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RIPEM

The formalization of the concept of Functions Defined by More than One Sentence in Mathematics textbooks

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Abstract: The general objective of this work is to analyze how the Mathematics textbooks, made available by the National Textbook Program (PNLD) 2021 — Object 2, approach the formalization of the concept of Functions Defined by More than One Sentence. Thus, an exploratory research with a qualitative approach was carried out. The theoretical notes are based on theorists and researchers who deal with the researched themes. The study revealed that textbooks present these functions in a contextualized way, exploring problem situations that enable students to recognize regularities in real phenomena and generalize observed laws or patterns. The examples used include topics such as income tax, production cost and body temperature variation, demonstrating the practical application of these functions. It was also identified that the formalization of the concept of these functions is done intuitively, without explicit mathematical rigor.

Keywords: Textbooks. Problem Situations. Functions.

La formalización del concepto de Funciones Definidas por más de Una Oración en los libros de texto de Matemáticas

Resumen: No se deben utilizar citas bibliográficas. El objetivo general de este trabajo es analizar cómo los libros de texto de Matemática, puestos a disposición por el Programa Nacional de Libros de Texto (PNLD) 2021 — Objeto 2, abordan la formalización del concepto de Funciones Definidas por Más de una Oración. Por lo tanto, se llevó a cabo una investigación exploratoria con un enfoque cualitativo. Las notas teóricas se basan en teóricos e investigadores que tratan los temas investigados. El estudio reveló que los libros de texto presentan estas funciones de manera contextualizada, explorando situaciones problemáticas que permiten a los estudiantes reconocer regularidades en fenómenos reales y generalizar leyes o patrones observados. Los ejemplos utilizados incluyen temas como el impuesto sobre la renta, el costo de producción y la variación de la temperatura corporal, lo que demuestra la aplicación práctica de estas funciones. También se identificó que la formalización del concepto de estas funciones se realiza de forma intuitiva, sin rigor matemático explícito.

Palabras clave: Libros de Texto. Situaciones Problemas. Funciones.

A formalização do conceito de Funções Definidas por Mais de uma Sentença nos livros didáticos de Matemática

Resumo: O objetivo geral deste trabalho é analisar a forma como os livros didáticos de Matemática, disponibilizados pelo Programa Nacional do Livro Didático (PNLD) 2021 —

Objeto 2, abordam a formalização do conceito de Funções Definidas por Mais de uma Sentença. Dessa forma, foi realizada uma pesquisa exploratória com abordagem qualitativa. Os apontamentos teóricos estão fundamentados em teóricos e pesquisadores que se debruçam sobre as temáticas pesquisadas. O estudo revelou que os livros didáticos apresentam essas funções de forma contextualizada, explorando situações-problema que possibilitam aos estudantes reconhecer regularidades em fenômenos reais e generalizar as leis ou padrões observados. Os exemplos utilizados incluem temas como imposto de renda, custo de produção e variação da temperatura corporal, demonstrando a aplicação prática dessas funções. Foi identificado também que a formalização do conceito dessas funções é feita de maneira intuitiva, sem um rigor matemático explícito.

Palavras-chave: Livros Didáticos. Situações-Problema. Funções.

1 Introduction[1]

The formalization of the concept of Functions Defined by More Than One Sentence in mathematics textbooks is the central theme of this paper. It is worth noting that, even in the digital age, physical textbooks play a fundamental role in education, especially in public schools with limited access to technological resources. These instruments provide guidance, practical examples and activities that help both teachers in the teaching process and students in understanding and applying mathematical concepts.

In the specific context of mathematics, textbooks are even more important. They are probably usable resources for the teaching and learning process, contributing to the construction of teaching strategies and facilitating the development of students' mathematical skills. One piece of mathematical content explored in basic education is Functions Defined by More Than One Sentence, which are used to model various real-life situations. In view of this, it is essential to investigate how textbooks approach the formalization of this concept and whether they present a clear and comprehensive understanding.

Based on these ideas, the general objective of this paper is to analyze the way in which mathematics textbooks, made available by the National Textbook Program (PNLD) 2021 — Object 2, approach the formalization of the concept of Functions Defined by More Than One Sentence. To this end, aspects such as the comprehensiveness and clarity of the explanations, the presence of practical examples and the suitability of the proposed activities will be investigated, with the aim of contributing to improving the teaching and learning of this important subject in accordance with the guidelines of the National Common Curriculum Base (BNCC), specifically in relation to skill EM13MAT404 (Brasil, 2018).

By analyzing textbooks, this research aims to contribute to the dissemination of the formalization of Functions Defined by More than One Sentence in Basic Education, in order to establish a solid and understandable basis for students. Thus, the relevance of this paper consists in the possibility of identifying possible improvements in teaching materials, in order to improve the process of teaching and learning mathematics. In addition, it seeks to stimulate reflection on the importance of this concept for teachers and students, highlighting how a proper understanding can strengthen the learning of the subject and develop essential mathematical skills.

[1] This paper is adapted from the master's thesis defended in the Graduate Program in Professional Master's in Mathematics in the National Network (Profmat) at the State University of Southwest Bahia, written by the first author and supervised by the second author.

2 Theoretical Notes

The textbook is a material that plays a fundamental role in disseminating socially accepted knowledge in various areas of knowledge, making it an indispensable resource for teachers and students in the public education system (Carvalho, 2018). Lajolo (1996) points out that, although it is not the only resource available, it stands out as a particular tool in the context of the classroom, being extremely important for collaborating with the dissemination of scientific knowledge. In a specific way:

> A [physical] textbook is an artifact printed on paper that conveys images and texts in a linear and sequential form. It is planned, organized and produced specifically for use in didactic situations, predominantly involving students and teachers and has the function of transmitting knowledge circumscribed to a school subject (Freitas, 2010, p. 268).

