Sociedade Brasileira de Educação Matemática

RIPEM

Prospective mathematics teachers' reflections on their strategies for solving a simple combination problem

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2238-0345

10.37001/ripem.v14i1.3618

Received • 12/09/2023
Approved • 06/12/2023
Published • 01/01/2024

Editor • Gilberto Januario

Abstract: This article aims to analyze and highlight prospective mathematics teachers' reflections on the strategies used to solve a simple combination problem. We carried out a qualitative study in the context of theoretical classes in an undergraduate course on the theory of problem solving. To this end, 11 students from Paraná solved a simple combination problem. The results showed that the participants believed the diagram and table strategies were the same. After the discussions, they understood that the strategies have their own representations but did not mention those linked to execution. We conclude that the reflections of the mathematics undergraduates provided an understanding of the representations of the strategies and that they began to value them when working in the classroom.

Keywords: Problem Solving. Combinatorics. Strategy. Representation.

Reflexiones de estudiantes de pregrado en Matemáticas sobre sus estrategias para resolver un problema de combinación simple

Resumen: El artículo tiene como objetivo analizar y evidenciar las reflexiones de los futuros profesores de matemáticas sobre sus estrategias para la resolución de un problema de combinación simple. Desarrollamos una investigación cualitativa en el contexto de las aulas teóricas de una asignatura de la carrera de teoría de la resolución de problemas, en la que 11 estudiantes de Paraná resolvieron un problema de combinación simple. Los resultados mostraron que los participantes creían que las estrategias del diagrama y de la tabla eran iguales. Después de las discusiones, entendieron que las estrategias mismas tienen sus propias representaciones, pero no mencionaron claramente las representaciones vinculadas a la ejecución. Concluimos que las reflexiones de los estudiantes de pregrado en Matemáticas lograron brindar una comprensión de las representaciones de sus estrategias y que comenzaron a valorar dichas representaciones en su trabajo en el aula.

Palabras clave: Solución de Problemas. Combinatoria. Estrategia. Representación.

Reflexões de licenciandos em Matemática sobre suas estratégias de resolução de um problema de combinação simples

Resumo: Este artigo tem o objetivo de analisar e evidenciar as reflexões de futuros professores de matemática sobre as estratégias na resolução de um problema de combinação simples. Desenvolvemos uma pesquisa qualitativa no contexto de aulas teóricas de uma disciplina do curso de licenciatura acerca da teoria da resolução de problemas. Para isso, 11 estudantes paranaenses resolveram um problema de combinação simples. Os resultados mostraram que os participantes acreditavam que as estratégias de diagrama e tabela eram iguais. Após as discussões, compreenderam que as estratégias em si apresentam as próprias representações, porém não mencionaram de forma clara as ligadas à execução. Concluímos que as reflexões dos licenciandos em Matemática proporcionaram a compreensão das representações das estratégias e que passaram a valorizá-las ao trabalho em sala de aula.

Palavras-chave: Resolução de Problemas. Combinatória. Estratégia. Representação.

1 Introduction

Problem solving is a way of teaching to help students develop competencies, skills, mathematical knowledge, and the relationship between mathematics and the world (Lester & Cai, 2016; Brasil, 2018; Proença, Campelo & Santos, 2022). Understanding problem solving for teaching implies developing knowledge inherent to the training of teachers who teach mathematics, such as mathematical, pedagogical, and curricular knowledge (Shulman, 1986; Ball, Thames & Phelps, 2008; Carrillo-Yañez *et al*., 2018).

A significant aspect of problem solving involves the process, which, from a cognitive point of view, guides someone to follow stages of thought. These can be understood as the non-linear sequence: representation (understanding the problem), planning (coming up with a strategy), execution (carrying out that strategy), and monitoring (reviewing the response and the resolution followed) (Proença, 2018).

Regarding the stage that involves presenting a problem-solving strategy, the literature shows that most studies focus on investigating the types of problem-solving strategies used by students and prospective teachers, and those that teachers actually use in the classroom environment (Pantziara, Gagatsis & Elia, 2009; Lockwood, 2015; Gomes & Viseu, 2017; Aydin-Güç & Daltaban; 2021; Fidelis *et al*, 2021; Martins & Martinho; 2021; Wu & Molnár, 2022). Only the study by Lockwood and Gibson (2016) emphasized understanding the specific strategy of using systematic and partial lists. The research by Oliveira and Proença (2022) focused on understanding the characteristics, potentialities, and limitations of using tables. Both studies involved prospective teachers. However, carrying out and expanding studies to differentiate problem-solving strategies is essential.

