓ, P.; KUZNIAK, A.; ESTRELLA, S.; MONTOYA, E. Análisis cualitativo de un aprendizaje estadístico temprano con la mirada de los espacios de trabajo matemático orientado por el ciclo investigativo. **Revista Educación Matemática**, v. 32, n. 2, p. 217-246, 2020.

Masami Isoda: Faculty of Human Sciences and Center for Research on International Cooperation in Educational Development- CRICED, University of Tsukuba, Japan. Brief description of academic background, professional activities, and the research field: Ph.D. in Mathematics Education. Professor at Faculty of Human Sciences and Director of CRICED, U. Tsukuba, Japan. He was Project Overseer on the APEC Lesson Study Project (2006-2018) having extended it beyond the APEC. He edited curriculum standards for mathematics in the ASEAN region under SEAMEO and Papua New Guinea, and several countries' textbooks. He received an honorary Ph.D. from KKU, Thailand, honorary Prof. from USIL, Peru for his contribution. Email: isoda@criced.tsukuba.ac.jp. ORCID: https://orcid.org/0000-0002-0017-3935. URL link for publication and CV: https://jglobal.jst.go.jp/en/detail?JGLOBAL_ID=200901091935440711

Raimundo Olfos: Pontificia Universidad Católica de Valparaíso- PUCV, Chile. Brief description of academic background, professional activities, and the research field: Associate professor in mathematics education at PUCV. Research in Curriculum, didactic and evaluation. Teacher training for teaching mathematics, in postgraduate and undergraduate programs of PUCV. Email: **raimundo.olfos@pucv.cl**. Orcid: https://**orcid**.org/0000-0002-9886-4282. URL link for academic information: http://ima.ucv.cl/academicos/raimundo-olfos/

Soledad Estrella: Pontificia Universidad Católica de Valparaíso - PUCV, Chile. Brief description of academic background, professional activities, and the research field: Ph.D. in Mathematics Education; she is professor at the Institute of Mathematics of PUCV-Chile, where she leads the Research Group of Early Statistics. Her research focuses on the practice of mathematics instruction, and on the improvement of teacher training and development through Lesson Study. She is currently Vice President of the Chilean Society of Mathematics Education, a member of

the International Council of the Mathematics Education Network of Central America and the Caribbean. E-mail: soledad.estrella@pucv.cl. Orcid: https://orcid.org/0000-0002-4567-2914. URL link for academic information: http://ima.ucv.cl/academicos/soledad-estrella/

Yuriko Yamamoto Baldin: Universidade Federal de São Carlos- UFSCar. Brief description of academic background, professional activities, and the research field: Ph.D. in Mathematics from UNICAMP, post-doctorate in the USA, scientific visits in Japan. She was a representative of Brazil to ICMI-IMU, a member of the Executive Committee of ICMI-IMU (2013-2020) and coordinated international projects. She is Senior Professor at UFSCar. Research in Mathematics Teacher Education at the Graduate Program in Mathematics Teaching (PPGECE-UFSCar), in the lines of Problem Solving, Lesson- Study, integrated teaching. E-mail: yuriko@ufscar.br. ORCID: https://orcid.org/0000-0001-7473-5657. URL Link to lattes: https://lattes.cnpq.br/8785562528979416