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Instrumental Meta-Orchestration: studies and outspreads

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Abstract: This paper discusses the trajectory of the instrumental meta-orchestration model (IMO) development, studies and results, and extensions. Instrumental meta-orchestration, developed as a solution for teacher training on instrumental orchestration (IO), relied on its development of concepts such as meta-situation, didactic meta-configuration, and meta-exploration model. An experimental study with its analysis led to the development of data visualisation instruments in an internal analysis context at the IO. These concepts and analysis instruments were used in research that followed that of the IMO, such as the extension of the IMO to interdisciplinary teachers' education. The concepts of ad hoc reaction, artefacts such as didactic webdoc, and analysis of events between orchestrations show their potential and subsidise new research for teacher training in integrating digital technologies.

Keywords: Instrumental Meta-Orchestration. Instrumental Orchestration. Digital Technologies. Teacher Education. Interdisciplinarity.

Metaorquestração Instrumental: estudos e desdobramentos

Resumo: O presente artigo discute a trajetória do modelo da metaorquestração instrumental (MOI), com ênfase para seus estudos, resultados e extensões. A metaorquestração instrumental, desenvolvida como solução para a formação docente a partir da orquestração instrumental (OI), contou com o desenvolvimento de conceitos, como a metassituação, a metaconfiguração didática e o metamodo de execução. Um estudo experimental com sua análise levou ao desenvolvimento de instrumentos de visualização dos dados em uma situação de análise interna à OI. Esses conceitos e instrumentos de análise foram utilizados em pesquisas que seguiram o modelo da MOI, como a extensão da MOI para a formação interdisciplinar. Os conceitos de reação *ad hoc*, artefatos como *webdoc* didáticos e análise dos eventos entre orquestrações mostram seu potencial e subsidiam novas pesquisas para a formação docente na integração das tecnologias digitais.

Palavras-chave: Metaorquestração Instrumental. Orquestração Instrumental. Tecnologias Digitais. Formação Docente. Interdisciplinaridade.

Meta-Orquestación Instrumental: estudios y desarrollos

Resumen: El presente artículo discute la trayectoria del desarrollo del modelo de meta-

orquestación instrumental (MOI), estudios y resultados, sus extensiones. La meta-orquestación instrumental, desarrollada como una solución para la formación de docentes en Orquestación Instrumental (OI), se basó en el desarrollo de conceptos como meta-situación, meta-configuración didáctica y meta-modo de ejecución. Un estudio experimental con su análisis condujo al desarrollo de instrumentos de visualización de datos en una situación de análisis interna en la OI. Estos conceptos e instrumentos de análisis fueron utilizados en investigaciones posteriores a MOI, como la extensión de la MOI a la formación interdisciplinaria. Los conceptos de reacción ad hoc, artefactos como webdoc didáctico, análisis de eventos entre orquestaciones muestran su potencial y subsidian nuevas investigaciones para la formación de docentes en la integración de tecnologías digitales.

Palabras clave: Meta-Orquestración Instrumental. Orquestación Instrumental. Tecnologías Digitales. Formación Docente. Interdisciplinariedad.

1 Introduction

One of the critical aspects of the effectiveness of digital resources in the teaching and learning process is integrating this theme into teacher education. In this sense, regarding the insertion of digital technologies into professional practice, Lucena (2018) takes instrumental orchestration — IO (Trouche, 2005; Drijvers, Doorman, Boon, et al., 2010) as a model that has helped in the preparation of classes that use different artefacts which help their apprentices in the instrumental genesis. However, according to her, this must be done based on reflective teacher education courses, particularly in mathematics, so that future teachers can build these instrumental orchestrations in their practice.

In her thesis, Lucena (2018) investigated a group of mathematics degree students while constructing an instrumental meta-orchestration (IMO) model. Among the various relevant results for teacher education, she introduces the ideas of the composition of Instrumental Orchestrations and some other elements, such as the undergraduates' reactions, called ad hoc reactions. She also analysed the didactic configuration between instrumental orchestrations, didactic meta-configuration, exploration meta-mode and ad hoc reactions taken in moments of instrumental orchestrations.

Many results and analysis techniques of the data generated by the instrumental meta-orchestrations became sources for new studies developed within the scope of the Study Group on Resources for Education — GERE. From this perspective, Morais (2021) expands the idea of instrumental meta-orchestration to investigate a model of interdisciplinary teacher education in the context of technical education integrated into high school. Moreover, Pontes (2022), inspired by the earlier mentioned model to generate an IO composition for teacher education, focuses on active methodologies. In the same direction, with the advent of the pandemic, Gitirana and Lucena (2021) developed an adaptation based on IMO and IO principles, culminating in the online IO model for teacher education and the online IO. Given this, this paper discusses some elements of the GERE studies path, which began with the works of Lucena (2018).

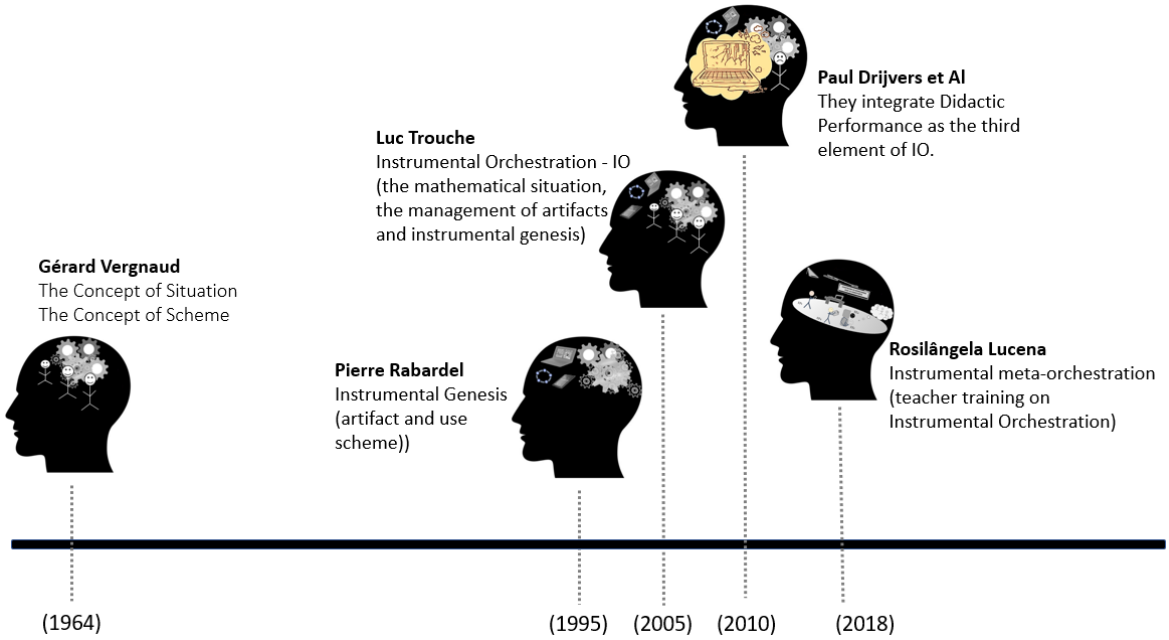
2 The instrumental meta-orchestration

The challenge of offering a differential to mathematics teaching with the integration of digital technologies inspired Trouche (2005) to develop a theoretical-methodological model capable of creating and managing a structure favouring teaching practice in mathematics classes. This differential does not emerge from the simple insertion of artefacts but from the integrated, intentional and systematised use of these means with the specific knowledge one

wishes to teach and learn.

Instrumental orchestration is a theoretical model (Figure 1) constituted by the notion of scheme and situation (Vergnaud, 1996b) through the concept of instrumental genesis (Rabardel, 1995) by the stages of didactic configuration and mode of execution, both created by Trouche (2005); and the stage of didactic performance, created by Drijvers *et al.* (2010).

Figure 1: The instrumental meta-orchestration path



Source: Lucena (2021, p. 310)

Integrating a mathematical situation and an artefact to help the learner solve the situation demands an architecture that promotes her/his instrumental genesis. In this direction, Trouche (2005) presents the metaphor of instrumental orchestration, which compares the classroom to an orchestra, in which the teacher is the conductor, and the students are the instrumentalists. In general lines,

> an instrumental orchestration is exactly the systematic arrangement by an intentional agent of the elements (artefacts and humans) of an environment to implement a given situation and, more generally, to guide the learners in the instrumental genesis and evolution and balancing of their instrument systems. The adjectives systematic and intentional are important:
> - systematic: "which proceeds methodically, in a definite order and for a determined purpose"; It also contains the idea of "integrated arrangement in a system";
> - intentional: an orchestration does not describe an existing arrangement (there always exists one) but designates the need for an a priori thought of such an arrangement.
>
> It is defined by didactic configurations (i.e., arrangements of environmental artefacts corresponding to each phase of a situation), their modes of exploitation and their articulation. (Trouche, 2005, p. 126, our translation)

In the context of instrumental orchestration, an arrangement is created by the conductor, consisting of a situation, a didactical configuration, an exploration mode, and a didactic performance. Once proposed to the students, the situation demands the subject's usage schemes to make its resolution with the support of the available artefact. The focus of instrumental orchestration is the students' instrumental genesis, composed by instrumentalisation and

instrumentation processes, which characterises how the subject transforms an artefact into an instrument to solve the situation — in this case, mathematical situations.