The context of this article took place in classes in a mathematics undergraduate course that dealt with the stages of problem solving, specifically identifying strategies for solving a combinatorial problem. During these classes, the prospective teachers pointed out that some strategies they considered were the same, which was a great motivator for this research. This work aimed to analyze and highlight prospective mathematics teachers' reflections on strategies for solving a simple combination problem. The structure of the article was designed to present problem solving and the strategies of problem resolution and their importance in teaching. The methodology, results, and discussion are then presented, ending with the conclusions.

2 Problem Solving

Authors such as Schoenfeld (1985), Mayer (1992), Polya (1994), Echeverría (1998), Sternberg (2010), and Proença (2018) discuss two theoretical aspects of problem solving. The first deals with what a problem is, i.e., the meaning of the term. The second deals with the problem-solving process to elucidate the cognitive path taken in solving a problem, which involves stages of resolution. Regarding what a problem is, Proença (2018) points out that

> a mathematical situation becomes a problem when the person needs to mobilize previously learned mathematical concepts, principles, and procedures to arrive at an answer. It is not, therefore, a direct use of a known formula or rule — when this occurs, the situation tends to be configured as an exercise (p. 17-18, our translation).

There is a difference between a problem and an exercise. While the former requires the creation of a resolution path, the latter presents an immediate means of finding the answer (Echeverría, 1998). In this sense, the problem-solving process, i.e., the path to be followed, goes through stages. Proença (2018) describes four stages in the problem-solving process: representation, planning, execution, and monitoring.

Representation is related to the interpretation and understanding of the problem, involving: a) linguistic knowledge associated with the terms and expressions of the mother tongue that are part of the statement; b) semantic knowledge that corresponds to the mathematical terms that are part of the problem; and c) schematic knowledge that consists of recognizing the nature of the problem, i.e., whether it is linked to algebra, geometry, arithmetic, among others.

In the second stage, *planning*, a resolution path is established, and it requires strategic knowledge, which consists of organizing a path (strategy) to reach the answer. In the *execution* stage, the established strategy is put into practice. This involves procedural knowledge, including performing mathematical calculations and creating schemes, diagrams, and drawings. Finally, *monitoring* involves reviewing the solution, correcting possible errors, validating the answer, and analyzing whether it matches the context of the problem.

3 Problem-solving strategies and their importance in teaching

Specifically concerning the planning stage, the use of resolution strategies is a relevant point. Among the possibilities of problem-solving strategies, the literature highlights several, such as trial and error; making a drawing, a diagram, a graph, a scheme, an organized list, a table; discovering a pattern; working from the end to the beginning; using logical deduction; reducing to a more straightforward problem; making a simulation (Krulik & Rudnick, 1982; Chi & Glaser, 1992; Polya, 1994; Vale & Pimentel, 2004; Posamentier & Krulik, 2009; Proença, 2018).

According to Chi and Glaser (1992), creating a strategy means searching for the solution space. For the authors, every problem has an initial state and a desired state, and the strategy is the path taken between these two states. In this way, Vale and Pimentel (2004) define strategies as thinking devices to solve a problem.

In this sense, Pozo and Angón (1998) argue that the use of strategies is based on the psychological processes of the individual, such as metacognition, which implies a reflection on the problem and the way the person solves it and what they call basic processes, which are the thinking schemes developed by the person. Corroborating this view of the individual, Sternberg (2010, p. 387, our translation) explained that "a good strategy depends both on the problem and on personal preferences regarding problem-solving methods".

In the case of following the diagram strategy, it is possible to think of a count or the well-known Probability Tree Diagrams, which corresponds to the person's preference and is linked to their way of thinking about the problem. Diagram strategies, generally presented in textbooks, correspond to types of symbolic representations, as shown by Lima and Borba (2022).

Dreyfus (1991) explained that symbolic representations are the first step in an advanced mathematical thinking process, along with the cognitive aspects of the person. These representations are required in the problem-solving process. As Proença (2022) explained about the planning stage, the person must mobilize the mathematical ability to think in mathematical symbols.

