

Analysis of pedagogical approaches in two collections of Mathematics curriculum materials

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Abstract: Assuming that the curriculum material is a tool to support the work of planning and implementing classes, the study presented was guided by the objective of *discuss the approach to the organization and role of students and the source of knowledge in two collections of curriculum materials for Mathematics*. Teacher's Manuals were selected from two collections of materials for the Final Years of Elementary School, in which Algebra tasks and related teaching guidelines were mapped. Based on pedagogical approaches as a theoretical framework, a documentary analysis was carried out considering the organization and role of students, and the source of knowledge. The results indicate that the role that students are expected to play in solving the tasks gives them little authorship in the learning process, with roles as respondents prevailing. As for the organization, the indication of individual resolution prevails, which disregards interactions in class. The source of knowledge is centered on students, teachers and curriculum material, with teachers standing out as the source.

Keywords: Teacher-Curriculum Relationship. Pedagogical Approaches. Student Organization. Role of Students. Source of Knowledge.

Análisis de enfoques pedagógicos en dos colecciones de materiales curriculares de Matemáticas

Resumen: Partiendo del supuesto de que el material curricular es una herramienta de apoyo al trabajo de planificación e implementación de las clases, el estudio presentado tuvo como objetivo *discutir el abordaje de la organización y rol de los estudiantes y la fuente del conocimiento en dos colecciones de materiales curriculares de Matemáticas*. Se seleccionaron Manuales del Profesor de dos colecciones de materiales para los últimos años de la Escuela Primaria, en los que se mapearon tareas de Álgebra y orientaciones didácticas relacionadas. Tomando como marco teórico enfoques pedagógicos, se realizó un análisis documental considerando la organización y papel de los estudiantes, y la fuente del conocimiento. Los resultados indican que el papel que se espera que desempeñen los estudiantes en la resolución de tareas no les confiere autoría en el proceso de aprendizaje, prevaleciendo el papel de encuestados. En cuanto a la organización, prevalece el indicio de resolución individual, que prescinde de las interacciones en clase. La fuente de conocimiento se centra en los estudiantes, docentes y material curricular, destacándose como fuente los docentes.

Palabras clave: Relación Profesor-Curriculum. Enfoques Pedagógicos. Organización de los Estudiantes. Papel de los Estudiantes. Fuente del Conocimiento.

Análise das abordagens pedagógicas em duas coleções de materiais curriculares de Matemática

Resumo: Assumindo que o material curricular é uma ferramenta de suporte ao trabalho de planejar e implementar aulas, o estudo apresentado se orientou pelo objetivo de *discutir a abordagem da organização e papel dos estudantes e da fonte do conhecimento em duas coleções de materiais curriculares de Matemática*. Foram selecionados Manuais do Professor de duas coleções de materiais

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para os Anos Finais do Ensino Fundamental, nos quais foram mapeadas tarefas de Álgebra e orientações de ensino correlatas. Com base nas abordagens pedagógicas como referencial teórico, foi realizada uma análise documental considerando a organização e papel dos estudantes, e a fonte do conhecimento. Os resultados indicam que o papel que os estudantes são perspectivados a desempenhar na resolução das tarefas pouco proporciona, a eles, a autoria no processo de aprendizagem, prevalecendo papéis como respondentes. Quanto à organização, prevalece o indicativo de resolução individual, o que desconsidera as interações em aula. A fonte do conhecimento é centrada nos estudantes, professores e material curricular, sobressaindo-se os professores como fonte.

Palavras-chave: Relação Professor-Currículo. Abordagens Pedagógicas. Organização dos Estudantes. Papel dos Estudantes. Fonte do Conhecimento.

1 Context of the study

The teaching of Mathematics involves different types of resources and support, ranging from manipulative resources, games, concrete materials to software that implements technology in the teaching and learning processes. In the considerations of Remillard and Kim (2020), these resources include curriculum materials, which are a wide range of programs and tools, printed or digital, available to teachers or generated by them to support teaching in the classroom.

In the studies we have been conducting, we use the term *curriculum materials* to refer to materials that translate prescriptions into various tasks to promote teaching and learning in Mathematics (Lima; Januario; Perovano, 2024; Soares, 2024, Rocha, 2025). Among the different types of materials, we focus on textbooks.

These materials, especially the Teacher's Manual, play an important role in the teaching process, since they incorporate texts and tools that are designed to meet the needs of teachers and their practices when implementing tasks in class. The Manual incorporates exercises, problems, and other tasks, whose approach includes definitions, examples, illustrations, and guidelines to support and communicate to teachers the conceptual and methodological approach, in addition to possibilities for development and monitoring of learning. Due to these characteristics, the Teacher's Manual supports the work of planning classes and other pedagogical actions (Remillard; Kim, 2020; Soares, 2024; Souza, 2024).

In Lima and Januario (2021) and in Lima, Januario, and Perovano (2024), it is asserted that these materials have been seen as vehicles that contribute to the promotion of pedagogical changes in Mathematics classes, that is, these materials incorporate innovations when implementing curricular reforms or when addressing content and presenting it in tasks.

Innovations can also be expressed in the guidelines for teachers, whether in the

introductory part of the Manual or in those guidelines in *L format*⁴ that accompany the reproduction of the pages of the Student's Textbook in some materials.

When interacting with the curriculum materials, teachers read and interpret the teaching guidelines and evaluate and select the tasks to then implement them in class. The tasks, in the words of Remillard and Kim (2020), serve to indicate what students need to do, serving as a context in which the content and teaching approaches are explored.

