IS THE MATHEMATICS EDUCATION A PROBLEM FOR THE SCHOOL OR IS THE SCHOOL A PROBLEM FOR THE MATHEMATICS EDUCATION?[1]

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ABSTRACT

The purpose of this article is to bring school mathematics education to the Wittgensteinian "therapeutic divan." School mathematics education is depicted as a "disease" that cannot receive a "democratic cure" in light of liberal-meritocratic projects that conceive of school as a preparatory cultural detour to a supposedly qualified insertion into social and professional lives. The recommended therapy is based upon a dialogue between Wittgenstein's unsystematic, aphoristic-therapeutic, post-epistemological views on language, meaning, mathematics, and learning, and Jean Lave's research on cognition and situated learning.

Keywords: school mathematics education; situated learning; Ludwig Wittgenstein; Jean Lave.

RESUMO

O propósito deste artigo é o de levar a educação matemática escolar e, com ela, a própria escola, ao 'divã terapêutico' wittgensteiniano. Em consonância com esse propósito, iremos tratar a educação matemática escolar como uma 'doença' que não pode receber uma 'cura democrática' à luz de projetos liberal-meritocráticos que concebem a escola como um desvio cultural preparatório a uma alegada inserção qualificada na vida social e profissional. Essa terapia será conduzida com base no diálogo cruzado que estabeleceremos entre o modo assistemático, terapêutico-aforístico e pós-epistemológico de Ludwig Wittgenstein falar sobre linguagem, significado, matemática e aprendizagem, e os estudos investigativos sobre cognição e aprendizagem situadas, conduzidos por Jean Lave.

Palavras-chave: educação matemática escolar; aprendizagem situada; Ludwig Wittgenstein; Jean Lave.

The title of this article suggests not only that schools can hamper the promotion of a democratic and socially emancipatory mathematics education, but also that mathematics education can occur beyond school walls. My purpose is to bring current educational

[1] Title of the lecture given by the author at the closing panel entitled "*Mathematics Education: What to teach? Why learn?*" during the Fifth National Symposium on Mathematics Education and eighteenth Regional Symposium on Mathematics Education, promoted by the Institute of Geosciences and Exact Sciences, Passo Fundo University (RS-Brazil), that occurred in the period of 05 to May 7, 2014.

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policies that keep school mathematics education centered upon notions of disciplinarity or interdisciplinarity to the Wittgensteinian "therapeutic divan." Since the late 19th century, such policies have been based on the principles that underpin the project of the republican school: a liberal-meritocratic, developmental, propaedeutic, stageist, progressive, selective, standardizing institution that is also, by extension, exclusionary and undemocratic.

As a political project, the republican school is conceptually governed by the Aristotelian belief that the creativity and productivity of individuals are not habilities constituted and developed at work, but rather arise from the enjoyment of "dignified idleness" and from their radical apartness from the issues and concerns that manifest in activity outside of school. It is this belief that underpins the conception of education as a necessary intellectual detour from all fields of human activity. This detour, according to Charlot (1986), is based on beliefs that were defended by Jean Chateau: "[Education] does not prepare one better for life as it is lived other than by intellectual detour"; "To understand the real, it is necessary at first to turn one's back on it"; and "It's not in everyday life that you prepare yourself better for everyday life, but by this abstract and artificial detour that is the educational detour." For Charlot, however, this idea of detour "is a theorization, by the pedagogy itself, of its own [ideological] procedure" (1986, pp. 38–39).

The title of this article also evokes Seneca's aphorism *Non scholae sed vitae discimus*[3] ["We live not for school, but for life"]. The ethical-political tone of this aphorism contradicts the kind of supposedly progressive primary and secondary schooling processes that are propaedeutic and self-encapsulated. Such schooling does not establish any dialogue with the ways that life occurs and proceeds in other contexts of human activity. In the name of preserving the illusory ideal of equal opportunity for all, the ethical-political project of republican schools has led authorities, parents, teachers, and students to interpret the mobilization of school practices of mathematical culture in ways that are uniform, unvarying, dogmatic, and authoritarian. Thus the political project of the liberal-meritocratic republican school cannot be effectively destabilized without radical investigation of the relationship between school and non-school mathematics learning. In addition, based on the dialogue established herein between the works of Lave and Wittgenstein, I intend to lead toward deconstructionist therapy, not only the mobilizing practices of mathematical culture (both inside and outside of school), but also the disciplinary principle that has guided this mobilization and, by extension, the ways that schools typically conceive of mathematics and mathematics learning.

Since the 1980s, several researchers have investigated the problem of the relationship between mathematics learning within and outside of school. Similar concerns have also been expressed by most of the researchers who consider issues of ethnomathematics and mathematical modeling. Although these studies may be based on different political and epistemological arguments, the common optimistic belief that seems to have guided the work of many of these researchers is that bridges and links between school and non-school practices of math learning can be productively established, both in terms of

[1]I found this aphorism in Engeström (1991), "*Non scolae discimus sed vitae*: Towards overcoming the encapsulation of school learning." This title attests to his concern with the problem of opening the school to non-school worlds. He defended this approach with what he termed "the theoretical perspective of expansive learning" (Miguel & Mendes, 2010).

facilitation and of student involvement with the so-called "school mathematics." Even when they have questioned the nature of school mathematical practices, however, the revisions proposed by these researchers only included replacement of the old practices with "new" ones haunted and overshadowed by the "eternal" light of typically disciplinary or interdisciplinary school mathematical contents.

Some notable exceptions deserve to be highlighted here because they have not judged dialogue between the inside and outside of school as productive. For example, Lins (1994, 1997) came to recognize the differences between the legitimate subject of "school mathematics" and what he called "street mathematics," as well as the ineffectiveness of attempts to transpose them. It seems that he did not view "street mathematics" as anything but an indistinct amalgam of individual daily practices (as opposed to school mathematical practices, which he agreed are desirable preparation for the appropriation of mathematics as a domain of knowledge practiced by professional mathematicians). A similar objection by Walkerdine (2004), who also criticized the possibility of bridges between these two worlds, was based on the political argument that school mathematical practices can only function as ideological simulacra of corresponding non-school practices.

In reference to the Law of Guidelines and Bases of National Education currently in force in Brazil, the "new" official discourse (which was also the basis of the old guiding principles of the liberal-meritocratic school) has been disseminating the belief among teachers, students, and parents that disciplinary school education has the power to develop students' "competencies" and "skills" as appropriate inclusions in social life and, particularly, in the different fields of human activity that students must be qualified to enter. The way such notions are expressed in official discourse is illustrated by the Basic Document of the National Exam for High School (ENEM) produced by the Ministry of Education:

> Competencies are the structural arrangements of intelligence, or rather, actions and operations that we use to establish relationships with and between objects, situations, phenomena and people whom we wish to know. The skills are consequences of the acquired competencies and refer to the immediate plan of the "how to do." Through the actions and operations the skills are perfected and articulated, enabling a new reorganization of competencies. (MEC/INEP, 2000, p. 5)

A possible criticism of this new official educational policy, which is based on the notions of competencies and skills, has to do with its non-problematized assumption that there is innate separation between "to know," "to know to do," and "to know to be." This assumption is no doubt inspired by the slogans "Learning to know," "Learning to do," "Learning to live together," and "Learning to be" in the UNESCO document on the challenges for education in the twenty-first century. The fixed dichotomy between "to know" and "to know to do" would correspond to another fixed dichotomy between a "competency" (i.e., a knowledge set that would condition on *a priori* "to know to do") and a "skill" (i.e., a result of a repeated practice of a "competency"). Accordingly, the absolutist illusion that guides this technicist paradigm of skills and competencies is that in school one acquires a generic and abstract competency—a "structural form of intelligence"—that is provided by appropriation from school knowledge, and then repeatedly applies this competency in other contexts of human activity in order for it to become a praxiological skill. Thus everything happens as if, for example, once one has

acquired a scholastic competency afforded by learning the laws of balance in physics class, one has also acquired the ability to not only ride a bike but also to apply and exercise this competency in other situations that would require the demonstration of physical balance but have nothing to do with bicycles at all.

Lave and other researchers involved in the theory of activity, by contrast, have investigated the problem of learning in "trans-school worlds" more systematically instead of considering a single undifferentiated, homogeneous world. Moreover, they have viewed life outside school not as epistemologically inferior to the world of school, but as a set of different fields and contexts of human activity in which knowledge is also produced, mobilized, and practiced. Nonetheless, most of these researchers have continued to group these different fields of human activity into the "world of practice," the locus of application of scientific knowledge that exists in opposition to or in dialectic interaction with the so-called "world of theory." The former is seen as a set of delimited disciplinary fields within which occurs the production of knowledge that is said to be "scientific" (i.e., specialized, accurate, and reliable). In turn, this "world of theory" should be legitimately and didactically translated to elementary school classes as well as professional courses. These investigators have not, however, questioned the ways that mathematical knowledge has typically been mobilized in school or if it should (or even could) be mobilized in other ways. Such questioning would also require researchers to problematize the conception of mathematical knowledge seen in their conflictual dualisms, such as "pure" versus "applied" and "theoretical" versus "practical," as well as to question the very conception of learning established by these dualisms.

