MATHEMATICAL MODELLING IN TEACHER EDUCATION COURSES: CONCEPTIONS AND TENDENCIES IN THE INTERNATIONAL COMMUNITY - ICTMA

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ABSTRACT

This paper presents the mapping of mathematical modelling (MM) pedagogical practices in teacher education courses done from publications available in the books organized by the International Conference on the Teaching of Mathematical Modelling and Applications – ICTMA. Mapping refers to the identification, survey, organization, classification, and data analysis of forty-two papers about teacher education. This study aimed at understanding how knowledge was built by society and became common part of the group, and not at analysing the quality of knowledge departing from immutable criteria. The analysed productions revealed that these researchers constitute a collective conception indicating the concern with and the support to the official documents and to the issues related to teacher education. The collective conception about modelling in initial and continuous teacher education courses seems to follow institutionalized standards.

Keywords: Mathematical Modelling, Teacher Education, Conception, Tendency, ICTMA.

RESUMO

Neste artigo, apresenta-se o mapeamento de práticas pedagógicas de modelagem matemática (MM) em cursos de formação de professores a partir de publicações apresentadas em livros organizados pela International Conference on the Teaching of Mathematical Modelling and Applications – ICTMA. Mapeamento refere-se identificação, levantamento, organização, classificação, análise de dados de quarenta e dois sobre formação de professores. Objetivou-se compreender como o conhecimento é constituído por uma comunidade e torna-se parte de um grupo. As produções analisadas revelaram que esses pesquisadores constituem uma concepção coletiva sobre modelagem em cursos de formação professores de matemática e parece seguir padrão institucionalizado.

Palavras-chave: Modelagem Matemática, Formação de Professores, Concepção, Tendência, ICTMA

1. Introduction

Mathematical modelling (MM) refers to the process towards the elaboration or the creation of a mathematical model about a problem-situation of some knowledge area, by making use of mathematical theories not only for the solution of a particular situation, but as support to other areas (Blum, Niss and Huntley, 1991). This process, applied in all sciences, has contributed extraordinarily to the evolution of human knowledge. According to Bassanezi (2002, p.19, *my translation*), “when it seeks to reflect upon portion of reality, in an attempt to understand or act on it, the process involves selecting arguments of reality or essential parameters and formalize them through a model".

In modelling, the procedures are basically the same employed in scientific research: problem delimitation; familiarization; hypothesis; model formulation; problem resolution; interpretation; and model validation. According to Biembengut (2003), such procedures require from the modeller mathematical knowledge and problem-situation knowledge, skills to read, describe and refine the data obtained from the phenomenon under the light of math, and creative and critical senses in the formation, the resolution and the evaluation of the model elaborated. Modelling is rooted in the human creative process and it encompasses scientific research.

In the last four decades in several countries, the defence for this process or method is increasing in the teaching and learning of math, in all levels of schooling. The main justification relies in offering students the opportunity to make use of math in order to comprehend a problem-situation of other knowledge area and know how to solve it. It means that teachers are able to provide students with opportunities to integrate math to other knowledge area, in particular, an area that students display interest in knowing more about. Osawa (2007) claims that the studied problem-situations give rise to knowledge learnt from experience, to comprehension reached through mathematical proofs, judgment, thought and foundation. According to Herget (2007), MM still facilitates students' comprehension of what is abstract (symbolic), by changing their perceptions in relation to the entities that surround them and in different ways, by stimulating their interests and, as a consequence, their learning.

MM for Education has emerged in the 1970s, impelled by the dissatisfaction and criticisms professors and businessmen interwoven in relation to the education of undergraduate students. For instance, Andrews and McLone (1976) explain that recent undergraduates in mathematics were good for solving problems, had reasonable knowledge of the literature and the mathematical techniques, however, they were not good enough to formulate, plan and make critical assessments of the problems encountered. The situation instigated many professors to find ways to justify math in the schools' curricular program. Among them, David Burghes in 1978, while working with high school teachers, in Cranfield University, produced papers and books about modelling, offered teachers workshops, and organized the first two International Conferences on the Teaching of Mathematical Modelling and Applications – ICTMA, in Exeter University, the United Kingdom, in 1983 and 1985. Since then, every two years, there is an expressive conference with a great number of participants and representatives worldwide. David Burghes is considered ICTMA's father - *International Community of Teachers of Mathematical Modelling and Applications* (Biembengut, 2003).

