Sociedade Brasileira de Educação Matemática

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Registers Mobilized by Pedagogical Students in the Introductory Study of Probability

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Abstract: This text aims to identify the difficulties and possible misunderstandings of probabilistic content among Pedagogy students, through different registers obtained from the resolution of three activities involving probabilistic problems. Methodologically, an interpretative qualitative description was adopted by analyzing the responses and reflections of teachers in initial training expressed during the development of the proposed activities. This case study was carried out with a class composed of 14 academics studying the Degree in Pedagogy at a public university in the State of Pará, Brazil. The results indicate that the teaching of probability depends on the teacher's familiarity with the particularities of this content and on the recognition, by part of the student, of the subject covered, as well as the objects mobilized in the study. The analysis of the protocols, with the observation of the discussions carried out by the research subjects during the resolution of the probabilistic problems, allowed us to verify that the students had difficulties in converting the register from the fractional form to the decimal form or from the decimal form to the percentage. In this sense, it was found that students discuss the procedures studied when they can suggest other ways of organizing the situation presented. Therefore, the context in which the activity is structured needs to be able to receive interventions from the student and only after starting discussions on procedures and calculations to be developed.

Keywords: Teaching probability. Training multipurpose teachers. Registers of semiotic representation.

Registros mobilizados por estudantes pedagogos(as) no estudo introdutório de probabilidade

Resumo: Este texto tem o objetivo de identificar as dificuldades e eventuais incompreensões de conteúdos probabilísticos de estudantes de Pedagogia, por meio de diferentes registros obtidos a partir da resolução de três atividades envolvendo problemas probabilísticos. Metodologicamente, adotou-se a descrição qualitativa interpretativa, analisando as respostas e as reflexões de professores em formação inicial expressas durante o desenvolvimento das atividades propostas. Esse estudo de caso foi realizado com uma turma composta por 14 acadêmicos cursando Licenciatura em Pedagogia em uma universidade pública do Estado do Pará. Os resultados indicam que o ensino de probabilidade depende da familiaridade do professor com as particularidades desse conteúdo e com o reconhecimento, por parte do aluno, do assunto tratado, bem como dos objetos mobilizados no estudo. A análise dos protocolos, com a observação das discussões feitas pelos sujeitos da pesquisa durante a resolução dos problemas probabilísticos, permitiu constatar que os estudantes apresentaram dificuldades em fazer a conversão do registro da forma fracionária para a decimal ou da forma decimal para a percentual. Nesse sentido, constatou-se que os alunos discutem sobre os procedimentos estudados quando podem sugerir outras formas de organização da situação apresentada. Portanto, o contexto em que se estrutura a atividade precisa estar apto a receber intervenções do aluno e só após terem início as tratativas sobre procedimentos e cálculos a serem desenvolvidos.

Palavras-chave: Ensino de probabilidade. Formação de professores polivalentes. Registros de representação semiótica.

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Registros movilizados por estudiantes de Pedagogía en el estudio introductorio de probabilidad

Resumen: Este texto tiene como objetivo identificar las dificultades y posibles incomprensiones de contenidos probabilísticos entre estudiantes de Pedagogía, a través de diferentes registros obtenidos a partir de la resolución de tres actividades que involucran problemas probabilísticos. Metodológicamente, se adoptó una descripción cualitativa interpretativa, analizando las respuestas y reflexiones de docentes en formación inicial expresadas durante el desarrollo de las actividades propuestas. Este estudio de caso fue realizado con una clase compuesta por 14 académicos de la Licenciatura en Pedagogía de una universidad pública del Estado de Pará. Los resultados indican que la enseñanza de la probabilidad depende del conocimiento del docente con las particularidades de ese contenido y del reconocimiento por parte del alumno de la materia tratada, así como de los objetos movilizados en el estudio. El análisis de los protocolos, con la observación de las discusiones realizadas por los sujetos de investigación durante la resolución de los problemas probabilísticos, permitió verificar que los estudiantes tuvieron dificultades para convertir el registro de la forma fraccionaria a la forma decimal o de la forma decimal al porcentaje. En este sentido, se encontró que los estudiantes discuten los procedimientos estudiados cuando pueden sugerir otras formas de organizar la situación presentada. Por lo tanto, el contexto en el que se estructura la actividad necesita poder recibir intervenciones del estudiante y sólo después de iniciar discusiones sobre los procedimientos y cálculos a desarrollar.

Palabras clave: Enseñanza de la probabilidad. Formación de docentes polivalentes. Registros de la representación semiótica.

1 Introduction

This research aims to contribute to the training of teachers in the early years of Elementary Education regarding probabilistic thinking. The purpose is to identify the difficulties and potential misunderstandings of probabilistic content among this audience. The goal here is not to quantify the errors or successes of multipurpose teachers, but to identify the registers mobilized in the introductory study of probability, in light of Raymond Duval's Theory of Registers of Semiotic Representation (2003, 2009).

