CHALLENGES AND DIVERSITY OF INITIATIVES FOR THE IMPLEMENTATION OF LESSON STUDY FOCUSED ON THE CONTINUING EDUCATION OF MATHEMATICS TEACHERS

Desafios e diversidade de iniciativas para a implementação de Lesson Study com foco na formação continuada de professores de matemática

Yuriko Yamamoto Baldin

Aparecida Francisco da Silva

Maria Alice Veiga Ferreira de Souza

Abstract

Comparing the development of Lesson Study (LS) in the Japanese educational system with the structure of the current educational system in Brazil allowed identifying differences that challenge the implementation of LS in the Brazilian context. The article presents a study of initiatives to implement Study Groups of Lesson Study (GLS), through a scheme of the organization of the educational system in Brazil, focused on the agents that connect the levels of the system, in two groups of GLS. The analysis aimed to answer the questions: "Who are the agents that mediate a GLS in the system? What actions of these agents promote continuity and effective results of a GLS?" The Groups performed a qualitative analysis of the actions of the agents in different contexts, but interpreted in the basic steps of LS, which resulted in the identification of their roles in LS as part of continuous education of mathematics teachers.

Keywords: Lesson Study; Continuing education; Intermediary agent in the educational scheme; Problem Solving in teacher training; Transformation of practice in the classroom.

Resumo

Comparar a trajetória de desenvolvimento da Lesson Study (LS) no sistema educacional japonês com a estrutura do sistema educacional atual no Brasil permitiu identificar diferenças fundamentais que desafiam a implementação de LS em contexto brasileiro. O artigo apresenta um estudo de iniciativas de implementação de Grupos de Estudo de Lesson Study (GLS), por meio de um esquema da organização do sistema educacional no Brasil, o que permitiu destacar os *agentes* que conectam os níveis do esquema em dois grupos distintos de GLS. O estudo buscou responder às questões: "Quem são os agentes intermediadores de um GLS no sistema? Que ações desses agentes promovem continuidade e resultados efetivos de um GLS?" Os Grupos realizaram uma análise qualitativa das ações dos agentes em contextos distintos, porém tais ações foram interpretadas nas etapas básicas de LS, que resultou na identificação de seus papéis na LS como método de formação contínua de professores de matemática da educação básica.

Palavras-chave: Lesson Study; Formação contínua de professores; Agente mediador de Lesson Study no sistema educacional; Resolução de Problemas na formação do professor; Transformação da prática na sala de aula.

Introduction

This article results from a deepening of the contributions of two Lesson Study Groups in Brazil, presented at Table 2 of the International Lesson Study Seminar in Mathematics Teaching - Seminário Internacional de Lesson Study no Ensino de Matemática (SILSEM), whose theme *focused on initiatives for the implementation of Lesson Study as an activity of continuing education for mathematics teachers*. The participants of Table 2 presented several perspectives on the experiences of implementing Lesson Study, bringing to the fore the need to discuss in more detail how the cultural character and structure of educational systems influence the initiatives of implementation of Lesson Study (LS) in schools, in order to favor the continued education of teachers, in mathematics. The expanded abstracts of the presentations of Table 2 (ESTRELLA, 2021; SOUZA, 2021; SILVA; BALDIN, 2021) bring important aspects that underlie the sections

of this article, while the experience in Chile (ESTRELLA, 2021) was incorporated into the article that deepened the theme of SILSEM Table 1, from the perspective of internationalization of the implementation of LS. This article presents an analysis of the initiatives of the Lesson Study Groups (GLS) in the states of São Paulo and Espírito Santo, highlighting the structural conditions of the educational system, interpreted in local contexts, which enable the implementation of Lesson Study with continuous and significant results.