Given this, the importance of problem-solving strategies in the classroom lies in the (prospective) teacher developing the knowledge that a strategy is a way of solving a problem, with several possible strategies. This is important and essential for teachers who want to approach problem solving in their teaching (Krulik & Rudnick, 1982; Proença, 2018). Therefore, they should realize that strategies have different forms of symbolic representation: a diagram, a table, a drawing, and a mathematical formula are different from each other. In addition, they should know that the choice of a particular strategy depends on the students' preferences, which involves their choices about knowledge. Above all, addressing and discussing students' strategies in the classroom can lead to the development of mathematical problem-solving skills, the usage and handling of mathematical symbols, and the construction of mathematical thinking.

4 Methodology

This article is classified as qualitative research, which, according to Gerhardt and Silveira (2009, p. 31, our translation), "is not concerned with numerical representativeness, but rather with deepening the understanding of a social group, an organization, etc." In this sense, to constitute the social group studied to achieve the research objective, 11 students participated. They were all from the undergraduate mathematics program of a public state university in Paraná.

The students were studying Supervised Curricular Internship III (Estágio Curricular Supervisionado III, in the original), which is part of the seventh semester of the course. During the classes, they were involved in studying the theory of problem solving. They undertook the research by agreeing to the Free and Informed Consent Form. The initial activity consisted of solving the following mathematical situation (possible problem):

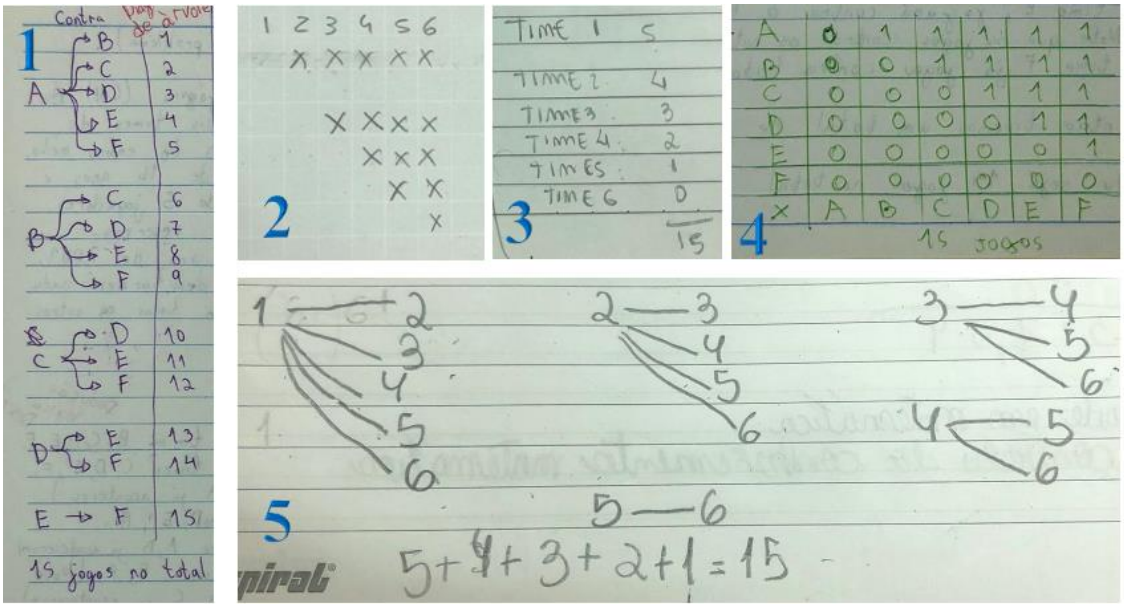
> *At the Colégio de Aplicação Pedagógica in the city of Maringá, Paraná, there are six indoor soccer teams of first-year high school students. These students have an average age of 16, and each team must consist of five players plus a maximum of five reserve players. There will be a tournament at the end of 2021, so in the planning of this tournament, each team will only play all the others once. How many matches should there be?*

They were asked to solve the problem in as many ways as possible. They subsequently presented them on the board. We also demonstrated two strategies that the students had yet to anticipate: drawing a hexagon and the formula for the diagonal of a polygon (for the hexagon). We then explained what a strategy is and classified the strategies they used, delimiting them into a tree diagram, logical deduction, a table, and the mathematical formula (Simple Combination).