In this sense, it is important that teachers who teach Mathematics pay attention to how the tasks are presented and what is provided in terms of teaching guidelines, which can determine the source of knowledge and imply interactions in classes. Remillard (2012, 2018) and Remillard and Kim (2020) consider that, in curriculum materials, classes, tasks and units are carefully organized to promote student learning.

The authors understand that, when using a certain material to teach Mathematics, teachers need to make sense of the sequence provided, that is, how concepts and ideas are constructed from the content. Therefore, it is necessary that the material helps teachers understand the scope and guidelines contained in it, in an objective and transparent way, presenting sufficient explanations for teaching and learning to take place.

The notice for the *Programa Nacional do Livro e do Material Didático* [National Book and Teaching Material Program], PNLD 2022, defines that the Teacher's Manual is the material used, “in correspondence with the Student's Textbook, to improve, expand studies, prepare lesson and assessment plans (formative and large-scale) and overcome students' learning difficulties” (Brasil, 2023, p. 19).

The study reported in this article⁵, developed in the Research Group on Curriculum in Mathematics Education (GPCEEM), was guided by the objective of *discussing the approach to the organization and role of students and the source of knowledge in two collections of Mathematics curriculum materials*. We present an analysis of the pedagogical approaches expressed in the Teacher's Manuals of two collections of Middle School curriculum materials. Implicit in the objective is the purpose of critically analyzing the pedagogical organization and

⁴ The *L format* in teacher manuals refers to a specific visual arrangement of the teaching guidelines corresponding to the tasks. In this configuration, the guidelines for teachers are presented around the page, forming the letter L, occupying the side and bottom margins of the page, while the main content remains centered. This format makes it easier for teachers to consult the guidelines simultaneously while implementing the tasks in class, allowing an integrated view of the content and the guidelines for its approach.

⁵ The article is part of Rocha's master's dissertation (2025) defended in the Postgraduate Program in Education at the State University of Montes Claros, organized in a multipaper format, written by the first author, supervised by the second author and co-supervised by the third author.

the prospective roles that students assume, as well as the sources of knowledge, evaluating how these approaches influence the development of mathematical abilities and student authorship in the learning process. In the following sections, we present what constitutes the pedagogical approaches, the methodological design, the analysis and the considerations.

2 Pedagogical approaches

Curriculum materials, more specifically the Teacher's Manual, are used by teachers to support lesson planning and implementation, as they allow teachers to understand and define how the content will be treated and addressed in order to promote learning for their students.

When interacting with the Manuals, teachers establish an interaction with them, being guided to adopt strategies that direct both teaching and learning (Lima; Januario, 2021). In this interaction, teachers can reproduce or adapt, often motivated by what students demand or by considering previously developed teaching objectives.

When interacting with the Manual, teachers can change mathematical tasks so that they can meet the specific needs of their class. They can adapt them so that the student can better understand the statement. They can also interpret more complex statements and guidelines in order to provide students with more understandable and easy-to-interpret language.

For Collopy (2003), teachers change what is proposed in the curriculum materials due to the knowledge or hypothesis they have about the needs of their students. This practice encompasses beliefs and knowledge that teachers have about the content, its teaching and about how students can influence their responses in relation to the materials, including how they use them and what they learn from them (Souza, 2024; Soares, 2024).

Curriculum materials include a series of tasks — exercises, problems, investigations — and additional guidance and suggestions for teachers, who need to determine which tasks are appropriate for students or how they can be adapted (Collopy, 2003). Teachers need to understand the complexity and difficulty inherent in the tasks. To do so, they can categorize and organize them, identifying whether or not there are aspects that may hinder or confuse students' reasoning.

In summary, according to Collopy (2003), reading, interpreting, and reasoning about mathematical concepts as they are incorporated into curriculum materials represents an important aspect of teachers' work, since it is their responsibility to provide learning opportunities in accordance with what is proposed in the material.

In this context, teachers adjust mathematical tasks so that they promote more

meaningful learning, in addition to presenting other strategies for learning to occur. We call pedagogical approaches the set of practices and resolution strategies, types of approaches to content, expected interactions between teachers and students; and where the source of mathematical knowledge is located, which guide the teaching and learning processes in the curriculum materials.

According to Remillard and Kim (2020), when considering pedagogical approaches, we question how and who is responsible for structuring students' experiences. If the understanding is that teachers are responsible, it is up to them to define how and when tasks are implemented; if the understanding is that curriculum materials are responsible, it is up to them to structure and implement teaching practices. Regardless of who is responsible, students are given different types of participation in their learning process, which structure their experiences and lead them to assume different roles in such a process.

For the authors, pedagogical approaches are the expectations and assumptions underlying the curriculum materials, which shape students' learning opportunities during Mathematics teaching. This means that pedagogical approaches contribute to the integral development of students, considering cognitive, emotional, social and physical aspects, enabling the development of knowledge, valuing diversity and the formation of critical and participatory students. They also contribute to the interactions that teachers can develop in teaching practices.

Remillard and Kim (2020) state that prominent pedagogical characteristics, that is, central aspects in relation to pedagogical approaches, include how and with whom students are expected to interact during classes, that is, they determine how knowledge is transmitted, the role of the teacher and the student, the organization of the student, the organization and the way in which new concepts are sequenced.

The role and organization of students are related to what is expected of their participation in a given task, that is, whether they participate by responding and writing down what is explained by the teacher, or whether they participate by presenting the reasoning mobilized in solving the task. Similarly, in the development of tasks, students are expected to be organized to develop the solution, whether individually, in pairs, groups or collectively.

In the study reported here, we considered and analyzed pedagogical approaches, which include the *organization and role of students*, and the *source of knowledge*. In the following section, we present the procedures for data collection.

