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The anthropological theory of the didactic as a methodological proposal to analyze digital resources

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Abstract: This article presents an analysis of digital resources for mathematics teaching and learning based on the anthropological theory of the didactic as a methodological proposal. Thus, data were collected from the general information available on the PhET digital platform: general descriptions of the digital resources *Equality Explorer: Basics* and *Equality Explorer*, teaching resources, and activities proposals sent by teachers. Furthermore, digital resources were handled to determine their mathematical praxeologies: the types and subtypes of tasks and their respective techniques and technologies. Combining ostensive aspects is highlighted among the research results as indispensable for working with mathematical praxeologies involving digital resources. In short, the results reveal that adopting the ATD as a methodological possibility allows analyzing not only the mathematical organization but can also support reflections about the didactic praxeology and the personal and institutional relations around the mathematical object.

Keywords: Anthropological Theory of the Didactic. Mathematics Praxeology. Ostensives. Digital Technologies in Mathematics Education. School Algebra.

La teoría antropológica de la didáctica como propuesta metodológica para analizar los recursos digitales

Resumen: El objetivo de este artículo es analizar los recursos digitales para la enseñanza y el aprendizaje de las matemáticas, presentando la teoría antropológica de la didáctica como propuesta metodológica. Para ello, se recolectaron datos de la información general disponible en la plataforma digital PhET: descripción general de los recursos digitales *Explorador de Igualdades: Básico* y *Explorador de Igualdad*, recursos didácticos y propuestas de actividades enviadas por los profesores. Además, se manejaron recursos digitales para determinar sus praxeologías matemáticas: los tipos y subtipos de tareas y sus respectivas técnicas y tecnologías. Entre los resultados de la investigación, se destaca que la combinación de aspectos ostensivos es indispensable para trabajar con praxeologías matemáticas que involucran recursos digitales. En definitiva, se considera que adoptar la TAD como posibilidad metodológica permite analizar no solo la organización matemática, sino que puede sustentar reflexiones sobre la praxeología didáctica y las relaciones personales e institucionales en torno al objeto matemático.

Palabras clave: Teoría Antropológica de la Didáctica. Praxeología Matemática. Ostensivos. Tecnologías Digitales en la Educación Matemática. Álgebra Escolar.

A teoria antropológica do didático como proposta metodológica para analisar recursos digitais

Resumo: O objetivo deste artigo é analisar recursos digitais para o ensino e a aprendizagem de Matemática, apresentando a Teoria Antropológica do Didático como proposta metodológica. Para tanto, os dados foram coletados a partir das informações gerais disponíveis na plataforma digital PhET, a saber: descrições gerais dos recursos digitais *Explorador da Igualdade: Básico* e *Explorador da Igualdade*, materiais de ensino e propostas de atividades enviadas por professores. Ademais, os recursos digitais foram manuseados, a fim de determinar suas praxeologias matemáticas: os tipos e subtipos de tarefas e suas respectivas técnicas e tecnologias. Dentre os resultados, destaca-se que a combinação dos aspectos ostensivos é indispensável para o trabalho com as praxeologias matemáticas envolvendo os recursos digitais. Em suma, considera-se que a adoção da TAD como possibilidade metodológica permite analisar não apenas a organização matemática, como pode também subsidiar reflexões acerca da praxeologia didática, das relações pessoais e institucionais em torno do objeto matemático.

Palavras-chave: Teoria Antropológica do Didático. Praxeologia Matemática. Ostensivos. Tecnologias Digitais em Educação Matemática. Álgebra Escolar.

1 Introduction

Teachers' use of digital and non-digital resources in their educational practices can favor student learning as long as it aligns with school knowledge and teaching methodologies. It is with this understanding that the role of resources (such as textbooks, manipulative materials, mathematical games, etc.) in mathematics teaching and learning processes has been the object of investigation in some research in mathematics education (Trouche, Gueudet & Pepin, 2020; Bittar, 2017, 2022; Almouloud, 2015; Grando, 2015; Miranda & Adler, 2010).

According to Gueudet and Trouche (2016), teachers select, adapt, and conceive artifacts specific to their professional development, called *resources*. Linked to this notion, we take the concept of resource, in a broad sense, as proposed by Adler (2000): from the English verb *re-source* (to source again or differently), *resource* can be understood as everything that *replenish* or *reconfigures* teaching work to enable student learning. Therefore, based on the author, resource is both a noun (object) and a verb (action).

From this perspective, Adler (2000) advocates that the teacher's activity involves a set of resources that are classified into three categories: (i) *human resources*, which include teachers' actions and their knowledge; (ii) *cultural resources*, which concern the concepts made available in the culture and tools such as time and language; and (iii) *material resources*, for example, educational software, manipulative materials, and school programs, among other tangible objects that can be used in teaching and learning processes (Miranda & Adler, 2010). In this article, we emphasize the analysis of material resources, particularly digital ones.

With the advent of remote teaching caused by the COVID-19 pandemic, all those categories of resources continued to be used in configurations and exploratory modes for each specific reality, whether online or offline or synchronous or asynchronous interactions. Thus, we design *digital teaching resources* as everything that can replenish and reconfigure teachers' professional performance, being available and shared essentially through the internet, to be used with didactic intent.

Furthermore, based on the development of a research project over two years (2020-2022) on resources for the study of mathematical equality relations, we consider that not only the analysis of the implementation of digital resources in mathematics classes is relevant but also the systematic evaluation of these resources, in a stage before their use in the classroom – based on theoretical-methodological assumptions from the field of mathematics didactics.