During this moment, when the prospective teachers presented their strategies, they curiously said that their colleagues' strategies were the same. This drew attention, as a table is not a diagram, for example. Clarification was sought as to the reason for these answers, and we discussed and reflected on the ideas. In this context, in order to understand the students' view that the strategies were the same and to highlight the reflections that came out of the classes, an online questionnaire was drawn up with the strategies they said were the same and three questions about them, as shown in Table 1.

Table 1 Online questionnaire

Consider the following strategies designed to solve the situation of the indoor soccer teams. Answer below:



Each classmate who displayed their strategy on the blackboard was asked who had done it differently. In general, your class felt that the strategies were the same. Explain why this was your first impression.
After the strategies were presented to everyone, the following provocation was made: in fact, these strategies are not the same. Explain your understanding/reflection based on your new view of these strategies.
What is your understanding of the importance of teachers knowing problem-solving strategies like these, given what students can do to solve a math situation? Explain.

Source: Survey data (2023)

To delimit the data, the training context that generated the students' reflections was considered, and three general axes were established: a) Initial understanding of the strategies, b) Reflections on the strategies, and c) Reflections on the importance of teachers knowing resolution strategies. Thus, the analysis of the prospective teachers' responses was carried out using Content Analysis, in accordance with the assumptions of Bardin (2011), based on the following stages: 1) Pre-analysis, in which the data was organized to form the *corpus* of the research; 2) Exploration of the material, with in-depth reading to establish the categories, obtained *a posteriori* as a result of the recording units; and 3) Treatment of the results, with discussion and interpretation of the data, based on research into problem solving.

5 Results and Discussion

Table 2 shows the results of the prospective students' explanations of their *initial understanding of the strategies*, which led them to say they were the same.

Table 2: Initial understanding of strategies being the same

Categories	Registration Units

Strategies use the same argument	PT4 — At first, it was natural to think that the strategies were the same since everyone came up with the same answer. In addition, **the argument for solving the question is similar in most cases**, which gives the impression that the strategies are the same.
	PT7 — They are essentially the same, with a few representative differences, but **they have the same principle**: describe each game and write down the total. For example, 1 and 5 make the same diagram of the tree, and 3 writes down the result of each diagram. On the other hand, 2 and 4 are practically identical, except for the notational representations; they distribute the tree diagram in table form.
	PT8 — I think they looked the same **because they were both related since it was the same problem. Also, because they were both arranged in tables and diagrams, they may seem like the same strategy at first**.
	PT9 — The first impression I got was that **strategies 1, 2, 3, and 5 are the easiest to understand, they are similar to the strategy I adopted**. Strategy 4 is very good, but it didn't cross my mind to set up a board.
	PT10 — At first, the strategies seem to be the same **because we already know that the content is about combination,** and since all the paths use concepts of **combination,** we believe they are the same.
	PT11 — (...) we had to **do some kind of counting or addition to find the solution to this problem situation,** also the analysis was done logically, game by game, for each indoor soccer team. This is very evident when we analyze Strategy 1, Strategy 2, Strategy 3, and Strategy 5, we can see that the strategies are similar (...).
Strategies only present different organizations	PT1 — At first, the strategies seemed the same because **they all arrived at the same solution to the mathematical situation, and also, there were similarities in the way they were written**, so before understanding why they were different, the first impression is that they were all the same.
	PT2 — At first, it may seem that the strategies are the same because we look at them and see that they **all clearly arrive at the same result, that is, without using a very different method.** We get the impression that they are the same; for example, with strategies 1 and 5, which are extremely similar, strategy 4 can look like the previous two but only represent them in a table. So, without prior knowledge, we can say they are all the same.
	PT3 — We commented that the strategies were similar because, **looking visually at examples 1, 3, and 5, we noticed a similarity in the resolution**, and so we are led to understand that the strategies can be considered the same or similar.
	PT5 — You can see that the strategies look the same, especially 1, 3 and 5 and 2 and 4, so much so that at the start of the discussion, we believed **they were the same strategies but written differently, changing the names of the teams (from letters to numbers) and arranging them on the paper in similar ways**, but not the same.
	PT6 — The impression that strategies 1, 3, and 5 are the same arises because

	they are diagrams that have just been organized in different ways. The same happens for strategies 2 and 4, where tables have been drawn up to organize the possible games, which are **again just organized in different ways**.