The objective of this introductory activity was to explore different registers that were mobilized throughout the resolution of probabilistic problems by Pedagogy students. In this investigation, the responses and reflections of teachers in initial training expressed during the development of the proposed activities were analyzed. Throughout the analyses, excerpts from audio recordings and written responses of future multipurpose teachers are presented. Fragments of the resolution processes are highlighted to justify the inferences made. The use of oral and written production in the analysis, according to Duval (2011, p. 105), “does not have the same roles in becoming aware [...] of mathematically relevant units of meaning in a representation”. Thus, the individual resolutions of participating students were analyzed based on their probabilistic thoughts, in the form of discursive registers, in light of the Theory of Registers of Semiotic Representation.

Therefore, this text is structured as follows: after the introduction, the constructs of the Theory of Registers of Semiotic Representation are briefly presented; following that, the methodology is outlined, and subsequently, the results are showed, which leads arguments to discussions.

2 The registers of semiotic representation

In Raymond Duval's theory (2004) of registers of semiotic representations, they are defined as “productions constituted by the use of signs belonging to a system of representation, which have their own difficulties of meaning and functioning” (Duval, apud Damm, 2002, p.143). According to Duval (2004, p. 43), "the formation of a semiotic representation is the use of a sign to update the vision of an object or to substitute the vision of that object”. Thus, we understand that, in the learning of probabilistic thinking, Pedagogy students are introduced to a new, conceptual, symbolic, and above all, representative world in solving problems and understanding probabilistic texts. Such cognitive activities require the use of representation systems different from natural language or images (Duval, 2003). According to Duval (2003), there are four very different types of registers of semiotic representation, as presented in Table 1:

Table 1 – Classification of the different registers mobilizable in mathematical functioning.

	Discursive representation	**Non-discursive representation**
MULTIFUNCTIONAL REGISTERS: treatments are not algorithmizable.	*Natural language* Verbal associations (conceptual) Forms of reasoning: –Argumentation based on observations, on beliefs; –Valid deduction from definition or theorems.	*Flat or perspective geometric figures* (configurations in dimension 0, 1, 2 or 3). -Operative and not just perceptual apprehension; -Construction with instruments.
MONOFUNCTIONAL REGISTERS: treatments are mostly algorithms.	*Writing systems* –Numeric (binary, decimal, fractional, among others); –Algebraic; –Symbolic (formal languages). Calculation.	*Cartesian graphics.* - Changes of coordinate systems; - Interpolation, extrapolation.

Source: Duval (2003, p. 14).

From Table 1, it can be inferred that monofunctional registers are those with their own algorithms in their structure, such as systems of numerical, algebraic, and symbolic writing; Cartesian graphs; changes of coordinate systems; interpolation and extrapolation, which we refer to as monofunctional registers in discursive representation. On the other hand, multifunctional registers are those in which treatments are not algorithmizable, that is, they have natural language as their discursive representation. They also appear in non-discursive form, such as flat or perspective geometric figures (Duval, 2003).

Duval (2003) presents the hypothesis that understanding in Mathematics, in this case, the probabilistic thinking of Pedagogy students, assumes the coordination of at least two registers of semiotic representation: treatment and conversion. Regarding treatment, Duval (2004) defines it as the transformation of a representation within the same register, for example, “performing a calculation strictly within the same system of writing or representation of numbers; solving an equation or a system of equations; completing a figure according to criteria of connectivity and symmetry” (Duval, 2003, p. 16). Talking about conversion, the researcher views it as the transformation of the register, that is, the conversion of a representation of the object into another register (or system), preserving the same mathematical objects. An example of this is the transition from a fractional representation to a decimal one or from a decimal representation to a percentage (Duval, 2004).

It is agreed with Duval's argument (2003, 2004) that for mathematical understanding, the mobilization and coordination of two or more different registers of semiotic representation (figure, natural language, among others) are necessary. Hence, in this research, it is sought to explore the greatest possible variety of representational systems for the mathematical object “probability” (Duval, 2012). Therefore, “this recognition is the fundamental condition for a student to transfer or modify formulations or representations of information during problem-solving” (Santos; Bianchini, 2012, p. 39).