Motivation and research questions

The first step in the analysis of the initiatives was to bring up the essential foundation of the implementation of a GLS that is characterized as a professional development activity of teachers of Basic Education, in our case, in the discipline Mathematics. To understand the fundamentals of LS as a method that positively influences the continuing education of mathematics teachers in Brazil, we seek to look at the historical and cultural contexts of the structure of Japan's differentiated educational system, which was established in the second half of the 19th century, after the opening of the country to interact with other countries, in the event defined as The Meiji Restoration, 1868. An organized education system with the establishment of a school structure took place together with the creation of the Ministry of Education in 1873 (ISODA; STEPHENS; OHARA; MIYAKAWA, 2007; SHIMIZU; CHINO, 2015). A new teaching and learning environment in the classrooms that were organized in schools implied the need to take differentiated teaching and learning methods, as well as managing the dynamics in classrooms and established programs of curriculum content. This revolutionary change in education, which broke with the tradition prevailing before the Meiji Restoration, was part of Japan's modernization policy in a way that would allow it to communicate with school models from Western countries. The origin of the Lesson Study methodology takes place at that time, coinciding with the beginning of Japan's new educational policy and also with the establishment of the first official teacher training school, in which Lesson Study emerged as a research activity of teachers in practice, since its origin. (ISODA *et al*., 2007; SHIMIZU; CHINO, 2015; STIGLER; HIEBERT, 1999; FERNANDEZ, YOSHIDA, 2004; SOUZA; WROBEL; BALDIN, 2018). Therefore, the perception of the temporal and cultural character of the implementation of the Lesson Study in Japan, together with the understanding of how LS has evolved for 150 years following the improvement of teacher education, is an important factor to reflect on the continued education of Basic Education teachers in Brazil, in mathematics, especially nowadays when the implementation of school curricula aligned with the National Base for Common Curriculum -BNCC is in course (BRAZIL, 2018).

In the analysis of the development of the school system in Japan, a fact stands out about how the hierarchical organization of the system has occurred since the establishment of the centralizing *Ministry of Education* in 1873. From the ministerial organization, there were studies for the promulgation of curricular codes, the school establishment with its administrative and pedagogical staff, and standards for the professional development of its teaching staff, which illustrates a *top-down scheme at* the levels that constitute the system (SOUZA *et al.*, 2018). However, the study groups of teachers that constituted, at the same time, from the training schools and in the schools where they worked, were the beginnings of the Lesson Study Groups - GLS, which contributed with their research activities on disciplinary content and methodologies in the classroom, for the elaboration of didactic sequences, teaching materials, and the concept of school curriculum. Then the study groups promoted the basis for solid professional development, through continuous teacher training activities. (ISODA *et al.*, 2007; SHIMIZU; CHINO, 2015). The development of Japanese teachers' research in GLS over the years has always been centered on the fundamental principle of *achieving effective learning of students in the classroom.* Thus, the investigative

activities on teaching and learning and, the learning of how to teach, did not come as normative impositions of higher bodies, but were worked as initiatives of teachers and teacher training school, from the planning of lessons and the practice in the classroom. This phenomenon characterizes a *bottom-up aspect* in the organizational system of school education, which constitutes a great differential with the Brazilian context (SHIMIZU; CHINO, 2015; SOUZA *et al.*, 2018). The development of the Japanese Lesson Study has always had incentives and official support from educational system bodies throughout the history of its evolution, with the parallel walk of school education standards and curriculum with the research of the LS Study Groups (GLS) by teachers, focusing on practices in the classrooms. Therefore, a reflection on the challenges of bringing Lesson Study potential in teacher education in cultural realities different from Japan's implies the need to understand and to analyze the challenges of implementation processes of a Lesson Study. In fact, importing only ideas and processes that are successful in the student learning process, without analyzing the key points of the process, leads to difficulties not expected or not understood. Watanabe (2018) points to a reflection by Chokshi and Fernandez (2004) on four misconceptions that "imports and implementations", without a careful study of the meanings of LS in different cultures, can bring, so must be avoided:

> 1- LS is about creating q unique, original, or never-seen-before; 2- There will be no benefit from just a few lesson study lessons; it´s important to conduct LS for as many lessons as possible; 3- LS is about perfecting a single lesson; 4- LS is about producing a library of tried-and-tested lessons for others to use. (WATANABE, 2018, p.4)

This means that initiatives to implement LS that effectively contribute to improvements in mathematics and learning in Brazilian schools should be understood within the educational system, and not restricted to experiments of methodologies for some lessons or some classes tested in practice and (or) their repetitions as teaching techniques.

In this scenario, to identify the protagonists in the Brazilian school system that can contribute to the initiatives for GLS implementation, we chose to analyze the organization of the educational system under its identification as a complex system (MITAL; MOORE; LLEWELLYN, 2014) with its parts and agents that connect with differentiated roles. Mital *et al.* (2014) describe an agent-based-modeling (ABM) for a model to frame the analysis of an educational system that is being used in the United States for education in Science, Technology, Engineering and Mathematics, which acronym STEM is widespread in educational media. Although the cited reference argues that this model is generally used in *top-down analyses* to investigate impacts on educational policy by the agencies involved, we highlight the statement that "education researchers began to analyze the effects of collaborative networks of teachers in the implementation of educational reforms" (MITAL *et al.*, 2014, p. 371), which justifies the choice of this model in our study to identify the agents of the Brazilian educational system and their roles in the Collaborative Studies Groups of LS, in SP and ES, as cases discussed in this article. On page 372 of the reference, there is a brief description of what can constitute agents (or class of agents) in the ABM model of a network of employees of an educational project, whose attributes to connect the levels of the system emerge through their functions in the network. For example, they can be agents in an educational project network, according to the nature and objectives of the project: school leaders, teachers (or instructors) of the school, collaborators (or coordinators), monitors (or tutors), graduate researchers, strategic partners (institutional or expert investigators, intervention advisors), students, educational agents (local, municipal, state, federal or network individuals, etc.) and the list is not exhausted.

