

Historical studies on mathematical education from the perspective of the teaching practice¹

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ABSTRACT

This work presents some theoretical and methodological considerations regarding why and how the history of mathematics and mathematical education contributes in developing the capacity of teachers to analyze their own practice. It supports the idea that, to advance towards such purpose, history must be *situated* in pedagogical manner and it presents the case of the appropriation of historical knowledge over mathematical formalism. Finally, the importance of comparative historical studies over the pedagogic *domestication* of formalisms is illustrated under different social and cultural contexts.

Kew words: historical studies, mathematical education, teaching practice

RESUMO

Este trabalho apresenta algumas considerações teóricas e metodológicas sobre por que e como a história da matemática e educação matemática contribuem no desenvolvimento da capacidade dos professores para analisar sua própria prática. Ele suporta a ideia de que, para avançar para esse fim, a história deve estar situada de forma pedagógica e apresenta o caso da apropriação do conhecimento histórico sobre o formalismo matemático. Finalmente, a importância de estudos comparativos históricos sobre a domesticação pedagógica de formalismos é ilustrada em diferentes contextos sociais e culturais.

Palavras-chave: estudos históricos, educação matemática, prática pedagógica

Pedagogical requirements of history as a resource for teaching practice.

There are three pedagogical “axioms” that are the general guide for historical mathematical studies. First of all, the objects of historical study must be “placed” alongside the means and purposes of the teaching practice and the particular conditions regarding the generation of mathematical thought in the students (Guacaneme, 2010). On the other hand, the nature of teaching is producing, not mathematical knowledge per se, but pedagogical and didactic knowledge instead. Third and last, the professional knowledge of the teacher points towards his intellectual autonomy as a consequence of the ideal practice of *knowing how to analyze* his own experience.

This practice would then lead to the teacher questioning the knowledge that mobilizes from his own experience (including the knowledge acquired from his own education), with the purpose of establishing to what extent he recognizes himself as a part of such or to discern if he must then produce a personal knowledge that is autonomous and interrogative (Perrenoud, 2005).

On principle, what is expected from this question on the disciplinary and professional aspects of the teacher’s experience is that it translates in the “improvement” of his practice. Regarding the first aspect, it enables a global understanding of the factors that intervene in any kind of thought development concerning an object under study such as (Perrenoud, 2001):

- Academic knowledge (about the purposes and contents of the disciplines, their historical formation, the personal dimension of the student, the institutional dimension of the school, the teaching modes), and

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professional knowledge (that define their relationship with the school system, peers, parents, other education social actors, their work and intellectual status).

- Informal arguments that circulate through communication.
- Intersubjective agreements that intervene in cooperation.
- Ideals and values that mobilize critical thought (on the objects of study, their social relevance, the individual and his relationship with others, universal human values).
- Specialized competences and integration Meta-competences (e.g., for the education of creative individuals that construct knowledge through cooperative learning).

Considering the liberating question on the experience of the teacher, knowing how to analyze is something that is only seen in the disciplinary or objectual dimension of the act of educating. It also has to do with the new “realities” in the field of education and pedagogy, which affect the professional dimension of the teacher:

- The de-localization and virtualization of the School.
- The fragmentation of the field of education with the emergence of new social actors.
- The reorientation of the mission of education according to the necessities of the market (in general, education participates in the unbalance between social movements and financial capital of this new age). Or its correlation with:
- The naturalization in the field of pedagogy, of a discourse on the *challenges* imposed on formal education by subjects that serve the new realities of a globalized world (the *accelerated rate of change* in the knowledge society, competitiveness and innovation, learning how to learn, learning to un-learn, relevant knowledge, etc.).
- The effects of the crisis of the historical relations between market, society and politics, and its influence in the field of education and pedagogy.

Facing this scenario of complex modes of questioning professional purpose and practice, the following question is befitting:

What type of history is useful to help in developing a pedagogy of knowing how to analyze?

- More than a single erudition is required regarding the event behind specific knowledge: who did it, when, in what time or under what mathematical context?
- An epistemological history on the academic wisdom is not enough in such itinerary of ideas until composing the wisdom that “really exists” (formal discourse).
- An education on the “positive development” of concepts, theories and trial methods, does not necessarily reveal the *reasons for being* of the theories that interest the teacher.
- Experience shows that this type of history can be *accommodated* well to the “transmissionist” model of teaching-learning...
- But be badly *placed* in the direction of pedagogy of knowing how to analyze (significant problems of the teaching practice).