It should be noted that "the textbook essentially assumes three major functions: to provide information, to structure and organize learning and, finally, to guide the student in the process of understanding the outside world" (Santos & Carneiro, 2013, p. 206). According to Oliveira (2021, p. 224), "this educational tool must be able to promote reflection on the multiple aspects of reality and stimulate the investigative capacity" of the public that uses this resource. In this way, as well as contributing to the organization of the activity, it is seen as a source of expanding knowledge.

It is important to note that, even with the advance of technology and the expansion of pedagogical resources available in the teaching and learning process, the textbook is still one of the most accessible resources used in Brazilian schools. However, Oliveira (2021) points out that, unlike the current scenario, access to this resource was the privilege of the most elite classes before the Technical Book and Textbook Commission (Colted). Education was restricted to a portion of the population and was a benefit granted to only a select group.

The paradigm shifts related to textbooks were only consolidated in the 1990s, a period marked by a series of transformations, educational reforms and the emergence of new educational parameters, such as the PCNs (Brazil, 1997, 1998, 2000). These milestones were responsible for driving significant changes in the design, content and approach of textbooks, making them more inclusive, diverse and aligned with society's educational needs.

Textbooks are currently made available by the PNLD, a public policy run by the National Education Development Fund (FNDE) and the Ministry of Education (MEC). This program supports the teaching and learning process in beneficiary schools. Initially, until 2004, textbooks were only distributed to public elementary schools. However, in 2004, the National Textbook Program for Secondary Education (PNLEM) was created, which represented an important milestone in this scenario.

The textbooks are chosen by the teachers who teach the subjects corresponding to these printed materials, thus promoting democratization in the choice of these materials. To help these professionals select teaching materials, the PNLD produces and makes available the National Textbook Guide. This guide presents reviews and information on each collection, providing an evaluative analysis of the theoretical-methodological context and structure of each work made available. At this point, it is up to the teacher to carry out a critical analysis of the guide and explore possible textbooks, and then make their choice collectively, together with other colleagues in the area, and indicate two collections to the manager of the public school where they work so that the request can be made.

In the field of Mathematics Education, the Mathematics Textbook (LDM), as part of the PNLD, plays a significant role in the education of both students and teachers (Carvalho & Lima, 2010). These books are widely used in the school environment, forming part of planning and teaching practice, and contribute to the teaching and learning process of the subject. Dante (1996) points out that mathematics is essentially sequential, in which one subject depends on another, and the textbook provides a useful support in this approach. Thus, it is understood that these materials can help math teachers develop didactic-pedagogical strategies for teaching.

As argued by Santos and Silva (2018), the LDM must be versatile in nature, capable of adapting to the different practices and contexts present in the classroom. These authors address the criticisms often leveled at math textbooks, pointing out that they can sometimes be difficult to interpret. However, these authors emphasize the importance of reconfiguring and understanding these materials in order to adapt them to the various pedagogical approaches employed by teachers. For Oliveira (2023), this implies transforming the textbook into a more efficient tool, aligned with the particularities and diversities of educational practices.

In the research conducted by Carvalho (2018), it was found that over twenty years of evaluations (1997-2017), there has been a gradual improvement in the quality of LDMs. However, he points out that these materials are not perfect and that the quality of teaching and learning is not guaranteed by good textbooks alone. This highlights the importance of the teacher's role in the skillful use of these resources.

It is important to note that the current LDM collections have been organized to cover the specific competences and skills of the area of Mathematics and its Technologies, in addition to meeting the general competences proposed by the BNCC (Brazil, 2018). It is therefore essential to understand that the BNCC establishes a basic set of skills that all students should develop throughout high school.

Specifically, this learning is geared towards the development of competencies. According to the BNCC (Brazil, 2018):

> [...] competence is defined as the mobilization of knowledge (concepts and procedures), skills (practical, cognitive and socio-emotional), attitudes and values to solve the complex demands of everyday life, the full exercise of citizenship and the world of work (p. 7).

The BNCC plays a crucial role as a guideline for all basic education in Brazil. However, it is important to consider the particularities of each school, as well as social and regional aspects when designing local curricula. The aim of this document is to promote an integral education that contributes to building a just, democratic and inclusive society (Brasil, 2018). In order to achieve this goal, it established ten general competences for the whole of basic education.

In addition, the BNCC, as a national reference, also plays a fundamental role in articulating and coordinating educational policies and actions developed at the federal, state, municipal and Federal District levels, especially with regard to defining teaching resources. As a result of this new educational context, the publishers responsible for producing the textbooks that compete in the PNLD selection processes have had to align their works according to this document, emphasizing both the general and specific competences of each area of knowledge.

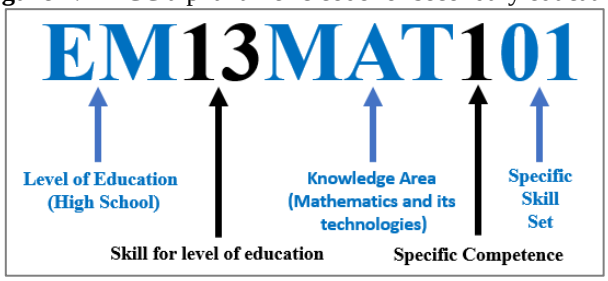
The BNCC has also established specific competences that give concrete form to the general competences in each area of knowledge, which are incorporated into the textbooks for the New High School. These specific competences are combined with the specific competences

of the elementary school stage, with the necessary adjustments to take account of the particularities of the student's education in a progressive manner. In order to guarantee the development of these competences, each one is related to a set of knowledge that represents the essential learning to be ensured to all students.

In order to highlight the interconnection between this knowledge, specific skills were created. Each skill is identified by an alphanumeric code that indicates, respectively, the stage of education, the period of the year of education, the area of knowledge, the specific competence and the set of specific skills.

Figure 1 illustrates this alphanumeric code. For further detail, we would point out that the first set of letters represents the stage of education, with "EM" for High School, while the first pair of numbers indicates the skills described, which can be developed in any year of High School, according to the definition of the curricula. Specifically, these numbers refer to the period from the first to the third year of this stage of education. Then there is a sequence of triple letters representing the area of knowledge or curricular component, "MAT" being for Mathematics and its Technologies. The next-to-last number refers to the specific competence, and the last pair of numbers indicates the set of specific skills related to the specific competence.