This interest in the ICTMA international conferences gave rise to an organized community that, besides promoting the conferences, is part of the International Congress on Mathematical Education – ICME. In the period 1983-2013, there were 16 conferences, and also, the Study Group 14 Conference of the ICME on modelling and applications (M&A). The studies presented in these conferences are published in printed books two years later. There are 16 books published until 2013 (from Conferences 1-15), about 500 papers on M&A with different focuses and from these, 51 were identified by being pertinent to pedagogical practices in MM in teacher education courses. In this corpus, there are applied and theoretical studies. From the applied ones, the focus is on the different schooling phases: from the early years of Elementary School to the end of graduation, and on the initial and continued education of teachers. According to Blum, Niss and Galbraith (2007), in the period 1965-1975, research suggests that M&A promoted arguments in favour of the inclusion of Mathematical Education; in the period 1975-1990, studies are characterized by the development of curricula and instructional materials to encompass the components of M&A; and from 1990 on, empirical studies on the teaching and learning of M&A have been added to the theoretical emphasis of research of the previous phases.

MM for Education has been stimulated and sustained by the gradual establishment of math teachers' communities, as well as the study groups and research groups that followed the proposals for the teaching of math. Worries about what, how, how much, and to whom teach math have contributed to the strengthening of the studies in MM for Education. These studies lead to research whose results imply proposals for teaching and learning that, in a cyclical process, promote new studies. The studies presented in conferences bring a conception that when shared, make ideas circulate and produce a collective conception. Conception here is defined as what comes from the beliefs, the knowledge acquired through the experiences and interactions of the individuals with the environment that surrounds them.

According to Thompson (1992), conceptions are meanings, concepts, propositions, rules, mental images, and preferences that a person has or a group of people have. The establishment of conceptions entails a process, both individual and social. Individual as a result of the activities and experiences a person goes through, and social as a result of the confrontation, the interaction with other people and other entities in the environment. This set of entities, the individual and the social dimensions and their closely associated relationships form a more or less interrelated and interdependent functional unit. Universally, conceptions tend to spread, even in the same generation, from the 'centers' to their 'margins', the meaning of existence and way of being of these relations are spread, as Linton (1971) explains. Consequently, in the various activities of the society, from the conceptions of several groups arise tendencies that, according to Ferreiro (1988), manifest themselves in different ways, and are renewed by the cohesion of its members through education and re-education of the people involved.

Education also contributes to each teacher's tendencies, in relation either to formal knowledge or to the procedures and methods learnt and/or experienced. These conceptions and tendencies may incur many implications, for example, they may instigate different understandings and lead to different criteria to determine the details and the relevant characteristics of educational activities and as the involved entities and relations see their meanings changed - different meanings from the previous one -, they support a growing change. Accordingly, the expression of each teacher and/or researcher MM in Education, whether in lectures or in written productions, brings her/his conception of modelling, based on knowledge or understanding that she/he has coming from experience, living and studying. This conception of the teacher/researcher, in particular, when expressed and disseminated, refers to what Fleck (1979) called style of thought.

According to Fleck's theory, each study expresses the researcher's style of thought from the type of accomplishment and/or the collective of people that have a similar style of thought constituted by knowledge and/or shared practices. Consistent with this theory, known as Compared Epistemology, style of thought "consists of a determined attitude and a type of accomplishment that completes it". And it is characterized by "common problem features that interest the collective of thought, by the judgments the collective thought considers evident and by the methods employed as means to know" (Fleck, 1979, p.145). Fleck proposed a theory about knowledge focused on heuristics and his theory allows for a historical and epistemological analysis to distinct areas of knowledge. It is centred in the analysis of academic productions, oriented by socio-historical studies to comprehend the interaction between the scientific practice and the contexts in which they occur. According to Delizoicov, Castilho, Cutolo, Da Ros, and Lima (2002), it is an interactive model of the knowledge process, connected to the social, historical, anthropological and cultural presuppositions and conditionings, that as is processed, transforms reality.

Salles (2007) explains that throughout time, the action of researchers and the social factors that interfere with the constitution of science construct a specific trajectory, with no determined beginning or end. It means that the most diverse historical, social and epistemological factors and entities interfere in the generation of scientific knowledge, that in turn requires reflection upon the facts and entities involved, careful and accurate historical analysis so that the common traces in the process of constitution of scientific concepts are identified.

Mathematical modelling, a research method particularly used in the sciences, has been proposed in Mathematics Education in the recent decades. The educational effort to provide better teaching of math culminated with the development of research on MM for education worldwide. As this defence for M&A in the ICTMAs has been on for about three decades, it is considered that conceptions - *styles of thought* - about MM for education are formed due to the idea circulation from a collective conception and, this way, understandings change or vary permeating discussions about teacher education in several countries. This collective conception contributes to generate different understandings and therefore different tendencies. In this realm, it is feasible to inquire: *what are the conceptions about mathematical modelling in courses of initial and continued teacher education in the international scene*?