Lave tried to escape these dualisms, as well as the one in which they are anchored: the one that sets body and mind in opposition. To properly situate the way she raised and dealt with the problem I am now taking to the divan, I begin therapy by inviting the reader to imagine and try to guess what the two persons referred to in the following text would be doing:

> Initially, a beacon is nailed together and behind Stake B. The first operator, called Man Aft, holds a beacon on Stake A, and with it, one of the handles of the chain. The second operator, named Man Forward, is holding another beacon, a packet of records, and the other handle of the chain. Holding the beacon about 20m from Point A, he asks Man Aft to supply an alignment. Man Aft, standing behind his beacon and looking for the beacon placed at Point B, through gestures seeks Man Forward's guiding beacon, so that it is on the same line as the other two. Then, he holds the handle exactly along the axis of his beacon. Man Forward stretches the current until he gets it to stay within a relatively small catenary. It is considered normal that a chain of 20m has a catenary with a central arrow of about 30 or 40cm. There is no need to make the arrow perfectly straight [as it would mean the realization] of an effort above normal. Stretching the current, Man Forward, always watching the alignment, brings his beacon into the position of the handle. The chain should be horizontal. To accomplish this, when the terrains are sloped, the operator who is on the lowest level of terrain lifts the handle while the operator who is at the highest point of the terrain holds the handle closest to the ground. The operator holding the handle far above the ground should stand sideways to the direction of the line, in order to control the verticality of the beacon in the direction that matters most. When the beacons are inclined laterally, and not forward or backward, the resulting errors are relatively small. (Borges, vol. 1, p. 16, 1977)

According to Borges, author of *Topography applied to civil engineering*, from which the excerpt above was obtained, these two people are measuring the horizontal distance between points A and B that are uneven in the physical space; that is, not situated in the same plane tangent to the earth's surface that passes through one of these points.
The reader is now invited to imagine a school scene in which a teacher asks students to use a graduated scale to measure the distance between two points, A and B, which are marked on a sheet of paper. Suppose that the children have successfully performed this practice of measurement in the school context. Could it then be said that they have actually learned to measure? Have they acquired the ability or competence (i.e., the competency known as "measurement") to measure in any situation? Or would it be more prudent to say that they have just learned to measure the distance between *those* two points on *that* sheet of paper, using a graduated scale? Could these children transfer this learning to measure the distance between a point on the ceiling and another on the floor of their classroom? Leaving school, could they transfer this learning to measure the distance between two points of a sloped and rough terrain, as the surveyors in the above example are doing? Could they transfer this learning to measure the distance between Campinas and São Paulo, or between Earth and the Moon? Could they also transfer this learning to understand and discuss how the problem of measuring distances related to, for example, the demarcation of borders (i.e., a measurement that constitutes a social, economic, and political problem of conflict involving different communities, possibly from ancient times to the present) ended up producing different measurement practices, each governed by a different set of rules that are not limited to those imposed by the situated topographical features of the land where the measurements are practiced?

Several other examples of the same kind, taken from the context of school mathematics education, could be provided in order to illustrate the plausibility of Lave's finding that little or nothing of what we learn of math in school can be transferred to other contexts of human activity. This finding would have led her to the notion of *situated learning* as well as to engage herself with experimental studies that can be *ecologically validated*. Such studies led her to conclude that

> There would be no question about the validity of extrapolating laboratory findings to other settings. But if math practice takes form in situationally specific ways (the very term "ecological validity" introduces this possibility), it implies that the formal mathematical properties of potential problems are not sufficient to determine what problems will emerge in practice. Other factors in the situation shape problems: ongoing activities, the structure of the setting, and their relations. (Lave, 2003, p. 101).

To explain the no-transfer of mathematics learning found in her ecological studies, Lave was forced not only to blame the failure of school-taught "formal mathematics" to deal with situated problems that are supposedly analogues of situations that manifest in other contexts, but also to suggest that *math practice* can take various forms. In other words, she sought to "overcome" the tension between the admission of unity or a plurality of mathematics, assuming the existence of a supposed variety of ways that the same math (the "formal" sort) could take. This distinction is best characterized in the following passage, in which Lave distinguished between math as a knowledge domain and math-in-practice:

> [A] distinction must be made between math-in-practice and math conceived as a system of propositions and relations (a "knowledge domain"). The term

> "knowledge domain" conotes a body of knowledge structured *as such*, a bounded "conceptual space." In practice, this abstraction has enabled and legitimized the analysis of processes of problem solving as if they were poorly realized or simplified versions of a putative knowledge structure. (2003, p. 97).

As a knowledge domain, math also has its form, which is constructed (albeit with significant differences) by the community of professional mathematicians as well as the school community. But speaking in a variety of ways about practicing the same mathematics, which is taken as a primary reference, is also to suggest that the possibility of seeing mathematics in these different ways, and thereby to see different ways to practice it, is ensured by the contents that we are used to seeing as "math." We are not forced to see these contents as "math" by force of a widespread tradition, but by the power of a community to elevate its own way of practicing math such that its way is the only "legitimate" way to practice it. Certainly, Lave not only identified and recognized the power of so-called "scientific" mathematics—which, in turn, nourishes within oneself the power of a "form" of practicing it that is also said to be "scientific," but also sought to challenge this constitutional power of scientific discourse by seeing it as one among numerous other everyday ways of producing or practicing this knowledge.

> Science features everyday practice in such terms that inevitably end up reinforcing its hegemonic role. To investigate science as everyday practice is essential to analyze how this scientific discourse builds the everyday. One and the other are constructed, in part, by contrast they establish among themselves. To analyze one is to question them both. Therefore, there is one aspect of the anthropology of science which implies the investigation of those practices that for a long time and in an entirely uncontroversial way have been given the label of "the everyday"—that is, something that is put directly in contrast to "science," which for this reason is presumed not to be "the everyday." Among the contexts in which the juxtaposition of "science" and "everyday," of "scientists" and "ordinary people" are more common are discourses on the mind, cognition, problem solving, logic, mathematics, specialized scientific thinking, and effects of the school. It is legitimate, therefore, to ask: "What happens to 'everyday' practices of the everyday when science itself is seen as a form of everyday social practice?" (Lave, 1996, pp. 109–110)

However, the argument about the recognition of the diversity of ways to practice a "same" content (i.e., a "same" referential mathematics that is seen as an area of knowledge or abstract contents) used by Lave to explain the no-transfer of school mathematical learning to other contexts could refer us to the question of the relevance or legitimacy of seeing a field of knowledge through the reductive lens of the binary opposition of "form" versus "content." In what sense, for example, would it be legitimate to see and extract a content (particularly a "unique" content) of the sequence of bodily actions (e.g., the ones performed by the two surveyors in the description above) to measure the distance between two points, and see that content as "the same" as content that supposedly manifests in the sequence of actions that children perform to measure the distance between two points marked on a sheet of paper? Moreover, in what sense would it be legitimate to see these contextually distinct sequences of actions (on the part of "surveyors" and children) as having "different forms?"

Sontag (1966) criticized specialized analyses of artistic-visual and literary works that are based on the possibility of identifying in them a supposed "content" that can be

detached from a supposed "form." She did so by blaming the hermeneutic interpretation project for this inconvenient separatist and abstractionist act which reinforces the illusion that there really is such a thing as "content":

> The fact is, all Western consciousness of and reflection upon art have remained within the confines staked out by the Greek theory of art as mimesis or representation. It is through this theory that art as such—above and beyond given works of art—becomes problematic, in need of defense. And it is the defense of art which gives birth to the odd vision by which something we have learned to call "form" is separated off from something we have learned to call "content", and to the well-intentioned move which makes content essential and form accessory. [...] Whether we conceive of the work of art on the model of a Picture (art as a Picture of reality) or on the model of a statement (art as the statement of the artist), content still comes first. The content may have changed. It may now be less figurative, less lucidly realistic. But it is still assumed that a work of art is its content. Or, as it's usually put today, that a work of art by definition says something. (What X is saying is ....", "What X is trying to say is …", "What X said is …" etc., etc.). [...] What the overemphasis on the idea of content entails is the habit of approaching works of art in order to interpret them that sustains the fancy that there really is such a thing as the content of a work of art. (1966, pp. 4–5)

Sontag's questioning of the separatist, abstractionist, and also inconvenient strategy that directs the viewer's gaze toward an artistic-visual or literary work in order to set up an opposition between "form" and "content" can also be applied to how people are accustomed to observe or reference cultural practices, regardless of whether or not the practices are passed along by scientific-academic or school processes of disciplinarization. In this sense, rather than see the actions of the two "surveyors" in the above example as decoupled in "form" and "content," we could see them as one or more sociocultural practices being performed in a certain context of a field of human activity. This field could be civil construction or another that could be specified.

We talk here about *sociocultural practices*[4] - or simply about *practices* - in analogous manner to Schatzki:

> Practices are organized nexuses of activity. Examples are cooking practices, rearing practices, political practices, farming practices, negotiation practices, banking practices, and recreational practices. Each, as an organized web of activity, exhibits two overall dimensions: activity and organization. A practice is, first, a *set of actions.* For instance, farming practices comprise such actions as building fences, harvesting grain, herding sheep, judging weather, and paying for supplies. Generally speaking, moreover, the actions that compose a practice are either bodily doings and sayings or actions that these doings and sayings constitute. By 'bodily doings and sayings' I mean actions that people directly perform bodily and not by way of doing something else. […] Examples are hammering, handing over money, turning a steering wheel, running, watching, looking, uttering words, and writing them. Examples of the actions these bodily doings and sayings might

[4] A sociocultural practice is cultural because it is always a bodily-rule-governed set of individual or collective symbolic actions that take place within a given time-space and can be significantly shared by members of one or more communities of practice who perform it on the basis of shared purposes. However, a sociocultural practice is also always social because even when it can be directly performed by a single person, it takes place on the basis of normative purposes and constraints—certain variables, in other words—of one or more contexts from a field of human activity that are seen as an open and historically situated form of organization of human interactions, socially established: a *form of life*, in Wittgensteinian terms.