Figure 1: BNCC alphanumeric code for secondary education



Source: Researcher (2023)

This structure of specific competences and skills in textbooks aims to promote an integrated approach to knowledge, allowing students to understand how the different contents are interconnected and how they can be applied in real situations. In this way, more meaningful and contextualized learning is sought, in line with the educational objectives established by the BNCC (Brazil, 2018).

In this line of thinking, it is possible to see that the specific Mathematics competencies for the New High School form an interconnected whole. Within this logic, it is important to emphasize that the skills come from five thematic units in the area in question, which are: Numbers, Algebra, Geometry, Quantities and Measures, and Probability and Statistics. However, considering the need to structure these units in such a way as to maintain the connection between the various fields of school mathematics and establish an integrated view of the subject when applied to reality, the BNCC introduced the possibility of creating three groupings: Numbers and Algebra, Geometry and Measures, and Probability and Statistics (Brazil, 2018). These groupings allow for a more integrated approach to content, providing students with a more complete and contextualized understanding of mathematics.

In general, this articulation is based on integration between the topics covered in previous years and also within the same year of schooling, mobilizing the knowledge already built up by the students. This curricular organization is assumed in the proposal for the New High School and, therefore, should be present in the current textbooks of the PNLD 2021 - Object 2. However, it is important to recognize that the BNCC (Brasil, 2018) faces criticism on several fronts. Some of these criticisms highlight the challenge of balancing the large amount

of content and skills proposed within the limited timeframe of basic education, including training itineraries. In addition, there are concerns about how this basis adapts to the various regional and school realities, as well as the possibility of overloading teachers and students with an excessive load of content. Although this is not the main focus of this paper, these aspects show that the implementation of the BNCC and its integration into textbooks are not without challenges and require critical reflection.

Among the skills provided for in the aforementioned basis and associated with the object of study in question, the EM13MAT404 skill stands out. This skill guides students to analyze Functions Defined by One or More Sentences, such as income tax tables, electricity, water and gas bills, among others. The analysis should also involve the algebraic and graphical representations of these functions, identifying domains of validity, image, growth and decrease. In addition, students should be able to convert these representations from one to the other, with or without the use of digital technologies, according to the BNCC document (Brazil, 2018).

The skills and competences established in the BNCC document, related to the study of Functions Defined by More Than One Sentence, direct both the exploration of concepts and the practical application of these concepts in various contexts, such as physical, biological and social phenomena. This approach allows students to understand the importance and usefulness of functions in the real world, enabling them to analyze and interpret concrete situations through algebraic and graphical representations.

It should be noted that a function is defined by more than one sentence when each of them is associated with a $D_1, D_2, D_3, \dots, D_n$ subdomain and the union of these n subsets form the D domain of the original application, i.e. each D_i domain, with $i \in \{1, 2, 3, \dots n\}$, is a subset of D. In functions of this type, the transformation law depends on the value of x. Along these lines, the $f: D \to \mathbb{R}$ function can be algebraically represented as follows:

$$f(x) = \begin{cases} g_1(x), if\ x \in D_1 \\ g_2(x), if\ x \in D_2 \\ g_3(x), if\ x \in D_3 \\ \vdots \\ g_n(x), if\ x \in D_n \end{cases}$$

As noted by Stewart (2014), these distinct formulas $(g_1, g_2, g_3, \dots, g_n)$ in different parts of the function's domain are what characterize Piecewise Defined Functions. This approach makes it possible to more accurately capture the variation of the phenomenon under study and adapt the function to the different conditions or behaviors present in each specific interval.

The study of functions, as highlighted by Abrantes, Serrazina and Oliveira (1999), provides relevant opportunities to establish connections between different areas of mathematics. Tables of values, graphs and analytical expressions, naturally related to the study of these mathematical objects, are associated with numerical patterns, geometric representations and algebraic methods (Abrantes, Serrazina & Oliveira, 1999).

These authors stress the importance of formulating and communicating generalizations, as well as recognizing and representing relationships between variables, as essential processes of mathematical thinking and their application in interpreting situations and solving problems in various subjects and in everyday life. Understanding formulas, constructing tables of values based on a given relationship and interpreting graphs are fundamental elements in this process (Abrantes, Serrazina & Oliveira, 1999).

Thus, the concept of generalization stems from the recognition that there are phenomena

that occur regularly and can be generalized. The study of functions allows students to observe patterns, establish relationships and make generalizations, developing critical thinking and problem-solving skills.

From this perspective, Tinoco et al. (1996) state that the ability to generalize is important for making mathematical abstractions. Based on this understanding, it is common for students, when observing regularities in certain situations, to extrapolate these observations and conclude that a specific law applies to all similar phenomena. However, it is essential that they develop the ability to present arguments coherently, using clear and concise language, in order to demonstrate the validity of the law formulated on the basis of their observations.

3 Methodological Procedures

The study carried out can be characterized as exploratory research with a qualitative approach, using documentary procedures. In this context, qualitative research employs various forms of knowledge, research strategies and data collection and analysis methods. Qualitative methods are based on text and image data, have specific data analysis stages and use a variety of research strategies (Creswell, 2010, p. 184).

Following this perspective, the qualitative approach is essentially explanatory, which means that researchers interpret data in the light of their worldview. This involves developing descriptions of people or environments, investigating data (primary or secondary) to identify themes or categories, and finally developing a personal and theoretical interpretation of their meaning, drawing conclusions, citing lessons learned and formulating new questions.

In this regard, an exploratory study was carried out to investigate conceptual aspects related to the topic. Thus, we sought to identify which volumes of the didactic collections of Mathematics, made available by the PNLD 2021 — Object 2, deal with Functions Defined by More Than One Sentence. The research was restricted to the volumes that present this content in their respective summaries. Thus, the exploratory research with a qualitative approach was conducted in two stages: a) to identify the volumes of the mathematics textbooks made available by the PNLD 2021 — Object 2 that deal with Functions Defined by More than One Sentence; b) to investigate the approach used by the textbooks to conceptualize Functions Defined by More than One Sentence.