In the ICTMAs there is circulation of ideas from *conceptions*. Each teacher/researcher comes from a social and historical context (active connections) and each teacher/researcher perceives the reality in such a subjective way that nurtures his/her research (passive connections). Therefore, teacher/reseacher's interactions allow for the establishment of conceptions, and by recurrence, collective conceptions about M&A for Mathematical Education.

In this ambit, this study aimed at analysing the productions published in the ICTMA's books about pedagogical practices on MM in teacher education courses. These productions, as shared by the scientific community, as ICTMA's, consolidate scientific knowledge. The intention in developing this research on international productions is justified by: (1) considering modelling a consolidated practice in basic education; and (2) highlighting the relevance of showing the community of researchers in mathematical modelling part of what was produced in ICTMAs and the contributions these productions can offer teachers and researchers.

2. Methodological Procedures

This study is bibliographical since the data base consists of 42 papers about the pedagogical practices in MM in teacher education courses. From these, 35 were published in nine ICTMAs books (published until 2011) and seven were published in the Study Group[1] 14 book (2007). Although the ICTMAs began in 1983, research focused in teacher education began to be presented from 1993 conferences on, whose publications were released from 1995 on. For this reason, the sources come from the period 1995-2011 (I did not analyse papers of the 15th Conference because there were only published in October 2013). It is essential to highlight that this kind of scientific production brings a set of studies conducted by M&A researchers and that these studies generated knowledge accepted by the scientific community. As stated by Biembengut (2008), studies based on

[1]MM was the 14th theme for the study of the International Commission on Mathematical Instruction (ICMI), initiated in 2001 and published in 2007.

bibliographical documents may offer a map about the theme of the problem or hypotheses to conduct the verification by other means. Consistent with Lüdke and André (1986), the chosen method resides in identifying factual information in the documents, from issues that allow affirming or evidencing something.

Therefore, the papers, written by whom participated in one or more ICTMAs, constitute a natural source that reveal different contexts (institutions and countries) and enable different interpretations and analyses. In this research phase, the focus resides in identifying the conceptions and tendencies about modelling the authors of these papers display. I expect in the following phase, to interview these authors to better understand their MM practices. This study was developed in two stages, named identification map and recognition map, as follows. These stages did not occur separately.

2.1. Identification map

This phase consisted of identifying the field in which the object is inserted in. The 42 papers (appendix - table 2) analysed belong to 32 "first authors"[2] of 12 countries of five continents. Following, I identified the countries, the number of papers and the number of same authors(s) and different authors (d): Australia (4: 2s, 2d), Brazil (6: 2s, 3s, 1d), Canada (2d), China (1), Denmark (1), Germany (5: 2s, 3d), Mexico (2d), Portugal (1), South Africa (2s), Sweden (5s), United Kingdom (1), USA (12: 2s, 10d).

In the first moment, I opted to assemble the papers into three groups, published between the years 1995-2011: the first (1995, 1997 & 1999), the second (2001, 2003 & 2005) and the third (2007, 2009, 2010 & 2011). I hypothesized there would be some changes throughout the years. But, departing from a careful study of all papers, I could not identify significant differences. In each paper, the researchers identified and classified the questions, sources and methods upon which the data were obtained. A summary of each paper was done and common topics were highlighted. As all the papers present: (1) justification, (2) process, (3) possibilities and (4) difficulties in making use of modelling for Education, I considered these topics as categories of analysis. These data allowed me to have a system for recognition. In Appendix (table 1), I present the similarities identified in the articles according to each category and each group. Due to space limits, the summaries of the papers and the resulting maps will not be offered in this article.

2.2. Recognition map

I sought to get acquainted with Fleck's Comparative Epistemology (1979) to have a better glimpse of the conception about the pedagogical practices in MM in teacher education courses worldwide. According to Fleck, a *style of thought* is understood as shared practice and knowledge. Based in Fleck, when this *style* or conception is shared by a researcher's group, this one displays a collective conception. After attentively reading the papers, I identified possible similar conceptions in MM, traces that were recognized and valued by the community. In addition to that, I sought to comprehend the studies departing from confluent and/or indicative places that suggest conceptions in a community of modelling for Mathematical Education. In the following section of this work, I sought to make the data explicit such that an illustrative image of the studies and the results could be produced.