> constitute are building a house, paying for supplies, making a left turn, hurrying home, whiling away time, checking for faults, ordering someone to stand, and composing a poem. To say that actions are 'constituted' by doings and sayings is to say that the performance of doings and sayings amounts, in the circumstances involved, to the carrying out of actions. (2001, p. 56)

This way of speaking in practices, as direct bodily performances, resembles how Wittgenstein described his "language games": "*I shall also call the whole, consisting of language and the activities into which it is woven, a "language-game*" (Wittgenstein, 2009, PI-7, p. 8[e]). It also resembles the way he spoke of the human body as a symbolic body, a body-language: "*The face is the soul of the body*" (Wittgenstein, 1998a, p. 26). It still resembles the way he used the word "action" as utterances or statements staged or performed by the symbolic-affective body:

> In this way, I'd like to say, the words "Oh, if only he'd come!" are charged with my longing. And words can be wrung from us a like a cry. Words can be *hard* to utter: those, for example, with which one renounces something, or confesses a weakness. (Words are also deeds). (Wittgenstein, 2009, PI-546, p. 154[e] -155[e], author's italics)

In an aphorism that evokes traces of meaning in Goethe's *Faust*, Wittgenstein not only clarified the ties of continuity and inseparability between language games and bodily actions but also suggested the primacy and connection, however remote, of these actions in relation to supposedly bodiless and abstract forms used in the practice of language: "The origin and the primitive form of the language game is a reaction; only from this can the more complicated forms grow. Language—I want to say—is a refinement, 'in the beginning was the deed' [1998a, p. 36] [...] "and we will sometimes speak of a primitive language as a language-game" [2009, PI-7, p. 8[e]]. This Wittgensteinian way speaking about language appears shortly after an example of a language game in which every word uttered by a mason is followed by an action of his helper that consists of passing him the building material requested: "In the practice of the use of language one party calls out the words, the other acts on them" (Wittgenstein, 2009, PI-7, p. 8[e]).

The Greek words *praxis* [practice] and *pragma* [action] refer to the same ancient Greek verb *πρασσω* (*prasso*), which meant "to practice," "to perform," "to execute," "to do," and "to experience personally"[5] – an equivalent, therefore, in English, to verbs such as "to perform," "to carry out," "to act," "to stage," and "to represent." Thus, language seen as praxis suggests that practicing a language game resembles directly[6] performing a staging; that is, participating directly in a scenic-bodily representation. What Wittgenstein was saying with this is that we always practice the language with the whole body and not just with culturally ruled vibration sounds emitted by our vocal cords (Miguel, 2015, in press).

In paragraph 23 of the *Investigations*, Wittgenstein presented verbs that express symbolic and direct bodily actions. By doing so he suggested that language is not used in a unitary and uniform manner but rather in unlimited, heterogeneous, and not-always-

[5] According to (Freitas, 2007, p. 13) and the Bible dictionary of Hebrew and Greek available at: http://www.dosenhor.com/grego/g-pi/g4238.

[6] The adverb "directly" is being used here in the sense of bodily actions not exclusively mediated by verbal (oral or written) performance of language.

ruled ways[7] that indicate (without necessarily prescribing or determining), in each contextual situation in which they are practiced, the variable meanings of words, objects and own bodily actions involved in each game. These include "*giving* orders, and *acting on* them; [...] *reporting* an event; *speculating* about an event; [...] *acting* in a play; *singing* rounds; etc." (Wittgenstein, 2009, PI-23).

In this sense, to stage or perform a practice is the same as staging or performing a ruled language game; that is, both endeavors involve disciplining the body in order to make it follow the rules of that game. In turn, the passages 201 to 206 of the *Investigations* suggest that, following the rules of a language game is not the same as interpreting them. These passages also show that Wittgenstein meant the word "practice" as a direct symbolic-bodily performance of rules that are not open to interpretation because they are based in common ways of doing and saying by human beings; that is, they are forms of organization instituted by human communities that share *forms of life*—purposes and ways of seeing and acting in shared activities (Miguel, Vilela, & Moura 2010, pp. 152–153).

> For what we thereby show is that there is a way of grasping a rule which is *not* an interpretation, but which, from case to case of application, is exhibited in what we call "following the rule" and "going against it". That's why there is an inclination to say: every action according to a rule is an interpretation. But one should speak of interpretation only when one expression of a rule is substituted for another. That's why 'following a rule' is a practice. And to *think* one is following a rule is not to follow a rule. And that's why it's not possible to follow a rule 'privately'; otherwise, thinking one was following a rule would be the same thing as following it. […] Suppose you came as an explorer to an unknown country with a language quite unknown to you. In what circumstances would you say that the people there gave orders, understood them, obeyed them, rebelled against them, and so on? Shared human behaviour is the system of reference by means of which we interpret an unknown language (Wittgenstein, 2009, PI-202-206, p. 87^{e}-88^{e}).

In this passage, Wittgenstein's mobilization of the word "interpretation"' is analogous to Sontag because, for him, interpreting a practice (which, for her, meant interpreting a work of art or a literary text) equates how this activity can be expressed in other ways: that is, the same way its direct bodily performance can be translated to other forms of performing it. However, according to Sontag, the "inconvenience" of interpretation did not prevent the establishment and perpetual reappearances of the hermeneutic project in history:

> The task of interpretation is virtually one of translation. The interpreter says, Look, don't you see that X is really – or, really means – A? That Y is really B? That Z is really C? What situation could prompt this curious Project for transforming a text? History gives us the materials for an answer. Interpretation appears in the culture of late antiquity, when the power and credibility of myth had been broken by the "realistic" view of the world introduced by scientific enlightenment. Once the question that haunts post-mystic consciousness – that of the seemliness of religious symbols – had been asked, the ancient texts were, in their pristine form, no longer acceptable. Then interpretation was summoned, to reconcile the ancient texts

[7] An example of anomic language game would be: An example of anomic language game would be: "In a conversation: One person throws a ball; the other does not know: is he to throw it back, throw it to a third person, or leave it lying, or pick it up and put it in his pocket, etc." (Wittgenstein, 1998a, pp. 84, 102).

> to "modern" demands. […] Interpretation thus presupposes a discrepancy between the clear meaning of the text and the demands of (later) readers. It seeks to resolve that discrepancy. The situation is that for some reason a text has become unacceptable; yet it cannot be discarded. Interpretation is a radical strategy for conserving an old text, which is thought too precious to repudiate, by revamping it. The interpreter, without actually erasing or rewriting the text, is altering it. But he can't admit to doing this. He claims to be only making it intelligible, by disclosing its true meaning (Sontag, 1966, p. 5-6).

Thus, even if the performance of interpretive or hermeneutic practices appears to be convenient or even necessary in certain situations, a practice cannot be interpreted or translated without transgressing the rules that guide its direct performance—that is, without replacing them with rules of other language games. But the practice that is directly performed, and which can be repeatedly interpreted, is not itself an interpretation (Miguel & Vilela & Moura 2010, p. 152-153). Therefore, the interchangeable use of the terms "language game" and "bodily or performative staging of language" is not arbitrary. In fact, we have pointed out in another context (Miguel, 2013) that Wittgenstein was reader and admirer of Georg-Christoph Lichtenberg (1744-1799), the German philosopher, astronomer, and mathematician who busied himself with issues such as the relationships between body and mind, reason and feeling, language and thought, man and woman, idea and reality. Lichtenberg undertook such reflections through a literary output loaded with refined humor and subtle irony, as shown by the following aphorism: "J'ai bien souvent remarqué que j'ai mal à la tête quand je me suis long-temps contemplé dans un miroir concave"[8] (Lichtemberg, 1966, p. 43). Nor did the mathematics and mathematicians of his time escape his keen critical eye: "Dans la loi 2 fois 2, 4 ou 2.2 = 4, il y a vraiment déjà quelque chose de la parallaxe du soleil et de la terre en forme d'orange[9]" (Lichtemberg, 1966, p. 48).

So, is not surprising that, after the anthropological turn[10] in his way of seeing the language and processes of meaning, Wittgenstein himself undertook to disturb fundamentalist conceptions of mathematics as a static domain of knowledge, formally systematized. Indeed, in an introductory speech to a course (which he taught at Cambridge from 1929 to 1944) about the foundations of mathematics he stated:

> I am proposing to talk about the foundations of mathematics. An important problem arises from the subject itself: How can I—or anyone who is not a mathematician—talk about this? What right has a philosopher to talk about mathematics? One might say: From what I have learned at school—my knowledge of elementary mathematics—I know something about what can

[8] "I have often noticed that I have a headache when I have long looked into a concave mirror."

[9] "In the law of 2 times 2 equals 4 or 2.2 = 4, there is truly already something of the sun and the earth parallax in the form of [the color] orange."

[10] The defense of an anthropological turn in Wittgenstein's thought—that is, the turn to a substantive and practical conception of the world that would have been strengthened, especially by the "Gramsci connection"—comes from the German philosopher and linguist Gunter Gebauer, based on the view that "in the phenomenological conception of the *Tractatus*, it was the I on the edge of the world which with constant look and an extramundane criteria, looks inside the world. In the second philosophy, the world is conceived from the inside; this philosophy needs therefore of innerworldly methods and criteria of equality. Wittgenstein finds them in the rules of 'grammar of behavior,' in the use and language games. [...] Apparently, in his frequent conversations with Sraffa, Wittgenstein developed a look at the social praxis and their interactions" (2013, pp. 65–66).

> be done in the higher branches of the subject. […] [P]eople who have talked about the foundations of mathematics have constantly been tempted to make prophecies—going ahead of what has already been done. As if they had a telescope with which they can't possibly reach the moon, but can see what is ahead of the mathematician who is flying there. That is not what I am going to do at all. In fact, I am going to avoid it at all costs; it will be most important not to interfere with the mathematicians. I must not make a calculation and say, 'That's the result; not what Turing says it is.' […] [O]ne might think that I am going to give you, not new calculations but a new interpretation of these calculations. But I am not going to do that either. I am going to talk about the interpretation of mathematical symbols, but I will not give a new interpretation. Mathematicians tend to think that interpretations of mathematical symbols are a lot of jaw—some kind of gas which surrounds the real process, the essential mathematical kernel. A philosopher provides gas, or decoration-like squiggles on the wall of a room. I may occasionally produce new interpretations, not in order to suggest they are right, but in order to show that the old interpretation and the new are equally arbitrary. I will only invent a new interpretation to put side by side with an old one and say, 'Here, choose, take your pick.' I will only make gas to expel old gas. (1976, pp. 13–14).