Content analysis was used to analyze the data. This technique consists of a set of systematic procedures and objective descriptions of the content of the message, allowing inferences and conditioning of production/reception variables, with the aim of obtaining quantitative or qualitative indicators (Bardin, 2016).

In this research, our data source was limited to mathematics textbooks for the first year of the New High School, available in the ten collections approved under the PNLD 2021 — Object 2, in accordance with the Call Notice of the General Coordination of Book Programs (Cgpli) 03/2019, which refer to didactic textbooks by areas of knowledge and specific didactic textbooks. It should be noted that in the New High School, teachers have the freedom to choose which textbooks they want to use in each year or semester. Previously, the content of functions was traditionally taught in the 1st year of high school, but this sequence may have changed due to new curricular approaches. However, in the Teacher's Manuals of all the books analyzed, there are clear indications as to which year or semester each volume can be adopted.

Thus, in the pre-analysis of the ten collections, we identified that only six of them met the selection criteria. It is worth noting that each collection has six volumes, and we only analyzed those intended for or directed to the 1st year of High School. So, our research analysis

consists of six LDMs. Table 1 below shows the codifications and identifications of each book selected for this research.

Table 1: Identification of the Mathematics textbooks analyzed

Code	Identification
LDM1	Collection *Conexões — Matemática e suas Tecnologias* (Publisher: Moderna), with the subtitle *Quantities, Algebra and Algorithms*. Responsible Editor: Fabio M. de Leonardo.
LDM3	Collection *Interação Matemática* (Publisher: do Brasil), with the subtitle *The treatment of information and problem solving through the function of the 1st degree*. Authors: Adilson Longen and Rodrigo M. Blanco.
LDM4	Collection *Matemática em Contextos* (Publisher: Ática), with the subtitle *Affine function and quadratic function*. Authors: Luiz R. Dante and Fernando Viana.
LDM5	Collection *Matemática Interligada* (Publisher: Scipione), with the subtitle *Affine, quadratic, exponential and logarithmic functions*. Responsible editor: Thais M. de Andrade.
LDM8	Collection *Prisma — Matemática* (Publisher: FTD), with the subtitle *Functions and Progressions*. Authors: José R. Bonjorno, José R. G. Júnior and Paulo R. C. de Sousa.
LDM10	Collection *Ser Protagonista — Matemática e suas Tecnologias* (Publisher: SM), with the subtitle *Numbers and Algebra*. Authors: Katia S. Smole and Maria I. Diniz.

Source: Researcher (2023)

4 Analysis and Results

In the textbooks analyzed, with regard to the part that is the object of study in this research, we highlight that they incorporate the idea of formalizing Functions Defined by More than One Sentence in a contextualized way. In this way, these materials converge with the ideas proposed by Santos and Carneiro (2013), since they not only perform the functions of informing, structuring and organizing learning, but also act as guides for students in the process of understanding the outside world. In this way, they provide relevant information, promote reflection on multiple aspects of both theory and reality, stimulate investigative capacity and contribute to broadening students' knowledge.

In order to show how these textbooks are approaching our research topic, we will carry out a thorough exploration of the selected textbooks. This detailed analysis has allowed us to gain a deeper understanding of how each of them approaches the concept of Functions Defined by More Than One Sentence. In addition, it was possible to describe a comprehensive view of the pedagogical strategies adopted and how these textbooks can improve their performance as auxiliary tools in the process of grasping knowledge.

In LDM1, the content related to Functions Defined by More than One Sentence is approached as a topic, in an intuitive and contextualized way. The concept is introduced by means of a table on the monthly incidence of Personal Income Tax (IRPF), which serves as a mathematical model to illustrate the situation. This approach highlights the exploration of social practices as a starting point for contextualizing mathematical content, using tables that are later converted into algebraic expressions. Table 2 in the book illustrates this situation.

Table 2: Formalization of the concept according to LDM1

Information	To find out what an individual's income tax (y) is, we apply to the monthly income (x) the calculations defined by the table established by the government. See the table for 2019.

<table>
<tr><td>Table</td><td>
<table>
<tr><th colspan="3">Monthly Income Tax for Individuals</th></tr>
<tr><th>Calculation Basis (Reais)</th><th>Tax Rate (%)</th><th>Portion to be Deducted from Income Tax (Reais)</th></tr>
<tr><td>Up to 1.903,98</td><td>–</td><td>–</td></tr>
<tr><td>From 1903,99 to 2.826,65</td><td>7,5</td><td>142,80</td></tr>
<tr><td>From 2.826,66 to 3.751,05</td><td>15,0</td><td>354,80</td></tr>
<tr><td>From 3.751,06 to 4.664,68</td><td>22,5</td><td>636,13</td></tr>
<tr><td>Over 4.664,68</td><td>27,5</td><td>869,36</td></tr>
</table>
</td></tr>
<tr><td>Abstractions and the Resolution Process</td><td>For a monthly income whose calculation base x is equal to $R\$\ 1.500{,}00$, the taxpayer is exempt, i.e. the tax is zero real.
For a monthly income whose calculation basis x is equal to, for example, $R\$\ 3.000{,}00$, the tax y payable is: $y = 3.000{,}00 \cdot 0{,}15 - 354{,}80 = 450{,}00 - 354{,}80 = 95{,}20$. So, the monthly tax payable is $R\$\ 95{,}20$.</td></tr>
<tr><td>Formalizing</td><td>We can write this situation mathematically by a function, with domain in a subset of the non-negative real numbers, given by more than one sentence. See.</td></tr>
<tr><td>Algebraic Representation</td><td>$$f(x) = \begin{cases} 0, if\ 0 < x \leq 1.903{,}98 \\ 0{,}075 \cdot x - 142{,}80, if\ 1.903{,}98 < x \leq 2.826{,}65 \\ 0{,}15 \cdot x - 354{,}80, if\ 2.8226{,}65 < x \leq 3.751{,}05 \\ 0{,}225 \cdot x - 636{,}13, if\ 3.751{,}05 < x \leq 4.664{,}68 \\ 0{,}275 \cdot x - 869{,}36, if\ x > 4.664{,}68 \end{cases}$$</td></tr>
</table>