3 Methodological design

Scientific knowledge is formulated based on a rigorous and systematic research process. This process involves various stages, ranging from the formulation of hypotheses to the analysis of data, seeking to answer specific questions or propose new questions. The act of research is, according to Gomes (2001), a path to follow that helps people satisfy their natural curiosity and, with this, produce knowledge, that is, research is investigating something or someone with the purpose of producing something new, new knowledge.

In the process of the study reported in this article, it can be said that the act of research involved exploration and investigation that went beyond the act of searching for answers. It was a *dive* into the Teacher's Manuals, reading, interpreting, questioning, reflecting and analyzing teaching guidelines and tasks. More than that, the act of research was, for us, the construction of a dialogue, establishing an ongoing *conversation* with works by researchers who discuss what we were seeking to understand.

Thus, the act of researching in Mathematics Education, more specifically the research of pedagogical approaches in curriculum materials, is pertinent to discussing the teaching and learning processes and the interactions that teachers and students are expected to develop.

Based on the discussions of Fiorentini and Lorenzato (2012), and considering the objective proposed for the study presented here, the research is configured with a qualitative approach, characterized as a documentary analysis, since, for data collection, Teacher's Manuals were selected from two collections of curriculum materials for Middle Education.

As a first step, the choice of the two collections (Table 1) was made based on the fact that they were those used by the students who attended the private classes taught by us (first author), most of whom were from the 6th to the 9th grade. Both collections were evaluated and distributed by the National Book and Teaching Material Program, 2022 edition.

Table 1: Identification of the collections analyzed

ID	Collection	Author	Publisher	Year of Publication
CM	A Conquista Matemática [<i>The Mathematical Conquest</i>]	José Rui Giovanni Júnior	FTD	2022
MR	Matemática e Realidade [<i>Mathematics and Reality</i>]	Gelson Iezzi, Osvaldo Dolce and Antônio Machado	Saraiva	2022

Source: Own elaboration based on data from each collection.

In the next step, we chose the Algebra thematic unit, which refers to the main doubts of

students in private classes. Afterwards, we consulted the corresponding abilities in the *Base Nacional Comum Curricular* [National Common Curriculum Base — BNCC] (Brasil, 2017). With the Teacher's Manuals in hand, we proceeded to the third step, which is to map all the tasks related to the abilities, totaling 664. In addition to identifying the correlation, we read the task statements and the respective teaching guidelines for the teacher, in L format. From the reading, we identified evidence of the organization and the expected roles that students assume in the resolutions, as well as the source of knowledge.

The option to analyze the Manuals was due to the fact that they reproduce the pages of the Student's Book, in addition to incorporating guidance texts for teachers, which support the work of planning and implementing teaching practices. The guidelines in the introductory part of the Manuals, the tasks proposed for the students and the corresponding teaching guidelines, written in L format, were analyzed.

4 Analysis

The analysis of the tasks and teaching guidelines incorporated into the Teacher's Manuals focused on pedagogical approaches, based on three categories: *student organization*, *student role*, and *source of knowledge*. In the collection *A Conquista da Matemática* (CM), 394 tasks were identified, while the collection *Matemática e Realidade* (MR) presented 270 tasks. The analysis of the pedagogical approaches is presented below.

4.1 Student organization

In the context of this category, the analysis of the tasks mapped in the two collections of curriculum materials considered terms, commands, and expressions present in the statements, as well as the guidelines for the teacher in L format. The organization of the students is inspired by Remillard and Kim (2020) and Januario, Perovano, and Lima (2024): *individual*, *pair*, *group*, and *collective*. According to the authors, these structures characterize the expected participation of students in solving the tasks, including the frequency and variation with which they solve them.

The first organization considered, *individual*, is the one in which the proposal is for students to solve the task individually, without interacting with peers, that is, the student works on the task independently, with or without clarifying doubts with the teacher. This organization allows each student to develop their own strategies and assume full responsibility for their proposed resolution.

We noticed that terms and expressions are common in the guidelines for teachers as suggestions on how to organize students — *solve individually*; *do it individually*; *let students do it alone*; and *organize students individually*. In this category, it was possible to map 264 tasks in the collection *A Conquista da Matemática* and 219 tasks in the collection *Matemática e Realidade*, totaling 483 tasks in which there are indications for students to solve them individually, as can be illustrated in the following guidance for the teacher: “*It is important that students carry out these activities individually and write down the topics they find doubt or difficulty with*” (A Conquista da Matemática, 2022, 6th year, p. 74). Figure 1 illustrates one of the tasks mentioned in the guidance.

Figure 1: Example of a task associated with the individual organization of students in the Teacher’s Manual

1. Gustavo bought his son's school supplies on credit. He paid a down payment of R\$230.00 and divided the remainder into two equal installments. If the supplies cost R\$870.00, the value of each installment, in reais, is:

a) 300. c) 318. e) 330.
b) 315. d) 320.

Source: A Conquista da Matemática (2022, 6th grade, p. 74).

The guidance given to the teacher for the set of tasks is that students solve them individually, so that they can write down and understand their doubts and difficulties. Remillard and Kim (2020) consider that, in this dimension, students are expected to communicate mathematical ideas in writing or other forms of representation. The example in Figure 1 shows that this is a task in which the student will mobilize procedures related to fundamental operations, and may resort to a numerical expression. This type of task reinforces the idea that it can be completed independently, ensuring that the student develops their mathematical abilities and understanding, without intervention from other participants.

With the analysis, it is possible to observe that, in the tasks mapped with this indication, students are guided to participate in classes, communicating their strategies and resolutions to the teacher, expressing their doubts and understandings. The teacher is responsible for conducting the class.