In this scenario, we pose the following question: How can the anthropological theory of didactics methodologically guide the analysis of digital resources? Intending to answer this question, we focus on the theme of school algebra, which, according to Bosch (2019, p. 52), "it has been at the core of the ATD development since its very beginning and can provide a rich illustration of the different treatments this research framework proposes".

Therefore, we aim to analyze digital resources for mathematics teaching and learning, presenting the anthropological theory of didactics as a methodological proposal. In the subsequent topics, we outline the theoretical foundation and discuss the methodological dimension and the analysis of the results, considering the general panorama of the discussions and reflections presented in this article.

2 Anthropological Theory of the Didactic

The anthropological theory of the didactic (ATD), inserted in the field of the didactics of mathematics of the French tradition, was developed by Yves Chevallard as an extension of his theory of didactic transposition (Chevallard, Bosch & Kim, 2015; Bosch & Gascón, 2014). The extension between those theories was motivated by the *ecological problem*[1], which enabled the debate about the conditions established regarding the different objects of knowledge to be taught (Araújo, 2009; Barbosa, 2017).

By theorizing about the anthropological dimension of mathematics (and didactic phenomena), Chevallard (1996) goes beyond the *teacher-student-knowledge* didactic system proposed by Brousseau, and inserts didactics in the field of anthropology, seeking to focus on *didactic organizations* related to the teaching and learning process of *mathematical organizations*.

Chevallard (1999) constructed the presentation of the ATD in an axiomatic way. In general, the ATD provides theoretical apparatuses that support investigations on "the conditions of possibility and functioning of didactic systems, understood as subject-institution-object relations" (Almouloud, 2007, p. 111).

At first, the theorist relied on these three primitive concepts: *objects*, *people*, and *institutions*. Besides that, there are the relations, concepts that are also essential for the ATD, which enable the transformation of the constituent elements of this tripod and allow them to continue to exist, or not, considering the time factor. Chevallard (1999) classifies them as *personal relations* of a *subject* with an *object* and *institutional relations* of an *institution* with an *object*.

At the ATD, everything is *object* (O), people included, i.e., "the object is any material or immaterial entity that exists at least individually" (Chevallard, 2018, p. 31). The theoretician states that any product of human activity endowed with intentionality is an *object*. Another fundamental concept is the notion of *person* (X), which the object of the common allusion "every individual is a person" should not be made because, to Chevallard (2018), the person changes over time based on their relations, i.e., objects come into existence to X, others change or even cease to exist. In this transformation, the *person* changes; the *individual* remains invariant. So, the *personal relation* of a *subject* X with an *object* O is designated in the system by R(X, O), which concerns all possible interactions of X with O at specific historical moments of X.

In turn, the *institution* (I), says Chevallard (2018), consists of a "complete" social device

[1] "The ecological problem is immediately presented as a means of questioning reality" (Araújo, 2009, p. 32).

that can have a very short extension in the social medium, allowing and imposing on its subjects their ways of doing and thinking, conceived in the ATD as praxeologies, in addition to enabling the existence of particular knowledge. Thus, all knowledge exists at least in one institution I. Examples of institutions are the classroom, the school establishment, departments of education, and the Ministry of Education, among others.

The *institutional relation* of an *institution* I with an *object* O is designated by R(I, O), concerning the existence of an *object* O for an *institution* I, when O is known by I. In this case, O is an *institutional object*. According to Chevallard (2018), an *institutional relation* is ideal when there is conformity between the *personal* and *institutional relations*. In other words, when R(I, O) resembles the R(X, O) of the "good subject" of I. This does not necessarily imply that the "best" *institution* is the one constituted by "good subjects" but rather the *institution* whose *subjects* allow transformations in *institutional relations*.

Still, in agreement with Chevallard (2018), a *person* X becomes the *subject* of an *institution* I, so the *object* O existing in I will also exist for X under the requirement of $R_I(O)$. In this sense, R(X, O) is constructed or changed through the requirement of R(I, O). It is with this view that Chevallard defines learning. "To him, there is learning from the moment the personal relation R(X, O) of an individual X with an object O changes" (Araújo, 2009, p. 35).

Moreover, an *object* O can exist in different *institutions* or can be seen distinctly in different *institutions*. O*bject* O can also change, evolve, age, or disappear over time in an *institution*. The institutions, as well as people and knowledge, are therefore mutable devices, depending on time and the historical, social, and cultural context.

To better illustrate the concepts discussed so far, let us briefly consider the mathematical object *equality relations* (O_{ER}), one of the fundamental knowledge for introductory work with school algebra. This mathematical content is an object for the flowing institutions: National Common Core Curriculum (Base Nacional Comum Curricular — BNCC) (I_{BNCC}) and Pernambuco Curriculum (Currículo de Pernambuco) (I_{CPE}) from the Brazilian federal government and the Pernambuco state government, respectively, since it exists in the curriculum guidelines in the area of mathematics proposed in these official documents (Brasil, 2018; Pernambuco, 2019), specifically in elementary school skills. In other words, in this context, there are *institutional relations* $R(I_{BNCC}, O_{ER})$ and $R(I_{CPE}, O_{ER})$.

Furthermore, the *object* O_{ER} must still exist in other institutions, such as the school and the classroom, so that the subjects (teachers with the purpose of teaching and evaluating, and students with the purpose of learning) belonging to these institutions also establish *personal relations* $R(X, O_{ER})$ with that object. Therefore, the "good subjects" in the current educational system in Brazil are those who know the O_{ER} and develop skills as per the I_{BNCC} and I_{CPE}.