In this perspective, Rabardel (1995) presents the artefact as something material or symbolic, produced for the subject or others. The subject's use schemes associated with the artefact allow its selection and discovery of functions and deviations, characterising the instrumentalisation process. In an intertwined way, as the subject seeks to know the artefact and transform it through its use schemes, they also undergo transformations caused by the accommodation, coordination and reciprocal assimilation of the already constituted schemes, a process called instrumentation.

In other words, when a conductor creates an instrumental orchestration, their didactic choices are guided by the situation they proposed. It is relevant to consider this aspect because when the subject is in action to resolve the situation, they are guided by their schemes of use, which are related to the available artefact, a fact that may result in its instrumental genesis. Vergnaud (1996b) states that there are no situation without schemes, nor schemes without situation. So, once the situation exists, the subjects' use schemes, correlated to the artefacts integrated into it, whether they are already existing schemes or those that develop throughout attempts at resolution, are the guiding threads of the instrumental genesis.

In this context, Vergnaud (2013) defines a scheme as an invariant activity organisation for a particular class of situations. It is a dynamic and functional totality formed by four components:

- objective, with sub-objectives and anticipations;
- rules of action, also taking information and control;
- operational invariants, which are the concepts-in-action and theorem-in-action;
- possibilities of inferences in the situation.

In this concern, Lucena (2018) warns about the challenge for anyone investigating a subject's schemes since observing them is more complex. Vergnaud (2013), in turn, explains that this is not an easy task, as the scheme components are only sometimes explicit, so much information about this scheme is implicit. Therefore, observing the subject's actions and confronting them about such actions can help to identify their schemes or their components.

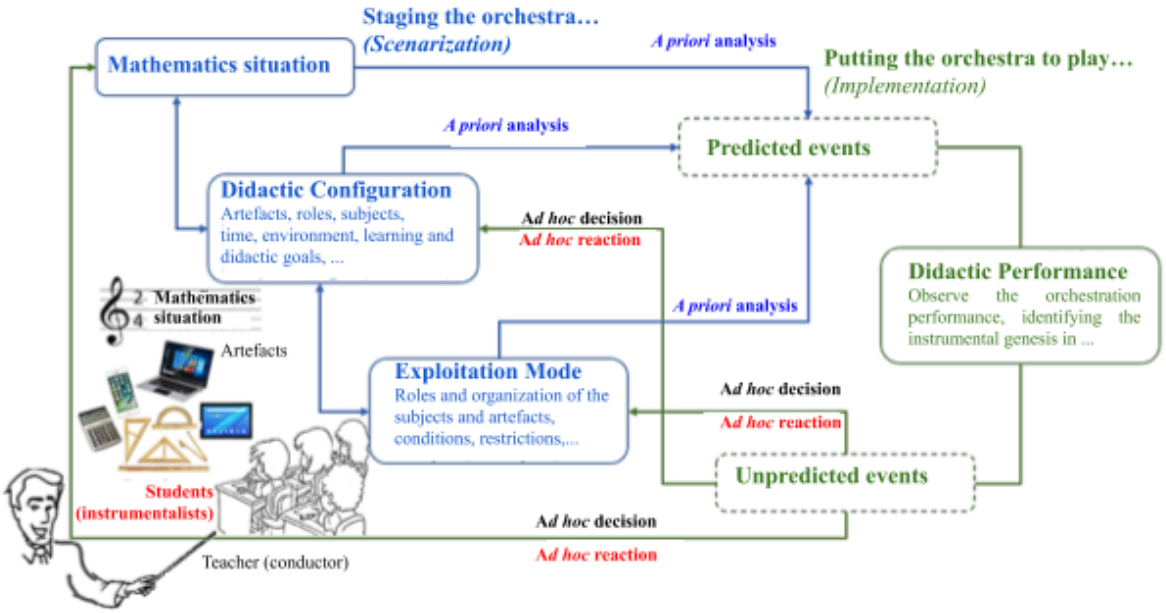
In a mathematics classroom, it is up to the teacher to create instrumental orchestrations (Figure 2) that favour students' instrumental genesis and their own analyses of how this happens. Thus, it is necessary to propose a situation integrated into one or more artefacts for its resolution and the organisation of the instrumental orchestration components.

In the didactic configuration, the conductor initiates the organisation of the arrangement based on decisions and didactic choices that involve the proposed situation, its conditions and restrictions. Moreover, the technological devices, the time and the organisation of the subjects to solve it are included. In the execution mode, the teacher should conduct a priori analyses to anticipate students' strategies and difficulties in solving the situation and problems regarding using the technological devices. Finally, the functions and roles of the teacher and the students are defined, and a "B plan" for possible foreseen intercurrences is elaborated.

Once in execution mode, the instrumental orchestration will allow the conductor to verify his didactic performance, which consists of monitoring the execution of the instrumental orchestration in full development to verify its success. It is at this stage that unforeseen events can be observed, for which ad hoc decisions are in order, that is, punctual decisions by the

teacher to resolve events that may jeopardise the smooth running of the orchestra. The ad hoc reactions taken by students are also observed when they are surprised by events that they did not expect to occur while solving the situation.

Figure 2: Instrumental orchestration diagram



Source: Lucena, Gitirana and Trouche (2021, p. 314)

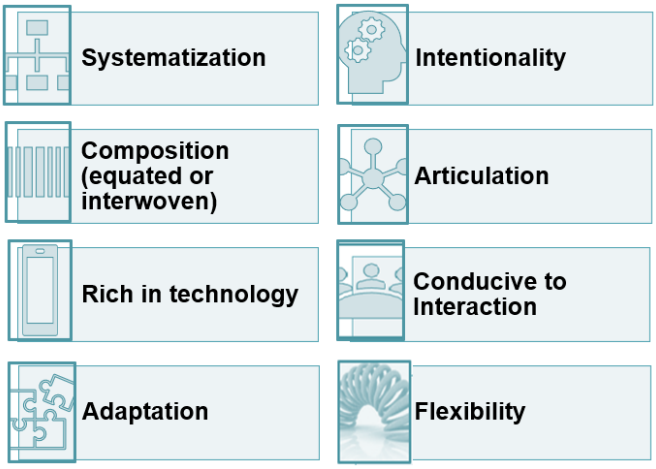
The didactic performance plays an essential role in data collection by those involved in the orchestra. This component enables recording the first impressions and unforeseen events, facts that occur but are not captured or remembered by the subjects, and can be recorded by photographic cameras, computer software or camcorders. This data analysis enables observing the subject's relationship with the artefact, its use schemes or its components, favouring the inference regarding the subject's instrumental genesis and the actual contributions of the orchestra.

Realising the relevance of the theoretical model of instrumental orchestration for integrating digital technologies into mathematics teaching, Lucena (2018) decided to develop a model to train mathematics teachers on the IO goal — instrumental meta-orchestration — defined as the systematic and efficient management intentionally, by an agent (teacher educator(s)) of artefacts and subjects (teachers and prospective teachers) confronted with a meta-situation, to appropriate the concept of instrumental orchestration. A meta-situation can be understood as a complex situation that can be analysed from a combination of the nature and difficulties of the IO involved. It is worth mentioning that the notion of situation is attributed to Vergnaud (1996a).

In this way, Lucena (2018) organises a composition of instrumental orchestrations, considering the model's inherent characteristics (Figure 3). It is systematic, as it allows structured organisation according to pre-established parameters; intentional, since it guides the decisions of those who orchestrate towards the objectives that one wants to achieve; composed, that is, formed by orchestras that can be sequenced or overlapped; articulator, preventing orchestras from becoming isolated and stagnant activities; rich in technologies, since they are at the heart of the instrumental genesis; conducive to interaction, since the orchestras can be performed individually or collectively, cooperatively or collaboratively; adaptable, since it allows changes to be made at any time during the execution of any of the orchestrations; and, finally, it is flexible concerning internal or external changes to the instrumental orchestrations

that make up the model, including about the training content.