As Lave did, Wittgenstein also established a distinction between mathematics as a domain of propositional and conceptual knowledge and mathematics as a diverse and heterogeneous set of performative practices. Wittgenstein did this through a set of sparse and unsystematic aphorisms which has opened the perspective of seeing mathematics in a way irreverently unique, as *mathematics in action*[11], — that is, as heterogeneous and dynamic sets of ruled symbolic stagings of the human body:

> […] Of course, in one sense, mathematics is a body of knowledge, but still it is also an *activity*. […] (Wittgenstein, 2009, PI-349 Part II, p. 238[e], author's italics).

> If mathematics is a game, then playing some game is doing mathematics, and in that
> case why isn't dancing mathematics too? (Wittgenstein, 1998b, p. 258).

> Why should I not say that what we call mathematics is a family of activities with a family of purposes? (Wittgenstein, 1998b, p. 273).

> There is no religious denomination in which so much sin as it has been committed through the misuse of metaphorical expressions as in mathematics (Wittgenstein, 1998a, p. 3).

> Don't ask: "What goes on in us when we are certain that . . . ?" but: How is 'the certainty that this is so' manifested in people's action? (Wittgenstein, 2009, PI-339 Part II, p. 237[e]).

[11] In the 1930s, this original performative way to see mathematics had other supporters. According to Taylor (2000, pp. 101–102), the German artist Hans Bellmer, who was also a reader and admirer of Lichtenberg, held the view that "all kinds of symbolic production are ultimately rooted in corporal experience"; this would include reflexes, gestures, sounds, words, graphics, and objects. According to him, figures of speech such as hyperbole and metaphor do not belong only to literature, but also to the very human body, the same occurring with the "mathematical processes that operate not just in the abstract domain, but in the flesh." To suggest that the body participates in the intellectual life and that is not the head alone inventing mathematics, he invoked Lichtenberg's aphorism that our belief about the validity of an algebraic equation "resides in the brain, but also, to some extent, on the thumb."

> Once I have exhausted the justifications, I have reached bedrock, and my spade is turned. Then I am inclined to say: "This is simply what I do." (Wittgenstein, 2009, PI-217, p. 91[e]).

Many contemporary readings (Gottschalk, 2004, 2007; Shanker, 1987) of Wittgenstein's unsystematic reflections about mathematics have pointed out, rightly, that the originality of these reflections has been, primarily, their contribution to the emergence of a normative conception of mathematics that cannot be made compatible with logicist, intuitionistic, formalistic, or conventionalist conceptions, or even with some recent anthropological conceptions. In addition, most of these readings seem to have taken as uncontroversial certain assumptions, namely that mathematics is exclusively a domain of knowledge and that as such it possesses uniqueness and universality, as well as the conception of mathematics in which it is solely the activity of a particular community of professional experts who have dealt "scientifically" with the subject throughout history. No doubt, such readings have found support in Wittgenstein's thought. In Brazil, however, they have in fact contributed to the establishment of a fruitful albeit incipient debate in the research community in mathematics education, within which formalist, empiricist (inductivist or fallibilist), and constructivist conceptions of mathematics, among others, still seem to exert considerable fascination.

However, the type of reading of particular interest here is the triple inflection that operates in the latter Wittgenstein's thought about mathematics. According to the first, although math can be seen as a "doctrine" it also comes to be seen as an "activity", taking this term in its double meaning of performance or direct bodily staging that operates on signs and social praxis of different communities of practice. According to the second, math is no longer seen as an unitary, structured, and logically consistent domain of knowledge but rather as a heterogeneous and self-sufficient collection of ruled ways of performative staging of language. In other words, mathematics as plural––no longer concerned with discarding old flags or ideals of unity, or unity in diversity—come to be seen as a set of practices, even as language itself loses its unity and fixity and is seen, in its different uses, as a heterogeneous and unlimited set of language games. Wittgensteinian thought allows us to move in this direction:

> Systems of communication [...] we shall call 'language games.' They are more or less akin to what in ordinary language we call games. Children are taught their native language by means of such games, and here they even have the entertaining character of games. We are not, however, regarding the language games which we describe as incomplete parts of a language, but as languages complete in themselves, as complete systems of human communication. […] When the boy or grown-up learns what one might call special technical languages, e.g., the use of charts and diagrams, descriptive geometry, chemical symbolism, etc., he learns more language games. (Wittgenstein, 1969, p. 81)

In turn, according to the third inflection that properly characterizes an anthropological turn of material and praxiological nature in Wittgenstein's thought, mathematics is no longer seen as an exclusive activity of a "scientific" community of experts, whether or not they are included in the category of "professional mathematicians," and is seen also as an activity that is intimately woven into different cultural forms of community life or social organization: "When I talk about language […], I must speak the language of every day" (Wittgenstein, 2009, PI-120, p. 54[e]).

Based primarily on these three points of inflection, we think that, for Wittgenstein, seeing mathematics in ruled bodily performance as guided by normative purposes is optional, but not arbitrary. This change in perspective

> allows uncoupling the mathematics of the exclusivist disciplinary picture—school or academic—in which it is trapped. Helps us to see it democratically as "mathematics in the plural," [that is], as what people can do, can learn to do or really do, even unknowing what they do when they practice certain ruled language games geared to achieve unequivocal and unambiguous purposes. Lets us see the mathematics education itself otherwise and look critically at what has been done for centuries in the schools in the name of we have called "school mathematics education." Allows us to consider other unlimited ways people are educated mathematically in all contexts of human activity. (Miguel, 2015, in press)

Thus it becomes possible to see a "complete mathematics" in the ways that workers involved in the "world postal," based on the "number" of postal codes,[12] perform with their bodies practices of spatial orientation and localization so that a letter can arrive unequivocally at the addressee's address. This is because, in the normatively ruled "game" of the CEP, the rules governing the meaning that *should be given* to the "number" that participates in that game are the same rules that should also guide the ruled bodily performances of the postman so that a letter can arrive, unambiguously, at the address indicated on the envelope (Miguel, 2015, in press).

Another "complete mathematics" could also "be seen" in the ways that the workers involved in the "world of trade" (based on the "number" and "rectangles" that make up the barcodes) perform bodily practices of inventory control of goods in order to identify, quantify, and regulate the buying and selling of different products and their suppliers unequivocally and unambiguously. This is because, in the normatively ruled game that involves barcodes, the rules governing the meaning that should be given to the "number" and "rectangles" involved in that game (the "barcode game") are the same rules that should also guide the bodily performances of the workers so that control of the inventory can be performed unambiguously, to achieve its purpose (Miguel, Vilela, & Moura, 2012).

The fact that such ruled games can be guided by unequivocal purposes does not mean that a letter sent by post cannot be lost or that errors in inventory control of goods do not occur. But if such errors do occur, we would tend to investigate their "empirical grounds" and not to doubt the normative rules that guide the CEP and barcode language games—games that were designed to meet unequivocal and unambiguous social purposes. It is also important to note that Wittgenstein allows us (in case we deem it necessary or even desirable) to see "complete mathematics" in these workers' bodily performances. We are given this potential not because there are "numbers" on the envelope (strictly speaking, there are not) or "numbers" and "rectangles" in a barcode (strictly speaking, there are not). Instead, we can see "complete mathematics" in these workers' bodily performances because their actions must be guided by unequivocal and undoubted meanings that they should assign, in each case, to these graphic signs so that they are able fulfill the social purpose assigned to them. Thus, although these graphic signs can be seen, *a priori* and invariably, as "mathematical objects," we do not need to

[12] ZIP code in the USA and CEP (Código de Enderaçamento Postal or postal adressing code) in Brazil.

see them this way and we need even less to see them as "mathematical content." According to Wittgenstein:

> […] it is essential to mathematics that its signs are also employed in *mufti*. It is the use outside mathematics, and so the *meaning* of the signs, that makes the sign-game into mathematics (Wittgenstein, 1998a, p. 257, author's italics).
>
> Am I less certain that this man is in pain than that $2 \times 2 = 4$? Is the first case therefore one of mathematical certainty? - 'Mathematical certainty' is not a psychological concept. The kind of certainty is the kind of language-game (Wittgenstein, 2009, PI-332, p. 236[e]).
>
> If these people believed that numbers were spirits and that they were exploring the domain of spirits by means of their calculations, or compelling the spirits to manifest themselves – would this now be arithmetic? Again – would it be arithmetic even the case where these people used the calculations for nothing else (Wittgenstein, 1998, p. 273-274).
>
> The comparison with alchemy seems natural. We might speak of a kind of alchemy in mathematic (Wittgenstein, 1998, p. 274).
>
> Is it already mathematical alchemy, that mathematical propositions are regarded as statements about mathematical objects, - and mathematics as the exploration of these objects? (Wittgenstein, 1998, p. 274).

Wittgenstein's post-epistemic, performative, and praxiological manner of Wittgenstein speaking about math and knowledge opens a similar perspective—albeit a more radically transgressive than the one suggested by Lave in relation not only to the ways of understanding mathematics education in any of the contexts of human activity in which it takes place, but also to the understanding of the difficulties involved in school mathematics education. Perhaps, after all, Lave did not completely deconstruct the dualisms "theory versus practice" and "body versus mind", because it seems that she did not actually consider how such dualisms are involved with the problem of language. To understand this relationship, it would have been necessary to admit that practicing language is not one among other practices. Nor is it a practice that can establish a bridge or mediation between a supposed mental cognitive world, independent of language, seen as internal to a subject who has the power to signify a supposed pre-significant external world. Instead, language is a practice that constitutes human bodies themselves as bodies of social and cultural subjects.