A part of the ATD refers to the development of the notion of *praxeology*, which allows modeling social practices, particularly mathematical activities. In the subtopic below, we delve deeper into this concept.

2.1 Praxeological Organization

As advocated by Chevallard (2018, p. 34), "the notion of praxeology is the heart of the ATD". A *praxeological organization* or *praxeology* concerns a type of *task* (T), a *technique* (τ) to perform a specific task (t) of type T, a *technology* (θ) that explains and justifies the technique used to perform T-type tasks, and, finally, a *theory* (Θ) that underlies technology (and all elements of the praxeological organization) (Chevallard, 2018). Thus, the *specific praxeological structure* [T, τ, θ, Θ] (which takes T as a starting "point") is composed of two parts: a *practical-technical block* [T, τ], or the "know-how-to-do" (praxis); and a *technological-theoretical block* [θ, Θ], or "knowledge" (logos).

Still based on Chevallard (2018), we point out that an essential fact in the notion of praxeology is that, from an anthropological point of view, no praxis is detached from logos. However, regardless of the institutional position of the observer (researcher facing the praxeology of the teacher and the students, teacher facing the praxeology of the students, etc.), the technological-theoretical block is hardly visible, seemingly absent.

According to Chevallard (1999), it is important to establish a distinction between *mathematical praxeology* and *didactic praxeology*. The *mathematical praxeology* covers the structuring of the mathematical reality for the classroom, composed with a focus on the types of tasks (T) — mathematics performed —, techniques (τ) — mathematics explained —, justified technologies (θ) and theories (Θ) relating to mathematical objects to be studied or constructed (Araújo, 2009).

On the other hand, *didactic praxeology* happens from the execution of a mathematical organization. This praxeological organization no longer arises to perform a mathematical activity, such as, for example, determining the unknown term in an equality relation, but with the concern of studying how to teach how to calculate the unknown term in mathematical equality. To this end, Chevallard (1999) distinguishes six didactic moments that make it possible to analyze didactic praxeology: (i) the first encounter with mathematical praxeology; (ii) exploration of the type of task and development of a technique; (iii) the elaboration of the technological and theoretical environment; (iv) institutionalization; (v) the work with the technique; and (vi) the evaluation[2].

In this work, we take further the notion of mathematical praxeology to highlight the tasks, techniques, and technologies identified in the use of digital resources.

2.2 Ostensives and Non-Ostensives

We have seen so far that, in the ATD, every human activity carried out in an institution can be modeled by the praxeological quartet [T, τ, θ, Θ] (Chevallard, 2018). As stated in Kaspary and Bittar (2018), these components can be identified and manipulated through some material manifest, designated by *ostensive*.

By manipulating or recognizing the ostensives, such as sounds, symbols, graphics, gestures, and drawings that we build, we access and touch a representation of things that live in a world of ideas, such as concepts and theorems, designated by *non-ostensives* (Chevallard, 1994). To facilitate the understanding of this fundamental concept in the analysis of mathematical activity, let us consider the non-ostensive object *equality relations*. In that case, an ostensive object widely used to represent this idea in mathematics classes by the teacher, or even in the textbook, is the two-pan balance.

We highlight that "we assume the ostensives as the primary ingredient of the technique" (Kaspary & Bittar, 2018, p. 406). That is, we highlight the ostensive aspects of digital resources to describe the techniques for carrying out their mathematical tasks.

> Resuming the fundamental notions of the anthropological approach, we will say that applying a technique translates into manipulating ostensives regulated by non-

[2] "From two aspects: the evaluation of personal relations and the evaluation of the institutional relation, both in relation to the built object, which are articulated with the moment of institutionalization, allowing to relaunch the study, demanding the resumption of some of the moments and, eventually, of the set of didactic path" (Barbosa & Lima, 2019, p. 1361).

> ostensives. The ostensives constitute the perceptible part of the activity, that is, when performing the task, these objects can be seen by both the observers and the actors. In the analysis of the mathematical work, the ostensive elements are part of the empirical reality, accessible to the senses. On the other hand, the presence of such or such non-ostensive in a given practice can only be induced or assumed from the manipulations of institutionally associated ostensives (Bosch & Chevallard, 1999, p. 11, authors' highlights).

Finally, we must note that an ostensive has two facets in an activity: the *semiotic* and the *instrumental*. The *semiotic dimension* refers to the meanings evoked by a specific ostensive, while the *instrumental dimension* refers to its operational function (Chevallard, 1994). In this study, we place more emphasis on the instrumental character of the ostensives, not discarding the relevance of the semiotic dimension.

2 Methodological aspects

We characterize this research as documentary since, according to Bailey (1994) and Mogalakwe (2006), it is a method that seeks to analyze the information contained in the documents about the phenomena that we intend to investigate. In this context, the researcher must use techniques to select, interpret, categorize, and identify the potential and limitations of material resources, commonly more written documents from the public or private domain (Payne & Payne, 2004; Mogalakwe, 2006). Since we seek to understand the studied phenomenon through the collected documents, taking into account the information available at the source, the research approach is qualitative (Patton, 2005).