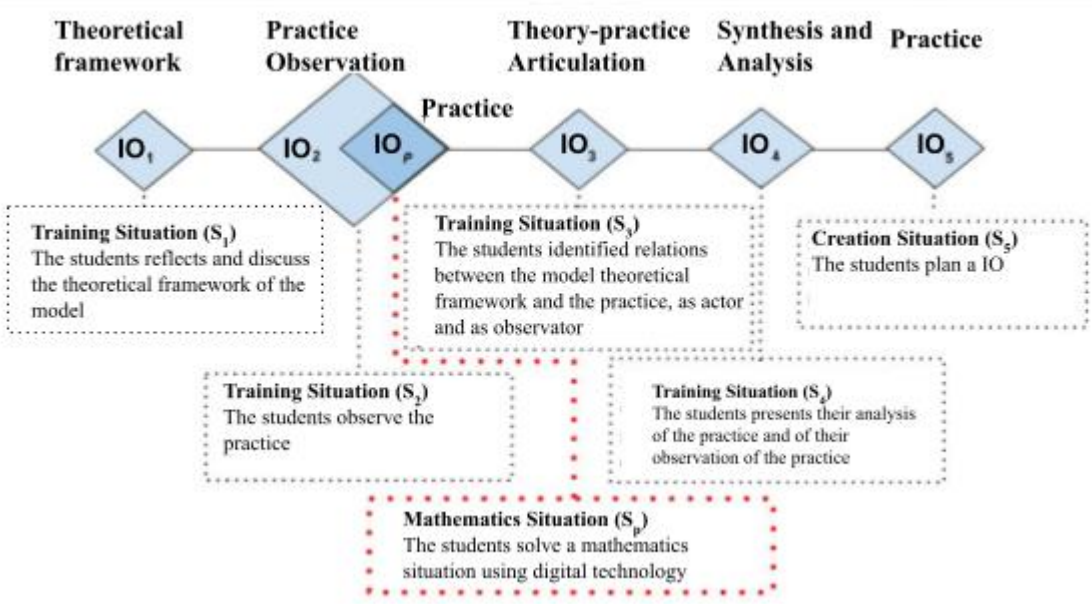
Figure 3: Instrumental meta-orchestration dimensions



Source: Lucena, Gitirana, and Trouche (2021, p. 316)

The external design of the instrumental meta-orchestration (Figure 4) reveals the entire model formation structure and highlights the nature of each instrumental orchestration and its respective situations.

Figure 4: Instrumental meta-orchestration characteristics

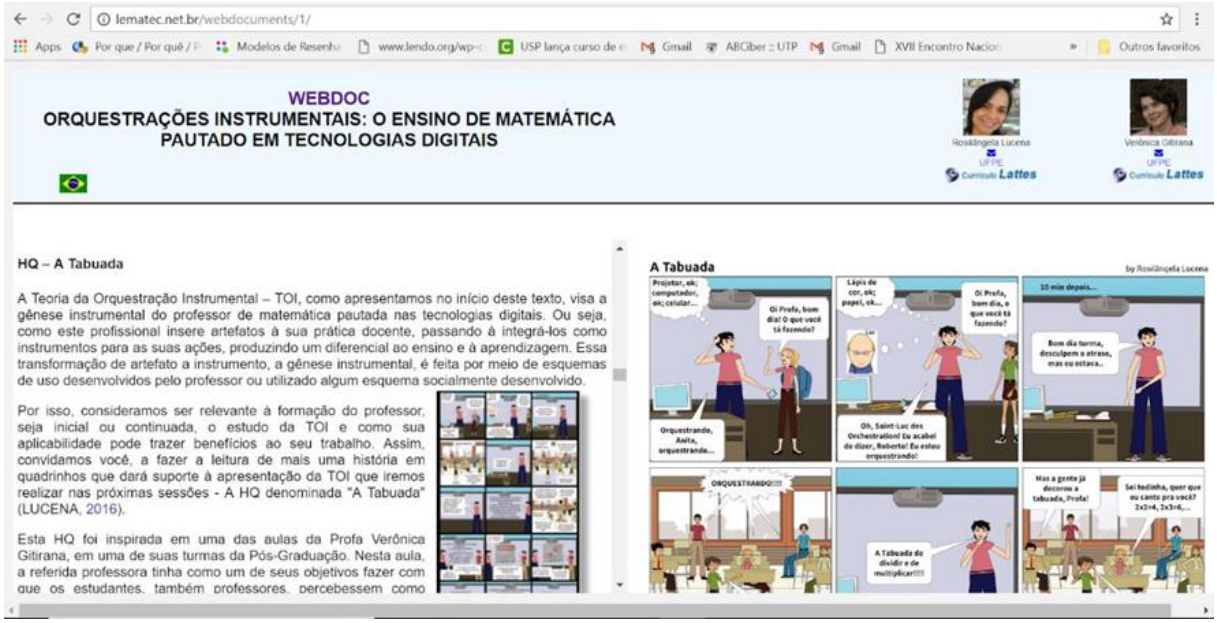


Source: Lucena, Gitirana, and Trouche (2021, p. 319)

IO_1, based on theory, presents a situation that allows reflection and discussion, in a forum, on concepts related to IO, explored in a webdoc. IO_2, for observation of practice, is used to observe, monitor and record what happens at IO_p, a central orchestra from which practical examples of concepts studied in the webdoc and discussed at IO_1 emerge. IO_2 and IO_p, in turn, are experienced simultaneously. IO_3, of theoretical-practical articulation, presents a situation of reflection and discussion, in a forum, on concepts and experiences experienced in previous orchestrations. The IO_4, of synthesis and analysis, allows participants to access the data generated by themselves to analyse them and share their results with the support of a presentation resource. Finally, IO_5, of a practical nature, proposes a creation situation for the training participants, when they are challenged to create their instrumental orchestration.

The instrumental orchestration composition is characterised by a solid articulation for understanding the IO concept and components. Available artefacts and protocols created by participants can be used or transformed. An example is the webdoc (Figure 5) made available in the first IO but used throughout the training by the participants. The webdoc is a multimedia document developed for didactic purposes and for a specific audience — future mathematics teachers who aim to appropriate the theoretical model of instrumental orchestration. It is an attractive text that increases the participants' chances of reading, indispensable for comprehending concepts.

Figure 5: Partial interface of the didactic webdoc on instrumental orchestration

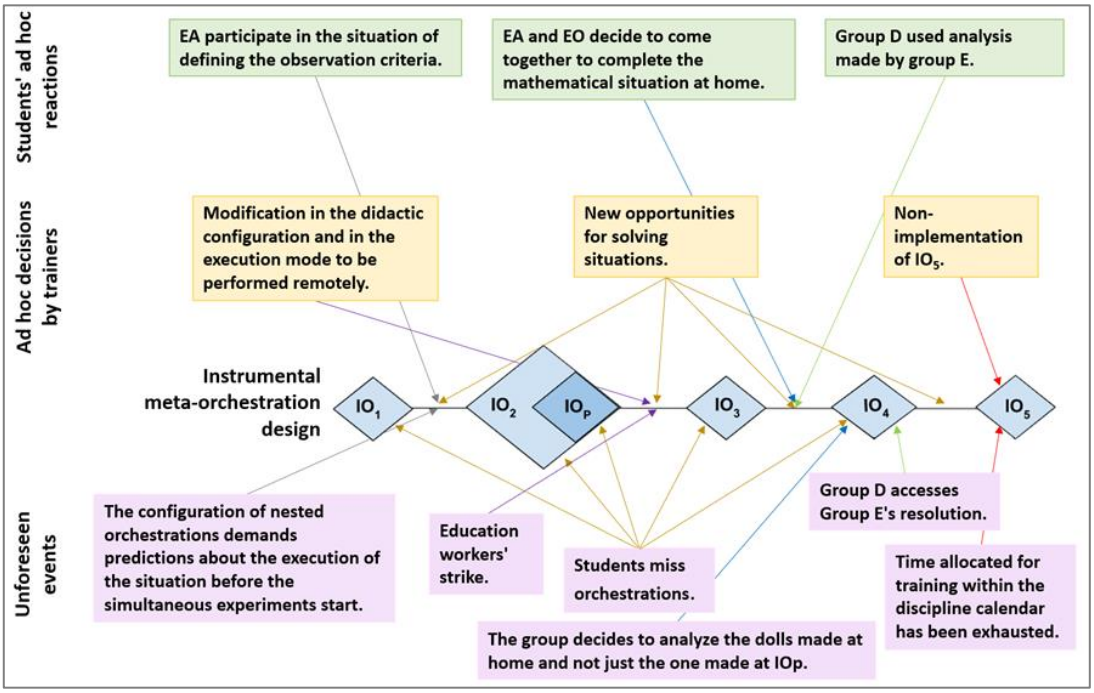


Source: https://geregroup.site/webdocs/webdoc3/

The webdoc in Figure 5 discusses concepts that underlie instrumental orchestration, such as scheme and situation, according to Vergnaud (1996b), the artefact-instrument relationship, the instrumental genesis and the instrumentalisation and instrumentation processes. It also discusses the didactic configuration, the exploitation mode and the didactic performance. All content is heavily exemplified through illustrations, videos, comics and images. The webdoc was created to encourage reading the text and to facilitate understanding the studied concepts. This artefact can be accessed free of charge through the access link provided by the Study Group on Resources for Education — GERE – UFPE.