In the school structure in Brazil, especially public schools, it is not easy in the time being to spontaneously organize GLS initiatives aimed at the professional

development of teachers that directly impact the learning activities of students within the classrooms. The few records we have are generally from individual initiatives or groups outside the school. In an educational system scheme, it is imperative to identify agents and their roles that can maintain a collaborative network to organize actions towards the educational objectives of the network. In this sense, we bring the concept of *intermediate agent* in mathematics education projects, which was part of the discussions of Panel 1 of ICME14 (RUIZ, 2021), as a bridge between mathematical researchers and mathematical teachers (educators) in a collaborative network of Mathematics Teacher Education. In Jaworski and Potari (2021) we find considerations about the tensions that may arise in a collaborative learning community if the objectives and roles of agents in the network are not harmonized. So, agents that can mediate the distinct approximations in a network, with special interest for the continuous education of teachers, is a current theme of research in Mathematics Education.

In this sense, this article seeks to answer the following research questions, considering the Brazilian context: *Who can be the intermediate agents of a Lesson Study Group and what is their role? What characteristics of intermediate agents allow continuity and satisfactory results of a collaborative Study Group?*

The following sections identify the organized structure of collaborative lesson study group, in two cases of the states of SP and ES, highlighting the *role* of intermediate agents in the LS projects and their implementations, considering the distinct regional contexts. As the methodology of analysis, the two groups focus on the crucial phases of Lesson Study to the continued training of the mathematics teacher, and on the Problem-Solving Methodology.

The evolution of LS in the organizational structure of Study Groups in the public educational system in SP

The structure of the state public education network, managed by the State Department of Education of the state of São Paulo - SEE-SP (https://educacao.sp.gov.br) involves a subdivision into 91 Regional Boards of Education -DRE, distributed in 15 regional centers in the state. To identify the structural chain that is focused on this article, we highlight, among the various sections (coordination centers) that make up the administrative management of each DRE, the Pedagogical Center Coordination - NP, in each DRE, in which are allocated the Coordinating Teachers of the Pedagogical Center (PCNP), for each curricular discipline or area of activity within the school curriculum. Among the many specific attributions of a PCNP, two stand out that show the direct relationship with the functions of intermediate agency in the process of establishing a GLS:

> to implement pedagogical and educational support actions that guide teachers in conducting procedures related to the organization and functioning of the curriculum in teaching modalities; and to identify needs and propose continuing education actions of teachers and coordinating teachers within the area of activity that is proper to it. (free translation) (https://dejosebonifacio.educacao.sp.gov.br/nucleo-pedagogico-2/)

The initiative to organize a GLS in the state of São Paulo as an activity of DRE José Bonifácio -SP emerged from a pilot project for in-service teacher training in partnership between the Brazilian Public Schools Mathematics Olympiad – OBMEP (https://www.obmep.org.br) and SEE-SP, the Project of Professional Training Workshops (PROF-OBMEP), in the years 2013 to 2015. Silva and Baldin (2021) point out that, from the PROF-OBMEP project, the managers and teachers of the Pedagogical Center of SEE-SP, and especially the direction and PCNP of the DRE of José Bonifácio who actively participated in the project, identified the need for continuous training of teachers for the development of the curriculum, from the perspective of the Problem-Solving Methodology. The essential objective of the

PROF-OBMEP project was to train basic education teachers in the Problem-Solving Methodology as a strategy for Teaching Mathematics in classrooms that develops Mathematical Thinking, in addition to identifying the connection between the contents of OBMEP's original problems and the contents of the school curriculum. The experience brought evidence to motivate the organization of a Study Group in more permanent conditions to transform the "Hands-on Workshops" with OBMEP´s problems of the previous PROF-OBMEP project into Lesson Study Groups - GLS, allowing an investigative analysis that identifies the needs of initial and continued education of teachers, as well as the possible conditions to disseminate the Lesson Study methodology based on Problem Solving (SILVA; BALDIN, 2021).