In the scenario of this investigation, for the production and analysis of data, we focus on the digital resources *Equality Explorer: Basics* (EEB) and *Equality Explorer* (EE), available on the digital platform Physics Education Technology (PhET). The PhET, linked to the University of Colorado Boulder, was developed by Nobel laureate Carl Wieman in 2002 and offers simulations in the area of mathematics and science in a free, playful, and interactive way; guided by educational research carried out with students and teachers (University of Colorado Boulder, 2022).

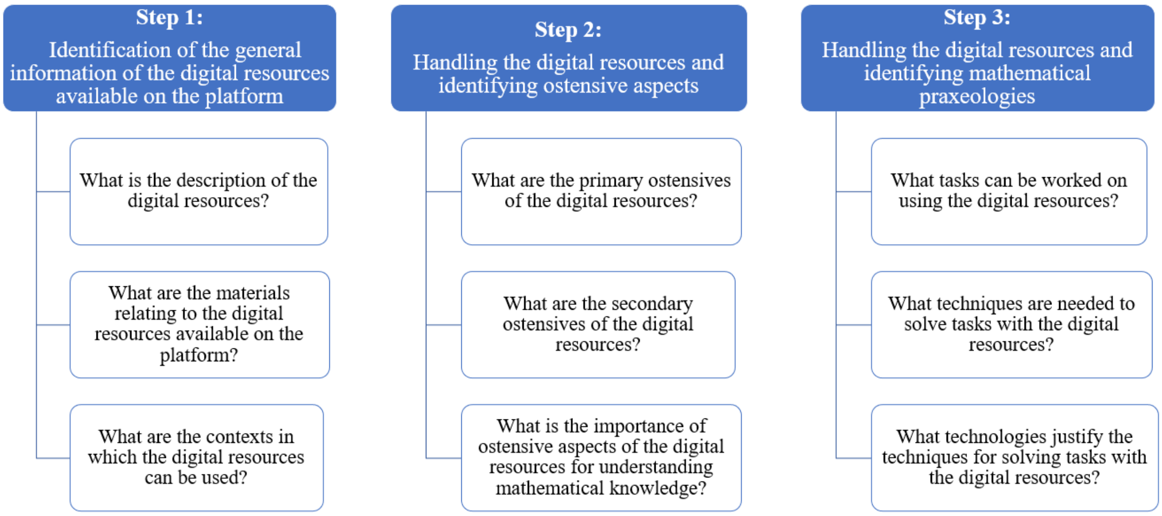
Regarding the data analysis methodology, we first selected general information presented in the PhET regarding EEB and EE, namely, a description of digital resources (mathematical topics that can be covered and learning objectives) and teacher tips. Then, in the second moment, we tried to handle the digital resources and identify their ostensive aspects. Finally, in the third moment, we performed interactive simulations with digital resources to determine the mathematical praxeologies that concern them.

In Figure 1, we illustrate the methodological organization of the research and the specific questions that guided the analyses at each stage.

In *Step 1 — Identification of the general information of the digital resources available on the platform*, the researcher (or teacher) must raise the descriptions, the linked materials, and the contexts of application discussed by the proposing platform of the digital resources. Without loss of generality, in our context, digital resources are interactive simulations. Nevertheless, it could be an interactive video or a digital game, for example.

In *Step 2 — Handling digital resources and identifying ostensive aspects*, the researcher (or teacher) must try to categorize the primary ostensives (fundamental for the study of mathematical knowledge, i.e., the understanding of non-ostensive elements) and the secondary ones (tools that help improve the way of using digital resources, without a direct relation with the mathematical object), in addition to questioning the relevance of ostensives as indispensable ingredients for mathematical activity — a kick to the last step.

Figure 1: Methodological organization of the research



Source: Self elaboration

In *Step 3 — Handling digital resources and identifying mathematical praxeologies*, the researcher (or teacher) must determine the tasks and subtasks that can be worked with digital resources, the techniques that enable the resolution of these tasks, and the technologies that help in understanding the justification for the use of the techniques.

We emphasize that, as mentioned in the previous section, we see the ostensive features of digital resources in an instrumental dimension, given that we aim to understand their possible operationalities in educational processes.

3 Results and discussion

By analyzing the general information of the digital resources *Equality Explorer: Basics* (EEB) and *Equality Explorer* (EE), we can identify the institutional relations $R(I, O_{IS})$ between the University of Colorado Boulder and *interactive simulations* (IS), as well as the personal relations $R(X, O_{IS})$ between teachers and IS, through the analysis of lesson plans published on the PhET platform.

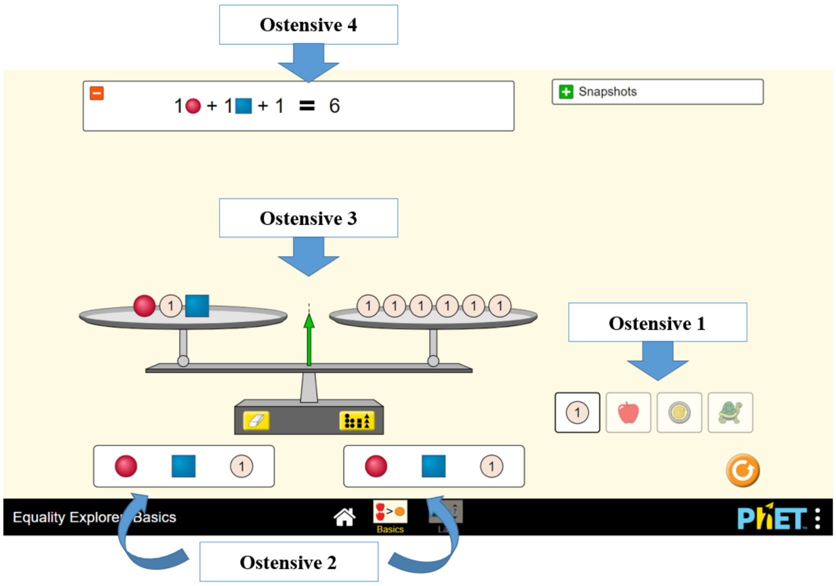
In this study, considering the institutional relations $R(_{IBNCC}, O_{ER})$, that is, the curriculum guidelines proposed by the recent normative document of Brazilian education about equality relations in school algebra teaching, we seek to establish personal relations, as researchers, with the digital resources mentioned. In this way, we focus on the results produced from Setps 2 and 3 of the methodological organization of this research (see Figure 1 previously exposed).