The instrumental meta-orchestration presents events external to the orchestrations in the time intervals in which each one is performed. These events are generally not foreseen and may lead the teacher educator to change the situation, or the elements previously defined for instrumental orchestration. These are *ad hoc* decisions or reactions taken by the teacher and the students, which need to be recorded and analysed to improve the training model, considering the results obtained from the adaptations and modifications carried out in the situations and orchestrations instrumental. Figure 6 shows the instrumental meta-orchestration's design and the record of unforeseen events (pink box) between the instrumental orchestrations throughout the training. Such events demanded *ad hoc* decisions from the teacher educators (yellow box) and *ad hoc* reactions (green box) from the students.

Figure 6: Events between instrumental orchestrations



Source: Lucena (2018, p. 342, our translation)

It is worth mentioning that unforeseen events related to the work carried out by the pre-service teachers may occur far from the teacher educator's field of observation and may not even be recorded by cameras. However, it is possible to question the conduct of the participants and the strategies for resolving the situation to identify such events and the decisions taken to resolve them.

3 The main study and the data analysis structure

The instrumental meta-orchestration is a theoretical-methodological model that presents diverse data, considering the different natures of the instrumental orchestrations that compose it and their respective situations. In this way, it is natural that the data that emerge from the execution of these instrumental orchestrations demand different collection and analysis instruments that contemplate their characteristics. After carrying out a preliminary study, a refinement of the model above led to the main study, which allowed the creation and adaptation of collection and analysis instruments, which were used to compose the analysis structure of this study and others that have been developed. In this text, we will present an excerpt from the analysis structure of instrumental meta-orchestration, contemplating only IO_1, IO_2 and IO_p since, in IO_3 and IO_4, we used the same or similar instruments, considering the nature of the data.

The main study was carried out in a public institution of higher education in the third period of a mathematics pre-service teacher education class. The training took place in a discipline that included, in its program, the teaching of didactic theories of mathematics. Situations were proposed to be experienced in person and others at a distance. The group of 24 enrolled undergraduates participated in the training, with varying attendance. The students were organised into six groups, with the data from Group D chosen for analysis, given that all participants had experienced the training in full, in addition to the excellent quality of audio and image of the videos of this group.

The instrumental meta-orchestration class took six face-to-face hours, of which two

hours were devoted to the IO_2 and IO_p experiments and four to the IO_4 presentations. The IO_1 and IO_3 were carried out at a distance, and the data – the participants' questions and answers — were posted on a Google Groups forum regarding the reading of the webdoc in the IO_1 and those that correlated the events of the IO_2 experiments and IO_p with reading the webdoc. In the IO_2, IO_p and IO_4, the data emerged from the footage, as well as from the observation protocols (IO_2), from the GeoGebra files and screen capture (IO_p) and from the slideshows, used to present the analysis of the IO_p (IO_4).

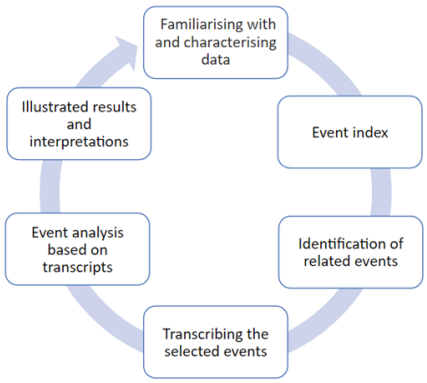
The analysis structure of the instrumental meta-orchestration contemplates the internal and external events that emerge from the performance of each instrumental orchestration. Thus, the collection, organisation, characterisation and analysis of events that reveal aspects of the participants' instrumental genesis are the primary focus for verifying the apprehension of these elements in instrumental orchestration. The didactic performance of each orchestration will allow not only the inference about the instrumental genesis of the participants throughout the meta-orchestration but also about the actual contribution of the IO as support. The microgenetic analysis technique supported by videography was chosen to guide data collection, organisation, characterisation, and analysis.

Videographic microgenetic analysis is a technique structured in stages (Figure 7), which were strictly followed. At that moment, the data was familiarised and characterised so that it was necessary to watch the videos repeatedly to write down and identify events related to the research problem. In addition, an event index was created based on themes relevant to the research, identified in the familiarisation stage. Significant events related to the research problem were also identified based on an interpretative and rigorous analysis of these events. Literal transcriptions of the audio of the events and detailed descriptions of the actions and gestures of the subjects present at the selected events were also made. Finally, exhaustive analyses of the transcripts were carried out, aiming at coherent interpretations of the micro-processes related to the activity, its characterisation, and the illustration of the results and interpretations.

According to Lucena (2018), this technique allows for analysing individual and group behaviours, mapped in detail in the events related to the research problem. For Meira (1994),

> The videography (study of activity through video footage) and Microgenetic Analysis (a detailed study of the evolution of relationships between agents and situations) combine to form a data collection and analysis model that allows for a robust and consistent interpretation of the psychological mechanisms underlying human activity. (Meira, 1994, p. 59, our translation).

Figure 7: Structure of the videographic microgenetic analysis

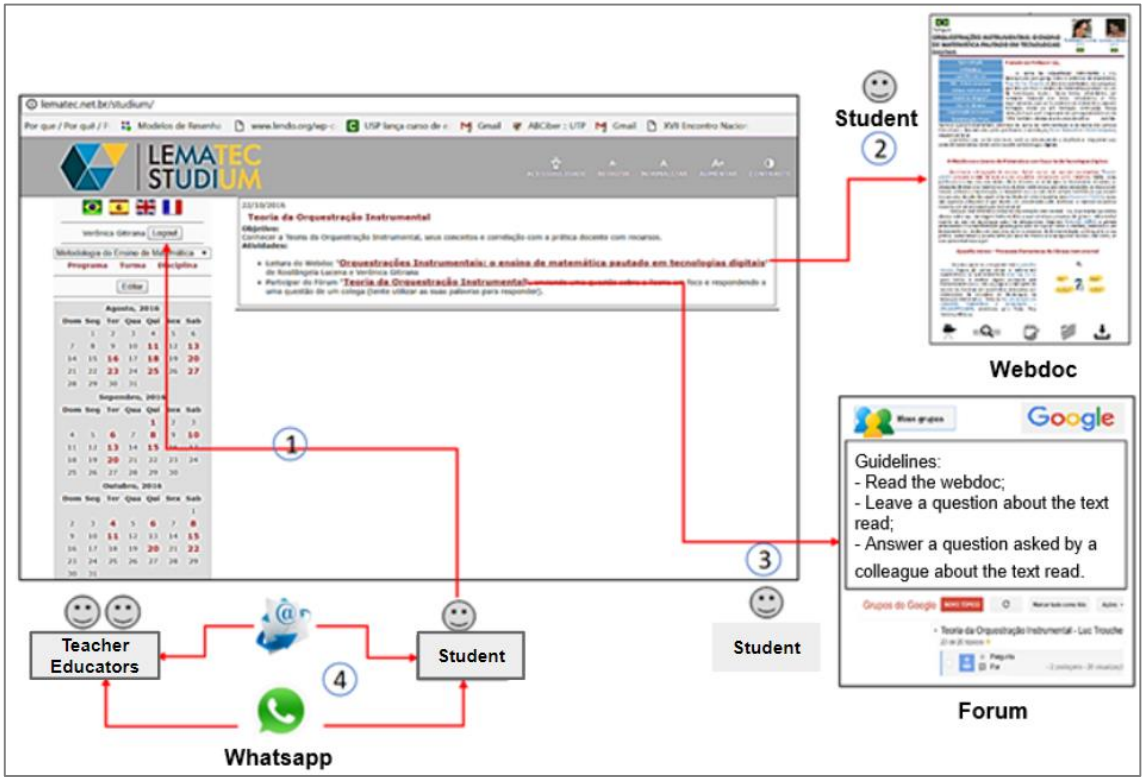


Source: Lucena (2018, p. 67, our translation)

The videographic microgenetic analysis was used to guide the creation of instruments for data collection, organisation and analysis, considering their specificities. In this text, it is relevant to present a synthesis of how the analyses of the instrumental orchestrations that make up the instrumental meta-orchestration were carried out. For this, we will highlight clippings of the performance of the EA_1 student throughout the training to apprehend the concept of didactic configuration.

In the IO_1, the proposed situation aimed to introduce and discuss the theoretical model of instrumental orchestration in a forum based on reading a webdoc with examples. The design (Figure 8) below highlights aspects of the didactic configuration and way of performing this IO. When accessing the course platform (1), the student must carefully read the guidelines on the IO training situation 1 and click the link to access the webdoc (2). After reading it, they should access the second link, which corresponds to the Google Groups forum (3), where they will leave a question and an answer to a colleague's question. If someone needs help, they can contact the teacher educators by email or WhatsApp.

Figure 8: Design of the IO_1



Source: Lucena, Gitirana, and Trouche (2021, p. 320)

To analyse the data in the form of a question and answer, it was necessary to create an instrument that could organise this content. In Table 1, we have an example of how the theme "didactic configuration" was exploited by participant AS_1 (Group D) to ask a question after reading the webdoc. There is a description of the question and the answer of two components of AS_1 schemes (Group D), that is, rules of action and operative invariants, and, finally, the characterisation of the mode of engagement with the webdoc, that is, the way they used the artefact to accomplish the task.