This study did not aim at analysing the quality of knowledge produced departing from immutable criteria. Instead, it aimed at comprehending how knowledge was constructed by the community and

[2] I use the expression "first authors" due to the fact that I counted 'one' for the papers that have more than one author.

how it was integrated in the common pile of the group in the period 1995-2011. I admit the coexistence of different knowledge models and educational development so that I am able to recognize the similar points among these studies and the possible factors conditioned by the scientific communities. I assume that these processes do not express people's neutrality. I understand that scientific knowledge does not emerge in the methodological order of observation/experimentation in a distinct perspective from the empiricist conception. It is not neutral from the conception of who studies, of who prescribes results.

3. Results & Discussion

The analysed 42 papers on pedagogical practices in MM in teacher education courses bring the historical, social and educational context in which they were produced. All of them present applied research. They describe the process of modelling and use a group of students, research collaborators, future teachers or teachers in continued education as participants to collect data. These studies depart from specific experiences lived in the classroom and contribute to understand the different issues involved. The authors defend the primacy of MM in the development of Basic Education teachers. According to Salles (2007), these activities contribute in such a way that the researcher learns to identify stable elements in the research object and, establish facts tacitly accepted by the collective of conception that permeates the international community.

Although curricular programs are similar in the 12 countries from the five continents that the authors of the studies analysed belong to, it is not possible to deny that the studies occurred under the cultural reference and own meaning that these curricular contents have to the respective population; ways that guide the ones interested in identifying conceptions. Meanwhile, when organizing the text statements into categories, it was possible to identify that some occurrences and reflections are common in the publications, even considering the elapsed time between one conference to the other and the participants of different countries. These similar reflections suggest the circulation of ideas, allowing for the establishment of a collective conception.

According to item 2.1, the established categories were: (1) justification, (2) process, (3) possibilities and (4) difficulties in making use of modelling for Education. In what follows, I turn to the reflections upon each category, considering the styles of thought present in the productions and that commune with the collective of thought.

(1) The authors' *justification* is based on the understanding that MM for Education allows the student in each schooling phase: (a) to learn the mathematical concepts better; (b) to interpret the meanings of the mathematical concepts; (c) to use technological resources to solve problems; and (d) to make students aware of social and environmental issues. This justification, in the 42 papers, is endorsed by the criticism to the style of teaching still in vigour in almost all the countries. This still effective teaching, according to Delizoicov, Castilho, Cutolo, Da Ros, Lima (2002), an indoctrination, dominated by a purely dogmatic teaching, also in teacher education courses, contribute so that students/future teachers imitate the model, contribute to the theoretical foundations and the way teaching operates.

Two common justifications in relation to school education are apparent in the expressions of the authors: (1) the promotion of knowledge to students that may be useful to them through the various disciplines or areas of knowledge; and (2) the inability recent undergraduates display to use academic knowledge in their work, a fact mentioned in almost all sectors of society. The justification of the majority of authors - departing from the criticism to the current teaching and departing from the defence for modelling - suggests a conception in this community.

This conception shows that *MM for Education allows teachers and future teachers to become aware of the various issues of society and of the results the current schooling structure produces*. This consciousness can display another conception about the way to teach; guiding them to commit themselves to make use of pedagogical practices that promote better formation of students in Basic Education. This understanding requires knowing how and when to approach curricular contents, but also knowing how to make the students from Basic Education, in particular, draw on this knowledge in moments beyond the school limits. It is a conception legitimized by the community that works with modelling for Education.

(2) The *process* of MM defended by the authors is that the teacher should: (a) depart from a subject or problem-situation of any area of knowledge that is interesting to the group of students; (b) ask the group to look for data that give rise to issues and then, seek for the problem solution; (c) orient the students to formulate these data making use of any mathematical structures (concepts, definitions, properties); and (d) guide them to solve the issue and evaluate the results. This conception about how to do modelling in sciences is maintained in the process of school teaching; it is how students are prepared for searching. It is characterized by the common traces of the problem-situations that the majority of papers bring as examples. The proposed problem-situations conduct students to a collective conception about MM as a process or method that enables knowing and developing competence to deal with issues of their surrounding environment.

The main argument is that modelling provides students with opportunities to make connections between the language present in their surroundings and the mathematical language. During this process, students improve their conceptual structures, their understandings of the mathematical concepts, and more, their critical and creative senses are improved in the formulation of data and in the evaluation of results. This argument indicates a conception that considers *MM for Education a way to surpass the linear process of teaching that is decontextualized from students' experience.* It is considered that each student has his/her own cultural spectrum that is revealed in his/her doings, outside the school context. When s/he experiences the modelling process in a triad: research-school-reality, the student can perceive his/her talent, his/her interests and even the importance of the school in this process of literacy to scientific and professional issues or issues related to companionship.