3.3 Equality Explorer: Basics

When analyzing the material resources (subject to virtual manipulation) of the EEB, we identified four types of ostensives in *basic* mode, which directly contribute to the study of equality relations: O_1 — *Types of objects*; O_2 — *Objects to be inserted into the two-pan balance scale*; O_3 — Two-pan balance scale; and O_4 — *Mathematical sentence with the equal sign*.

Taking into account Figure 2, we point out that the O_3 — *Two-pan balance scale* can contribute to understanding mathematical equality as a notion of equivalence, i.e., the equal sign can be understood from a relational perspective. The analogy established between O_3 and the non-ostensive *equality relations* happens through the relation between each side of the balance and each equality member. Thus, when the two-pan balance scale is balanced, we have an equivalence relation. This notion is verified through the O_4 — *Mathematical sentence with the equal sign*, which explains the symbology "=" when there is a balance in O_3.

Figure 2: Ostensives in the EEB screen in *basic* mode

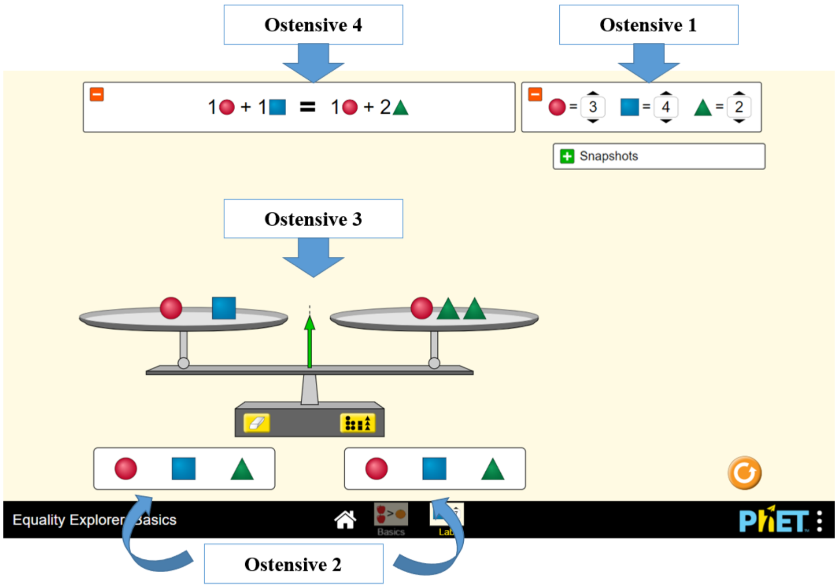


Source: Adapted by the authors from the PhET platform

To visualize these ostensives in the study of the theme at stake, we need, first of all, to insert or remove objects from the balance scale, mobilizing the O_2 — *Objects to be inserted into the two-pan balance scale*, which refers to operations performed on an equality. The inserted and retained objects are influenced by the O_1 — *Types of objects*, which allows choosing which types of objects (geometric solids, fruits, coins, and animals) to be used in the simulations.

In *lab* mode (see Figure 3), unlike the *basic* section, the O_1 — *Types of objects* allows students to, in addition to choosing the type of object (sphere, cube, and pyramid) to be mobilized in the simulation, assign the values of these objects, which vary from 1 to 20.

Figure 3: Ostensives in the EEB screen in *lab* mode



Source: Adapted by the authors from the PhET platform

Another aspect to be observed is the existence of other secondary ostensives that facilitate the manipulation of the *Equality Explorer: Basics*, for example: “eraser” ( ), which removes all objects from the balance scale if students want to propose another situation and “object stacker” ( ), which stacks each type of object on top of the other, organizing the objects inserted in the two-pan balance scale. We will not emphasize the manipulation of these and other tools as it is not part of the objective of this study.

We emphasize that the four main ostensives of the EEB described above are permeated in the analysis of mathematical praxeologies, as they are essential for mobilizing techniques in solving tasks using the referred digital resource. As stated before, we agree with Kaspary and Bittar (2018) that the ostensives are the primary ingredient of the technique.

Chart 1 depicts six subtypes of tasks related to task ***T*** — *Create and solve problems using the Equality Explorer: Basics.* The task subtypes are not explicitly arranged in the PhET platform. They were identified by the researchers through constant analyses of the EEB functionalities.