In Table 1, EA_1 (Group D) does not use the webdoc excerpt to formulate a question but as the body of a statement to the question. Reading the text or part of it to choose an affirmative passage and transforming it into a question is a form of AS_1's engagement with the webdoc. It

is essential to state that the webdoc contained the answer to EA_1's question, which may indicate their intention to solve the task and not necessarily share a doubt. All questions and answers from participants registered in the IO_1 and IO_3 forums were organised in this instrument model and subsequently analysed, as shown in Table 1.

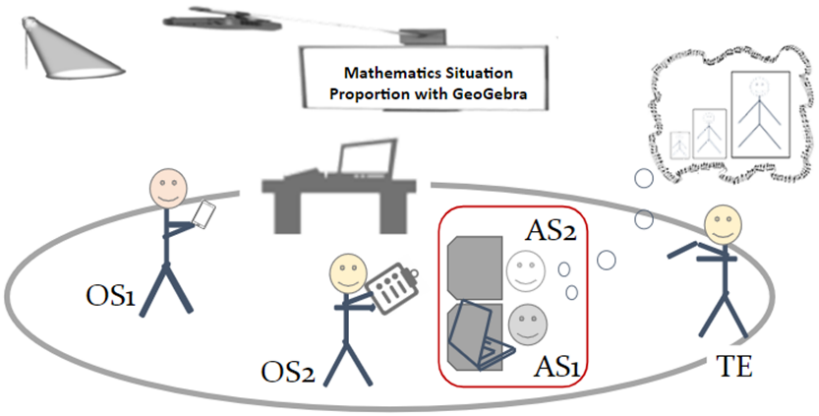
Table 1: EA_1 posts (Grupo D) about the didactic configuration

Event (Thematics)	Question	Answers	Rules of Action	Operative Invariants (knowledge)	Modes of engagement
Didactic Configuration	**AS_1/D -** *In the didactic configuration, the teacher needs to dedicate time to predict what might happen during the execution of the orchestration. What is the purpose of this?*	**AS_1/C -** The purpose is to avoid surprises and unforeseen circumstances in the classroom, as the teacher has to be concerned that all students understand the orchestration.	**AS_1/D -** - Read the *webdoc*. - copy and paste an excerpt from the *webdoc* turning it into a question, posting it in a new forum post. **AS_1/C -** - Read the *webdoc*. - answer based on the *webdoc* reading - excerpt on didactic performance. - post the formulated answer, in line with the question on the subject.	**AS_1/D -** - The teacher needs to predict what can happen during the performance in the exploitation mode of the orchestration. - Each element of the orchestration has a purpose. **AS_1/C -** - he understands that the purpose of the predictions made by the teacher in relation to the exploitation mode is to avoid unforeseen events that could prevent the students from not understanding the orchestration.	**AS_1** - Read text to choose an affirmative passage and turn it into a question.

Source: Lucena (2018, p. 174, our translation)

We can observe that instrumental orchestration 2 co-occurs with IO_p (Figure 9), whose situation consists of defining criteria considering the theoretical model of instrumental orchestration for observing an experiment based on these criteria. In other words, while the AS duo* solve the mathematical situation, the actor-students observe and record the significant events. For this reason, the OS was assigned the task of creating observation criteria before the IOp started.

Figure 9: Didactic configuration scenario of IO_2 and IOp[1]



Source: Lucena (2018, p.186, our translation)

However, an unexpected situation during the performance of IO_2 divided the execution of the training situation into two moments. In the first, called $IO_{2.1}$, the group decided not to follow the rules of the guidance protocol (Figure 10) intended only for student-observers, which consisted of sharing, realising the situation, and defining the role of each participant in the group, as an observer-student and actor-student, as well as the criteria that would guide the observation. In the second moment, called $IO_{2.2}$, there is the observation of the IO_p itself.

[1] Duo (AS_1 e AS_2): two people (students) interacting to perform an activity (mathematical situation) collaboratively.

Figure 10: Guidelines for OS

Training situation: create observation criteria (notation and filming) of the mathematical task, based on the Instrumental Orchestration (TROUCHE, 2005).
I - Consider, for the criteria elaboration, the instrumental genesis processes, the schemes of use, the management aspects that can interfere (contributing or not) with the performance of the mathematical situation (conditions given by the teachers to carry out the situation, organization of the group, choice of resources, the actions of each actor-student, etc.);
II - Consider, during the observation and the annotation, the criteria elaborated by you and the guidelines;
Observation Criteria:
(a) _____
(b) _____
(c) _____
(d) _____
(e) _____

Source: Lucena (2018, p.254, our translation)

In $IO_{2.1}$, a discussion between the members of Group D to decide who would assume the role of student-observer and student-actor was recorded on video, which was observed, transcribed, and analysed. In Passage 1, we can see AS_1's interest in deciding who gets the role of student-observer. The criterion that AS_1 uses is verifying who read the webdoc.

Passage 1: AS_1's speech about the webdoc reading.

AS_1: I do it like this [...] since he read the text ... did you read a lot [ask the OS_2]?
AS_1: You read the text more than I did.
OS_1: I read the text, but it has been a while. I have to reread it.
AS_1: So, do I.
OS_1: I know one or two things here.

Source: Lucena (2018, p. 261, our translation)

The content of Section 1 was organised in Table 2, considering components of the AS_1's scheme, action rules, operative invariants and inference possibilities. This organization facilitated the analysis of the discussion on the definition of the role of each member of Group D.

Table 2: Definition and use of the criterium "have read the webdoc to select the OS"

Subgoal	Rules of action	Operative Invariants	Possible inferences
Distribute the roles between the group members.	**AS_1** - Define as a choice criterion to assume the role of observer-student - having read the *webdoc*.	**AS_1** - Based on a criterion, a group can be classified into two subgroups.	**AS_1** - Three of the students have read the text but all of them want to stick with the task, so it's better to choose who will work with GeoGebra.
	AS_1 - Probe who in the group has read the webdoc to take on the role of observer-student.	**AS_1** - If someone knows the webdoc, s/he can assume the role of observer.	
	AS_1 - Compare who read more than the others to assume the role of observer-student.	**AS_1** - If someone knows the elements of instrumental orchestration, s/he can assume the role of observer.	

Source: Lucena (2018, p. 261, our translation)

The explicit idea in Passage 1 and Table 2 is that whoever claims to have read more will be the observer, given the need to master the IO better to define the observation criteria and observe guided by such criteria. Whoever claims to read less will be the actor-student. However, the students disagreed, so other criteria were created and discussed. In this context, it is relevant to highlight the importance of a functional instrument, as in Tables 1 and 2, which contributed to a finer analysis of the events.

After defining the role of each group member, the components set out to define the observation criteria (Passage 2), inspired by the content of the webdoc.

Passage 2: Observation criteria definition

TE_1: Yeah, [...] do you remember... did you read the text?
OS_2: Yes.
TE_1: What is most important in instrumental orchestration? What are the key elements?
OS_2: Artefact and environment!?
TE_1: Artefact, instrument, ...
AS_1: We already put it here.
TE_1: [TE_1: starts to read the paper they wrote down] [...] relationship between artefact and instrument, what else?
AS_2: The schemes we will use... [lowers his voice at the end of the sentence].
TE_1: The group's schemes.

Source: Lucena (2018, pp. 265-266, our translation)