3 Methods

This research is guided by a qualitative approach of an interpretative nature (Minayo, 2011). The methodological choice is for a case study (Yin, 2001), that is, the research presents characteristics of an investigation of a contemporary phenomenon within a real-life context (Yin, 2010). Such a choice is justified because it involves the analysis of a particular and unique situation, especially used when the boundaries between the phenomenon (teaching probability) and the context (Pedagogy students' class) are not clearly evident. In this situation, the purpose is to understand, through an activity in the context of the National Plan for Formation of Teachers for the Basic Education (PARFOR)[4], how Raymond Duval's Theory of Registers of

[4] The National Plan for Formation of Teachers for the Basic Education (PARFOR) is an initiative of the Coordination for the Improvement of Higher Education Personnel (CAPES) aimed at contributing to the adaptation of the initial training of in-service teachers in the public basic education system by offering degree courses corresponding to the area in which they work.

Semiotic Representation contributes to and aligns with Initial and Continued Teacher Training in an introductory study on the content of probability. The study participants were 14 students from the Degree in Pedagogy at a public university in the State of Pará, Brazil. The selection of the class and the respective participants was facilitated by the fact that the professor/researcher was teaching the subject "Theoretical and Methodological Foundations of Mathematics Teaching" in the second semester of 2022 at the mentioned university.

In order to maintain the anonymity of the study participants, they were numbered from A1 to A14. The research activity consists of four simple probability problems. The registers are obtained through participant observation, as well as through audio and video recordings. For registering the resolution of activities, oral and written justifications were requested because, according to Duval (2011, p. 99), "thinking in Mathematics always mobilizes at least two registers". The activity involved introductory concepts of probability, and, therefore, the resolution could be carried out without the need for knowledge of mathematical algorithms. Table 2 relates the set of activities that make up the proposed didactic sequence for the Pedagogy students:

Table 2 – List of activities proposed to the group of undergraduates

Activity 1: If you randomly point to a letter in the alphabet, what is the probability of that letter being a vowel?
Activity 2: What is the probability of randomly drawing a card from a deck, and that card being an ace?
Activity 3: A fair die is rolled. Determine the probability that the face facing up is:
a) An even number.
b) A number greater than 4.
c) A multiple of 3.

Source: Developed by the authors (2023).

The implementation of this sequence took place in the months of July and August 2022, involving the participation of the 14 students. The activities were carried out individually during the regular class period of the "Theoretical and Methodological Foundations of Mathematics" subject. All participating students declared their willingness to take part in the investigation. Therefore, the productions and dialogues of the students were analyzed not only in terms of the mathematical knowledge they should mobilize to solve probability activities but also in terms of the dialogical actions of the registers that probabilistic resolutions suggested mobilizing in a coordinated manner. To achieve this, it was necessary to identify, in the resolutions of the activities, the conversion and treatment registers according to the registers mobilized by the participants in the research.

Therefore, in identifying understanding, the mobilization of registers was analyzed because, according to Duval (2011), the cognitive process involved in understanding the mathematical object mobilizes at least two registers of representation. This mobilization "requires a ceaseless activity of conversions, which remain implicit but must be more or less spontaneous" (Duval, 2011, p. 116). Given this recognition, a selection was made of the procedures and inquiries mobilized in the process.

4 Results and Discussions

The analyses were conducted in light of the Theory of Registers of Semiotic Representation (Duval, 2009) by identifying which processes of probabilistic thinking were present in the students' dialogues and protocols. It all began with the presentation of the training proposal; then, the first activity was written on the board. At that moment, it was observed that the class fell silent, and gradually, whispers among the students and laughter in the background began, as can be seen in the first dialogical action presented in Table 3:

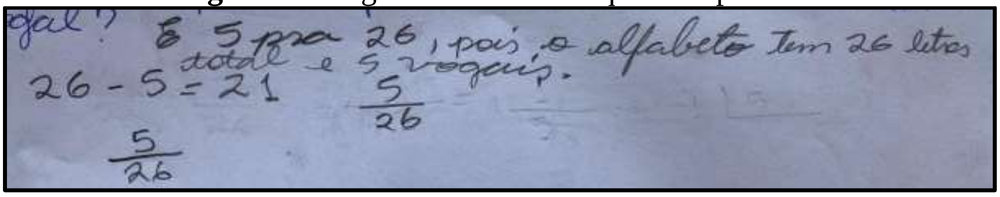
Table 3 – Identification of procedures and (mis)understandings of the students

Student: Description of the action
Student A2: I wrote it, but I did not understand it (laughs).
Student A5: I said this subject would crush our dreams.
Student A12: Calm down... people.
Professor/researcher: Read carefully, and any doubts will be resolved.
Student A9: I bought a new notebook for this subject (laughs).
Student A2: Almost all of us (laughs).
Student A7: I did not understand that word there.
Professor/researcher: Which one?
Student A7: Probability. Does it have to do with that thing of succeeding or not?
Student A1: Perhaps with the idea of a game or hitting or missing.
Student A6: Winning or losing (laughs).
Student A10: Of getting this or that.

Source: Developed by the authors (2023).