In the process of creating the GLS in José Bonifácio, initiated in 2016, the conviction of the directing board and PCNP of DRE-JB who had participated in PROF-OBMEP was the driving force for the implementation of a GLS. The synergy between the research in Mathematics Education of the authors Baldin and Silva and the Pedagogical Center points to the DRE and PCNP as *crucial* intermediary agents with their actions based on their attributions.

Transformation of a pilot project to GLS-SP

The gradual process of transformation of problem-solving workshops as strategies for teaching and learning mathematics in classrooms was consistently proceeded from the adoption of Lesson Study fundamentals by basing the planning of GLS activities, following a script with studies on the meaning of Lesson Study steps. The first stage of a Lesson Study begins with the definition of a topic or topics within the curriculum, and proceeds with the study of the teaching material guided by the school curriculum, to sustain a lesson planning, including the research-lesson that is one of the key stages of Lesson Study. The understanding and interpretation of the phases of the Lesson Study involve due adaptations to the cultures and the operative school structures in different countries, or even in local contexts such as in the case of states and regional educational boards, for example, in the state educational network of São Paulo. Attention to this aspect is a challenge for all who are involved in the process of implementing new forms of teacher training. In this scenario, the difficulties that arise include the actions or orientations of educational bodies that do not presuppose the Lesson Study within school activities. The research-lesson activities and investigations that result from it for the direct improvements of the practice inside the classrooms constitute a differentiated characteristic of the Lesson Study, and the lack of structural and cultural conditions to work the LS methodology in schools can cause administrative as well as organizational obstacles in schools that participate in a GLS. Silva and Baldin (2021) warn that initial difficulties of this nature can prevent the proper development of forms of continuing education that demand constant and planned activities of study groups on content, teaching and learning methodologies, and efficient evaluation.

Thus, the role played by DRE-JB and its PCNP in the areas of mathematics and science in the initiative of organizing a GLS, systematized in the meetings planned by the DRE, through the official convocation of teachers (registrants, volunteers) of schools subordinated to the DRE for their technical training schedules, highlighted the importance of an *intermediary agency* to the establishment of a more stable and continuous model to the GLS. The Problem-Solving Methodology, which is central to the Japanese Lesson Study, was already worked on in the PROF-OBMEP Project and, the continuity to work on Problem Solving as a strategy for the development of LS in the group, is in accordance with the precepts of BNCC (BRASIL, 2018) as well as they are explicit in the current curriculum of the state of São Paulo. The adoption of Lesson Study, with the perspective of being an effective means for continuing teacher education, which encourages them to research on teaching materials and to study them for the

implementation of curricula, is supported by increasing results registered outside the Japanese culture. The experience in Chile, in the words of Mena, Montoya and Navarro (2012, p.327), "Lesson Study offers tools to advance the specific and methodological knowledge of teachers and also in the evaluation and knowledge of students' possibilities", corroborates this statement.

Silva and Baldin (2021) analyze and discuss the journey of the LS implementation process at DRE-JB since 2016, including, in addition to other structural aspects, cultural considerations that can halt the constitution of effective Groups of LS. Especially highlighted are the aspects, generally isolated, of teachers´ work to transpose the curricular guidelines for practice in the classroom, amid the lack of self-confidence in establishing the connection between their own knowledge and the school content that must be learned by their students. In this respect, Silva and Baldin (2021) emphasize the role played by universities, with their researchers, in consolidating the GLS of the DRE-JB, when, with their contributions in elucidating the connections between the mathematical discipline and the pedagogical knowledge of school content, they allowed establishing a solid **partnership between educators** (university professors and researchers) and **managers**, in this case represented by the Regional Board of Education and its Pedagogical Center with the pedagogical coordinators-PCNP. The importance of the partnership between managers and educators finds it also parallel in the analysis of the implementation of Lesson Study in Chile (ESTRELLA, 2021). Still in this line, the recognition of PCNP in the DRE-JB as an *intermediary agent* in the educational system chain also allows identifying the role of educators external to the basic education system (researchers and universities) as an *intermediary agency*, essential to foster and stimulate the intermediation that leads to the continuity and consolidation of a GLS.