Source: Organized by the researchers (2023) from LDM1 (2020, pp. 82-83)

Table 2 shows an illustrative example of the approach to formalizing Functions Defined by More Than One Sentence. In this respect, the textbook explores the IRPF table, providing relevant information about the income brackets and the respective tax rates. This analysis allows students to understand how the tax base affects the monthly contribution, establishing a clear relationship between the independent variable (tax base) and the dependent variable (monthly contribution). Thus, the algebraic representation of the Function Defined by More than One Sentence is shown, demonstrating that different sentences are used for different ranges of values.

However, it is important to mention that LDM1's approach could be even closer to the idea of generalization presented by Abrantes, Serrazina and Oliveira (1999). Thus, it would be enriching if the example provided a more detailed explanation of how the different laws of formation of the functions are related to the different sections of the domain, establishing a more explicit connection between the data presented in the tables and the formulation of the algebraic expressions.

We suggest that LDM1 needs to improve the sequence of presentation, placing greater emphasis on the relationship between social practices and algebraic expressions. This could be done through a more detailed analysis of the patterns observed in the data and the identification of common elements in the different parts of the function. In this way, students would have a clearer understanding of the transition from contextualization to mathematical formalization. We believe that this possibility could be an indication for the teacher who uses this or a similar book.

Although LDM1 addresses the Function Defined by More Than One Sentence, it is important to note that the observation made in Table 2 does not specifically explore all the aspects mentioned in skill EM13MAT404. Thus, it failed to mention the analysis of the function's domains of validity, image, growth and decrease, aspects that can be addressed at this

point.

In LDM3, the subject of this research is covered in a subtopic entitled "Functions defined by more than one sentence", which is part of Topic 4 - Affine Function, belonging to Unit III - Affine Function. The introduction to this topic highlights the existence of contexts in which a function models a situation using more than one law of formation, considering different parts of the domain. In this sense, the textbook uses the reading of a text and the Income Tax (IR) table as a resource to introduce the research topic. From there, the functions that define each tax band are presented and then an intuitive association is made with Functions Defined by More than One Sentence, accompanied by their algebraic representations. Table 3 in the book illustrates this situation.

Table 3: Formalization of the concept according to LDM3

<table>
<tr><td>Information</td><td>How to calculate income tax?
There are many bases for calculating income tax, depending on how the person is registered in the Receita Federal system. The most common way of calculating income tax is for salaried workers, called Withholding Income Tax (IRRF). This calculation is made by the employer, who collects part of the employee's salary and sends it directly to the IRS. To determine how much of the salary will be withheld at source, the calculation is based on the following table:</td></tr>
<tr><td>Table</td><td>
<table>
<tr><th>Calculation Basis (Reais)</th><th>Rate</th><th>Portion to be deducted from Income Tax (Reais)</th></tr>
<tr><td>Up to 1.903,98</td><td>free</td><td>free</td></tr>
<tr><td>From 1903,99 to 2.826,65</td><td>7,50%</td><td>142,8</td></tr>
<tr><td>From 2.826,66 to 3.751,05</td><td>15%</td><td>354,8</td></tr>
<tr><td>From 3.751,06 to 4.664,68</td><td>22,50%</td><td>636,1</td></tr>
<tr><td>Over 4.664,68</td><td>27,50%</td><td>869,4</td></tr>
</table>
</td></tr>
<tr><td>Additional Information</td><td>• The monthly calculation base is given by the amount of the registered salary, minus other taxes (e.g. INSS).
• The rate is the percentage of the salary that will form the Income Tax.
• The portion to be deducted is an amount to be subtracted from the rate.</td></tr>
<tr><td>Abstraction and the Resolution Process</td><td>Using the table is simple
1. Look at how much is left over from your salary after taxes such as INSS have been deducted: for example, R$2,000.00.
2. See what band this amount is in: R$ 2,000.00 is on the second line.
3. Calculate a percentage of this amount according to the rate indicated in the line of the table: looking at the second line, the percentage is 7.5%; calculating 7.5% of R$ 2,000.00, we get R$ 150.00.
4. Subtract from the result the amount of the portion to be deducted by looking once again at the line in the table: looking at the second line, the deduction is R$ 142.80; making 150 – 142.8 = 7.2, we conclude that someone who receives R$ 2,000.00 as their basic salary will have R$ 7.20 withheld at source when they receive their salary.</td></tr>
<tr><td>Formalizing</td><td>Here's how we can represent the Income Tax function using the symbology used for functions defined by more than one sentence. Note that the laws of formation of this function belong to different classifications. While the expressions that determine the value of the function for x > R$ 1,903.98 are functions of the 1st degree, the expression that determines the values of the function for the first part of the domain is a constant function.</td></tr>
</table>

Algebraic Representation	$f(x) = \begin{cases} 0, to\ 0 < x < 1903{,}99 \\ \frac{7{,}5}{100}x - 142{,}8, to\ 1903{,}99 \leq x < 2826{,}66 \\ \frac{15}{100}x - 354{,}8, to\ 2826{,}66 \leq x < 3751{,}05 \\ \frac{22}{100}x - 631{,}1, to\ 3751{,}05 \leq x < 4664{,}68 \\ \frac{27{,}5}{100}x - 869{,}4, to\ 4664{,}68 \leq x \end{cases}$

Source: Organized by the researchers (2023) from LDM3 (2020, pp. 33-34)

Table 3 shows the approach to the Function Defined by More Than One Sentence, using the example of calculating income tax. In this context, the book explores the different tax brackets and their respective rates, relating them to the tax base. The algebraic representation of this function is also presented, showing how the laws of formation vary depending on the value of the tax base.