Lave did recognize, properly, that a person thinking alone in a forest does not cease to be involved in social cognition. However, she would do well to admit that, whether or not this person is alone in the forest, he or she is only capable of being involved in social cognition by the fact of having appropriated socially shared language games. Wittgenstein expressed this dependence in a concise and beautiful way: "The world is my world because of the language" (1961, p. 82e). With this relationship in mind, if—as stated by Lave—it seems indispensable to understand and interpret learning and cognitive processes as situated practices in communities, I think we should also add that the language shared by the community is the determining reference system not only for this understanding and interpretation but also for the learning of any other community practice. Not to admit this would be to put oneself in the same situation as St. Augustine, as noted by Wittgenstein:

> Augustine describes the learning of human language as if the child came into a foreign country and did not understand the language of the country; that is, as if he already had a language, only not this one. Or again, as if the child could already *think*, only not yet speak. And "think" would here mean something like "talk to himself" (Wittgenstein, 2009, PI-32, p. 19e, author's italics).

Lave, while recognizing the situated character as well as the affective, social, and cultural nature of cognitive processes, seems to have failed to get rid of an "I"—a "self" or a "mind" that, as a ghostly clone of the symbolic body, has both the role and the power to act as a second disembodied mediated instance of meaning between public language games and the symbolic body that acts and means. According to Schatzki, "performative Wittgenstein" has the power to declare that

> Signifying must not be understood on the order of an imaterial mind that controls the body as a captain steers a ship. *A person is his or her body*. Once it is signified to someone that such and such an action is to be carried out, he therewith performs the bodily action that constitutes the signified one. For it is the same person signified to who, in being his body, carries out bodily activity. There is no need for him to seize, occupy, or activate his body in order for bodily activity occurs. An instrumental body does not imply that the body is an instrument. (Schatzki, 2003, p. 45, italics added).

In *The Blue Book* (1962), Wittgenstein himself discussed the dualism of "body versus mind" as follows:

> It is misleading then to talk of thinking as of a "mental activity". We say that thinking is essentially the activity of operating with signs. This activity is performed by the hand, when we think by writing; by the mouth and larynx, when we think by speaking; and if we think by imagining signs or pictures, I can give you no agent that thinks. If then you say that in such cases the mind thinks, I would only draw your attention to the fact that you are using a metaphor, that here the mind is an agent in a different sense from that in which the hand can be said to be the agent in writing. If again we talk about the locality where thinking takes place we have a right to say that this locality is the paper on which we write or the mouth which speaks. And if we talk of the head or the brain as the locality of thought, this is using the expression "locality" in a different sense (Wittgenstein, 1962, p. 6-7).

In an evocative and pithy aphorism, he repeated these conclusions in a subtly ironic and refined way: "I really do think with my pen, for my head often knows nothing of what my hand is writing" (Wittgenstein, 1998a, p. 24). Also in the *Investigations*, he continued to argue in the same direction:

> And we do here what we do in a host of similar cases: because we cannot specify any *one* bodily action which we call pointing at the shape (as opposed to the colour, for example), we say that a *mental*, *spiritual* activity corresponds to these words. Where our language suggests a body and there is none: there, we should like to say, is a *spirit* (Wittgenstein, 1979, PI- 36, p. 22e, author's italics).

These aphorisms suggest that it is no longer possible to draw a stark line between thinking and doing, between "to know" and "to know to do." Furthermore, they suggest that all knowledge is always a "to know to do"—that is, a knowledge that cannot be dissociated from a human body that mobilizes it, that practices it. For the purposes of this article, "to practice" is the same as to bodily participate in a language game and to

let oneself be guided by its rules, whether or not one is aware of them. In the following passages from the *Zettel* (1967), Wittgenstein seems to suggest a such post-epistemic conception of knowledge, not only because he breaks with the persistent and widespread belief that the subject who learns would be a "Cartesian self" that stands between the human body and the language game in which he or she participates, but also because the management of learning processes would be directly connected to the production of management and self-management policies of the human body itself. In turn, this body can only express, express itself, and therefore learn, by participating directly and effectively in public language games:

> The writing is certainly a voluntary movement, however, automatic. And of course you do not feel every movement when typing. You feel something, but it would not be possible to analyze this feeling. A hand writes; it does not write because you want to, but what you want to write. When you write, you do not look it or perplexed or forward; it not think: "what does it will write now?" But not because you just want that it should write that. Well, it is the very fact that I wanted to write what I want that could haunt me. (Wittgenstein, 1967, § 586)

> The child learns to walk, crawl, and play. She does not learn how to play voluntarily or involuntarily. But, what makes the game moves in voluntary movements? How would it be if they were unintentional? You could also ask: What, then, makes these movements in a game? Their character and scope. (Wittgenstein, Zettel, § 587).

From a Wittgensteinian perspective, the human body is always a expressive or symbolic body, a body that cannot express and express itself significantly independently of public language games in which such expressions may be intended. Thus, an expressive body is a body that has no autonomy to signify before or outside of a language game, but whose expressions only become significant when that body participates in language games. It is not an "I" that has a body that can express him or her. I am not an "I" that has a body that expresses me. I am my body, which is expressed and practiced in language games. Therefore, the dualism between mind and body, which is based on the distinction and disconnection usually established between *being a body* and *having a body*, should be seen as a learned habit. As noted by Schatzki (1996), such a feeling of disconnection can be enhanced by certain remarkable states of disorder experienced by the expressive body:

> Incidentally, the distinction between being and having a body cuts across the three dimensions of bodily expression. With a range of bodily expressions at her disposal, a person is usually at home in her body, automatically expressing mental and intellectual conditions and carrying out actions through effortless and unconsidered bodily performance. She is unproblematically a body. Breakdowns in any dimension of bodily expression, however, can impress upon her a distinction between her and her body. Examples of such disturbances are debilitating depression, uncontrollable crying or laughter, chronic or acute bodily pain, paralysis, nervous tics, and the effort required in learning complex, subtle, and coordinated activities such as piano playing or skiing. When such disturbances occur, the body either ceases or fails to express certain conditions of life spontaneously. [...] With the frustration, encumbrance, and deformation of expression, the unquestioned coincidence of a person and her body is broken (Schatzki, 1996, p. 46).

One of the repercussions of this way of breaking free of the dualism "body versus mind" in the field of (school) education is that the act of learning is seen as a situated relationship (i.e., in person or remotely) that is established between two or more bodily practitioners of a same language game. These practitioners involve themselves in the game, intentionally or not, in ways that are competent but also asymmetrical. Using a Wittgensteinian perspective, Gebauer (2013) magnificently illustrated this process of teaching and learning.

> A child learning to swim cannot conceive through thoughts how to swim when she swims. Instructions and explanations of behavior by adults are generally useless; they make the learning process even more difficult. But a teacher can show the technique to the child, who then directs the body behavior of the model and tries to imitate his movements. In water, however, the child feels that the movements are different than on earth. A good teacher is in the pool along with the student, takes his hand and leads them through the water with firm strokes. The student feels driving arm and hand position; feels like he can adjust his movements to buoyancy, until finally master the use of your body in the water. In interactive situation with the teacher, in addition to sight, there was also participation from other forms of sensory perception. Thus, the student feels on your skin and with your muscles the buoyancy and the effects of the swimming movements. Learning how to swim begins in the first phase, with the imitation of others; the model is then appropriated by the student with his movements and incorporated by him with the assistance of the teacher. In the learning situation the child performs a motor agreement with the teacher. For the learning process, it is essential that he can run the decisive step of learning—understanding how the technique works. [...] A striking example of this kind of imitation is the process in which arises the first means of communication between mother and child when the child responds to the mother's smile with his own mimetic form, which, in turn, is imitated by the mother. When the child has finally learned some techniques, she is in a position to acquire basic models of language games. *Wittgenstein describes this process as a kind of public performance, similar to a ball game learning*. (Gebauer, 2013, pp. 74–75, italics added).

Thus, in a Wittgensteinian perspective, learning processes are invisible through the fixed and invariable lens of the dualism "form versus content" and do not need to be organized based on a disciplinary bond. The focus of learning is no longer an "incorporeal *I*." Instead, the human body itself participates directly in language games; that is, a human body learns how to play by playing, by performing practices. To Lave, however, although the practices are viewed as direct bodily actions (a view with which I agree), they do not seem to be seen in connection with language and not, therefore, in connection with ruled language games. In her works, it is even possible to identify unjustified conflations of the words *activity* and *process*, neither of which are always identified as *practices*. This fact can be illustrated by her magnificent example of what I would prefer call two cultural practices being simultaneously performed: the practice of knitting and of reading:

> I can read and knit. Sometimes *the process of knitting gives shape* to the reading. I might read while knitting a row, but wait to turn the page until the row is finished, or stop reading in order to pick up a dropped stitch. At other times I read to the end of the page before starting a new row, knitting faster if the plot thickens, slightly tighter when it gets tense. Knitting projects look more promising if they don't require constant attention; hard-cover books appeal partly because their pages stay open better. Knitting is a structuring resource for the *process of reading* and reading provides structuring resource

> that *give shape* and punctuation to the *process of knitting*. They shape each other, but not necessarily equally. Usually one is the *ongoing activity*, the other is given shape more than it shapes the first (Lave, 2003, p. 98-99, italics added).