Silva and Baldin (2021) also argue about the importance of the partnership between institutions and their agents that facilitates achieving the results expected by the community, because in such partnership, the profile of the participants and how to enable the participation of the teacher are defined, without interfering in the planning and execution of the activities as usual duties of the participant school unit or the participants themselves. The results obtained by the GLS of the DRE-JB are in the cited reference. The GLS model of DRE-JB, which continues currently including other disciplines, or the pedagogical use of digital resources like scientific calculators, was consistent with the actions planned by other DRE in the region, so that the GLS of DRE-Jales is currently under development, which also works on the Problem Solving Methodology in accordance with the stages of LS for teacher training, on the research theme selected within the didactic material of the new high school curriculum of the state of São Paulo, formulated upon the concept of *competency and abilities* in the curriculum development:

> Study of **functions** to develop the ability **EM13MAT101**: To interpret critically economic and social situations and facts related to Natural Sciences that involve the variation of quantities, by analyzing the graphs of the functions represented and the rates of variation, with or without the support of digital technologies. (free translation) (GOVERNO DO ESTADO DE SÃO PAULO, 2020)

The structured model of the educational system as a complex system, analyzed from the perspective of ABM model, which identifies the agents and their attributions in the system, makes the analysis of results collected from GLS implementation projects in the context of the State of São Paulo more efficient to detect the different phases of project development. The project is currently in execution.

The COLABORA Study Group, from the state of Espírito Santo

The GLS of ES called COLABORA - Research Group on Collaborative Teacher Training emerged in 2014 in order to contribute to the development of a new

vision, a new professional posture, and why not say of a *new professional modus vivendi*, through continuous training of teachers, having as background the *improvement* of knowledge of school contents and their teaching practices, in the light of the Japanese Lesson Study.

COLABORA is composed of teachers of basic, technical, technological, higher and graduate education, primarily in the area of Mathematics, as well as researchers of national and *international stricto sensu* graduate programs. The meetings of COLABORA are held weekly in person and (or) remote, for about 2 hours or more, depending on the demand of the moment and according to the ongoing projects and studies.

In general, the actions of study, teaching, research, and extension of the GLS are conducted in a parallel way by the members who seek to contribute their theoretical and practical experiences and whose themes are concentrated, mostly, in the development of Lesson Study editions carried out together with education professionals in schools of the state of ES. Although the inspirations emanates from *the Japanese school modus vivendi*, we do not take our actions as imports of a model or method simply, as Watanabe (2018) argues, but as a starting point for realizations outside the Japanese context, and without falling out of the essence, in search of an identity of the Group itself, immersed in a system diverse of Japanese educational system.

In this sense, the editions of Lesson Study undertaken by COLABORA led us to understand that some adaptations and adjustments would be necessary, in view of differences between Brazilian and Japanese schools, after all, teaching is a cultural activity, agreeing with Stigler and Hiebert (2009) and Hargreaves (1996). In the search for the group's objectives, it is necessary to consider the performance of *intermediary agents* as relevant part of the process of implementation of the actions to be carried out and that we will begin to discuss.

The main intermediary agents of COLABORA are the educational, research and extension institutions: Federal Institute of Espírito Santo, Foundation for Research and Innovation Support of Espírito Santo, Secretary of Education of the State of Espírito Santo and the Municipal Secretariats of Espírito Santo, Federal University of Espírito Santo, Federal University of São Carlos, Paulista State University - São José do Rio Preto, University of Lisbon, Rutgers University - Newark, in the people of its managers, professors and technicians. In addition, family members of students and local communities are part of the process of building and participating in the complex educational system in which COLABORA is inserted.

Each intermediary agent has characteristics that act directly or indirectly in the actions and purposes of the COLABORA, either favoring, or disfavoring. A limitation imposed by educational institutions is the conception of the maximum workload of teachers in the classroom, to the detriment of stimulating collaborative and formative work for teaching. In this conception, the agents understand the teachers as ready and finished, after all, they have an undergraduate diploma that "fully enables" them to the profession. This point of view refers to the isolation of teaching work, which impairs (or makes impossible) the establishment of collaborative work - one of the pillars of the Lesson Study.

The culture of isolation for planning the lessons hinders opportunities to share experiences and, with this, improve professional development. The *Brazilian school modus vivendi* is organized in such a way that it does not yet benefit the approximation of teachers and, consequently, hinders collaboration. But the change of this *status quo* is neither simple nor immediate. COLABORA has been achieving timid but important ruptures in some educational spaces for conquest, never by imposition, nor let alone by speech disconnected from practice. It is necessary to live the benefits, and this occurs when teachers (or other education professionals) learn more about certain content and(or) by their teaching practice.