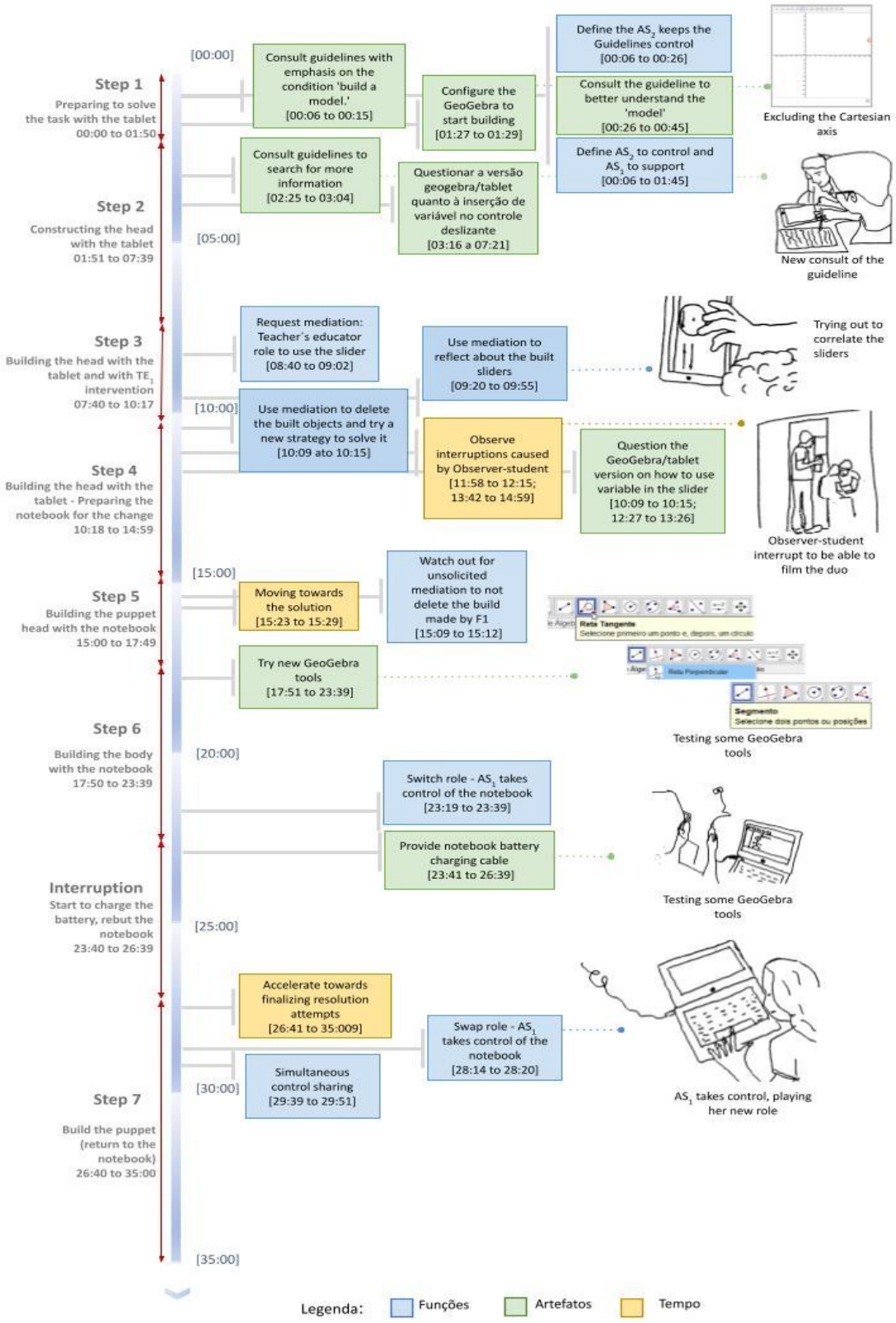
Thus, with the help of the teacher educator, Group D defines that the focus of observation will be on the events in which there is a close relationship with the artefact/instrument. Thus, they considered the processes of instrumentation and instrumentalisation. It also focuses on knowledge related to GeoGebra and mathematics in the schemes highlighted and the events related to the mathematical situation's resolution time. This instrument (Table 2) was used to organise the content of the observation protocol and to analyse how the observers related what they watched in the IO_p experiment with what they read in the webdoc and discussed in the IO_1 forum.

In the Pivot instrumental orchestration, a video of the interactions, actions and gestures of the student-actors when solving the mathematical situation, the screen capture that reveals the resolution of the situation in the Geogebra software and the observation protocol of the student-observers contributed to the creation of a bank of examples relating to terms and concepts related to instrumental orchestration, all organised by theme, which mark the trajectory of student-actors with events described and illustrated in timelines. Timeline 1 (Figure 11) highlights vital elements of the didactic configuration, with the use of colour legends: the roles of the participants (TE, OS, AS) in blue, available artefacts in green and the definition of time to carry out the proposed situation, under the determined conditions, in yellow.

In the case of IO_p, the only one with a mathematical situation, analysis of the participants' double instrumental genesis was required. One was carried out from the perspective of instrumental meta-orchestration, with thematic timelines, which chronologically highlight events related to the concepts to be apprehended by the participants, such as the one mentioned above in Figure 10. The other, in turn, was based on the perspective of doing mathematics with integrated technology in the context of the IO_p, with timelines related to the mathematical situation and the instrumental genesis that marks the integration of students' knowledge about mathematics and the GeoGebra software, demanded by the situation at stake. Analyses of interactions and schema components of actor-students complement those

performed on timelines of events related to IO concepts.

Figure 11: Timeline of the instrumental genesis related to didactic configuration.

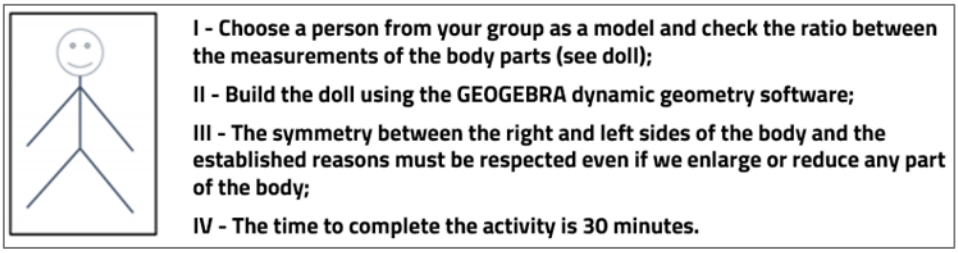


Source: Lucena (2018, p. 205, our translation)

When starting the IO_p, the actor-students must read the guidelines protocol and solve

the mathematical situation (Figure 12) in GeoGebra according to the conditions presented. As for the observer-students, they should observe and record the actor-students' interactions, gestures, resolution strategies, and actions.

Figure 12: Maths-didactics situation and the draw given by the teacher educators.



Source: Lucena, Gitirana, and Trouche (2021, p.323)

All materials collected by the observer-students — such as videos, screen captures, and cursive records of the mathematical situation resolution process by the ASs — were organised, classified, and analysed according to the structure proposed by the chosen analysis technique. In the case of IO_P, the events that give light to the mathematical situation and the instrumental genesis of the ASs (how they solve the mathematical situation with GeoGebra software supports) were chosen. Two timelines were organised to detail and chronologically illustrate all events aligned with these two themes.

We will present, below, an extract of the analysis of only one of the 19 events of the timeline — instrumental genesis, referring to the 1st stage of the seven corresponding to the resolution of the mathematical situation. This event focuses on one of the conditions of the situation, which is proportionality (Passage 3).

Passage 3: Scheme anticipations on solving the problem.

AS1: [...] So, it means that each part of the body has to have a ratio between them because it's saying that, if you increase or decrease any of the parts, the ratios have to be respected, so, you have to be proportionate. Like, arm with leg, with head. I don't know... there's the trunk. Now how are we going to do this? [...].
AS1: The head. But you have to think, right? Like, how is it going to be proportional?
AS2: [...] but I think the head will be what will be proportional, does it?
AS1: [...] it is like the radius of the head, we change the radius, I don't know, it was adjustable, the radius of the head, I don't know, a circumference? Or are you going to make a head […]?

Source: Lucena (2018, p.219, our translation)

In Passage 3, we can observe evidence of anticipations of the scheme to solve the problem. At this stage, the duo finds it challenging to articulate mathematical knowledge with the GeoGebra software to define a ratio that guarantees the proportionality of the puppet's body parts, allowing its enlargement and reduction. Therefore, the duo's knowledge of dynamic geometry is at stake in that first moment and compromises the entire construction of the puppet throughout the IO_P.

In Table 3, a possible process of anticipation of the AS_1 scheme is highlighted by stating that if the measurement of the head radius (circumference) is "adjustable", the proportionality between these measurements (radius and circumference length) is guaranteed. Thus, a clear definition of the magnification ratio can be observed directly with the simulation to enlarge and reduce the constructed object.

Table 3: Highlight possible anticipation in an EA_1 scheme

Rules of action	Theorems-in-action	Concept-in-action	Possibles inferences
AS_1: Before starting, reflect on how to arrive at the ratios between the parts of the puppet, which allows increasing and decreasing each one proportionally.	**AS_1:** maintaining the ratios guarantees proportionality between the parts of the puppet.	- ratio (as ratio between body parts).	**AS_1/AS_2:** The measure of the length of the circumference is proportional to the measure of the radius. If you connect the circle radius to the slider variable, the head will zoom in/out.
AS_2: Start by making a part that proportionally enlarges the head.	**AS_2:** if the head is a circle, then there is proportionality between its length and its radius.	- circle; - ratio; - proportionality;	
AS_1: Predict the head shape as a circle, so it's already proportional with the adjustable radius.	**AS_1:** if the radius of the circle has a variable as measure, you can increase and decrease the head proportionally.	- variable.	