In the context of the *pair* organization, we mapped 54 tasks, 41 of which are in the collection *A Conquista da Matemática* and 13 in the collection *Matemática e Realidade*. We identified some expressions present in the task statements, such as *in pairs*; *in advanced pairs*; *work in pairs*; *discuss together*; *solve in partnership*; *compare the answer with your partner*;

and *exchange ideas with your partner*. Such expressions suggest that the tasks be developed and discussed in pairs. As illustrated in the following excerpt: “*Emphasize, through the scavenger hunt, work in pairs. Students can perform the calculations together*” (A Conquista da Matemática, 2022, 7th grade, p. 146). The scavenger hunt is presented in Figure 2.

Figure 2: Example of a task associated with organizing students in pairs Teacher’s Manual

THINK AND ANSWER

The School on TV program organizes weekly competitions among students. In one of the programs, the following questions were presented. In one of the programs, the following questions were presented. Observe the options, the maximum time to answer, the score corresponding to each correct answer and participate in the competition by solving the questions in your notebook.

- What is the number whose three times plus 6 results in 21?
 - Table of answer options:

1	5	2	10
8	9	4	3

- Time to answer: 30 seconds
 - Score: 10 points
- Half of a number plus twice that number results in 20. What is that number?
 - Table of answer options:

5	7	24
12	15	16
18	6	20

- Time to answer: 1 minute
 - Score: 20 points
- How many points did you get in this competition?

Source: A Conquista da Matemática (2022, 7th grade, p. 146).

For the task (Figure 2), it is suggested that the task be solved in pairs, since it presents characteristics that indicate intense and focused participation by the student in partnership with another colleague. The process of solving the task in pairs allows students to work in a participatory and collaborative manner. In this type of organization, it is possible for them to decide among themselves how the dynamics of the pair will be. When the recommendation is for the organization in *advanced pairs*, the suggestion is that one student supports the other in solving the task, that is, the idea is that a student who has more learning can help the colleague who is having difficulty.

In the organization in pairs, the tasks mapped in the two collections are those that require greater mathematical reasoning in their resolutions, using different approaches, with expressions such as *combine your ideas and discuss your approaches*. These are expressions that indicate that the task can be solved in pairs. In addition, we observed that these are tasks that facilitate joint work, that is, most of them are challenging tasks that require the combination of different abilities.

By proposing tasks with this type of organization for students, the Mathematics teacher enables them to understand the importance of working together and helping each other, in

addition to promoting mathematical argumentation, since by suggesting the advanced pair, for example, the student with greater learning has the opportunity to explain to the classmate his line of reasoning for a given task, which promotes a challenging learning environment, where the student becomes an agent of his own learning and a collaborator in the learning process of a classmate.

In *group* organization, students are organized to solve tasks in small groups, in order to discuss among themselves the best resolution strategy, to negotiate procedures and to socialize discoveries. In the mapping, we found 54 tasks with the indication of this type of organization, 36 in the collection *A Conquista da Matemática* and 18 tasks in the collection *Matemática e Realidade*, with the highest incidence being in the 9th grade volume of both collections.

In the mapped tasks, we observed the presence of expressions such as *exchange ideas with colleagues*; *discuss in a group*; *solve with the class*; and *ask students*. These are tasks that promote collaboration and participation of several students; their statements are more complex and there is a combination of abilities related to them. Tasks with these characteristics are those in which students are invited to communicate their resolution strategies to the group or class, expressing their understandings or doubts (Januario, Perovano and Lima, 2024). The focus when proposing such tasks is on mutual help and collaboration, in addition to allowing students and teachers to exchange ideas. The following instruction to the teacher presents characteristics of this organization: “*To do this, divide the class into groups of four students: two will use one strategy, and the other two will use another. Students will choose the strategy to use to reach the resolution: numbers, diagrams, drawings of geometric figures or manipulative material*” (A Conquista da Matemática, 2022, 7th grade, p. 155). The task illustrated in Figure 3 refers to this orientation.

Figure 3: Example of a task associated with organizing students into groups in the Teacher’s Manual

7. Determine the root of the following first-degree equations with one unknown.

a) $\frac{x}{2} + 1 = \frac{x}{5} + \frac{1}{4}$

b) $\frac{x}{4} + \frac{x}{3} = x - 100$

c) $\frac{4}{5} + \frac{3x}{4} = \frac{1}{10} + x$

d) $\frac{1}{6} - \frac{x}{2} = -\frac{2x}{3} + \frac{1}{4}$

Source: A Conquista da Matemática (2022, 7th grade, p. 155).

Group resolution encourages collaborative participation and sharing of responsibilities.

In addition, it encourages student interaction, enabling the development of abilities, aiming for a solution with the participation of all those involved, as highlighted by Viana and Lozada (2020) in their study.

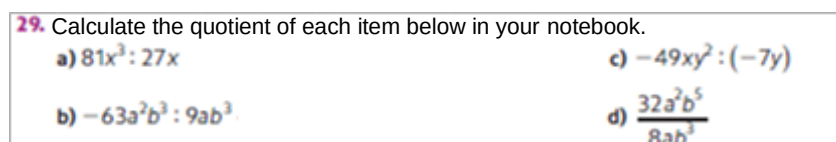
It is important to highlight that tasks that require students to be organized into groups need to be part of the pedagogical proposal of the teacher who teaches Mathematics, since when working in groups, students help each other, share responsibilities, work on different perspectives, and promote the debate of ideas, which enriches the class and provides moments of learning that are different from those usually proposed.