(3) About *possibilities*, the authors argue for MM as a means so that students may (a) construct their knowledge by understanding the concepts involved; (b) choose significant problems to the context; (c) become capable of explaining their reasoning with the correct use of mathematical language; (d) have better performance in mathematical modelling activities; and (e) know how to use it in their pedagogical practices, since these students are future math teachers. This defence, present in all papers, suggests that when doing, people get to know and when people get to know, they do. It is a cyclical process. Going from the simplest to the most complex issues and continuously revising can make modelling a more effective process to deal with the various issues that involve the living context. *MM for Education is a dynamic process: it can be modified whenever necessary so that students' knowledge can be improved.* This instance reveals another conception.

(4) The *difficulties* in the implementation of MM reside in the current educational structure: available timings and schedules, curricular programs divided into various subjects. Difficulties emerge for future teachers and in-service teachers. For future teachers the difficulties reside in knowing how to use the mathematical language to describe the problem-situations and, thus, resist to changes. And for in-service teachers, difficulties reside in the available time they have to get acquainted with the themes chose by the students to orient them.

The curricular model in teacher education courses is similar in the various countries. It consists of several disciplines, each one under the responsibility of a teacher and with a restricted number of class hours. Math, although present in all schooling years of Basic Education, follows, in general, the same process of teaching, without connection to the other disciplines of the curricular structure. This model contributes to the fact that the students from these teacher education courses, who experienced a ‘traditional’ way of teaching for more than 12 years, have difficulty in interpreting the context and data from a subject of any knowledge area; in recognizing the math required to interpret data; in formulating mathematically and in analysing it.

This difficulty, a consequence of the experienced education by these teachers, emerges when these students or even teachers become aware of MM. They learn about it in a single subject in an initial or continued teacher education course, with a limited number of hours if compared to the time experienced in ‘traditional teaching’. As a result, obtaining effective results depends on the interest the participants involved have in following the modelling pathway; that advances gradually departing from practical and conscious activities about the importance of learning.

The current educational structure in various countries does not lead to the integration among math and the other disciplines of the course, neither the diverse areas of knowledge. As a consequence, *there are difficulties students and also teachers have in making use of modelling in the classroom practices, particularly in the initial and continued education, due to the time spent in this educational structure.* Paraphrasing Salles (2007), MM will only reach a status of legitimized knowledge if it resists to the tests imposed by the group that ‘maintains’ the traditional process in the teaching of math. The incorporation of MM for education can establish a dynamic relation between the ‘traditional’ and the ‘innovations’. Allowing for a change in the conception of math teaching departs from reactive interactions among the teachers in the group. This kind of argument present in many these 42 papers suggests another conception.

Departing from the categories: *justification*, *process*, *possibilities* and *difficulties*, it may be claimed that the conception displayed by these 42 papers, of the 32 ‘first authors’ converge in the understanding that modelling can contribute not only to improve mathematical teaching and learning, but also to provoke reaction and interaction between the body of teachers and the body of students involved in the on-going and necessary production of knowledge. It is a mutual share of acquired experiences. These authors, participants of the ICTMAs, make explicit their knowledge gained in the interactions among the theories of MM for Education and their practices of modelling in classrooms. As Wenger (1998, p.45) points out, “this collective learning results in practices that reflect both the pursuit of our enterprises and the attendant social relations”. These interactions conduct to a conception.

According to Fleck (1979), so that a style of thought can be constituted, it passes through the phases of instauration, extension and transformation that occur by means of interactions of distinct groups in the circulation of ideas inter-collectively and intra-collectively. This circulation of intra-collective ideas occurs among experts and the circulation of inter-collective ideas among non-experts. This process implies seeking for knowledge that brings increment to the existing data and yet, creates a collective a thought. It is worth highlighting that these styles of thought, that come since the teachers-researchers of the first phase (1965-1975) in the classification proposed by Blum, Niss and Galbraith (2007), are present in the community, independently of the country, the geographical distance, the educational system.

These conceptions or *styles of thought*, denominated by Fleck (1979), may be considered the scientific knowledge of the ICTMA. This knowledge was consolidated by facts, theories and interpretations shared by the community of practice. Perceiving that the process involved in MM for education is even more relevant when dealing with issues that allow having a particular set of data that can be better studied using specific methods and as consequence, become charmed with the solution and the validity of this solution.