History will be better placed in accordance with the pedagogic ideal of knowing how to analyze, if, besides explaining the epistemology of disciplinary knowledge, meaning the statute of truth of the statements of a theoretical system, it were to explain the reasons for being, referring to the *how* of the event that constitutes an object, considering the human activity of reasoning under a determined historical and cultural context.

A significant case of history as a resource for pedagogical practice: mathematical formalism.

The pedagogical appropriation of mathematical formalism is a real *burden* of the teaching practice while its nature eludes the knowing how to analyze element. The resistance to change of the formalist attitude of teaching has been recognized as one of the causes for failure of modern mathematical reforms. As Choquet said (Rogalski, 2007):

Time and time again we succumb to the miracle of carefully conceived programs and we think, especially at University level, that the final goal of our pedagogy is a well structured course on a modernized program.

The professor thoroughly prepares a beautiful, rigorous course, clear as the water of a fountain, and is later surprised after testing that this water has turned into a murky and somewhat repulsive liquid. What happens is that the beauty of the subject that is taught and the clarity of the presentation are not enough, and are even not essentially indispensable.

Then it is necessary to ask for the type of historical studies that will serve the purpose of helping the teacher to tame mathematical formalism as part of his practice. In this case the *how* refers to questions of the following nature that escape the conscience of the teacher regarding the formal mathematical objects:

- The act of constituting formal objects and theories from questioning mathematical problems.
- The articulated series of modes of conscience displayed by the subject as part of such actions; the role of intuition in formal thought.
- The manner in which new objects become objects under study and are “legitimized” under a theoretical domain.
- The mathematical objectivity procedures, the thematic processes employed, the principles of abstraction.
- The characterization of the nature of the objects as a function of the methods employed in their constitution.

According to Brousseau’s program, the matter is about understanding how the axiomatic method works to better orient teaching from the perspective of *didactic transposition* (Brousseau, 1986):

“The axiomatic presentation is a classical presentation of mathematics. Besides the scientific virtues that it is known for, it seems wonderfully adapted to teaching. It allows each instant to define the objects that are studied with the help of notions introduced previously, and thus enables one to organize the acquisition of new knowledge with the help of previously acquired knowledge. Hence, it grants the student and his professor a tool to order his activity and accumulate, in a minimum amount of time, a maximum of “know-hows” that are close to the “highest knowledge”. Evidently, this must be complemented with examples and problems whose solution demands its employment.

“But this presentation completely obliterates the history of knowledge, understood as the succession of difficulties and questions that have caused the appearance of fundamental concepts, their use to present new problems, the influence of techniques and questions originated from progress in other areas, the rejection of some points of view considered false or ordinary and the countless discussions on such matters. It conceals the “true” way science works, impossible to communicate and to describe faithfully from the outside, to put in its place, a fictitious genesis.”

History as a pedagogical resource would explain to us that not everything in formal thinking is logic (Kitcher, 1984) and (Arboleda, 2008):

- That one thing is *mathematical formalism* regarding the solution options of problems, deciding on propositions, consistency and criteria of existence based on demonstration...
- And a completely different thing is the *formal mode of thinking*: systematic, automatic, and extensive behavior.
- That as Hilbert said himself: “certain extra-logical objects exist intuitively concerning immediate experience prior to all thought”.

The comparative historical studies and the *domestication* of formalism in different socio-cultural contexts.

Still, this type of history does not satisfy the purpose of aiding the teacher to domesticate formalism in his teaching practice. Even if it clarifies the conformation of objects under the frame of theories and reasoning modes about objects in the context of mathematical activities, this history continues to be incomplete, at least from the pedagogic standpoint. There is a missing history that explains the transformations suffered by the theoretical reference systems in the constitution of systems of appropriation techniques and for using such objects and modes of reasoning under different socio-cultural contexts. (Radford, 2006) and (Obando, 2011).

For example, the history of the transformations introduced on mathematical rigor and formality, in the practice of teaching Calculus and Analysis, until achieving an average level of thought in our universities in a medium term

period, may be useful for the teacher interested in knowing how to handle the topic of objectivity according to the particular conditions of his teaching (Arboleda, 2010).

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