The heated discussion in the face of the previously unknown mathematical object confirms the doubts and concerns of the students regarding the subject, as well as the difficulty in understanding the question, which poses a cognitive cost for performing mathematical procedures (Moretti; Brandt; Franco, 2012). However, with minimal intervention, through dialogues among the students, the initial registers were gradually being constructed and documented on the A4 sheets. Figure 1 illustrates one of these situations:

Figure 1 – Register of A6's first question protocol



Source: Researchers' archive.

The register presented by student A6 reveals indications of identifying the event and the sample space, meaning there is implicit information – treatment –, as it was identified that the alphabet has 26 letters (sample space or possible cases) and five vowels (favorable cases). In this response, one can observe the mobilization of the register in written natural language and the use of auxiliary representations, such as respecting fractional numerical information.

Thus, defining the sample space of a specific event is an essential step in establishing the probability of that event. In other words, "in many cases, it is the most important, as the solution is quite obvious to someone who knows all the possibilities" (Bryant; Nunes, 2012, p. 5).

In that regard, Duval (1993, p. 47) states that "any register is always incomplete in relation to the denoted object". Moretti, Brandt and Franco (2012) add that the articulation between different registers of semiotic representation is a condition for accessing the understanding of the mathematical object. In the second activity, presented in Table 4, the confirmation of this assertion occurs:

Table 4 – Identification of procedures and (mis)understandings of the students

Student A13: Let's go by logic: the alphabet has 26 letters, five of which are vowels, and the rest are consonants.
Student A6: I followed this logic and even wrote it here.
Student A12: I did not think about it that way.
Professor/researcher: Tell us how you thought.
Student A12: I was already thinking here while writing this "thing" on the board that it was 26 to five vowels, but I could not write it down.
Professor/researcher: Your thinking is very good. Someone else?
Student A2: I wrote something similar to Student A13.
Student A7: Me too, but like a fraction (laughs).
Professor/researcher: Leave your thoughts on the A4 sheet.
Student A7: I think I am starting to understand.

Source: Developed by the authors (2023).

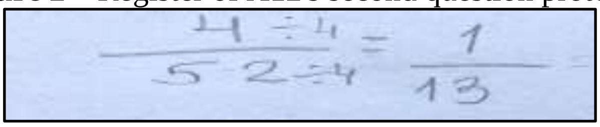
This dialogic excerpt, in addition to the register of student A6, indicates that the same mathematical object can be represented by different semiotic registers. The hypothesis was that students would have less difficulty solving this task, as the expected result was the fractional form 5/26; however, it became evident the difficulty they had in using the most appropriate representation. Thus, it can be inferred that the difficulty in solving the first question is nothing more than the difficulty of transforming the register of written natural language into a numerical register – the mobilization of the register of written natural language into fractional numerical. Therefore, it is necessary to mobilize different registers so that the student understands that there is not a single representation of a given mathematical object (Moretti; Brandt; Franco, 2012).

Campos and Pietropaolo (2013) reflect on the need for initial and/or continuing training courses to address the thematic axis “Probability and Statistics”, advocated in the National Common Curriculum Base (NCCB), with emphasis on problem situations involving the concept of probability and the identification of students' difficulties when starting the construction of probabilistic thinking.

In this regard, Fernandes, Serrano and Correia (2016, p. 84) argue that “the training of teachers in the early years of schooling is decisive and crucial to ensure the proper education of the student”. Therefore, it is necessary to introduce teachers in initial training to contact with random experiments in which they make use of probabilistic thinking, so that such experiences can be enhanced in the classroom and in their pedagogical practice (Pinheiro; Silva; Pietropaolo, 2018).

As can be seen in Figure 2, in the treatment and/or conversion registers, in the activity of drawing an ace from a deck of cards, it was found that the fractional representation determined by the students was processed appropriately, it means, they understood what was being asked in the question. An example of such a register is presented in Figure 2:

Figure 2 – Register of A12's second question protocol



Source: Researchers' archive.

Considering Figure 2, student A12 also resorted to the cognitive activity of treatment at the level of processing implicit information to determine the equivalent fraction: 1/13. The register presented by student A12 instigates implicit mathematical reflections that may go unnoticed by students as mathematical content, such as equivalent fractions. During the socialization moment, students presented their answers on the board, and then they were questioned about the results. Throughout the discussion, it was noted that 40% of the students attempted to establish an internal transformation – treatment in the form of fractional representation – by stating that the fractions 4/52, 2/26 and 1/13 are equivalent. Finally, when asked by the researcher if there was another way of representation, initially, some responded affirmatively, and later, they denied it. At this point, there was a certain lack of consistency regarding the understanding of equivalent fractions. After this stage, the third dialogic action was organized, as presented in Table 5:

Table 5 – Representative scheme of identifying procedures and (mis)understandings of the students

Student A13: This time it was not difficult to find the result.
Student A9: As I know the cards in the deck, I had no difficulty.
Student A12: I did it straight away because I play cards and know all the cards, and I immediately started calculating and simplifying the fractions.
Professor/researcher: How did you make these simplifications?
Student A12: I worked with this content some time ago, and then I remembered to simplify as much as possible.
Professor/researcher: I understood...
Student A5: I also simplified it, but I put the result as 1/13. Is it wrong, professor?
Professor/researcher: Do not worry, your register is very good.
Student A11: I only left it until the result 2/26.
Professor/researcher: Very good.