4. Final Remarks

The papers analysed reveal that these researchers who circulate in the conferences organized by ICTMA constitute a collective conception, indicating the concern and support to the official documents and to the issues related to teacher education. The collective conception about modelling in math teacher education courses follows institutionalized standards. It means that the events in the modelling process in courses for teachers or future teachers enunciate necessary changes in the educational structure, based on values, goals and other inspiring stimuli that come from the different people within the system.

To Fleck (1979), the researcher's conception designates the formal aspects of his/her research that comprises all ways of expressing the units associated in the process and in the results. The conception comes from shared practices and shared knowledge. When a certain conception is shared by a group of people, a collective conception – a tendency is established. It may be claimed that in the ICTMA conferences I encounter groups that share the same conception that is composed by collective conceptions. This sharing occurs when the researcher identifies in his/her research object traces that are recognized and valued by the community, that are present in other studies about similar themes. There are hues of this conception that arrange facts and fit in the MM theory for education dominant in the discussions of mathematical education. These various elements coalesce around different conceptions of knowledge production, but when confronted, as pointed out by Salles (2007), constitute an extension of the established conception.

This fact indicates that the style of thought about MM for Education that circulates in the ICTMAs is established in the knowledge of different groups of researchers. And according to Fleck's theory (1979), the changes in this area will become noticeable as a collective process, in which the transformations will be construed by the community tendency, when the ideas circulate in the ICTMA conferences. The importance of analysing scientific productions throughout time in MM in the international scene is justified by the panorama that this type of research results, providing a map of the contributions, needs and challenges related to MM. According to Witter (1996), it is by means of this type of research that a basis of scientific data may be formed. This basis consolidates certain knowledge and thus, allows for scientific advancement.

Investigating how each group's conception is adopted, how it is incorporated by the community, and how these conceptions are established and transmitted, constitute the beginning of another study. Diverse presuppositions may be formulated. Some may direct the researcher's steps to the origins of M&A in the teaching practices of the school system. It is relatively simple to identify how the ideas and proposals are disseminated. These proposals and ideas, written or verbalized, carry styles of thought, express attitudes or ideals that deal with experiences, beliefs, values and casual sequences, whose order is diverse.

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APPENDIX

Table 1: Similarities identified in the articles

Categories	Occurrences identified in the articles	Team
Objectives	Helping students to: - develop skills of reflection in MM activities for better understanding mathematical concepts; - interpret the meanings of mathematical concepts; - use technological resources to solve problems; - experience and understand the processes of problem solving; - do research departing from mathematical models of the real world; - use math concepts in social life. Checking and analysing: - how MM activities are being inserted in teacher education; - the students' competence and skills in developing MM activities; - how professionals from other areas develop MM activities; - the degree of development of mathematical concepts by means of MM activities; - how MM activities influence teachers' practice and their mathematical knowledge; - the interactions between teachers and students during the development of MM activities; - the processes and outcomes of a course on MM; - the students' difficulties with the use of technology in a course on MM.	 (1, 2, 3) (1, 2, 3) (1, 2, 3) (1, 3) (1, 3) (2) (1, 3) (2, 3) (2) (3) (3) (2, 3) (1, 3) (1)
MM process	The author(s) describe the modelling process	(1, 2, 3)
Possibilities	MM enables students to: - build their knowledge departing from the concepts involved; - choose meaningful problems from the context and be able to explain their reasoning with correct use of mathematical language; - write and express strategies and solutions; - recognize learning as a process of (re)construction of knowledge; - interact with the teacher and rely on her/his contribution during the MM process; - experience situations that can influence practice; - understand concepts and develop skills for solving problems; - show excitement and motivation with the proposals; - obtained good performance; - recognize MM as part of their training; - understand mathematical concepts and their responsibility for their own learning; - acquire knowledge and develop skills and attitudes to interact in society; - use MM in their pedagogical practices. It enables teachers to: - change attitude towards MM activities with their students; - assist students in the process of acquiring skills and abilities; - better organize the curriculum; - evoke interest, motivation, joy and success in teaching mathematics; - develop skills to improve the way mathematics is taught. Other possibilities: - identifying the possible implications for undergraduate programs in mathematics;	 (1, 2, 3) (1, 2, 3) (1, 2, 3) (1, 3) (1) (1) (1) (1, 3) (1, 2, 3) (1, 3) (1, 2) (1, 3) (1, 2, 3) (1) (1, 3) (3) (1, 3) (1) (1, 3)