Source: Lucena (2018, p.219, our translation)

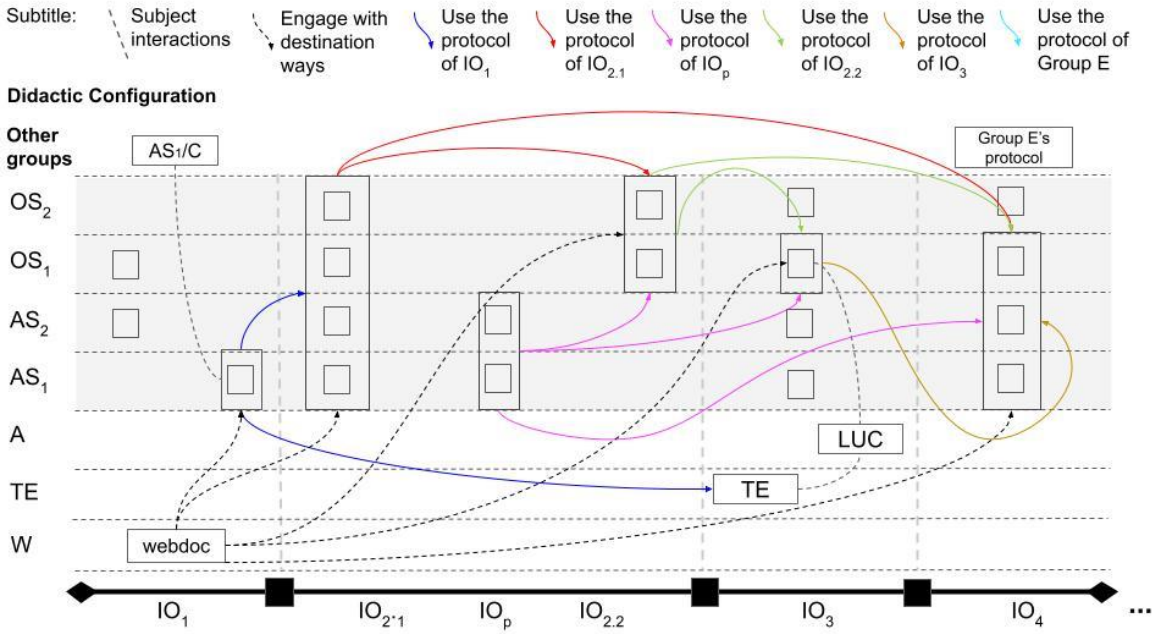
The events highlighted in Table 3 allow for the observation of evidence that the duo's scheme to build the head, with the possibility of enlarging/reducing, consisted of constructing the slider to the radius of the circumference. AS_1's focus is on reflecting on how to define the ratio between the puppet's body parts to ensure proportionality between them so that it is more focused on anticipating actions before trying. EA_2, on the other hand, starts with the construction and shows itself with a scheme of doing and thinking during the process. Both knew that the measure of the length of the circumference is proportional to the measure of the length of the radius and that if they could relate the measure of the radius to the slider, the head would increase/decrease proportionally and, consequently, they could apply this strategy throughout the process of building the puppet.

So far, we have presented a synthesis of how the data analysis was carried out within each instrumental orchestration. However, to infer the appropriation of the undergraduate students on the concepts of instrumental orchestration, it was necessary to observe the evidence of the instrumental genesis of each participant in the training, individually, with repercussions in the group. For this, we created an instrument that allowed us to highlight the interactions of each component of a group with the others, with the teacher educator, with the author of the theoretical model, and with created protocols. The instrument was used to record these interactions related to the following themes: didactic configuration, mode of execution, didactic performance, mathematical situation, and instrumental genesis. In it, there is a caption (Figure 13) that describes the individual trajectory of each participant in the group.

The caption (Figure 13) highlights the following items: the interactions between the subjects carried out in the IO_1 and IO_3 forums, grey line, dashed; student engagement records with webdoc content, black, dashed arrow; postings on IO_1 forums became a protocol used on IO_2 and IO_3, blue arrow; IO_3 forum posts also became protocols used in IO_4, brown arrow; the observation criteria of $IO_{2.1}$ generated the protocol used to carry out the cursive record and production of the video of the observation of the practice in $IO_{2.2}$, red arrow. In addition, written protocols and video were used in IO_4, green arrow; the events generated by the duo's work at

IO_p formed an event bank, one of the essential protocols at IMO, used at IO_4, pink arrow; and, finally, the discussions generated in the IO_3 forum articulated with those in the IO_1 forum and the IO_2 and IO_p experiments also became protocols used in IO_4, light blue arrow.

Figure 13: Appropriation of didactic configuration trajectory



Source: Lucena (2018, p. 361, our translation)

Figure 13 is an example of how the created analysis tool was used. Although it presents the interactions of all the group's components, we will focus on AS_1 to exemplify the analysis of the degree students' interactions. We can observe that the instrument allows us to read vertically — that is, the performance of AS_1 within each IO, individually — and horizontally, that is, the performance of AS_1 throughout the entire formation. The same can be considered for all members of Group D, emphasising its interest in the didactic configuration in each of the IO and for each participant, OS_1, OS_2, AS_1, and AS_2. Furthermore, the same considerations can be made when interest in the subject emerges from interactions with the webdoc read (w), with the teacher educators (TE), with the author of the theoretical model they seek to apprehend, Luc Trouche (Luc) or protocols created by the group in the experience of each orchestration.

We can see in the figure that students AS_1 and OS_1 showed greater interest in the didactic configuration theme than their other colleagues. We can also note that AS_1, for example, developed engagement with the content of the webdoc (IO_1) and used it as an artefact to solve the situation of $IO_{2.1}$, as well as the interaction with the teacher educator in order to decide with the group criteria of observation of the IO_p. It is important to remember that AS_1 and AS_2 experienced the IO_p, created to support the resolution of a mathematical situation by integrating the GeoGebra software, which generated a bank of examples on different aspects of instrumental orchestration. This experience strengthened AS_1's interest in the didactic configuration since the elements that make up this stage of the IO are perceptible when they are in execution.

As it constitutes a bank of examples of instrumental orchestration, especially in the eyes of those who observe the execution of an IO_p, it can be said that this also favoured the work of the observers in the elaboration of the $IO_{2.2}$ protocol, still with support from the reading of the webdoc and the criteria elaborated in the $IO_{2.1}$ protocol. In IO_4, AS_1 brings up the theme of

"didactic configuration" again during the presentation of the analysis of the IO_p, with a mention of the content of the webdoc, the observation protocols and the events of the experience in the IO_p. The analysis proposed in IO_4 highlights the IO_p as a bank of examples of the aspects that permeate the theoretical model of instrumental orchestration. It is relevant to note that, in each orchestra, the participants generate protocols that serve as artefacts in subsequent orchestrations to help them resolve their situations.

4 The interdisciplinary instrumental meta-orchestration

The study and creation of instrumental meta-orchestration gave rise to other studies that now extend IMO to other fields or training modalities, such as training for interdisciplinary education (Morais, 2021) and training on teaching methodologies (Pontes, 2022), sometimes takes elements developed at IMO to deal with online training (Gitirana & Lucena, 2021), carried out during the pandemic. In this article, we will discuss the extension to the formation of integrated technical education in greater detail.

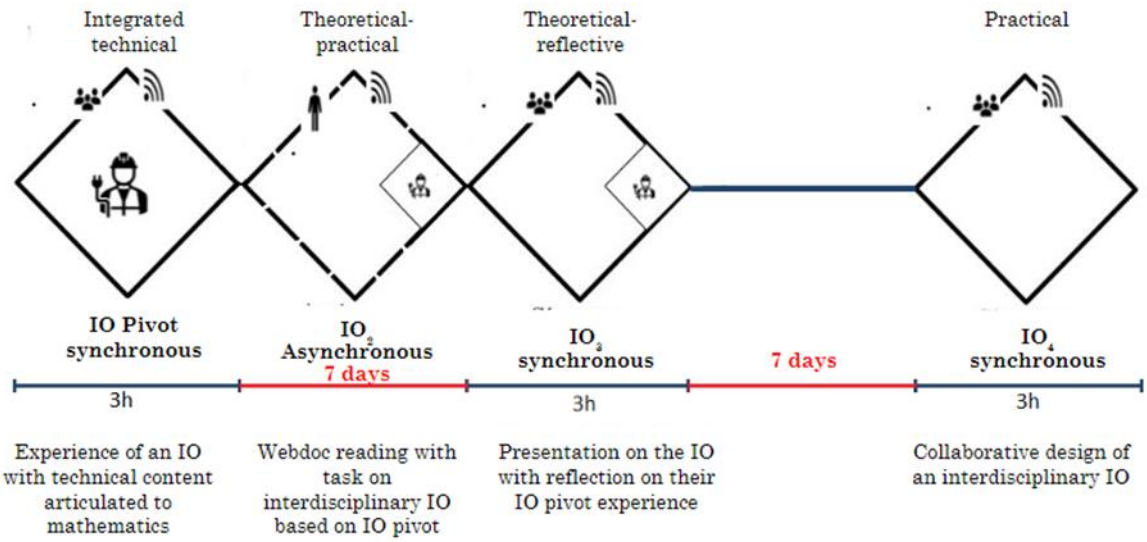
There are several contexts in which mathematics relates to other knowledge areas. In professional education, technical education integrated into high school encompasses professional disciplines related to the technical course and propaedeutic disciplines, such as mathematics. Since both areas need to be approached in a connected way, and considering the wealth of artefacts typical of technical education, Morais (2021) seeks to extend the model of instrumental meta-orchestration (Lucena, 2018) to an interdisciplinary perspective to form teachers from both areas of professional education — mathematics and technique — on the IO model (Trouche, 2005), considering the specificities of integrated technical education.