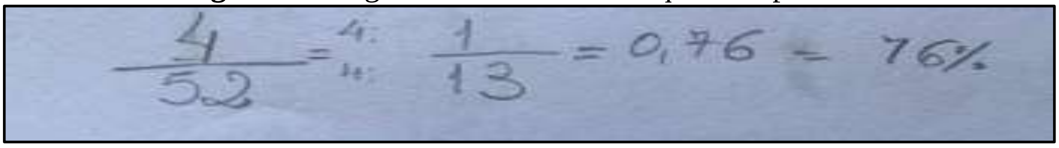
Source: Developed by the authors (2023).

It was found that 100% of the students identified the relevant data for solving the second question, while 72% identified the register in the form of fractional representation by presenting equivalent fractional forms of such data in their solutions. Regarding conversion, it was observed that 14% of the students were able to perform it, that is, for the transition from fractional to decimal form or from decimal to percentage form. Additionally, 29% did it partially, and 57% were unable to do it.

It is worth noting that, even though the answers were presented with 100% adherence to the fractional representation, the researcher, in this case, the professor of the subject, took the opportunity during the activity to request the conversion. Thus, it was clarified the need to describe the professor's attitude because this conversion was actively sought, meaning it would not have occurred if the activity had been completed only with the presentation of the fractional result. With this interest in mind, at this moment, the professor's action was described, since he needs to plan activities that contain spaces to accommodate other forms of presenting resolutions, making it possible to present other ideas that eventually arise from of students' resolution options.

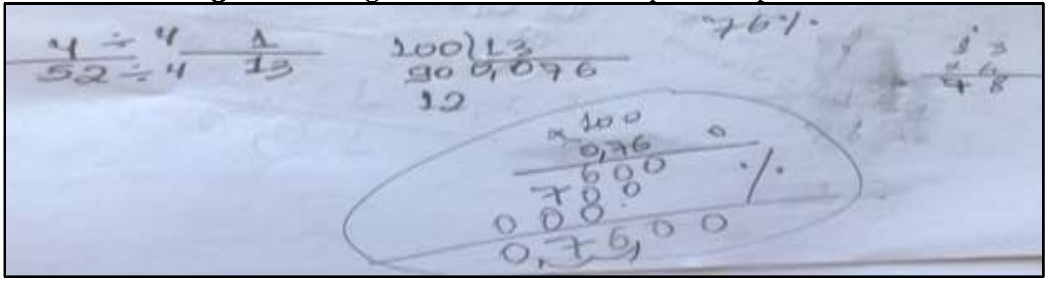
Seeking to identify conversion registers in the students' protocols, based on prior knowledge, the registers of students A12 and A5 are presented. In the first register, A12 mobilized the conversion from fractional to decimal representation and then from decimal to percentage representation; student A5, on the other hand, registered the conversions in a detailed manner. The conversion registers of the students related to this question can be seen, respectively, in Figures 3 and 4:

Figure 3 – Register of A12's second question protocol



Source: Researchers' archive.

Figure 4 – Register of A5's second question protocol



Source: Researchers' archive.

During the sharing of this question, students who answered correctly were asked to explain their strategies to those who had difficulty understanding; they were also asked to choose a new register that would address the question. However, few changes were identified in the registers, which allowed us to infer that this behavior shows the importance given by the students to the numerical value register, as well as the difficulties they have with operations involving decimal and percentage representations.

Pozo and Crespo (2009) argue that students immediately seek to find a numerical value as the answer to a given activity, which leads them to blindly apply an algorithm, without distinguishing an object from its representation. In this sense, it is necessary to enable students, through the use of different registers, to recognize mathematical objects in their various possible semiotic representations (Duval, 2011), since mobilizing different registers of semiotic representation allows us to understand the student's way of thinking mathematically, and not just the procedures specific to the content of probability (Almouloud, 2007).

Additionally, it was observed that the second question was well received by the students; the only complaint was regarding decimals and percentages, which, according to them, required a bit of time and attention, as they were not requested in the first activity. An example of this is the statement from student A2: "I have a lot of difficulty with values involving percentages".