	- teaching mathematics with meaning; - offering future teachers knowledge about MM, since practice is formed during training; - providing an alternative method to the traditional one; - enabling innovations that traditional assessment would not reveal; - using mathematical models in different situations, in new applications.	(1, 2, 3) (1, 3) (1) (1) (1, 2)
Difficulties	In relation to students, the difficulties encountered refer to: - resisting to change; - addressing issues in a non-linear fashion; - engaging in some stages of the course; - working in groups, the written and oral exposure; - having weak notions of elementary mathematics; - understanding the context surrounding a given situation; - the mathematical language; - not paying attention to important data provided by technological resources; - validating the model; - knowing how to relate the real world with the abstract model; - choosing the appropriate model for the studied situation; - understanding the relationship between the model and the real situation. Other difficulties encountered: - some necessary processes to carry out MM activities are not possible to be approached in the existing curriculum - the inclusion of this perspective in schools has been slow; - the lack of experience teachers have in MM, in the interpretation of the context, in the availability to carry out research, in the choice of subject and in group work.	 (1, 2, 3) (1, 3) (2, 3) (1, 2) (1) (1) (1, 2, 3) (2) (2) (3) (3) (3) (3) (3) (3)

Source: this author

Table 2: Articles studied: ICTMA, 6 to ICTMA

i	Event Year	Year of Publication	Title	Authors	Edition
Sloyer C, Blum W and Huntley ID, 1995, editors, Advances and Perspectives in the Teaching of Mathematical Modelling and Applications, Water Street Mathematics, Yorklyn, Delaware.					
1	1993	1995	Mathematical modelling as a context for preservice teacher education	Barry Shealy	ICTMA 6
Houston SK, Blum W, Huntley ID and Neill NT, editors, 1997, Teaching and Learning Mathematical Modelling, Albion Publishing Ltd, Chichester					
2	1995	1997	An introductory course on mathematical models and modelling: a constructivist approach for middle school teachers	Don Cathcart; Tom Horseman	ICTMA 7
Galbraith P, Blum W, Booker G and Huntley ID, editors, 1998, Mathematical Modelling, Teaching and Assessment in a Technology-Rich World, Horwood Publishing Ltd., Chichester.					
3	1997	1998	Physical modelling of mathematical concepts in subjects studied by pre-service undergraduate teachers: a pedagogical perspective	John Green	ICTMA 8
4	1997	1998	Prospective South African Teachers' handling of pedagogical activities related to the applications of mathematics	Cyril Julie	ICTMA 8
Matos J. F, Blum W., Houston K., Carreira S. P., editors, 2001, Modelling and Mathematics Education: ICTMA 9: Applications in Science and Technology, Horwood Publishing, Chichester.					
5	1999	2001	Mathematical modelling in pre-service teacher education	Jonei Cerqueira Barbosa	ICTMA 9
6	1999	2001	Mathematical modelling and technology in teacher education - Visions and reality	Thomas Lingefjärd; Mikael Holmquist	ICTMA 9
7	1999	2001	Mathematical applications and modelling: a case study involving first year higher education	Fernanda Tavares	ICTMA 9