When considering the *collective* type of organization, it was possible to map 53 tasks in the collection *A Conquista da Matemática* and 20 in the collection *Matemática e Realidade*, totaling 73 tasks with this indication. In the tasks, expressions such as *exchange of ideas; see with the class; remember with the class, collaboration and joint project; and exchange ideas with the class stand out*. Expressions of this type suggest interaction between students. In addition, expressions such as *compare your solutions and discuss the different strategies* appear in a way that we understand them as reinforcing the idea of collectivity.

We identified in the following guidance the potential for organizing students into groups: “*It is important to promote a space for correction of each item, valuing the students' socialization and argumentation. This is another opportunity to identify whether students have doubts or still make conceptual errors*” (Matemática e Realidade, 2022, 8th grade, p. 109). This guidance accompanies a set of tasks, such as the one exemplified in Figure 4.

Notably, the mapped tasks are those that require different strategies for resolution and in which situations from the students' possible daily lives are addressed, or are tasks that guide moments of discussion and correction with the class, as is the case of the example illustrated in Figure 4 and the guidance that accompanies it.

Figure 4: Example of a task associated with organizing students into groups in the Teacher's Manual



29. Calculate the quotient of each item below in your notebook.

a) $81x^3 : 27x$ c) $-49xy^2 : (-7y)$

b) $-63a^2b^3 : 9ab^3$ d) $\frac{32a^2b^5}{8ab^3}$

Fonte: Matemática e Realidade (2022, 8th grade, p. 109).

Furthermore, characteristics such as different perspectives and abilities, promoting communication, questioning, exchange of ideas and sharing of information, are present in tasks that incorporate an indication of collectivity, as being ideal for the resolution process. In tasks

with an indication of collective organization, the entire class participates in the lesson, enabling communication of the procedure, discoveries and results of the students, or allowing the expression of doubts in class. In this case, the proposal is that the students help each other.

When considering the four types of student organization indicated in the tasks or in the corresponding teaching guidelines — individual, pair, group and collective — we must recognize the need for us, teachers who teach Mathematics, to diversify teaching strategies and class organization so that, together with other factors, an environment can be created in which students have the opportunity to be agents of their learning process. In each of the types of organization, there is an underlying objective to be achieved: in individual organization, the main focus is related to the individual work on the task, promoting reflection and autonomy in the student. In tasks whose organization is indicated by pairs, the focus is on mutual help and partnership between students. In small group organization, the focus is on dividing tasks and responsibilities, exploring the different abilities to be developed by students. Finally, in collective organization, interaction between the entire class is possible, exercising a sense of community, belonging to the whole and joint collaboration. In addition to these organizations, the main focus is on promoting student learning.

In the next section, we explore the expected role that students play in solving mathematical tasks, highlighting their participation when relating to what is presented in the curriculum materials.

4.2 Role of students

In the context of this category, we consider the expected role that students play when engaging in mathematical tasks, as they assume different roles in class. We consider three dimensions to be analyzed: the student as a *respondent*, *describer*, and *arguer*. Each of these dimensions classified here has its own characteristics and has unique importance in the context analyzed.

Remillard and Kim (2020) considered, in this category, how student learning was conceptualized, based on the types of learning they had when engaging with mathematical concepts and doing mathematical work.

In the mapping done, considering the role of students, we cataloged 290 tasks as respondent, 50 as describer, and 70 as arguer, totaling 410 tasks in the collection *A Conquista da Matemática*. In the collection *Matemática e Realidade*, we cataloged 284 tasks, 223 tasks as respondent, 43 tasks as describer, and 18 tasks as arguer.

In the category of the student as respondent, we identified common terms and expressions in the instructions to the teacher, such as how the student will answer the questions; *identify*; *determine*; and *verify*. Remillard and Kim (2020) highlight that, in this category, there are tasks in which students present answers without necessarily giving explanations for them. Furthermore, we consider students to be in the respondent role when their answers are direct and quick, expressing or confirming their answers with those presented by the teacher. In some cases, students copy the answers from the board or from the material that was made available, without explaining or describing how they proceeded in their resolutions (Januario, Perovano and Lima, 2024). The task illustrated in Figure 5 suggests the student in the respondent role.

Figure 5: Example of a task associated with the student as a respondent in the Teacher's Manual

8. It is known that a and b are two natural numbers such that $a \times b = 27$. What is the value of the expression $b \times a$?

Fonte: A Conquista da Matemática (2022, 6th grade, p. 53).

The following excerpt is part of the guidance to teachers, which reinforces the role of students as respondents.

With the proposed activities, students can be asked to compare the Egyptian, Babylonian and Roman Numeration Systems and consolidate their knowledge about the characteristics, similarities and differences between the systems, establishing connections with the Decimal Numeration System. Encourage them to have an active stance to develop and solve the proposed questions (A Conquista da Matemática, 2022, 6th grade, p. 18).

In the examples, it is possible to see the role of the student as a respondent in the task and in the guidance to the teacher. In the task, the student needs to use the properties of equality to determine the value of the expression; while in the guidance, it is suggested that the teacher ask the students to compare the numbering systems. As a respondent, the student can present initial answers that will allow the teacher to identify points that contribute to discussions and to evaluate the learning process.

When working with mathematical tasks with this dimension, it is important that the teacher be careful so that the student does not focus only on the final answer, without necessarily engaging with the underlying concepts and ideas, responding in a hurry, without any adequate reflection of the processes involved.

Next, we analyzed the tasks in which the student is expected to play the role of a *describer*, that is, we considered the tasks in which the students are asked to list a set of

procedures in the resolution process (Remillard and Kim, 2020). It was possible to notice common terms and expressions in the tasks, such as *observe*, *describe*, *describe steps* and *determine*. In these tasks, students are asked to do something, identify steps and patterns, check resolutions, recognize some property or definition.