In other words, the difficulty in understanding the statement mentioned by the students is nothing more than the difficulty of transforming the fractional representation into a decimal or percentage representation. Regarding this, it is up to the professor to decide when to request both representations, as, according to the above, it is agreed with the students that such additions to the presentation of the answers may require more time. For this reason, aiming to contribute not to the quantity of representations, but to the understanding of the activity performed, it was opted to request only in the second activity.

According to Duval (2003, p. 24), “the cognitive phenomena revealing mathematical activity concern the mobilization of various registers of semiotic representation and the conversion of these representations”. Thus, after the socialization, through the intervention carried out, which consisted of working with decimal and percentage numbers, some of the students managed to achieve the intended conversion, presenting the registers used by the professor/researcher during the discussion. Here, we return to Duval's argument (2009) that different representations of the same mathematical object present different contents, that is, working with a fractional representation and a decimal or percentage number are different situations, although they represent the same mathematical object.

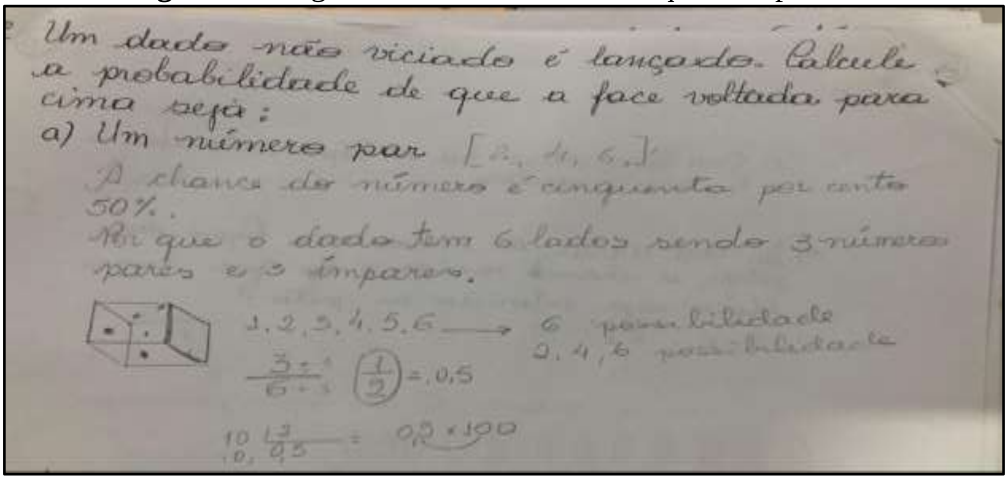
In short, the dialogues and registers of the second question present the same mathematical content as the previous one; however, it may present cognitive costs to students due to the different strategies for performing this conversion. Indeed, according to Duval (2009), in a conversion transformation, the probabilistic object remains the same, but its content is different when considering its form of representation. Thus, it becomes evident that the articulation of content in Mathematics classes still presents several weaknesses, especially when there is no subject that explores the thematic unit of “Probability and Statistics” in the Degree in Pedagogy program.

In order to conclude, a third activity was proposed in an attempt to initiate a generalization and solidify the understanding of the mathematical concept. The activity presented the following statement: “an unbiased die is rolled; determine the probability that the face facing up is: a) an even number; b) a number greater than 4; and c) a multiple of 3”.

The idea at this point was that students would perform similar calculations to those done in the previous two questions, only using different values. This way, different registers could be identified, leading them to generalize the relationship between fractional, decimal and percentage representation and, eventually, mobilize registers involving treatment and conversion.

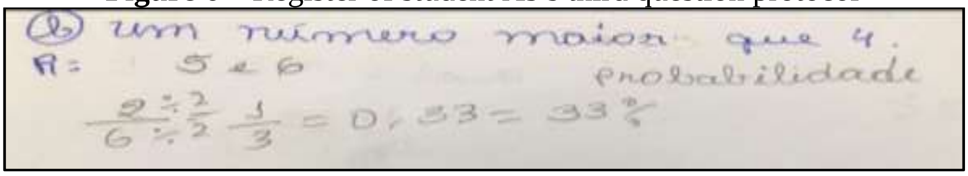
In the responses to this question, unlike what happened in the previous ones, students did not encounter major difficulties; they proceeded to solve it immediately, that is, mobilized at least one semiotic register and arrived at the result. Most successfully solved items "a", "b", and "c"; however, some students erred in their responses by making mistakes during the conversions. Such occurrences are illustrated in Figures 5, 6 and 7:

Figure 5 – Register of student A3's third question protocol



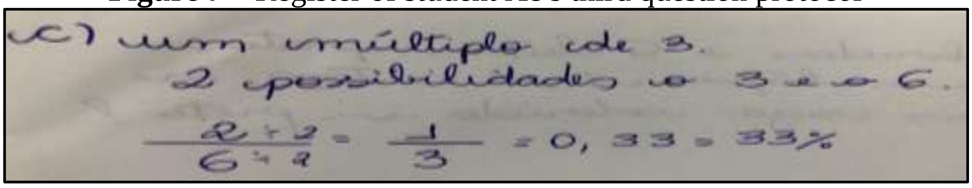
Source: Researchers' archive.