LDM3's approach is close to the ideas of Tinoco et al. (1996), by using the example of calculating income tax to illustrate Functions Defined by More Than One Sentence. By highlighting the different calculation bases and presenting the income tax table, this textbook establishes a connection between social practices and the mathematical formalization of these functions. However, in order to improve the formalization, it would be important to provide a more detailed explanation of the relationship between the different laws of formation and the sections of the domain.

In addition, it would be enriching if LDM3 provided a more detailed explanation of how the different formation laws are related to the different sections of the domain, establishing a more explicit connection between the data presented in the Income Tax table and the formulation of the algebraic expressions.

Overall, LDM3 partially meets skill EM13MAT404, by addressing the Function Defined by More Than One Sentence. Thus, the analysis carried out on the board allows students to analyze the graphical and algebraic representation of the function, relating them to the domain of validity and the laws of formation. However, the approach does not specifically explore conversion between graphical and algebraic representations.

In LDM4, the content related to Functions Defined by More Than One Sentence is presented in Chapter 1, entitled *Affine Function*. The book addresses this subject as a specific topic, allowing the concepts to be applied to real situations and facilitating the understanding of natural and social phenomena. In this sense, in the subtopic *Exploring functions defined by more than one sentence*, the book introduces the idea that there are functions composed of more than one sentence. It uses mathematical language in the *Explore to discover* section to approach this content. Thus, by locating the points on the Cartesian plane, students are encouraged to reflect on questions related to the applications f, g and h. Table 4 about the textbook illustrates this situation.

Table 4: Formalization of the concept according to LDM4

Information	In Explore to find out above, we have worked on the function f, which, as you concluded in items a to f, is not an affine function, but can be expressed using the laws of two functions g and h. The law of this function f can be expressed as follows:
Algebraic Representation	$f(x) = \begin{cases} g(x), if\ x \geq 2 \\ h(x), if\ x \leq 2 \end{cases}$

Formalizing	Functions like this are called **functions defined by more than one sentence**. In functions of this type, the law of the function depends on the value of x, as in the case of f, where $f(x)$ is given by one law if $x \geq 2$ and is given by another law if $x \leq 2$.

Source: Organized by the researchers (2023) from LDM4 (2020, p. 58)

In this Table 4, the concept of Functions Defined by More Than One Sentence is formalized. In this way, LDM4 works with the function f, showing that it is not an Affine Function, but can be expressed using the laws of two functions, g and h. The algebraic representation of this application is presented, in which $f(x)$ is equal to $g(x)$ if x is greater than or equal to 2, and equal to $h(x)$ if x is less than or equal to 2. This approach demonstrates that applications like these are called Functions Defined by More Than One Sentence. In this type of transformation, the law of the function depends on the value of x, varying according to whether the value of x is greater or less than 2.

However, it is interesting to note that, although the book initially contextualizes the teaching of mathematics in previous approaches to arouse students' curiosity in discovering the characteristics of functions that model real facts, the authors of LDM4 choose to use mathematical language and algebraic procedures to formalize the concept of Functions Defined by More Than One Sentence. This reveals a concern for mathematical rigor in the presentation of these concepts.

In this way, the formalization of the concept of Functions Defined by More Than One Sentence is close to the idea of mathematical rigor presented by Stewart (2014). The authors of LDM4 are concerned with using mathematical language and algebraic procedures to formalize functions, seeking to establish a precise and mathematically correct approach. This emphasis on mathematical rigor reflects the importance of presenting concepts in a precise and consistent way.

However, it is important to note that, in addition to mathematical rigor, it would be beneficial to provide students with a broader understanding of the meaning and applications of Functions Defined by More Than One Sentence. Exploring practical examples related to students' everyday lives could help them visualize and understand these functions, making learning more relevant (Oliveira, 2023).

In the context of skill EM13MAT404, LDM4 does not directly address this, as it does not present an explicit approach to Functions Defined by More Than One Sentence. Although Table 4 shows a function that is not affine and can be expressed through the laws of two functions, the formalization of this concept is done intuitively and does not explore aspects such as domain, image, growth and decrease. In order to fully meet this skill, it would be necessary to address these aspects in greater depth.

The LDM5: before introducing the concept of an affine function, in topic 4 of Chapter 2 — Notions of Functions, the textbook explores the formalization of Functions Defined by More than One Sentence. In this context, a problem situation is presented to illustrate that this situation cannot be represented by a single algebraic expression. Table 5 highlights this situation.

Table 5: Formalization of the concept according to LDM5

Problem-Situation	Take a look at the following situation. To produce up to 20 units of a certain package, an industry has an initial cost of R$240.00 plus a cost of R$5.00 per package. To produce more than 20 units, the raw material supplier offers a discount to the industry. As a result, the cost per package is R$4.50 plus the initial cost of R$240.00. What is the function that represents the total cost C of this industry as a function of the quantity x of packages produced?
Abstraction	According to the text, we can see that the cost of this industry depends on the amount of packaging to be produced. We can represent the cost of producing up to 20 units using the function C, whose law of formation is given by:
Algebraic Representation	cost to produce up to 20 packages; initial cost to produce up to 20 packages $C(x) = 5x + 240, if\ 0 < x \leq 20$
Abstraction	However, if the number of packages is greater than 20, we can represent the cost using the C function, given by: cost to produce
Algebraic Representation	cost to produce up to 20 packages; initial cost to produce up to 20 packages $C(x) = 4,50x + 240, if\ 0 < x \leq 20$
Formalizing	Note that this situation cannot be represented by a single sentence. That's why we've written two sentences that depend on the number of packages to be produced. This situation describes a function defined by more than one sentence.
Algebraic Representation	$C(x) = \begin{cases} 5x + 240, if\ 0 < x \leq 20, with\ x \in \mathbb{N}^* \quad (I) \\ 4{,}50x + 240, if\ x > 20, with\ x \in \mathbb{N}^* \quad (II) \end{cases}$

Source: Organized by the researchers (2023) from LDM5 (2020, p. 43)

The table 5 of LDM5 presents a problem situation which involves determining the total cost of an industry according to the quantity of packaging produced. In this connection, the book points out that, for up to 20 units, the cost is described by one function, while for more than 20 units, the cost is represented by another function. This approach illustrates the presence of Functions Defined by More Than One Sentence, in which the law of the function varies according to the value of x.