In this scenario, based on the definition, characteristics and design of the IMO, an interdisciplinary IMO was developed, defined as "the systematic and intentional management, by teacher educators, of artefacts and subjects (teachers from different areas of knowledge) confronted with a meta-situation, to appropriate the concept of IO based on an interdisciplinary context" (Morais, 2021, p. 117). Besides containing the eight attributes of the IMO (Figure 3), the interdisciplinary IMO had one more: interdisciplinarity. This inclusion is because all the IOs of this model are interdisciplinary, being planned and experienced by teachers from different areas (mathematics and technique) to implement an interdisciplinary situation.

Regarding the design of the IMO (Figure 4) and considering the peculiarities of technical education, some adjustments and modifications were made to elaborate the design of the interdisciplinary IMO (Figure 14), composed of four interdisciplinary instrumental orchestrations. All training had to be adapted for remote living, as it took place in July and August 2021, a period of isolation and restriction caused by the new coronavirus. Synchronous meetings took place via the Google Meet virtual room.

IO_p is the first of the interdisciplinary instrumental orchestrations, designed and taught by the professor of the technical area and centred on the training of the mathematics teachers regarding technical knowledge and artefacts of the professional course, in correlation with mathematical content (Morais, Gitirana, & Lucena, 2021). IO_2, in turn, consists of reading and interpreting an interdisciplinary webdoc asynchronously. In the document, IO_p events were used to exemplify elements of the IO model and its theoretical foundations and to discuss some basic concepts of the professional axis. In IO_3, there is a discussion about the concepts learned based on the experiences of previous instrumental orchestrations. Finally, IO_4 proposes a situation of creation in which professors from both areas must collaboratively build their own interdisciplinary IO.

Figure 14: Design of interdisciplinary IMO.



Source: Morais (2021, p. 118, our translation).

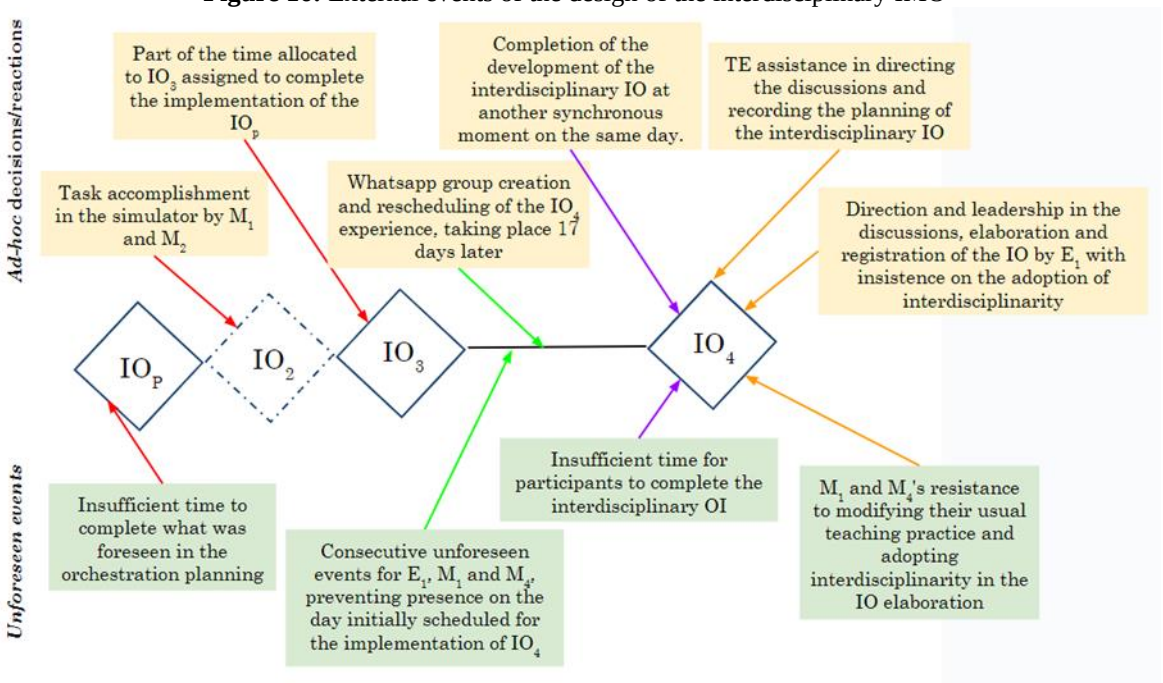
Unlike the IMO, in this model, the option was chosen to experience the IO_p right at the beginning of the training, when we verified (through a questionnaire initially applied to the mathematics teachers) these teachers' scarce knowledge of the use of artefacts of the professional course and even of its themes, which would be fundamental for the resolution of the technical-mathematical situation to be proposed. Furthermore, with this inversion of the order of orchestrations, it was possible to use an interdisciplinary IO (the IO_p) to support the interdisciplinary webdoc (Figure 15) based on a situation experienced by the training participants.

Figure 15: Partial interface of the interdisciplinary webdoc

Source: https://geregroup.site/webdocs/webdoc4/

After analysing the didactic meta-performance of the interdisciplinary IMO, some unforeseen events and *ad hoc* decisions/reactions were evidenced (Figure 16).

Figure 16: External events of the design of the interdisciplinary IMO



Source: Morais (2021, p. 195, our translation)

In some instrumental orchestrations, they extrapolated the time, a fact that required its conclusions at other times. It points to the need to extend the duration of the orchestrations and create complementary material on technical content (to be made available to mathematics teachers before experiencing IO_p) and an asynchronous moment for practising the use of professional artefacts (during the IO_2), a direct consequence of the interdisciplinarity of the IMO. This attribute also required the need to include a discussion of the topic (during IO_3), especially in the context of integrated technical education, since maths teachers showed significant resistance to integrating the different areas of knowledge. This need is also related to the training of each teacher participating in the interdisciplinary IMO.

The teacher in the technical area (with an interdisciplinary background) had to take on the role of teacher educator at times, which influenced his performance throughout the course. According to Lucena (2018, p. 352), in the IMO, the structure of analysis "considers the analysis of different types of instrumental genesis of the participants, in different roles/functions, in the execution of different situations". In the interdisciplinary IMO, this diversification also occurs, considering the dual role of one of the in-service teachers, who also acted as a teacher educator. Therefore, when analysing its didactic performance, each function must be considered when observing its instrumental genesis.

Finally, we conclude that the interdisciplinary characteristic of the IMO demands several adjustments and care throughout the process, both in structuring and analysis, to meet the model extension particularities.

5 Final Remarks

Knowledge develops in trajectories, which extend to contexts beyond those initially created, so they fork into new paths. From a concept, gaps are glimpsed, allowing new solutions and concepts to be traced and created. In this article, we discuss the trajectory of the instrumental meta-orchestration model, developed from the notion of instrumental orchestration, which, in turn, has already been supported by notions, approaches, and theories,

such as schemes, instrumental approaches, situations, instrumental genesis, etc.

In studies of instrumental meta-orchestration, in addition to the IMO concepts, such as meta-situation, didactic meta-configuration and meta-mode of execution, it is perceived the need to discuss the students' reactions that alter the didactic configuration or generate new modes of execution, called ad hoc reactions, in addition to those of the teacher.

An approach to teaching or teacher training does not consist of a single instrumental orchestration; they are always organised in chains or compositions of instrumental orchestras. Looking at the events between instrumental orchestrations is one of the essential results and leads to the need to create notions such as didactic meta-configuration and exploitation meta-model. The need to study events between the IOs and the need for alterations led to including characteristics of flexibility and adaptation to the instrumental meta-orchestration.

The internal analysis of a training approach based on the instrumental meta-orchestration model demanded the construction of visual instruments that would allow an overview and approaches for each of the selected themes, a fact that culminated in a timeline of foreseen and unforeseen events, instruments used in other studies later, given their analysis potential.

The analysis of the didactic performance and composition of instrumental orchestrations showed the need to understand the evolution and transformation suffered by an artefact. Student protocols that result from one IO become artefacts of others. This dynamic led to the development of another motion visualisation diagram, given its importance for the effectiveness of a composition of instrumental orchestrations and the necessary predictions in a priori analyses. In other words, if, in an IO, the use of the protocol of students in the previous IO is foreseen, it is necessary to foresee what to do for students who miss the previous IO or those who are not involved and do not do it.

Thus, the instrumental meta-orchestration was soon extended to the interdisciplinary training of teachers who work in technical education integrated into the scenario, an environment in which interdisciplinarity has its locus and importance for the training of technicians. Creating artefacts and typifying the orchestrations involved gain even more strength. In this process, there is a need to discuss the involvement of students and teacher educators. There is progress regarding the actors' perspective in instrumental meta-orchestration and the role-changing dynamics between actors. The didactic webdoc also assumes a unique role, mainly with the participation of the prospective teachers in its authorship.

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