In the analysis we performed, we catalogued 50 tasks in the collection *A Conquista da Matemática* and 43 tasks in the collection *Matemática e Realidade*, totaling 93 tasks in which students played the role of describer. By having students work on tasks with this approach, the teacher provides a type of learning, as the student needs to organize ideas to describe the resolution of the task, that is, he needs to understand the procedures.

On the other hand, when working with this approach, it is necessary that the teacher uses it with a frequency considered in relation to the others, preventing students from conceiving the tasks in a mechanical way, without giving them the importance they deserve. Let us look at an example of a mathematical task in which the student acts as a respondent (Figure 6).

Figure 6: Example of a task associated with the student as a respondent in the Teacher's Manual

8. The 40 students in a class sit in n rows of desks, each with $(n + 3)$ desks. In your notebook, answer the following: If there are no empty desks left, how many students are there in each row?

Fonte: *Matemática e Realidade* (2022, 9th grade, p. 79).

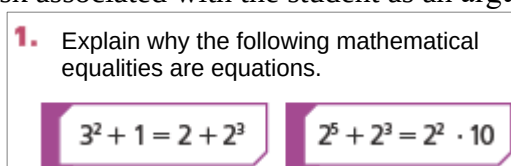
It is possible to see that, in this task, the student needs to observe and analyze that the number of rows multiplied by the number of desks will result in the number of desks, that is, he needs to solve the equation $(n(n + 3) = 40, \text{ logo } n^2 + 3n - 40 = 0)$ to complete the task.

In the analyzed Manuals, we identified a greater presence of tasks in this dimension in the 8th grade volume of the *A Conquista da Matemática* collection, with 20 tasks, and in the 6th grade volume of the *Matemática e Realidade* collection, with 14 tasks. The lowest index is in the 7th grade volumes of *A Conquista da Matemática*, with 5 tasks, and in the *Matemática e Realidade* collection, with 8 tasks.

In addition, we analyzed tasks in which the student is expected to act as an arguer. In this dimension, the student explains the procedures adopted, verbalizing his/her reasoning, informs what he/she identified in terms of relationships, properties, and characteristics, communicating his/her strategies and understandings. Remillard and Kim (2020) argue that in this dimension, students are asked to show the work they did to arrive at an answer using numerals or representations.

In the analysis performed, we identified the presence of terms and expressions in tasks, *such as show that, justify your answer, explain and discuss with*. There are 70 tasks with this approach in the collection *A Conquista da Matemática* and 18 tasks in the collection *Matemática e Realidade*, totaling 88 tasks indicating the student as the arguer. The greatest presence of this approach is in the 7th grade volume of *A Conquista da Matemática*, with 29 tasks, and in the 9th grade volume of *Matemática e Realidade*, with 9 tasks. Figure 7 illustrates a task that requires the student to act as an arguer.

Figure 7: Example of a task associated with the student as an arguer in the Teacher's Manual



Fonte: *A Conquista da Matemática* (2022, 7th grade, p. 142).

It can be seen that, in the task illustrated in Figure 7, the student needs to understand the ideas that justify whether or not a mathematical sentence is an equation, so that, when solving the task, he or she needs to justify why these sentences do not meet the definition of an equation.

In the following section, we present the role that the teacher plays in the tasks addressed in the classroom context.

4.3 Source of knowledge

The analysis of the mathematical tasks in the Teacher's Manuals of the two collections shows significant results regarding the centrality of the source of knowledge, in *teachers*, *students* or in the *curriculum material*. These were the three dimensions considered for the analysis, each indicating different implications for the teaching and learning processes.

Remillard and Kim (2020) highlight that the source of knowledge describes the types of tasks that are designed in daily classes that offer opportunities for students to interact with mathematical concepts and definitions, in addition to interacting with the teacher and other students.

It is understood that the source of knowledge is where the approach and treatment of the content are centered or where the content that enables the construction of knowledge is determined. In Remillard and Kim (2020) and Januario, Perovano and Lima (2024), there is an understanding that the source of knowledge is what reveals where the mathematical authority lies; the category considers the differences in how knowledge is generated and how it is

transmitted in the curriculum materials.

This focus may be on the student, when he or she acts with his or her prior knowledge and cultural experiences. It may be focused on the teacher, who acts as an interpreter of the content, addressing and transforming more scientific content into more pedagogical language. And it may be centered on the curricular material that systematizes and structures the content. In the tasks analyzed, it was possible to identify which sources of knowledge are predominant.

In the dimension in which autonomy is centered on teachers, we identified tasks in which these professionals are expected to assume the central role, dominating the dialogue and conducting the tasks, in addition to being responsible for interpreting and communicating concepts and definitions. In addition, it is the teachers who determine the tasks, the sequencing, the pace, and the type of involvement and participation of the students. These are tasks in which teachers are transmitters or facilitators, as we can see in the task illustrated in Figure 8 and in the corresponding teaching guidance.

It was possible to identify that approximately 86 tasks in the collection *A Conquista da Matemática* are teacher-centered, while 43 in the collection *Matemática e Realidade* have this approach. The tasks identified with this dimension are those that demonstrate a more traditional approach, in which teachers take control of the dialogue, directly guiding student learning.