In this regard, as an example, an experience draws attention. In an edition of Lesson Study on the concept of fractions

with second-year elementary school students, managers were members of GLS. These participants were convinced of the advantages of collaborative planning by seeing their teacher-pedagogues understand this content and verify the performance of the students in the class applied by one of them (Figure 1). Today, this municipality has reorganized itself in time-space and has been developing formations based on Lesson Study, for disciplines beyond mathematics. The rupture of the insulation should be gradual - it is part of the process. The collaboration must integrate *the school modus vivendi* as a *continuum* and little by little the group is convinced of the benefit of the work in partnership, and continues to break the culture of isolation. Here is a case of an intermediary agent who started collaborating with COLABORA.

Figure 1 - Collaborative planning meeting in Lesson Study on the concept of fraction

Source: the authors.

Figure 2 - Application of the lesson plan with students of the second grade of elementary school

Source: the authors.

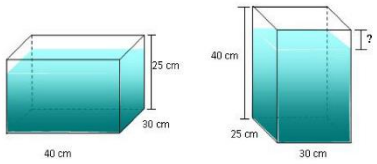
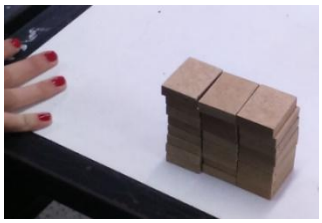
In addition to the organization of teachers' time in schools, isolation hides other factors that can justify it - the fear of the teacher showing what he knows (or does not know) about certain content he teaches. Lesson Study gradually assists in the overcome of this fear, in view of each other's support "in order to achieve common objectives negotiated by the collective, establishing relationships that tend to non-hierarchization, shared leadership, mutual trust and co-responsibility for the conduct of actions". (DAMIANI, 2008, p. 215). In COLABORA, the rule is to value contributions and discard all spirit of hierarchy and leadership. There are no differences by titles or by profession time. Everyone has something to contribute, especially regarding the content to be taught.

Intermediary agents do not always invest in incorporating the spirit that everyone has something to learn (or perfect) and, therefore, there is no need to fear in admitting ignorance of some content or how to teach it. An invisible way to reinforce these teaching fears is the tacit message left by the political organization of some Brazilian schools that still prioritize and attribute professionals by the time of teaching or the accumulation of academic titles. These institutions understand that know-more-who-has-worked-more.
Although professionals with longer time working in Education may have accumulated more experiences, it is not true that this baggage necessarily translates into potential learning of their students.

In this perspective, all editions of Lesson Study enabled teachers to learn about the mathematical content that

originated them. As an illustration, in the Lesson Study on the concept of volume, many teachers taught only procedural aspects (WANDERLEY; SOUZA, 2020). They knew how to calculate, but they were unaware, for example, that the concept of volume involves density knowledge, and that it is indicated that their teaching should be conducted by comparison, measurement, and production (Chart 1). The challenge was to learn concepts beyond procedures. And the help came from different fronts: scientific articles, books, research groups specialized in the subject, consultations to *knowledgeable others*, etc. The challenge here is to train professors-researchers and talk about research, which refers to investments in time for searches, readings, debates, etc., which usually Brazilian teachers do not have. Almost all projects in COLABORA were carried out with time outside the ordinary hours of the members.

Chart 1- Example of situations of (a) comparison, (b) measurement and (c) production, in the study of the volume concept

(a)	(b)	(c)
Imagine a sphere, a cube, and a pyramid of square basis as in the figure below. The diameter of the sphere, the side length of the cube, the side lengths of the squared base and the height of the pyramid all have the same measure.	Imagine a box in glass, completely closed and almost full of water, as in the figure. Observe that the level of water is 5cm below the maximum. Now, let us put the box upright so that the level of water is 40cm high. In this case, how below the maximum will be the level of water?	Construct a stacking with 24 units of volume. (24 u.v.).
		

Source: Wanderley and Souza (2020, p. 9)

In addition to the teachers' time, other deprivations occurred to achieve Lesson Study editions: (1) physical displacement for the planning, application, and observation meetings of the classes and, (2) the lack of pedagogical materials. In due course, sometimes, the GLS had the help of the Federal Institute for the loan of vehicles for travel for the application of lesson plans in some municipality far from the metropolis; and (or) financial resources originated from projects approved by the Foundation for Research and Innovation Support of Espírito Santo for the purchase of materials. Without the resources coming from these agents, many projects would have declined or occurred without proper records *(e.g.,*audio and video recording) for further critical reflection.