The approach adopted in Chart 5 is similar to the ideas presented by Tinoco $et\ al.$ (1996), who propose a problem situation that allows students to recognize regularities in a real situation and generalize the laws or patterns observed. The emphasis given to abstracting the data from the problem situation and explaining the Function Defined by More Than One Sentence promotes reflection on aspects of reality[2] and broadens students' knowledge by connecting mathematical content with practical everyday situations.

However, for a more in-depth understanding of the concept of Functions Defined by More Than One Sentence, it would be interesting if the textbook provided more examples and problem situations related to students' everyday lives. This would help make the connection between the mathematical content and its practical application, making it more meaningful and enabling a better understanding of the subject.

[2] The term "reality" is used to refer to the real contexts in which mathematical concepts are applied. A specific example, such as packaging costs, establishes a practical connection with the learning process. In this sense, textbooks emphasize concrete situations to illustrate the application of these concepts in practice. In this context, teachers play a fundamental role in adapting these examples to the students' reality, making learning more relevant.

In the context of skill EM13MAT404, the textbook helps by presenting a problem situation involving the total cost of an industry as a function of the quantity of packaging produced, which is a Function Defined by More than One Sentence. The table discusses the algebraic representation of this application, the domain of validity and highlights the characteristic of Functions Defined by More than One Sentence. However, the analysis does not explicitly explore the conversion between graphical and algebraic representations, a relevant aspect for the full development of the skill.

In Chapter 1, LDM8 addresses the topic of Functions Defined by More Than One Sentence, offering an opportunity for students to explore and understand this important class of functions. At the start of this content, LDM8 takes up the idea presented in the chapter's introductory text, which focuses on a mathematical model related to income tax. Using tables and algebraic expressions, the book explores how taxes are calculated and collected, providing a concrete example of the application of this mathematical object. Table 6 highlights this situation.

Table 6: Formalization of the concept according to LDM8

<table>
<tr><td>Information</td><td>We have seen that Personal Income Tax (IRPF) is a tax levied on income acquired from sources in Brazil by taxpayers resident in the country or abroad. This tax is levied according to a progressive table, indicating the rate corresponding to each calculation base, which depends on the income of each taxpayer.
See below for the monthly IRPF table in force in 2020.</td></tr>
<tr><td>Table</td><td>> Monthly incidence table in 2020
<table>
<tr><th>Calculation Basis (Reais)</th><th>Rate (%)</th><th>Portion to be deducted from income tax (Reais)</th></tr>
<tr><td>Up to 1.903,98</td><td>–</td><td>–</td></tr>
<tr><td>From 1903,99 to 2.826,65</td><td>7,5</td><td>142,80</td></tr>
<tr><td>From 2.826,66 to 3.751,05</td><td>15</td><td>354,80</td></tr>
<tr><td>From 3.751,06 to 4.664,68</td><td>22,5</td><td>636,13</td></tr>
<tr><td>Over 4.664,68</td><td>27,5</td><td>869,36</td></tr>
</table></td></tr>
<tr><td>Abstractions and the Resolution Process</td><td>Based on this table, we can calculate, for example, the tax levied on the income of a worker whose monthly tax base is R$3,350.00. In this case, we must apply the 15% rate to the calculation base and deduct R$354.80 from this amount.
Please note:
$R\$\,3.350{,}00 \cdot 15\% - R\$\,354{,}80 = R\$\,502{,}50 - R\$\,354{,}80 = R\$\,147{,}70$
Therefore, the income tax levied on a calculation base of R$3,350.00 per month is R$147.70.
The monthly income tax contribution, in reais, is a function of the tax base, also expressed in reais, because each value of the tax base corresponds to a single monthly income tax contribution. The tax base is the independent variable and the monthly income tax contribution is the dependent variable.
Read the mathematical definition of a function below.</td></tr>
<tr><td>Concept</td><td>Given two non-empty sets, A and B, a function of A on B is a relation that associates each element x of A with a single element y of B.</td></tr>
<tr><td>Algebraic Representation</td><td>To indicate a function from A into B, we can write $f: A \rightarrow B$ (read: f from A into B).

The function f transforms x of A into y of B, which can be written as $y = f(x)$ (read: y equals f of x).</td></tr>
<tr><td>Abstraction</td><td>In the situation we are studying, the values corresponding to the tax base can be considered elements of set A and the monthly income tax contribution values as elements of set B. Based on the monthly IRPF table in force in 2020, considering</td></tr>
</table>

	x the values corresponding to the calculation base and $f(x)$ the monthly income tax contribution, we can write a formation law to represent this function. Note:
Algebraic Representation	$f(x) = \begin{cases} 0, if\ \ x \leq 1903{,}98 \\ 0{,}075x - 142{,}80, if\ 1903{,}99 \leq x \leq 2826{,}65 \\ 0{,}15x - 354{,}80, if\ 2826{,}65 \leq x \leq 3751{,}05 \\ 0{,}225x - 636{,}13, if\ 3751{,}06 \leq x \leq 4664{,}68 \\ 0{,}275x - 869{,}36, if\ x > 4664{,}68 \end{cases}$
Formalizing	Functions such as the one that models the monthly income tax contribution according to the tax base are called functions defined by more than one sentence.

Source: Organized by the researchers (2023) from LDM8 (2020, pp. 13-14)

The Table 6 of LDM8 presents an approach to the Function Defined by More than One Sentence, using the example of the monthly income tax contribution based on the monthly IRPF table. In this context, it establishes a connection between mathematical concepts and the practical application of income tax, allowing students to understand how to calculate the monthly contribution based on their income bracket.