Figure 8: Example of a task associated with teachers as advisors or facilitators in the Teacher's Manual

2. Using two letters, x and y, for example, write an expression that represents: a) twice a real number added to twice another real number. b) the product of the sum and the difference of any two real numbers. c) the sum of the squares of any two real numbers. d) the difference of the squares of any two real numbers. e) the square of the sum of any two real numbers. f) the sum of the square root of a real number with the fifth part of another real number.	Use Activity 2 to discuss some situations with students, for example: Is the equality $x^2 + y^2 = (x + y)^2$ always true for any real numbers x and y? No, by assigning any real values to x and y, it is possible to see that the equality does not hold. The equality is valid for which real numbers x and y? For the equality to be true, x or y must be zero, that is, equal to zero. What is the general expression for an even natural number (n)? And for an odd natural number? Students are expected to identify $2n$ as even and $2n+1$ as odd. Discuss with students the meaning of a generalization and its importance for Mathematics and how algebraic language was decisive in this context.
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Fonte: *A Conquista da Matemática* (2022, 8th grade, p. 105).

The analysis also identified tasks in which the centrality of knowledge is seen in students, that is, tasks that require students to be producers of knowledge. In this dimension, they are expected to reason, reaching discoveries and making connections with the expected content. Here, we relate students as describers and arguers, when, for example, tasks such as “How many whole numbers are there between the roots of the equation $x^2 - 2x - 15 = 0$?” are

proposed, and guidelines such as *“In activity 4, the number of whole numbers between the roots of the given equation is requested, an important fact for students to analyze different sets”* (A Conquista da Matemática, 9th grade, p. 104).

We can see in the task and the corresponding teaching guide that, in order to solve it, the student, in addition to calculating the roots of the given equation, needs to determine a set of values to know how many integers there are between one root and another of the equation.

The authors Remillard and Kim (2020) argue that in this dimension, students generate knowledge from the exploration of properties and relationships, and from the development of diverse resolution strategies. In addition, they determine the correction through reasoning, which makes teachers and the curriculum material act in the presentation of proposals to students, who are encouraged to explore and discover. Students, as a source of knowledge, need to build understanding and comprehension in their own way, through dialogue and investigation, individually, in groups or collectively.

We identified that this approach is present in 74 tasks from the collection *A Conquista da Matemática* and 21 tasks from the collection *Matemática e Realidade*. It was possible to perceive that the tasks that shift the source of knowledge to the student are those with more interactive, investigative proposals, proposing learning focused on autonomy; they are tasks with approaches that propose the student at the center of the development of their own knowledge.

As a third dimension in relation to the source of knowledge, we consider tasks that highlight the centrality of the curricular material, or teaching guidelines, such as, for example, *“Explore the notion of the universal set and the solution set of an equation through examples presented (and others that you consider necessary to expand on the board), emphasizing that an equation can have a solution in one universe and not in another”* (A Conquista da Matemática, 7th grade, p. 144).

In this excerpt from the guidance, the centrality is in the curricular material, when it is suggested that the teacher explore the notion of the solution of an equation and the universal set through the examples presented in the material, even with the indication that the teacher is free to add other examples.

When analyzing the tasks that shift the source of knowledge to the curricular material, as well as to the related teaching guidelines, it is possible to observe that the materials appear with a lesson structure and tasks already organized, outlined to support teachers and students in the classroom context. To this end, it was possible to see that, in the collection *A Conquista*

da Matemática, there are 70 tasks; in the collection *Matemática e Realidade*, there are 45 tasks.

In summary, this set of tasks presents characteristics that indicate that knowledge, teaching strategies, and correct answers are determined by the material supporting curricular development. In this sense, the curriculum material becomes the main source of reference for teachers and students.

Remillard and Kim (2020) consider that, in this dimension, knowledge and correct answers are determined by the material, which suggests its prescriptive role. This centralization can ensure uniformity and consistency in the quality of teaching. On the other hand, conceiving of the material as the only possibility of generating knowledge can make the flexibility of planning and teaching practices, and the creativity of teachers and students, unfeasible. It can influence teachers' abilities related to decision-making regarding teaching contexts and the contextualization of content in integration with different types of knowledge, as discussed in Lima, Bianchini, and Gomes (2018).

5 Considerations

With the aim of *discussing the approach to the organization and role of students and the source of knowledge in two collections of Mathematics curriculum materials*, in Middle School, the research presented in this article shed light on the pedagogical approaches inherent to the tasks and related teaching guidelines, incorporated into the Teacher's Manuals. The dynamism and creativity of teachers in relation to teaching Mathematics are important aspects when planning and implementing classes, in order to promote an environment that provides relevant learning for students. This is why it is important to discuss teaching approaches in curriculum materials.

The three categories that guided the analysis — student organization; student role; and source of knowledge — proved to be relevant to the pedagogical approaches. The way in which students are organized and grouped influences how they interact with each other and with the tasks, whether in moments of individual, pair, group or collective resolution, which directly impacts collaboration and engagement in the learning process. Likewise, the role that students play in developing tasks can provide them with autonomy and authorship, as they can take on the process as active agents or as passive recipients of the intended knowledge.

The pedagogical approach related to the source of knowledge indicates where knowledge originates in the classroom context: whether from the teacher, the student or the curriculum material. The source of knowledge shapes the teaching dynamics, and may imply a

flexible, collaborative and participatory pedagogical practice, meeting the needs of students and valuing interaction between them.

In this category, it was possible to observe that knowledge can shift to the student, to the teacher and to the curriculum material itself, implying the approach to teaching and learning; once centered on the material, this can limit the pedagogical dynamics and the autonomy of those who are part of the process.

Furthermore, when we, teachers who teach Mathematics, recognize who assumes the role of source of knowledge, we have the opportunity to reflect on how these different approaches impact the autonomy and development of students' abilities.