As noted, Lave refers to the cultural practices of reading and knitting sometimes as *processes* and sometimes as *activities*, but not exactly as *practices* as I argue herein. Moreover, in the above example, the dichotomy between *form* and *content* reappears in the form of a dichotomy between *form* and *process*, or between *form* and *activity*. However, the performative use that I am applying to the notion of language games does not allow us to see a practice dichotomically (i.e., as a combination of *form* and *content*). Once a practice has been a ruled bodily performance of the language—a ruling that always takes place—it cannot, strictly speaking, be *interpreted*: it can only, as Wittgenstein would say, *be followed*. Thus, my performative and anti-interpretive way of seeing practices does not allow me to "explain" the issue of the non-transfer of math learning as Lave does; that is, by assuming the existence of a variety of ways a same referential mathematics (i.e., the "formal" one) could manifest. Alternatively, the everyday mathematical practices should not be seen as application, interpretation or variety of scientific or mathematical school practices, but are the latter that could come to be seen as abstract and separatist interpretations of the everyday mathematical practices. Indeed, in the example of our "surveyors" in action, it is the school practice of measuring, with a ruler, the distance between two points on a sheet of paper that should be seen as a practice of theoretical interpretation—and therefore, as an abstractionist, separatist, generalizing, normative, verbal, and conceptual practice—of measurement practices that are performed in other fields and contexts of human activity. This possibility revives the problem of dualism that has already been established between referential or mediated practices and direct or situated practices, as well as another more general: the dualism between theory and practice.

The latter dualism seems to have had its historical emergence in ancient Greece, around the transition from the sixth to the fifth century BC. This emergence, however, did not occur before the unusual perception – made possible by the advent of the cultural practices of speech in different languages and cultural practices of phonetic writing (alphabetic or not) – of the establishment of a disconnection between ancient existing *forms of bodily practicing knowledge* in different fields and contexts of human activity and the *content* that such bodily practices mobilized in other ways. In this sense, the very *form* of Euclid performing geometric content through the rhetorical practice of writing in ancient Greek followed, without doubt, in the wake of other previous forms of practicing knowledge that were not yet seen as decomposable and separable in terms of *form* and *content*, or even distinguishable and nameable as *geometric content*.

If it is possible to detect in history the occurrence of this disconnection, then, even after the advent of theoretical fragmentation and disciplinarization of practices, neither knowledge in general nor so-called mathematical knowledge could be viewed and practiced any longer as bodily knowledge. Therefore, it is necessary to distinguish among the different ways in which knowledge is directly, non-disciplinarily, and performatively practiced by and with the symbolic body, and the ways they are indirectly mentioned, cited, described, or theorized exclusively through cultural practices of speech and writing. From a Wittgensteinian perspective, this distinction can be illustrated by the example of our "surveyors." On the one hand, both of them are directly performing a situated practice of measuring in a context of the field of activity

of civil construction—a practice that was repeatedly performed and modified by many workers in this field, for centuries. On the other hand, Borges (the author of the text that refers to the "surveyors") described, through the direct performance of the practice of writing, the situated practice of measuring that is directly performed by the surveyors. So, even as the surveyors were directly performing a practice of measurement, Borges was directly performing a situated practice of writing that describes, quotes, or refers to a situated practice of measuring that he was not performing directly.

In turn, in a Wittgensteinian perspective, Euclid did not use language empirically to describe any situated practice in any field of human activity. But this does not mean that he did not use the language, or that he used it illegitimately. Instead, he normatively used the language to produce a "fictionalist theory" about practices that have already been directly performed in different fields of human activity. For this purpose he made use of rhetorical-linguistic resources, including the creation of fictional or conceptual entities (point, line, plane, angle, etc.); the unequivocal definition of how such entities should behave in the fictional language game they produce; the transformation of similar situated practices directly carried out in different contexts of human activity via strictly linguistic generic practices (which are often seen as "decontextualized practices" but can be seen as situated in the theoretical or scientific field of human activity). The movement of Euclid is therefore not of the concrete to the abstract or the abstract to the concrete, but rather of a situated plane fictionally transfigured to a generic plane logically conformed, and of the latter to a situated plane logically transfigured. This movement places us back at the dualism "situated practices versus generalizing practices," as well as with the effects of this dilemma in relation to the problem of learning and school mathematics education. According to Lave's notion of situated learning:

> Knowledgeability is routinely in a state of change rather than stasis, in the medium of socially, culturally, and historically ongoing systems of activity, involving people who are related in multiple and heterogeneous ways, whose social locations, interests, reasons, and subjective possibilities are different, and who improvise struggles in situated ways with each other over the value of particular definitions of the situation, in both immediate and comprehensive terms, and for whom the production of failure is as much a part of routine collective activity as the production of average, ordinary knowledgeability. These interrelated assumptions run deeply through the work presented here. (Lave, 2006, p. 17).

However, if the production of knowledge is always a dynamic process of mobilizing traces of its previous performances in different fields of human activity, then Lave's finding of no transfer of learning suggests not only that to speak of "situated learnings" is idiosyncratic, but also that knowledge itself should be seen not necessarily as a static and hierarchically organized body of generic and abstract content but rather as something dynamic that produces itself, reproduces itself, and transforms itself in contextualized bodily performances. From a Wittgensteinian perspective, the production of knowledge cannot be separated from the language games that constitute it; in this sense, language and knowledge constitute themselves mutually and simultaneously. Thus, knowledge flows in language games and is performed in them through a mechanism of iterations and compositions of other practices. Such compositions are both performatic and performative. Because they are always idiosyncratic, we say that such compositions are *performatic*, and due the fact they always manifest as direct bodily performances, we say they are *performative*.

In short: in a Wittgensteinian perspective, knowledge, values, purposes, desires, power relations, and so forth are not seen as previous, independent, or even separable from the language games that simultaneously and mutually constitute them. In addition, any alleged act of abstraction, separation, or extraction of these elements, with the pretense or intention of being in possession of "knowledge itself," "value in itself," "a purpose in itself," and so forth, appears illusory or impossible. This is so because such acts could only be affected at the price of making the supposed abstracted element participate in another scenic language game.

Lave also recognized the impossibility of speaking in terms of "decontextualized knowledge":

> Usually, contextualized learning is not discussed alone, but as part of a duality of which decontextualized learning forms the other half. But the theories of context discussed in the previous section are intended to apply broadly to *all* social practice: They claim that there is no decontextualized social practice. Such a claim commits us to explaining what has often been taken to be "decontextualized knowledge" or "decontextualized learning" *as* contextualized social practices (Lave, 2006, p. 22).

However, it seems to have been based on this abstractionist and separatist illusion that the school curricula that have transmitted mathematical culture were constituted in the history of our republican school. Thus, such schools have institutionalized the illusory belief of the relevance of teaching and learning a "mathematical knowledge in itself"; that is, a mathematical knowledge that can be, and is, disconnected from any cultural practices that would or could mobilize it, as well as from any field of human activity in which such practices could be enacted. Nonetheless, as Lave suggests, the institutionalization of this illusion was accompanied by an even greater illusion; namely that after we have transmitted or taught the "mathematical content in itself" (which, supposedly, is abstract, generic and formless content divested of its direct bodily performances) students would be in a state of preparedness to "transpose" them, "apply" them and, even more importantly, apply them "correctly" and "successfully" to any non-school contexts and situations of human activity.

A third and even more subtle illusion also persists in the field of school education activity, one that overlaps with the previous two: the refusal to see school practices of teaching mathematics as typically and only school modes to perform math. This observation follows from the fact of not recognizing that mobilizing the cultural practices widely seen as being mathematics were and continue to be carried out in different non-school contexts of human activity, before the very advent of schooling of these practices.

Thus, the school processes of mobilizing non-school practices of mathematical culture have done nothing more than elect certain practices to be performed in certain socially valued contexts of human activity, through a process of extracting from such perfomances a content deemed privileged and then re-performing them in a typically school way (i.e., by making reference to them exclusively through verbal and/or imagetic performances of language). In fact, it is Lave herself, based on historiographical research conducted by Patricia Cohen (1982) on the process of constitution of arithmetic as a school discipline in eighteenth-century England, who

identified this typically school type of disciplinary and abstractionist re-performances of non-disciplinary everyday practices:

> Arithmetic instruction was introduced into British elementary schools about 1750, brought into the school from the marketplace. The curriculum consisted of what Cohen calls "denominate math," systems of weights, measures and their equivalents, for different branches of commerce - the latter provided the structuring resources for the school curriculum. It organized teachers' and children's day-to-day activities. [...] By about 1820 the math curriculum in the US looked somewhat less like a survey of the quantitative practices of the craft/mercantile world and began to take on an institutionalized structure of its own (math lessons on addition, then subtraction, then multiplication, then the rule of three). The curriculum was no longer ordered specifically in mercantile terms (though it was still controversial because of its commercial connections (Lave, 2003, p. 99).

It seems also to have been based on that school typically reductionist mode of performing valued trade practices outside of school that Arithmetic would have constituted and participated such a school discipline both in teacher formation in the first Normal School in Brazil, during the imperial phase, as in the formation of children who attended the so-called Schools of First Letters, which functioned as schools attached School to the *Normal Schools*. This connection was noted by Farias (2014) in his doctoral dissertation, *Mobilizing practices of arithmetical culture in the teacher formation in the Normal School of Rio de Janeiro (1868-1889): listening to imperial spectra*.

We see from these examples that the typicality of mobilizing school practices of mathematical culture is characterized by a particularly abstractionist, reductionist, essentialist, generalist, and structuralist mode of performing certain non-school cultural practices which, in their respective contexts of direct performance, contemplate unequivocal and non-ambiguous social purposes. For first time, the school performance of a non-school practice isolated and abstracted it of one or more situated contexts of human activity in which it was, and is, directly performed. By ignoring these situated contexts, this school performance also ignores, by extension, the values, purposes, desires, expectations, power relations, and more that also mobilized in the different situated non-school performances of this practice.

Next, as enacted, this school performance has reduced this abstract practice to the strictly minimalist set of internal rules that constitute it as a "practice itself." Such a set of rules allows the automated performance of this practice in any activity contexts, based on other social purposes that may lead to its performance. Finally, the school performance of this practice adds subtly to this set of rules, disciplinarian values inherent in the political project of acculturation mass that define the identity and institutional roles to be fulfilled by the republican school in the geopolitical context in which it is situated.