Figure 6 – Register of student A9's third question protocol



Source: Researchers' archive.

Figure 7 – Register of student A8's third question protocol



Source: Researchers' archive.

It was noted that protocols 5, 6 and 7 contained a satisfactory solution, which mobilized different semiotic registers. Students A8 and A9 mobilized the registers of conversion from a fraction to a decimal and from the decimal register to the percentage. However, student A3 mobilized, in addition to these, also the registers in natural language, figural and discursive apprehension, by justifying that the result would be 50%, since the die has six faces, that is, three odd numbers and three even numbers. In the students' protocols, it is clear that the cognitive activity of treatment is in the fractional representation, that is, the construction of such a structure refers to the actions related to finding equivalent fractions, which was mobilized adequately by the students. According to Duval (2004), “treatment” is the cognitive activity in which the transformation of representation occurs within the same system.

By analyzing the actions responsible for converting to other representations at the processing level of the question, it can be observed that students A3, A8 and A9 performed the conversion activity adequately. This became evident when students were able to perform the transformations from fractional to decimal register, as well as from decimal to percentage, “different registers, but which represent the same concept, however, present different relative cognitive facilities” (Camargo Filho; Laburú, 2013, p. 55). Upon analyzing the presented registers, it was noted that 20% of the students still had difficulties in converting the registers; however, the others used auxiliary representations and performed the conversions correctly.

According to Duval (2009), the mobilization of registers of semiotic representation through conversion is not something so simple, and it can become even more difficult when the student has little knowledge about the mathematical object. It is believed that the level of difficulty of the third activity would be similar to the previous ones, and that the question should be tackled with more skill due to the fact that possible errors had already been corrected previously. According to Duval (2009), students have little difficulty with questions involving transformation through treatment, but struggle more with questions involving conversion.

Vizolli (2001, 2006) argues that students are able to represent decimal and/or fractional registers, however, they have difficulty in perceiving that 0,1 and 1/10 are representations of the same number or mathematical object (rational number). This difficulty becomes more pronounced “when teaching the concept of fractions converges to the meaning of the part-whole relationship, which prevents students from recognizing the same mathematical object in different representations” (Barros; Vizolli, 2021, p. 7).

Furthermore, it was also identified that when students were asked to engage in solving the third question in the classroom, they showed themselves favorable to this new challenge. An example of this is the following entry in the researcher's field diary, related to the statement of student A12: “I really understood this subject, I liked, professor, the way you taught us, without the need to memorize mathematical formulas”. This observation made by the student was uttered shortly after the completion of solving this question.

In terms of mobilization of registers, it was possible to perceive the use of figural and natural language registers in all three questions. These two registers were the most mobilized, occurring simultaneously, as students needed to transform the command of the question into a figural register, with the justification that this would facilitate their understanding; thus, consequently, they mobilized different representation registers associated with the same object

(Moretti, 2002). In reference to the presence of the figural register, we can highlight its use after considering the question of representation allied with simultaneous mobilization, as suggested by Duval and Moretti (2012, p. 3):

> There is an important and marginal word in Mathematics, it is the word "representation". It is mostly used in the verbal form "to represent". A writing, a notation, a symbol represents a mathematical object: a number, a function, a vector... Similarly, drawings and figures represent mathematical objects: a segment, a point, a circle. This means that mathematical objects should never be confused with the representation made of them. In fact, any confusion leads, sooner or later, to a loss of understanding, and the acquired knowledge quickly becomes unusable within its learning context: either because it is not remembered or because it remains as "inert" representations that do not suggest any treatment. The distinction between an object and its representation is, therefore, a strategic point for understanding Mathematics.

Regarding this, according to Duval (2013, p. 14), "the originality of mathematical activity lies in the simultaneous mobilization of at least two registers of representation at the same time, or in the possibility of switching between representation registers at any time". With the completion of the activities reported in this investigation, it is considered to have made progress in the development of probabilistic thinking of teachers in the early years of Elementary Education and, especially, in the construction of the autonomy of the different semiotic registers of future pedagogues.