Chart 1: Elements of mathematical praxeologies identified in the use *of Equality Explorer: Basics*

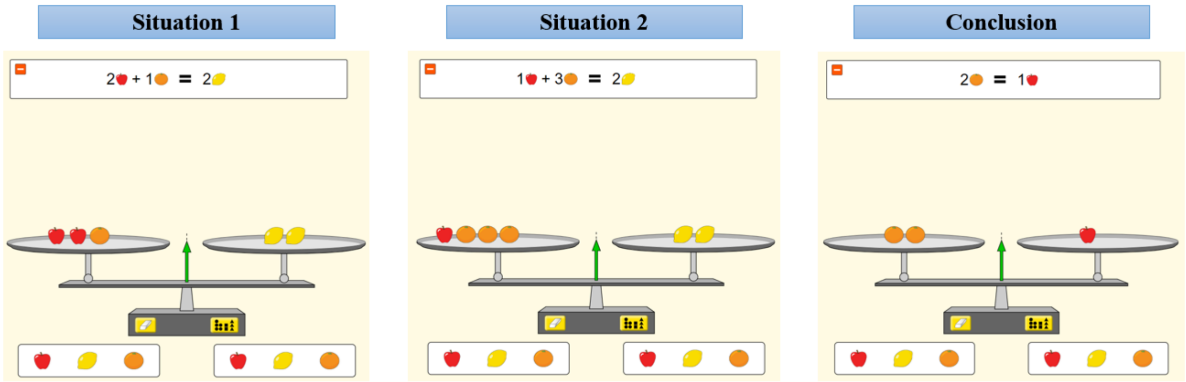
Types of tasks	Description of techniques	Technologies
t_1: Show an equality relation between equivalent amounts of objects through the interactive balance scale.	**τ_1:** Choose the type of objects (O_1) and place the objects (O_2) on both sides of the balance (O_3) of the EEB until reaching its equilibrium, which can be observed through equality (O_4).	**θ_1:** Notion of equality in a problem situation involving balance. **θ_2:** Interpretation of the equal sign as equivalence.
t_2: Determine, through the interactive balance scale, different sentences of object additions that result in the same sum.	**τ_2:** Apply τ_1. Relate the objects (O_2) displayed on each side of the balance scale (O_3) to their numerical values (O_1). Observe the mathematical equalities to verify the relations (O_4), which can be saved with the screenshot tool.	**θ_1, θ_2** **θ_3:** Identification of equivalence between equality sentences involving numerical addition operations.
t_3: Show, through the interactive balance scale, that the equality relation between two members remains when the same object is added to each member.	**τ_3:** Apply τ_1. Add the same type of object (O_1 and O_2) on both sides of the balance (O_3) to check their balance and check equality (O_4) between addition operations.	**θ_1, θ_2, θ_3** **θ_4:** Additive principle of equivalence.
t_4: Show, through the interactive balance scale, that the equality relation between two members remains when the same object is subtracted from each member.	**τ_4:** Apply τ_1. Subtract the same type of object (O_1 and O_2) on both sides of the balance (O_3) to check their balance and check equality (O_4) between addition operations.	**θ_1, θ_2, θ_3, θ_4**
t_5: Determine the unknown term in an equality relation involving addition and subtraction through the interactive balance.	**τ_5:** Place only on one or both sides of the balance scale the following objects: red sphere and/or blue cube (O_1 and O_2). Add spheres with the numeral “1” on one or both sides of	**θ_1, θ_2, θ_3, θ_4**

	the balance scale (O_3). Subtract the same number on both sides of the balance scale until you reach the value of the object (red sphere or blue cube).	
t_6: Determine the unknown term in an equality relation involving addition, subtraction, multiplication and division through the interactive balance.	**τ_6:** Place on one or both sides of the balance scale the following objects: red sphere and/or blue cube (O_1 and O_2). Add spheres with the numeral "1" on one or both sides of the balance scale (O_3). Subtract the same amount on both sides of the balance, checking the scale between them. Remove half of the mass from both plates, which equally represents the division of both members, to reach the value of the red sphere or blue cube objects.	**$\theta_1, \theta_2, \theta_3, \theta_4$** **$\theta_5$:** Multiplicative principle of equivalence.

Source: Adapted by the authors from Oliveira, Almeida and Espíndola (2021)

For illustrative purposes, Figure 4 shows how the task subtype **t_1**, displayed in Chart 1, can be explored to understand the use of the EEB in introductory teaching of school algebra, focusing on student learning about the theme of *equality relations*.

Figure 4: Examples of **t_1** using EEB in basic mode



Source: Adapted by the authors from the PhET platform

We elucidate that the task subtype **t_1** — *Show, through the interactive balance scale, a relation of equality between equivalent amounts of objects* — is fundamental to any experimentation with the EEB around the notion of equivalence (**θ_1, θ_2**), as students need to place equivalent objects on both sides of the two-pan balance (**τ_1**) so that it stays in balance. As shown in Figure 4, we have three equality relations equivalent to each other. In each of them, there is an equivalence between the objects in both equality members. In this example, starting from situations 1 and 2, we reach a conclusion.