These audio and video recordings have also been figured out as challenges for implementing some editions of Lesson Study by COLABORA. In all editions, approvals and signatures were requested in terms of free and informed consent of parents and guardians for the recording of audio and video of the students' productions in class. In these terms, some relatives of students from suburban schools refused to disclose their identifications and their national official documents, for reasons that were not convincing. In these cases, the directive agents of these schools acted along with the family members to ensure the suitability and ethical secrecy guaranteed in the terms. This was a relevant contribution from one agent to another to the realization of the planned classes in the light of Lesson Study.

In a way, achieving success in actions such as those developed in COLABORA is difficult without the effective participation of agents who constitute (or should be constituted) as partners. Often, the performance of these agents occurs in a silent and invisible way

for individuals who are not members of the GLS, but they form an important basis for the realization of the Lesson Study whole cycle, and for the development of educational cultures that help education professionals to evolve in their professions, after all, the school is also a teacher's place to learn. The aid from the agents is of different orders and ranges from psychological support to the material, from financial resources to the organization of time, from effort to *change the status quo of* the school to commitment to the professional training of all.

Concluding remarks

This article analyzes two Lesson Study implementation cases in different contexts in two states of Brazil. The history of the initiatives shows a rich scenario of possibilities in which the Study Groups must unfold to achieve the objective of continuing education of mathematics teachers that overcomes the procedural model that usually occurs in the implementation of diverse methodologies in the school context. The ABM model for the educational system, as a complex system (MITAL et al., 2014), allowed the identification of key agents in each GLS with its intermediate role between the parts of the system. The intermediary agents contributed to overcome the challenges that each group faced for its constitution, in addition to valuing the role and the protagonism of participants in the construction of groups that are evolving towards the model faithful to the principles of the original Lesson Study. In this sense, the study of curriculum material and specific content of mathematical knowledge through the Problem-Solving Methodology in teachers' meetings is shown to be a consistent strategy to achieve the objectives of good practice in the classroom, where students develop mathematical thinking, although each GLS, in different places and contexts, will plan and carry out proper Lesson Study editions through specific continuing education projects of mathematics teachers, using own resources and possibilities.

References

BRASIL. **Base Nacional Comum Curricular**. Brasília: Ministério da Educação. 2018.

CHOKSHI, S.; FERNANDEZ, C. Challenges to importing Japanese lesson study: Concerns, misconceptions and nuances. **Phi Delta Kappan**, v. 85, p. 520-525, 2004.

DAMIANI, M. F. Entendendo o trabalho colaborativo em educação e revelando seus benefícios. **Educar**, n. 31, p. 213-230, 2008.

ESTRELLA, S. Lesson Study en Chile: más de una década de avances y hallazgos. *In:* SEMINÁRIO INTERNACIONAL DE LESSON STUDY NO ENSINO DE MATEMÁTICA, 2021, Brasília. **Anais** […]. Brasília, 2021. No Prelo.

FERNANDEZ, C.; YOSHIDA, M. **Lesson Study:** A Japanese Approach to Improving Mathematics Teaching and Learning. Mahwah, N. J.: Lawrence Erlbaum, 2004.

GOVERNO DO ESTADO DE SÃO PAULO. Secretaria de Educação. Diretoria de Ensino da Região de José Bonifácio. **Núcleo Pedagógico 2**. 2020a. Disponível em: https://dejosebonifacio.educacao.sp.gov.br/nucleo-pedagogico-2/.

GOVERNO DO ESTADO DE SÃO PAULO. Secretaria de Educação. **Currículo Paulista, Ensino Médio**. Efape. 2020b. Disponível em: https://efape.educacao.sp.gov.br/curriculopaulista/ensino-medio. Acesso em: jan. 2020.

HARGREAVES, A. **Changing teachers, changing times: teachers' work and culture in the postmodern age**. London: Cassel, 1996.

ISODA, M.; STEPHENS, M.; OHARA, Y.; MIYAKAWA, T. **Japanese Lesson Study in Mathematics:** Its Impact, Diversity and Potential for Educational Improvement. Singapore: World Scientific, 2007.

JAWORSKI, B.; POTARI, D. Implementation of a developmental model of teachers´ and didacticians´ learning through inquiry: design, operationalization, and outcomes. **ZDM-Mathematics Education.** On-line publication: Springer 2021, p.1-12. DOI:10.1007/s11858-021-01290-x

MENA, A.; MONTOYA, M.S.; NAVARRO, S. Estudio de Clases en Chile: Talleres Comunales. *In:* ISODA *et al.* (Eds.). **El Estudio de Clases Japonés en Matemáticas:** su importancia para el mejoramiento de los aprendizajes en escenario global. 3a edición. Valparaíso: Ediciones Universitarias de Valparaíso, 2012, p.320-328.