Source: Survey data (2023)

The two categories — *Strategies that use the same argument* (PT4, PT7, PT8, PT9, PT10, and PT11) and *Strategies only present different organizations* (PT1, PT2, PT3, PT5, and PT6) — show that when the prospective teachers displayed them on the board, they apparently did not know they were different strategies. This is clear from the words of students PT6 and PT8, who replied: "(...) *again, just organized in different ways*" (PT6); "*Also, because they are both arranged in tables and diagrams, they may seem like the same strategy at first*" (PT8). The comment is contrary to what researchers such as Krulik and Rudnick (1982), Posamentier and Krulik (2009), and Proença (2018) have argued about the importance of clearly knowing different strategies to deal with them when addressing problem solving in the classroom.

This result reveals an aspect that has not been the focus of studies on problem-solving strategies in general and combinatorics in particular. Only the works by Lockwood and Gibson (2016) and Oliveira and Proença (2022) dealt with differentiation but with specific strategies. Thus, according to Gomes and Viseu (2017), when investigating the strategies of 12 prospective teachers, it is first necessary to involve the prospective mathematics teachers in training that deals with different strategies, as was done in this training context. Therefore, in addition to the participants having the opportunity to engage with different strategies in solving the simple combination problem, Table 3 shows the results of the *reflections on the strategies* after the collective discussion with the trainees.

Table 3: Reflections of the prospective teachers after discussion of strategies not being the same

Categories	Registration Units
The strategies differ according to their particularities	PT2 — After this provocation, saying that the strategies are not the same, we began to see the resolutions in a different way, knowing that every detail can make the strategy change completely because **you use different tricks that make them change and also change the way you interpret and analyze it** (...).
	PT10 — Each strategy **has its own individuality, differentiating it from the others. Although they have similarities (1 and 5 make a diagram, 2 and 4 make a table), they have different lines of thought, which creates different resolutions**.
	PT11 — (...) these strategies are related to **the way in which each student thought particularly** (...). Let's think, in this case, there were 6 indoor soccer teams, but if the problem situation asked for the number of games for 20 indoor soccer teams, the Strategies could be different. In this case, for example, I particularly wouldn't use Strategy 1, and Strategy 5, because it wouldn't result in an abbreviation of the mathematical thinking process (...).
Strategies differ according to their type	PT3 — When we look at the strategies again, we realize that they are not the same because **each one uses a different process,** for example, visual 5, tabular 4. And these processes use different mathematical concepts.
	PT4 — I realized that even though the thinking for solving the problem was similar in each case, **the way of doing it was different,** and therefore the strategies were different. So, we can't conclude that the strategies are the same

	just because the result was the same.
	PT5 — Concerning strategies 1, 3, and 5, notice that **in strategy 1 all the games were described, and then the number of each one was written down, one by one, while in strategy 3, for example, the number of games for each team was described and then added up**, so they are different strategies. About strategies 2 and 4, you will notice that strategy 4 is more explicit about the teams in each game, while in strategy 2, you just put the teams and an x, but note that in the end, strategy **2 counts the number of x's,** and **strategy 4 adds up the number of games from 1 to 1.**
	PT6 — After the explanation, you realize that the strategies are **different, especially when it comes to carrying out the planning** (...).
	PT7 — They are different because the way each student thought of solving is different. **Two counted the number of x's, while 4 added all the numbers in the table.** Both **1 and 5 made the tree diagram**, **but the first counted the number of games one by one, the other added up the number of games for each team, and 3 summarized 1 and 5 without the need for the tree diagram**.
	PT8 — In reality, the strategies used were different. **The first was done using an arrow diagram, the second with a table of checkmarks, the third with a vertical table of values, the fourth with a table of game possibilities, and the fifth with a diagram similar to the first**. Although they all look similar, you can't say they're the same.
Understanding the existence of types of strategies	PT1 — Understanding that strategies are not the same, however much they may seem so, before having this knowledge is to open up **a new perspective on different types of strategies that are** often unknown, broadening our knowledge base.
	PT9 — (...) I hadn't thought of other strategies, so **it was interesting to see the other methods**.