By bringing together the different categories analyzed, it becomes clear how each one contributes to the development of the teaching process. Each aspect, from the organization and role of students to the source of knowledge, functions as interconnected elements that together contribute to the design and implementation of a teaching practice that can reverberate learning processes for teachers and students. Understanding this allows teachers to create a learning environment that is not limited to meeting curricular prescriptions, but that also respects and responds to the individualities of students, their expectations and potential.

By adding the analysis to the pedagogical approaches analyzed here, it was possible to perceive the interrelationship between them, revealing how each one contributes to explaining how the Teacher's Manuals incorporate teaching tasks and guidelines. By analyzing the tasks and the corresponding guidelines, it was possible to infer the need to not only identify, but also balance the different approaches when planning and implementing lessons with the support of curriculum materials.

References

BRASIL. Ministério da Educação. Fundo Nacional de Desenvolvimento da Educação. **Edital de Convocação n. 02/2020 – CGPLI**. Edital de convocação para o processo de inscrição e avaliação de obras didáticas, literárias e pedagógicas para o Programa Nacional do Livro e do Material Didático. Brasília: MEC/FNDE, 2023.

BRASIL. Ministério da Educação. Secretaria de Educação Básica. **Base Nacional Comum Curricular**: Educação Infantil e Ensino Fundamental. Brasília: MEC/SEB, 2017.

COLLOPY, Rachel. Curriculum materials as a professional development tool: how a Mathematics textbook affected two teachers' learning. **The Elementary School Journal**, v. 103, n. 3, p. 287-311, jan. 2003. <https://doi.org/10.1086/499727>

FIORENTINI, Dario; LORENZATO, Sérgio. **Investigação em Educação Matemática**: percursos teóricos e metodológicos. 3. ed. rev. Campinas: Autores Associados, 2012.

GOMES, Alberto Albuquerque. Considerações sobre a pesquisa científica: em busca de caminhos para a pesquisa científica. **Intertemas**, v. 5, p. 61-81, nov. 2001.

JANUARIO, Gilberto; PEROVANO, Ana Paula; LIMA, Katia. Materiais curriculares (de Matemática) como gênero discursivo. In: **Anais do IX Seminário Internacional de Pesquisa em Educação Matemática**. Natal, 2024, p. 1-13.

LIMA, Gabriel Loureiro; BIANCHINI, Barbara Lutaif; GOMES, Eloiza. Conhecimentos docentes e o Modelo Didático da Matemática em Contexto: reflexões iniciais. **Educação Matemática Debate**, v. 2, n. 4, p. 116-135, jan./abr. 2018. <https://doi.org/10.24116/emd25266136v2n42018a06>

LIMA, Katia; JANUARIO, Gilberto. A relação professor-materiais curriculares e sua interface com o conhecimento profissional docente em Matemática. In: SILVA, Marcelo Navarro; BUENO, Simone. (Org.). **Estudos sobre currículos na Educação Matemática**. São Paulo: Livraria da Física, 2021, p. 147-167.

LIMA, Katia; JANUARIO, Gilberto; PEROVANO, Ana Paula. A relação de professores que ensinam Matemática e materiais curriculares. In: DUTRA-PEREIRA, Franklin Kaic; LIMA, Katia. (Org.). **Diálogos e Interfaces da Educação Matemática e da Educação Química**. 1ed. Cruz das Almas: EdUFRB, 2024, p. 21-38.

REMILLARD, Janine T. Examining teachers' interactions with curriculum resource to uncover pedagogical design capacity. In: GUEUDET, Ghislaine; In: FAN, Lianghuo; TROUCHE, Luc; QI, Chunxia; REZAT, Sebastian; VISNOYSKA, Jana. (Ed.). **Recent advances in research on Mathematics teachers' textbooks and resources**. New York: Springer, 2018, p. 69-88. https://doi.org/10.1007/978-3-319-73253-4_4

REMILLARD, Janine T. Modes of engagement: understanding teachers' transactions with Mathematics curriculum resources. In: GUEUDET, Ghislaine; PEPIN, Birgit; TROUCHE, Luc. (Ed.). **From text to lived' resources: Mathematics curriculum materials and teacher development**. New York: Springer, 2012, p. 105-122. https://doi.org/10.1007/978-94-007-1966-8_6

REMILLARD, Janine T.; KIM, Ok-Kyeong. **Elementary Mathematics curriculum materials: designs for student learning and teacher enactment**. Suíça: Springer. 2020. <https://doi.org/10.1007/978-3-030-38588-0>

ROCHA, Cleia Ferreira Niz. **Abordagem pedagógica em duas coleções de materiais curriculares de Matemática**. 2025. 96f. Dissertação (Mestrado em Educação) — Centro de Ciências Humanas. Universidade Estadual de Montes Claros. Montes Claros.

SOARES, Raíssa Caroline de Oliveira. **Relação professor-materiais curriculares e o conhecimento profissional docente em Matemática revelado no Estágio Supervisionado**. 2024. 155f. Dissertação (Mestrado em Educação). Universidade Estadual de Montes Claros. Montes Claros.

SOUZA, Iolanda Márcia de. **Relação professor-materiais curriculares e o conhecimento profissional docente sobre o campo conceitual aditivo**. 2024. 174f. Dissertação (Mestrado em Educação). Universidade Estadual de Montes Claros. Montes Claros.

VIANA, Sidney Leandro da Silva; LOZADA, Claudia de Oliveira. Aprendizagem baseada em problemas para o ensino de probabilidade no Ensino Médio e a categorização dos erros apresentados pelos alunos. **Educação Matemática Debate**, v. 4, n. 10, p. 1-28, 2020. <https://doi.org/10.24116/emd.e202017>