MITAL, P.; MOORE, R.; LLEWELLYN, D. Analyzing K-12 education as a complex system. **Procedia Computer Science**, v. 28, Elsevier B.V, p. 370-379. 2014. DOI:10.1016/j-procs.2014.03.046

RUIZ, A. Plenary Panel 1 Actors for Math Teacher Education: Joint Action versus Conflicts. *In*: **Plenary Activities of ICME14**, 2021. Disponível em: https: //icme14.org.

SHIMIZU, S.; CHINO, K. History of Lesson Study to Develop Good Practices in Japan. *In:* INPRASITHA, M. *et al.* (Org.) **Lesson Study:** Challenges in Mathematics Education. Singapore: World Scientific, 2015, p. 123-140.

SILVA, A.F.; BALDIN, Y. Y. Transformando Oficinas de Formação de Professores de Matemática na Metodologia de Resolução de Problemas em Grupos de Lesson Study. *In:* SEMINÁRIO INTERNACIONAL DE LESSON STUDY NO ENSINO DE MATEMÁTICA, 2021, Brasília. **Anais** […]. Brasília, 2021. No Prelo.

SOUZA, M. A. V. F. de; WROBEL, J. S.; BALDIN, Y. Y. Lesson Study como Meio para a Formação Inicial e Continuada de Professores de Matemática – entrevista com Yuriko Yamamoto Baldin. **Boletim Gepem**, v. 73, p. 115-130. 2018.

SOUZA, M.A.V.F.de **Lesson Study sem Fronteiras: limitações, desafios e algumas soluções de implementação**. *In:* SEMINÁRIO INTERNACIONAL DE LESSON STUDY NO ENSINO DE MATEMÁTICA, 2021, Brasília. **Anais** […]. Brasília, 2021. No Prelo.

STIGLER, J.; HIEBERT, J. **The teaching gap:** best ideas from the world's teachers for improving education in the classroom. New York: Free Press, 1999.

WANDERLEY, R. A. J.; SOUZA, M. A. V. F. Lesson Study como processo de desenvolvimento profissional de professores de Matemática sobre o conceito de volume. **Perspectivas da Educação Matemática**, v.13, n. 33, p.1-20. 2020.

DOI:10.46312/pem.v13i33.10302.

WATANABE, Y. Japanese Lesson Study in the United States: looking back and looking ahead. **Educational Designer**, v.3, n.11, 2018. p.1-13.

Yuriko Yamamoto Baldin: Universidade Federal de São Carlos- UFSCar. Brief description of academic background, professional activities, and the research field: Ph.D. in Mathematics from UNICAMP, post-doctorate in the USA, scientific visits in Japan. She was a representative of Brazil to ICMI-IMU, a member of the Executive Committee of ICMI-IMU (2013-2020) and coordinated international projects. She is Senior Professor at UFSCar. Research in Mathematics Teacher Education at the Graduate Program in Mathematics Teaching (PPGECE-UFSCar), in the lines of Problem Solving, Lesson- Study, integrated teaching. E-mail: yuriko@ufscar.br. ORCID: https://orcid.org/0000-0001-7473-5657. URL Link to lattes: https://lattes.cnpq.br/8785562528979416

Aparecida Francisco da Silva: Universidade Estadual Paulista "Julio de Mesquita Filho" – UNESP. Licenciatura em Matemática pelo IBILCE/ UNESP - mestrado e doutorado em Matemática no IMECC/UNICAMP; Atua como professora do Departamento de Matemática do IBILCE/UNESP, coordenadora da OBMEP e orientação de Grupo de Lesson Study. Área de pesquisa atual: Resolução de Problemas em Matemática, Lesson Study e Formação Inicial e Continuada de Professores de matemática. E-mail: aparecida_francisco57@hotmail.com. Orcid: https://orcid.org/ 0000-0001-5570-1232. Link para lattes: http://lattes.cnpq.br/9821391048399255.

Maria Alice Veiga Ferreira de Souza: Instituto Federal do Espírito Santo. Graduada em matemática (ufes), doutorado em educação matemática (Unicamp), pós-doutorado em resolução de problemas de matemática (Univ. Lisboa-Portugal), pós-doutorado em números racionais (Rutgers university - usa). Professora e pesquisadora do programa de pós-graduação em educação em ciências e matemática (educimat) atuando na formação de professores.E-mail: alicevfs@gmail.com. Orcid: http://orcid.org/0000-0003-2038-813X. Link para lattes: http://lattes.cnpq.br/2876710785262591