The school teaching of the division algorithm provides an illustrative example. First, school teaching practices often take it as a "content in itself"—that is, as disconnected from any directly situated cultural practices of division or sharing something sharable that could effectively occur in some context of human activity. Thus, school teaching practices consider the division algorithm as "something abstract and generic" that must

be learned so that, presumably, students can "apply it" in numerous non-school situations in which some practice of division must be performed.

A school teaching practice of the "division algorithm in itself" can also be seen as a normative language game whose rules should guide the bodily actions of the participants in the game in order to achieve the purpose being pursued. When such rules are followed properly, they enable the achievement of the desired purpose. Such rules are those that govern the practices of counting and recording of quantities based on our positional decimal numeral system. Other rules are those that guide the spatial arrangement of signs printed on a sheet of paper in accordance to the required mode by the very process of performing a division by the usual algorithm. It is clear that this arrangement could be different, and different rules should be explained and followed if it is.

There are, finally, other rules on which are based the kind of division that the usual algorithm performs. In this case, the purpose is to divide a given amount into equal parts, with the lowest possible remainder and without "breaking" the objects being divided. In a Wittgensteinian perspective, such rules, properly followed, should lead unequivocally to the correct result. This does not mean, however, that one cannot learn to mechanically perform a division with such an algorithm; that is, that one cannot learn to properly follow such rules and get the correct, desired result without being aware of why these rules always unequivocally produce the correct result. In other words, one can learn how to divide correctly without knowing how to justify the procedure via explanation. If this is pedagogically desirable, is a discussion of other nature.

What matters, in a Wittgensteinian perspective, is to be able to distinguish one thing from another—to be aware that to perform a particular division, by means of the division algorithm, is to participate in a language game that is different from the one that justifies how the rules are functioning, in that situated particular case, to produce the correct result. We would be participating in another very different language game if we wanted to provide a general proof of the "validity" of a proposition concerning the Euclidean division algorithm[13] for any integer quantities involved in any division.

Yet another distinction is required, however. Knowing how to divide in accordance with the rules upon which the division algorithm is based, but by using an electronic calculator, only means that we know how to use an electronic calculator for carrying out a usual division, and thus, that we know how to play a language game different from the one that enables us to perform a division "with paper and pencil," by the usual algorithm.

It should also be pointed out that, in a Wittgensteinian perspective, learning how to play any of these "games of division" is not a necessary condition for learning how to play any of the others. Furthermore, we can also learn to play any of them mechanically. For example, we can learn to divide with an electronic calculator without having to learn the rules that guide the written or printed performance of the division algorithm and without having to learn how an electronic device has been technologically designed and

[13] The theorem of Euclidean division can be stated as: "Given two natural numbers *a* and *b*, where $a < b$, there always exist also only another two natural numbers q and r, respectively named quotient and rest, such that $b = aq + r$, where $r < a$."

produced in order to follow these rules unequivocally for any case of division. This situation is comparable to the way a child can learn to speak his or her native language without needing to learn (or be aware that the language follows) rules for combining sounds of a language system in order to communicate, with sense, with the community of speakers of that language. To learn any of these "school games of division" must not be seen as a necessary prerequisite to learning any other, but this learning must also not either preclude or necessarily facilitate the learnings of any of them—even if these games present "family resemblances". In a Wittgensteinian perspective, therefore, Lave's finding of not-transfer learning could also apply to the most typical language games staged in school mathematics education.

At this point, let us cross the school walls in order to investigate how division is practiced in other contexts of human activity. The first belief that must be deconstructed tells us that if we do not learn the algorithm of division the way it is taught in school, we will be prevented from practicing division in other activity contexts. For example, a poultry worker who farms chickens, who has not been to school but who does need to pack a lot of eggs in containers that can hold one dozen eggs each, could perform this distribution without even needing to know the number of eggs to be packed. Even if the worker knows that number, it would not make sense to divide it by 12 previously, because such a division in no way could alleviate your work. Moreover, even if the worker wanted to estimate the number of egg cartons that would be required to hold all the eggs to be distributed, this could be done without necessarily resorting to the division algorithm, but only to an action based on proportionality, similar to the resource used by the ancient Egyptians, who calculated a division with repeated multiplications of the divider by 2.

The way I am framing the practice of egg distribution in packaging here looks more like a school problem that has been artificially assembled to "justify" the learning of the division algorithm in school; this school practice barely resembles the diversified ways that the division is bodily performed, nowadays, in different contexts of the field of chicken farming. For example, the Goretti Hensel chicken farm[14], located in the Brazilian city of Salvador do Sul, has more than 13,000 chickens that produce, per day, more than 10,000 eggs. A truck passes by the farm twice a week; each time the driver collects and transports about 40,000 eggs that then must be sorted at the warehouse. Machinery and trained employees do the cleaning and grading of the eggs. Of nearly 2.5 million eggs in the warehouse, half are exported.

Some countries require the expiration date to be printed on each egg, and this impression is made by a special machine. Another machine ensures that the eggs do not break by placing each one into its carton with the tip down. Another problem is the proper packaging of the egg cartons in sturdy boxes, because they must be transported to distant cities and may be in transit for up to 45 days. Because they are perishable, eggs cannot be stored for long. Furthermore, the practices of cleaning, sorting, disposal, division, packaging, packing, transport, and so forth are regulated by law by the Ministry of Agriculture, Livestock, and Supply. A list that is part of Ordinance No. 1 of February 21, 1990, issued by the Department of Inspection of Animal Product, defines general rules for the inspection of eggs and egg derivatives:

[14] http://sites.ruralbr.com.br/naestrada/2010/11/14/canal-rural-na-estrada-mostra-logistica-de-producao-e-distribuicao-de-ovos/

- "Egg." The name "Egg" means a chicken egg in its shell. Other kinds of eggs must be accompanied by an indication of the species they come from.
- "Fresh Egg" means the egg in shell that was not conserved by any process and fits in the classification established. This egg loses its designation of "cool" if intentionally subjected to temperatures below 8°C. The recommended storage temperature for fresh eggs is between 8ºC and 15ºC with a relative humidity between 70% and 90%.
- "Refrigerated Egg" means the egg in shell conserved by industrial cold according to the specifications of Article 725 of RIISPOA.
- "Pickled Egg" means the product resulting from the treatment of shelled egg or egg parts which have been frozen, salted, pasteurized, dehydrated, or have undergone other procedures duly approved by SIPA.
- "Integral Egg" means the natural egg, shelled, that preserves the natural proportions of yolk and clear. When mixed, resulting in a homogeneous substance.
- "Yolk" means the product obtained from the egg shelled and separated from the clear or albumin.
- "Clear" means the product obtained from the shelled egg and separated from the yolk.
- When varying proportions of clear and yolk are used, not observing the proportion of a natural egg, the product will be named "Mixed Eggs."
- The eggs must come from farms under official veterinary control and installed in a covered area protected from wind and direct sunlight. The site must have adequate capacity for the amount of eggs received. 13 boxes of 30 dozen per m^3 is the recommended proportion.
- It is prohibited to pack in the same container: 1) eggs from different species; 2) fresh and conserved eggs; 3) eggs from different classes or categories.
- Eggs considered inedible shall be placed in an identified container. Included in this category are eggs described in Article 751 of RIISPOA, including: 1) eggs with visible foreign matter; 2) eggs with part of the shell missing; 3) eggs with dirt stuck to the shell or with cracks.

Any resemblance between that legal text and the definitions found in Euclid's *Elements* is not mere coincidence. All of the performances of practices on any chicken farm in Brazil must be carried out in accordance with such a set of rules. Thus, in the "world of the chicken farm," an egg is not always edible. In other words, what we are accustomed to calling "egg"' in other contexts is not always seen as "egg," and any egg is not always equal to any another egg. In such a world, "eggs" are normative "objects" that should be normatively "manipulated"; that is, the norms of their manipulation should be based on rules that, in the *form of life* on the chicken farm, should be seen as a given: "What has to be accepted, the given, is—one might say— *forms of life*" (Wittgenstein, 2009, PI-345, Part II, p. 238[e]).

In a Wittgensteinian performative view of mathematics (which is to say, not in mathematics seen as a field of knowledge), in the world of the chicken farm, it makes sense (even if, in such a world, the word "mathematics" is not used in this way) to speak of mathematical practices as a set of language games that are normatively governed—that is, guided by unequivocal and non-ambiguous purposes that are performed by these games. However, such games should not be seen as a transposition of a "school or scientific mathematics" to the form of life of the chicken farm. Thus, in this world, the workers who perform normative practices, whether such practices are governed by rules laid down by legislation or by other means, would be involved in mathematical activity even though no (school or scientific-academic) content considered to be "typically mathematical" could be viewed or perceived in their bodily performances.

Moreover, in the world of chicken farms, there are many other "non-school rules" that should be taken into account for practicing the division of eggs in suitable packaging to avoid breakage and in additional packaging so that they may be sent to different countries. The more we investigate the situated rules that guide the different ways by

which workers perform diverse types of egg division in the field of chicken farming, we would find hardly any workers performing division in accordance with the school algorithm. This result would hold true even though the language game of division in equal parts, without "breakings" and with the lowest possible remainder, can also be practiced by some of them and/or by machines properly designed to perform such mechanical tasks quickly and correctly. Indeed, the Yamasa Machinery Industry[15] is one of the oldest and best-recognized Brazilian manufacturers of equipment for selection, classification, and processing of eggs. It produces modern machines that sanitize eggs, classify them by weight, detect impurities and cracks in the shells, and pack the eggs and egg products in kits and trays. These machines are capable of sorting and packaging from 30,600 to 180,000 eggs per hour.

After assessing the form of life of the chicken farm, we could investigate other fields of human activity in order to verify how dividing practices are carried out in them. We could, for example, investigate the field of business activity in order to verify how partners divide net income among themselves. We could investigate the field of jurisprudence in order to verify possible ways of dealing with the problem of the division of property among heirs. We could investigate the activity field of economic management of a nation in order to verify how in this context, the problem of income distribution among citizens is treated. And so forth.