This LDM8 approach is consistent with the ideas presented by Stewart (2014), as it recognizes and reinforces the use of multiple representations of a function. In this sense, there is a comprehensive pedagogical approach to teaching and learning this content. This perspective promotes a holistic view of Functions Defined by More Than One Sentence, stimulating critical thinking and the application of mathematical knowledge in different situations.

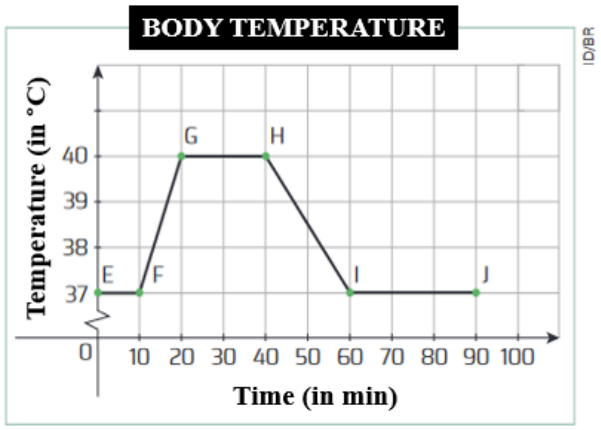
However, in order to improve understanding of the concept of Functions Defined by More Than One Sentence, it would be beneficial if LDM8 provided more practical examples and explored different problem situations related to students' everyday lives. These additional examples would help students to visualize the different ways of representing these functions and to better understand their usefulness and application in different real-life contexts.

In the context of skill EM13MAT404, LDM8 partially fulfills this by addressing the Function Defined by More than One Sentence in Table 6. The algebraic representation of the function is presented, allowing students to observe the different sentences that make up this mathematical object. However, the analysis does not delve into aspects such as the domain, image, growth and decrease of the application, which are mentioned in the skill. To fully meet the skill, it would be necessary to explore these aspects in greater depth and provide more comprehensive examples of Functions Defined by More than One Sentence in different contexts.

LDM10 presents the research topic in Chapter 5 - Other Functions, specifically in the topic Functions Defined by Parts. A problem situation involving the graphical sketch of body temperature and its algebraic representation is explored. This approach aims to demonstrate that this type of function is defined by more than one sentence. This explanation is shown in Table 7.

Table 7: Formalization of the concept according to LDM10

Problem-Situation	The human body has a normal temperature of between 36 °C and 37.5 °C. A person fell ill and their temperature rose to 40 °C, which means a very high fever. They took some medication and after 20 minutes their fever dropped back down to 37 °C. The following graph shows the variation in this person's body temperature over time.

Graphic Representation	
Abstraction	This graph can be described by different affine functions and constant functions. Calling f the function that encompasses them all and x the time, we have:
Algebraic Representation	$f(x) = \begin{cases} 37, if\ \ 0 \leq x \leq 10 \\ \frac{3}{10}x + 34, if\ 10 \leq x \leq 20 \\ 40, if\ 20 \leq x \leq 40 \\ -\frac{3}{20}x + 46, if\ 40 \leq x \leq 60 \\ 37, if\ x \geq 60 \end{cases}$
Formalizing	This is an example of a piecewise function.

Source: Organized by the researchers (2023) from LDM10 (2020, p. 143)

In Table 7 of LDM10, a problem situation is presented involving the variation of a person's body temperature over time. The abstraction made from the graph and the algebraic representation of the Function Defined by Parts allow students to recognize regularities in a real phenomenon, as highlighted by Tinoco et al. (1996). This approach demonstrates the generalization of different temperature situations in relation to time.

However, it is important to note that the formalization of the concept of Functions Defined by Parts in Table 7 of LDM10 is done intuitively, without explicit mathematical rigor. It would be enriching if the book provided a greater mathematical foundation for the definition and formalization of these functions, through formal definitions and specific properties. This would help students to better understand this type of function and its characteristics.

In relation to skill EM13MAT404, the book partially meets this requirement by addressing the Function Defined by Parts in Table 7. The problem situation presented involves the variation of body temperature over time and demonstrates the graphical and algebraic representation of this function. However, the analysis does not explore aspects such as domain, image, growth and decrease, which are mentioned in the skill.

5 Considerations

In light of the analysis of the PNLD 2021 Mathematics textbooks — Object 2, it was possible to observe different approaches in the treatment of the concept of Functions Defined by More Than One Sentence. Each book presents its own particularities in the way it explores this topic, seeking to meet the BNCC guidelines and promote a relevant understanding on the part of the students. Some books provide a more in-depth exploration, with practical examples, contextualized activities and discussions about the properties of functions. These books are concerned with relating the concept to everyday situations, making learning more meaningful.

However, we also identified a need for improvement in some areas. Some books could

be more mathematically grounded and rigorous in their formalization of Functions Defined by More Than One Sentence, highlighting the properties, domain of validity and behaviour of functions on different intervals. In addition, it would be beneficial to include more practical and diverse examples, covering different contexts and applications of these mathematical objects.

It is important to highlight the importance of textbooks as educational tools that provide information, structure and organize learning, as well as acting as guides for students in the process of learning. According to Oliveira (2023), these works have the potential to promote reflection on multiple aspects of reality, stimulating investigative capacity and contributing to the expansion of students' knowledge.

Looking to the future, it is hoped that textbooks will continue to evolve, increasingly seeking a balance between contextualization, mathematical reasoning and the applicability of Functions Defined by More Than One Sentence. In addition, it is desirable that these textbooks offer a variety of complementary resources, such as practical activities, challenging problems and the use of digital technologies, which can enrich the teaching and learning process in this specific area of mathematics.

In this way, this research contributes to a reflection on how textbooks approach the concept of Functions Defined by More Than One Sentence, highlighting their potential and suggesting ways to improve their pedagogical approach. A careful analysis of these books can help educators select materials that promote meaningful learning, stimulating students' interest in and understanding of this important mathematical subject.

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