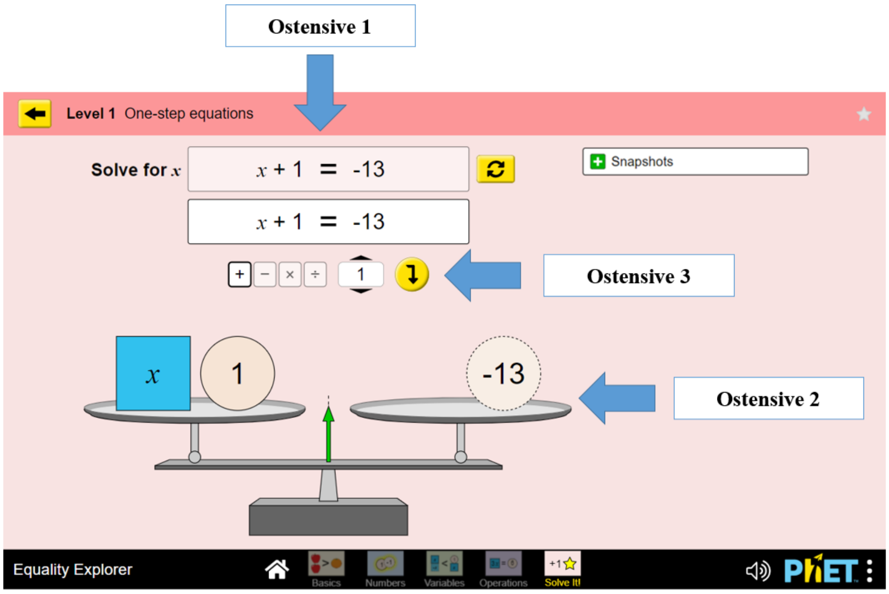
Through examples like this, we expected students would perceive, through different proposed situations, the underlying mathematical properties and be able to generalize them algebraically. For reasons of brevity, we have not listed the details of each task proposed in Chart 1. Despite this, we recognize that other simulations can be carried out in the same segment.

So, in the technical-practical block, we highlight six tasks and six techniques that, in turn, are based on five technologies. Certainly, we have not exhausted the possibilities of raising other types of tasks with the *Equality Explorer: Basic*.

3.4 Equality Explorer

As illustrated in Figure 5, in the *Equality Explorer* (EE), we identified three types of ostensives in the *Solve it!* mode: O_1 — *1st-degree polynomial equation in alphanumeric language*; O_2 — *Two-pan balance scale as a metaphor for the 1st-degree polynomial equation*; and O_3 — *Operators to solve the 1st-degree polynomial equation*:

Figure 5: Ostensives of the EE screen in *Solve it!* mode



Source: Adapted by the authors from the PhET platform

Unlike the EEB, the EE already presents the 1st-degree polynomial equations through the O_1 and O_2 ostensives when the user opens a specific level in the *Solve it!* section, while O_3 needs to be handled by the user to solve the problem situation proposed in O_1 and O_2.

To determine the subtasks related to this digital resource, we resorted to the results of the thesis produced by Araújo (2009), which deals with teaching equations based on the ATD.

In Chart 2, we have examples of the task $\boldsymbol{T_1}$ — *Solve 1st-degree polynomial equations using the Equality Explorer at levels 1, 2, 3, 4 and 5 in the Solve it! section* — regarding subtypes: $\boldsymbol{t_{1.1}}$ — *Solve an equation like* $ax + b = c$ and $\boldsymbol{t_{1.2}}$ — *Solve an equation like* $a_1x + b_1 = a_2x + b_2$ (Araújo, 2009). We point out that the *Equality Explorer* updates the equations at each access, that is, several situations are proposed. This fact made it impossible to quantify all the examples presented on the platform.

Chart 2: Examples of task subtypes identified when using the Equality Explorer in the Solve it!

Proposed levels on the platform	Examples	Task subtypes
Level 1 – One-step equations	10x = – 180	$t_{1.1}$
	3 = x – 4	
	(1/9)x = – 8	
	113 = x + 5	
Level 2 – One-step equations with	– x = 12	

negative coefficients	– (1/7)x = 4	
	4 = – (1/2)x	
	– 130 = – 5x	
Level 3 – Two-step equations	117 = 9x + 9	
	5x + 6 = – 19	
	– 216 = 10x – 6	
	– 7x + 2 = 198	
Level 4 – Multi-step equations with fractions	(– 2/7)x + 1 = 9/7	
	(3/10)x + 1/10 = 5/2	
	1 = (2/3)x + 5/3	
	– 136/3 = (8/3)x – 8/3	
Level 5 – Multi-step equations with variables on both sides	7x + 2 = 6x – 23	$t_{1.2}$
	– 3x – 101 = 6x – 5	
	7x – 8 = 5x – 14	
	– 9x – 7 = – 10x + 1	

Source: Self elaboration

As shown in Chart 2, from levels 1 to 4, the tasks refer to subtype $t_{1.1}$ and at level 5, the tasks refer to the subtype $t_{1.2}$. To solve these types of subtasks, students should be encouraged to use technique τ_{NTC}: Neutralize terms or coefficients (NTC), which is characterized by isolating the unknown, performing the same operation on both sides of the equation (Araújo, 2009). Therefore, the technique τ_{NTC} is justified by the following technologies:

- Principles of equivalence between equations with equal solutions or roots (θ_{PEE}):
- ➢ Additive principle: when we add (or subtract) both sides of an equation by the same amount, we obtain a new equation equivalent to the first;
- ➢ Multiplicative principle: when we multiply (or divide) the two sides of an equation if it is multiplied (or divided) by the same quantity (non-zero), we obtain a new equation equivalent to the first (Araújo, 2009).