Of course, in these cases, the constitutive rules of the language games that guide these practices differ from those that guide school practices that involve the division algorithm. However, returning to the problem of distance measurement that I cited at the beginning of this article, we recall the following aphorism of Wittgenstein:

> […] What "determining the length" means is not learned by learning what *length* and *determining* are; rather, the meaning of the word "length" is learnt by learning, among other things, what it is to determine length (Wittgenstein, 2009, PI-338 Part II, p. 236[e]).

To paraphrase this aphorism, we could say that what it means to "determine the outcome of a division" is not learned by the fact that one learns what is "division," what is "determine," and what is "outcome," but rather that one learns the meaning of the word "division" by the fact that one learns what it is to determine the outcome of a division. Perhaps Wittgenstein said here indicates that only *a posteriori* learning can be seen as a process of conceptual learning, and that this cannot be a process of conceptual learning unless it is undertaken on the basis of practices that mobilize such concepts in different contexts of use. In addition, none of these uses can be highlighted as privileged in relation to others, and not all of them converge to a common use from which all others can be derived (Vilela, 2013).

All these examples strengthen the Lave's finding of not-transfer of learning from one context to another. Nonetheless, I do not wish to suggest that the division algorithm should no longer be taught in school, or even that it cannot be presented and prove useful for practicing divisions or partitions in non-school contexts of human activity. One can, undoubtedly, learn to practice written or "mental" computation both in school and outside of it. Moreover, there is no doubt that we learn in all contexts of activity, from the time we are born and throughout our lives. But if learning is not a problem, we

[15]http://ubabef.com.br/siav/imprensa/70.

should ask why, at school, learning is seen as "the" problem. Perhaps this is because school contexts are politically instituted as intentional spaces of unified teaching and compulsory learning in which the ways practices are performed are, in most cases, dramatically different from the ways they are performed in other contexts. This difference arises because, in school, teaching practices are almost always indirect, referential, and remissive; that is, they are practices of conversation, reading, and writing that mobilize other practices that cannot be directly performed in school, but only through artificial simulations.

I am trying to indicate with these examples that students could be better prepared to understand the modes of organization and operation of different forms of community life—as well as to understand the asymmetric power relations that permeate them—if they could have the opportunity of, in school, discussing different ways of practicing division in different contexts of human activity. This would be a positive way as a Wittgensteinian perspective could pedagogically deal with the finding of not transfer of learning referred by Lave.

On the one hand, adopting such an approach would mean questioning authoritarian and dogmatic school curricula that consist of an endless chain of conceptual contents that are fixed, essentialist, and single sense and which are successively arranged on the steps of a supposedly solid ladder; but that, because in fact this ladder has not been designed for any non-school form of life, can find no support or hooks, either on the ground or in the sky (Rorty, 1988). On the other hand, such an approach would suggest an unprecedented democratization of the school curriculum, to the point of tossing so far over the school walls not only mathematics itself configured as a discipline, but also all disciplines configured as organized sets of "content in itself"—that is, content designed for the permanent encapsulation of students at the school.

After abandoning all such anchorless ladders[16], schools could come to acquire different faces according to the various political projects that may guide them. What I can see, in this broad horizon of possibilities, is that school can prove to be a space for community life in which teachers, children, and young people propose to collectively tackle the Wittgensteinian therapeutic-grammatical problematization in terms of an open, diverse, and heterogeneous set of non-disciplinary cultural practices that are directly performed in different non-school contexts of human activity. But for that transformation to occur, the notion of "school discipline" would have to be subjected to a grammatical therapy, with the purpose of investigating various ways to operate the deconstruction of dietary and unilateral curricula composed of still images, exclusivist visions, and unique meanings.

The passage I suggest from a curriculum based on a principle of disciplinary organization to another that is non-disciplinary or *indisciplinary* (Moita Lopes, 2006; Miguel, Vilela, & Moura, 2010) is not a passage that would abandon or erase

[16] This concept alludes to the therapeutic characteristic of philosophical activity practiced by Wittgenstein, as well as to their ethical and political attitudes toward life: "Each sentence that I write is trying to say the whole thing, that is, the same thing over and over again & it is as though they were as views of one object seen from different angles. I might say: if the place I want to reach could only be climbed up to by a ladder, I would give up trying to get there. For, the place to which I really have to go is one that I must actually be at already. Anything that can be reached with a ladder does not interest me". (Wittgenstein, 1998a, p. 22/p. 9).

disciplinary distinctions and typifications in name of the defense of a generalist, diffuse, and arbitrary mobilization of cultural practices in school. Rather, such a passage would be meant to promote a radicalization of the distinctions between practices that, within limits (if there are any), affirms the idiosyncratic and situated nature of any cultural practice. From this perspective, all language games (whose bodily performances are always radically situated in historical-cultural forms of life that constitute and transform themselves in time and space) are complete and distinct from each other. Yet, they can maintain family resemblances. If in this passage we also want to book a place for something that could resemble what I call "mathematics education," this something would be a set of language games designed to guide actions of humans and human artifacts toward the achievement of unequivocal or unambiguous social purposes, whether or not such purposes are ethically sound.

For example, a cultural artifact known as "GPS" (global positioning system) was initially designed in the context of war between U.S. and Iraq, so that bombs could unequivocally reach their targets, regardless of whether or not a target was visible and of weather conditions. Of course, based on such an unequivocal purpose, the design and construction of GPS devices were conducted from a set of rules of a language game that could prove suitable for this purpose. However, this example does not mean that human abilities can be automatically transferred to machinic artifacts supposedly endowed with enough "intelligence" to transpose or reproduce polymorphic human actions. In fact, based on their research on breadmaking machines, Collins and Kusch claimed that

> machinery and pieces of knowledge that are "explicit," such as instruction manuals and books, are misleading. Their meaning seems to be within themselves, but actually, it is given by us. Their potential lies in the tacit knowledge and social understanding involved in their uses, both by their producers as by their users. These skills are acquired through common enculturation and socialization within groups or similar forms of life. The question can be better understood when it is realized that there are two kinds of human action that are related in different ways to mechanization: the machines can mimic the mimeomorphic actions without loss, but they fail when one expects them to reproduce polimorphic actions. [...] Making bread by hand is a "world"; that part of making bread which is done by the machine is a "micro world." [...] Any "smart" or automatic machine is a "social prosthesis." [...] So, the study of a breadmaking machine, instead of being an example of the possibility of incorporating human skills in a machine, is an example of how to make a machine work, without incorporating human skills. [This example can also be used] to explain the difficulties of transferring the tacit knowledge of the technologies in the areas of steel and mining from Japan and Australia to Brazil (Collins & Kusch, 2010, p. 219; p. 234; pp. 240).

The passage I am suggesting from a disciplinary to an *indisciplinary* curriculum does not propose the abandonment of the idea of generalization in the name of an overvaluation of the situated character of practices. Instead, it suggests that the very notion of generalization is not seen as a "mental operation" or a good or useful "methodological procedure," but rather as a cultural practice that assumes meaning, relevance, and different political connotations depending upon the language games in which it is participating and upon the different contexts of the activity or activities in which it is practiced. Therefore, as a practice, it must be subjected to therapeutic treatment like any other, whether or not the treatment is seen as "scientific." Thus, this passage from disciplinary to *indisciplinary* curricula indicates a substitution of disciplinary distinctions that are based on criteria and conceptual hierarchies for

indisciplinary distinctions that are based on differentiating among the criteria of the grammars of the various language games, as well as upon praxeological criteria that characterize the forms of organization of the different fields of human activity in which such games are performed.

In the *Investigations*, Wittgenstein did not speak of *language games* and *forms of life* in a unique way. In addition to his examples of language games themselves, there is also a persistent purpose in "elucidating" - through analogous examples (while never freezing a definition) - with what a language game could resemble: sometimes with a scenic-corporal composition of the oral practice of the native language with other bodily actions (Wittgenstein, 2009, IF-7); sometimes with heterogeneous composite architectural materiality of an ancient city in its constant transformations (Wittgenstein, 2009, IF-18); sometimes with fictitious or simply imagined games.

Thus there is positive way by which a Wittgensteinian perspective helps to us to take seriously, on a political level, the problem of not-transfer of learning: by expanding, but without limiting the expansion, the meanings of the word "mathematics"; by opening the subject of mathematics to other forms of life beyond the one in which the professional mathematicians community lives and works; and by tearing the disciplinary veil that tightly binds the unique way that mathematics is presented in school. Lave seems to have been aware of the dangers of closing the school in order to dialogue with the different fields of human activity and—in the name of science and the supposed superiority of scientific discourse in relation to the various modes of practicing language in these fields of activity—of setting such fields in opposition, in terms of legitimacy as well as ideology, to the world of school.

> The school is itself often opposed to everyday life. [...] It looks, therefore, for learning, for thought, for knowledge, for information processing, to representations of knowledge, etc. as if it were something decontextualized from, universal, and ahistorical processes through which every human being works and has always worked in his personal negotiation with the world. Consequently, it increases the probability of any eventual distinctions at the heart of cognitive science between scientific thought and the thought of the "other" becoming naturalized or universal distinctions. (Lave, 1996, p. 111)

The possibility I have discussed of expanding the uses of the word "mathematics" is not intended to constitute a new definition of mathematics. Instead, it is offered as a politically consequential way to challenge the colonizer and exclusionary power. But it is also offered as a way to deconstruct the ideological role of social inequality that the liberal-meritocratic orientation of school mathematics education has been playing since national systems of education were first instituted: to, in the name of political purposes seen as "appropriate," form the man or woman who is said to be "civilized." After more than 150 years of installation of such dogmatic and undemocratic schooling, it is appropriate to ask whether mathematics education is a problem for schools or whether schools are a problem for mathematics education.

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