For Duval (2013), it is essential, in mathematical activity, to mobilize and coordinate different registers of semiotic representation, such as natural language, graphical, algebraic, among others. Therefore, in this study, we sought to explore the widest possible variety of representational registers for the mathematical object "probability". Thus, it is considered relevant to ponder, still according to the author, the nature of each register presented by the students, in order to provide suggestions for activities involving treatment and conversion registers among different semiotic registers.

In general, the participants of the research perceived the contribution of the subject and, especially, of the probability content, as expressed by student A5: "my notebook still has blank pages, let's continue the content, professor". Regarding this statement, Batanero, Contreras and Díaz (2011) argue about the fragility of the knowledge of teachers in the early years of Elementary Education, and, in this case, pedagogues in initial training, considering that the majority do not have specific training or have taken a course on teaching probability.

5 Final considerations

Initially, we would like to emphasize that we cannot fall into a confusion of naive statements, because “the ideological inversion becomes evident in the reductionist equation: poorly trained teacher + poor quality school + unprepared student = national poverty!” (Shiroma; Evangelista, 2014, p. 13). The authors argue that if this situation were described in this way, the entire resolution would be encompassed in teacher training; maintaining this approach, we would fail to see, through the lens of the above reference, that often students' difficulties enter classrooms occupying their time and thought. This situation can hinder the learning of a particular subject if it is presented without considering different forms of representation.

Therefore, undoubtedly, the improvement of education involves, among numerous aspects, investments in the initial and continuing training of teachers, but it goes beyond that. It is necessary to invest in ongoing training, especially for multipurpose teachers, as a necessary condition for teaching content. This provides teachers with access to a varied range of interpretations, representations, and also equips them to develop lesson plans that involve students in presenting solutions to activities in the classroom. It is known that the curricula of multipurpose teacher training programs lack subjects in the field of exact sciences, such as Mathematics, Physics and Chemistry. However, it is these teachers who will teach basic scientific and mathematical concepts to students in Elementary Education. Therefore, it is essential to invest in ongoing training for this segment of education workers.

"Probability" has its peculiarities because it presents some characteristics that other areas of Mathematics do not have: the fact that it deals with chance is what makes it different. Our thinking is, by default, essentially deterministic – we are not prepared to deal with contingency. Therefore, it is essential for both teachers and students to become familiar with randomness, as it is part of life.

However, Mathematics teachers generally do not have the skills to teach probability in Elementary and High School. This is because the National Curriculum Guidelines (NCG) for Mathematics courses do not provide for the teaching of statistics and probability in Mathematics degrees. Thus, even if an Elementary School student is accompanied by a Mathematics teacher, it does not necessarily mean that they will have a teacher with sufficient knowledge to overcome the difficulties of their teaching.

The Article 4 of the NCG for Pedagogy, in its single paragraph, section VI, establishes that the multipurpose teacher must: “teach Portuguese Language, Mathematics, Sciences, History, Geography, Arts, Physical Education, in an interdisciplinary manner and suitable for different stages of human development” (NCE, 2006). It is not necessarily the case that a single teacher will handle the teaching of all these subjects, but generally, they will be responsible for most of them. If a Mathematics teacher, who undergoes at least three years of exclusive subjects in their field, may not have had contact with "probability" content, what can be inferred about a multipurpose teacher tasked with all the aforementioned areas? Indeed, it should also be noted that pedagogical and didactic knowledge alone is not sufficient; there must be an understanding of the subject matter and its context. Subject matter knowledge can be acquired through self-awareness or additional training, but understanding the context only comes with experience and a deep knowledge of the subject matter that goes beyond the school curriculum.

According to Lopes (2008, p. 66), "the central element of the teacher's professional knowledge is undoubtedly the didactics of the content, but it is not enough", emphasizing the importance of "pedagogical and didactic knowledge of how to teach it". The author further adds that "an appropriate combination of knowledge about the mathematical content to be taught and pedagogical and didactic knowledge of how to teach it is necessary" (Lopes, 2008, p. 60). According to the author, achieving these specifics is the greatest challenge because reaching such knowledge requires an investment in continuous training that goes beyond mere meetings – it is knowledge acquired through experimentation, reflection, and evaluation of what works and what does not. In this sense, when seeking to describe each activity and all the students' comments, it was possible to observe the attempts that motivated the resolutions, and also the fact that when the manipulated object is familiar, there is an approximation of the student with the question. After all, by knowing it, the only doubt is about how to reach the correct procedure.

When the student is able to comment on their ideas, it is possible to notice that attempts are made to combine the new content with something already known. Therefore, situations such as "what can be observed when dealing with a deck of cards" can become attractive, instead of situations created from, for example, the game of chess. Thus, what is taught has a close relationship with the proposed situation, and this can generate exchanges of ideas among students. Such exchanges may even be an opportunity for the students themselves to create other situations and, in this way, explore other moments in the study of probability.

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