			students		
Ye Q, Blum W, Houston SK and Jiang Q, editors, 2003, Mathematical Modelling in Education and Culture: ICTMA 10, Horwood Publishing, Chichester					
8	2001	2003	A mathematical modelling course for pre-service secondary school mathematics teachers	Zhonghong Jiang, Edwin McClintock e George O'Brien	ICTMA 10
9	2001	2003	Mathematical modelling in teacher education	Mikael Holmquist e Thomas Lingefjärd	ICTMA 10
10	2001	2003	Two modelling topics in teacher education and training	Adolf Johannes Riede	ICTMA 10
Lamon S., Parker W., Houston K., editors, 2003, Mathematical Modelling: A way of life ICTMA 11, Horwood Publishing, Chichester.					
11	2003	2003	Learning mathematics using dynamic geometry tools	Thomas Lingefjärd; Mikael Holmquist	ICTMA 11
Blum, W.; Galbraith P.L.; Henn, H.W.; Niss, M. Modelling and Applications in Mathematics Education: The 14th ICMI Study					
12	2004	2007	What knowledge do teachers need for teaching mathematics through applicatjons and modelling?	Helen M Doerr	Study Group
13	2004	2007	Classroom activities and the teacher	Soeren Antonius; Chris Haines; Tomas Hojgaard Jensen; Mogens Niss; with Hugh Burkhardt	Study Group
14	2004	2007	Mathematical modelling in high school mathematics: teachers' thinking and practice	Olive Chapman	Study Group
15	2004	2007	mathematical modelling in teacher education - necessary or unnecessarily	Thomas Lingerfjard	Study Group
16	2004	2007	Modelling in teacher education	Thornas Lingefjärd	Study Group
17	2004	2007	Moving the context of modelling to the forefront: preservice teachers' Investigations of equity in testing	Katie Makar; Jere Confrey	Study Group
18	2004	2007	Mathematical modelling of social issues in school mathematics in South Africa	Cyril Julie; Vimolan Mudaly	Study Group
Haines C.; Galbraith P.; Blum W.; Sanowar K, editors, 2007, Mathematical Modelling ICTMA 12- Education, Engineering and Economics, Horwood Publishing, Chichester, UK.					
19	2005	2007	Exemplar models: expert-novice student behaviours	Rosalind Crouch	ICTMA 12
20	2005	2007	Exploring university students' competencies in modeling	France Caron e Jacques Bélair	ICTMA 13
21	2005	2007	Modelling and the critical use of mathematics	Jussara de Loiola Araújo	ICTMA 12
22	2005	2007	Teacher-student interactions in mathematical modeling	Jonei Cerqueira Barbosa	ICTMA 12
23	2005	2007	Modelling as an integrated part of the class on calculus	Adolf Johannes Riede	ICTMA 12
24	2005	2007	Modelling for pre-service teachers	Susann Mathews; Michelle Reed	ICTMA 12
LESH, Richard, et al editors, 2010, Modeling Students` Mathematical Modeling Competencies. ICTMA 13, Springer, Londres.					
25	2007	2010	Modeling as isomorphism: using new technologies in mathematics teacher education	Sergei Abramovich	ICTMA 13
26	2007	2010	An investigation of a team of teachers' shared interpretations of the teacher's role in supporting and enhancing group functioning	Betsy Berry	ICTMA 13
27	2007	2010	Insights of teachers' unconscious behaviour while dealing with modelling problems in the classroom	Rita Borromeo Ferri e Werner Blum	ICTMA 13
28	2007	2010	Teachers' probabilistic reasoning in context and the measurement heuristic	Lynn Carlson	ICTMA 13

29	2007	2010	Secondary teachers learn and refine their knowledge during modeling in a learning community	César Cristóbal Escalante	ICTMA 13
30	2007	2010	Future teachers' professional knowledge on modelling	Gabriele Kaiser	ICTMA 13
31	2007	2010	Tensions revealed by teachers when conducting Mathematical Modelling	Jonei Cerqueira Barbosa e Andréia Maria P. de Oliveira	ICTMA 13
32	2007	2010	Mathematical modelling for teachers: it has to be modelling and teaching	Craig Pournara e Erna Lampen	ICTMA 13
33	2007	2010	Integrating a models and modeling perspective in mathematics teacher education: how can it be accomplished and what does it look like?	Kelli Thomas and Juliet Hart	ICTMA 13
34	2007	2010	Scientific modeling for inquiring teachers network (SMIT'N): the relationship between elementary teachers' views of scientific modeling and nature of science	Orvil White, Valarie L. Akerson, Huseyin Colak e Khemmawaddee Pongsanon	ICTMA 13
35	2007	2010	Taiwan mathematics teachers remodel their teaching model via doing and conducting modeling activity in classrooms	Shih-Yi Yu, Ching-Kuch Chang, Lee-Meei Lin	ICTMA 13
36	2007	2010	Mathematical modeling: implications for teaching	Maria Salett Biembengut; Nelson Hein	ICTMA 13
37	2007	2010	A professional development course with content and pedagogical knowledge and with an introduction of models and modeling in science to physics teachers	Genaro Zavala; Hugo Alarcon; Julio Benegas	ICTMA 13
KAISER, Gabriele, et al editors, 2011, Modeling Students` Mathematical Modeling Competencies. ICTMA 14, Springer, Londres.					
38	2009	2011	Mathematical Modeling in Theacher Education - Overview	Jill P. Brown	ICTMA 14
39	2009	2011	Models and Modelling Perspectives on Teaching and Learning Mathematics in the Twenty-First Century	Helen M. Doerr; Richard Lesh	ICTMA 14
40	2009	2011	Mathematical Modelling in a Distance Course for Theachers	Maria Salett Biembengut; Thaís Mariane Biembengut Faria	ICTMA 14
41	2009	2011	In-Service and Prospective Teachers' Views About Modelling Tasks in the Mathematics Classroom – Results of a Quantitative Empirical Study	Sebastian Kuntze	ICTMA 14
42	2009	2011	Pre-Service Secondary Mathematics Teachers' Affinity with Using Modelling Tasks in Teaching Years 8-10	Gloria Stillman; Jill P. Brown	ICTMA 14