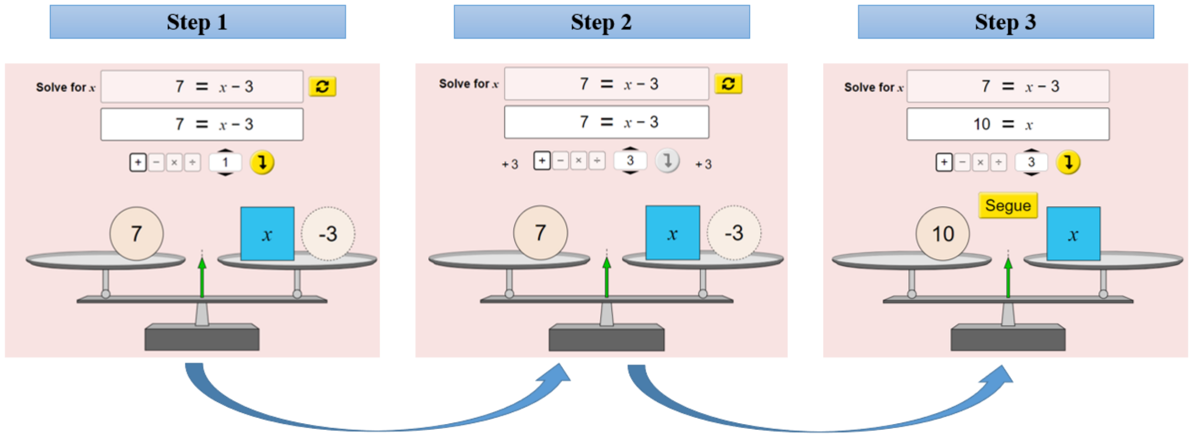
We point out that technique τ_{NTC} to solve task subtypes $t_{1.1}$ and $t_{1.2}$ in the use of the *Equality Explorer* requires detailing, especially if students use only the ostensive digital resource to solve the proposed situation. We reinforce that it is through the O_3 that students can apply τ_{NTC}.

For the sake of brevity, we will discuss below (see Figure 6) an example regarding the task subtype $t_{1.1}$, highlighting the practical-technical block [T, τ] and the technology we identified through the manipulation of the EE.

As mentioned above, when opening the screen in level 1 of the *Solve it!* mode, students are faced with a 1st-degree polynomial equation to solve with at least one stage. In the case of Figure 6, to solve the equation $7 = x - 3$ (introduced in Step 1, via O_1 and O_2) in one step, students need to neutralize the term -3 to isolate the unknown x. To do this, they must operate + 3 on both sides of the equation (proposed in Step 2, through O3). The additive principle of equivalence between equations justifies this neutralization of the term. Finally, we get the solution to the equation (see Step 3), which is $x = 10$.

It should also be noted that the operators of O_3 belong to the set {– 10x, –10, – 9x, – 9, – 8x, – 8, – 7x, – 7, – 6x, – 6, – 5x, – 5, – 4x, – 4, – 3x, – 3, – 2x, – 2, – x, – 1, 0, 1, x, 2, 2x, 3x, 4, 4x, 5, 6, 6x, 7, 7x, 8, 8x, 9, 9x, 10, 10x} and, depending on the term in the equation, students need to neutralize it by parts. In the case mentioned above, the operator + 3 belongs to the set, which allowed solving the equation in just one step. So, although *Equality Explorer* can contribute to the understanding of equality as an equivalence using O_3 — as it is necessary to operate on both sides of the equations presented on the platform — the set of operators is restricted, prolonging the steps to solve specific tasks.

Figure 6: Example of the subtype $t_{1.1}$ of the EE at level 1 of Solve it!



Source: Adapted by the authors from the PhET platform

Another limiting factor of the digital resource is the specific technique for solving tasks on the platform. In this sense, other techniques commonly used by students — such as Equality testing by trial and error and the transposition of terms or coefficients with inverse operations — can be mobilized from other teaching approaches, encouraging the use of different resources, for example, pencil and paper.

In general, aiming to expand the study of this mathematical object to the context of the use of digital resources in mathematics education, such as we did in the previous subsection, we evoke in this subsection some elements of mathematical praxeology in the teaching of 1st-degree polynomial equations, proposed in Araújo's (2009) thesis. This movement allowed us to see the possibility of two task subtypes, whose 1st-degree expressions are reducible to canonical form. These subtasks can be resolved with the technique of neutralization of terms or coefficients (τ_{NTC}), justified by the equivalence principle between equations (θ_{PEE}).

4 Final considerations

Intending to analyze digital resources for mathematics teaching and learning, presenting the anthropological theory of didactics as a methodological proposal, we list some examples for illustration purposes. Roughly speaking, we have not exhausted the discussions and reflections on the use of the digital resources *Equality Explorer: Basics* and *Equality Explorer* in the field of school algebra. Other examples can be found in works by Oliveira, Almeida, and Espíndola (2021) and Almeida, Espíndola, and Oliveira (2022).

In this article, we establish personal relations with digital resources when we focus on ostensive and mathematical praxeology concepts advocated in the anthropological theory of the didactics. Such articulation encouraged us to verify the need for simultaneous work with the ostensives of the digital resources as a possibility to develop students' understanding of mathematical content at stake through mathematical praxeology, considering the practical-technical block and the technology.

We believe that other aspects of digital resources can be investigated from the ATD perspective. As for implementing these resources in the classroom, the researcher can seek to analyze the personal relations that teachers and students have with these digital resources. Concerning the curricular demands of the school system in each socio-cultural context, one can investigate the institutional relations established by the normative documents on the mathematical knowledge at stake and the integration of digital resources in mathematics teaching to understand how they can be explored from such perspectives.

We consider we have not exhausted the methodological possibilities of systematically analyzing digital resources based on the ATD as the theoretical framework. Furthermore, we believe that other theories can be added to the analysis of digital resources for didactic purposes.

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