Source: Survey data (2023)

The three categories — *The strategies differ according to their particularities* (PT2, PT10, and PT11), *Strategies differ according to their type* (PT3, PT4, PT5, PT6, PT7, and PT8), and *Understanding the existence of types of strategies* (PT1 and PT9) — reveal that the prospective teachers understand the differentiation between their strategies. This leads to the inference that they may have come to understand the types of strategies presented, which authors such as Krulik and Rudnick (1982), Posamentier and Krulik (2009), Vale and Pimentel (2004) and Proença (2018) have pointed out in the literature.

This understanding can be illustrated by student PT10, who replied: "*Each strategy has its own individuality, differentiating it from the others. Although they have similarities (1 and 5 make a diagram, 2 and 4 make a table), they have different lines of thought, which creates different resolutions*". It also reveals a reflection aimed at pointing out the thinking involved. In this sense, it is possible that understanding the types of strategies also broadens the participants' understanding that there is a preference and a psychological process driven by the person's thinking schemes and symbolic representations, according to Dreyfus (1991), Pozo and Ángon (1998) and Sternberg (2010).

These results show that the prospective teachers were able to broaden their understanding of the types of strategies, revealing the differences that occurred to the prospective mathematics teachers in the studies by Lockwood and Gibson (2016) and Oliveira and Proença (2022) for specific strategies. This reveals progress in research that has already been done, such as work focused only on showing the strategies and performance of prospective teachers, such as Lockwood (2015) and Wu and Molnár (2022), involving combinatorial problems; and Gomes and Viseu (2017), on a geometric problem.

Therefore, if prospective teachers do not develop a consistent understanding of the types of strategies and their characteristics and differences, it is possible that, in the classroom, teachers will just bring different strategies to their students to use directly and not reflect on the differences and similarities, as happened in the study by Aydın-Güç and Daltaban (2021). Given this reflection and understanding of strategies that the participants experienced in their classes, Table 4 shows the results of the *reflections on the importance of the teacher knowing resolution strategies* to use in the classroom. The first two categories relate to the teacher, and the last two focus on the students.

Table 4: Reflections on the importance of addressing strategies in the classroom

Categories	Registration Units
Encouraging the teacher's knowledge of problem-solving strategies	PT3 — I think it's important for the student and for **the teacher to be prepared for the possible answers that may arise to direct the student in the strategy** when necessary.
	PT8 — The problem-solving approach **helps to develop [the teacher's] logical and strategic thinking,** as it helps them to understand each stage of solving a mathematical situation.
	PT9 — The teacher must be prepared for the questions the students will ask while solving the problem. **Knowing all the strategies behind a problem helps the teacher be prepared.**
	PT10 — We need to know some strategies (at least the most common ones) so that **it's possible to imagine what the student will develop and how to work with this as part of the teaching-learning process**, planning to build new concepts using previous knowledge.
Introducing students to problem-solving strategies	PT11 — (…) like every teacher, we want the student to build their knowledge, so even if the student doesn't yet know these strategies (1, 2, …, 5, …, n), where they only know how to use the simple combination formula, it**'s up to us as teachers to have this knowledge of various strategies for this problem statements to be presented to the students so that they already start to have this contact** (…).
Valuing students' thoughts	PT4 — (…) if the teacher recognizes the different strategies, they **value the students for their thinking**. What's more, there's no risk of the teacher claiming that a student's solution is incorrect simply because they don't know that strategy.
	PT5 — By knowing the possible strategies that the students can take, **the teacher can deal with the situation in the best way by knowing if the students' thoughts are correct or not**.
	PT7 — The teacher needs to be prepared for the most diverse strategies because **the teacher needs to understand the logical thinking used by the student** to know the student's level of understanding of the content.

Valuing students' strategies	PT1 — If the teacher knows different ways of solving a given math situation, **he or she can "accept" the students' solution in different ways,** and this shows that he or she is well-prepared in a way, not assuming that only his or her answer is right!
	PT2 — (…) in the classroom, students can solve in different ways and have different thoughts to arrive at the same result, and **the teacher has to have the discernment not to point out that the student solved it the wrong way just because the student solved it in a different way from the one he/she had prepared**, he must explain and point out that a problem has several solving strategies to arrive at the same result.
	PT6 — It's important for the teacher to know the strategies because students don't always see the possible paths that the teacher had thought of, so **they can mediate in a more assertive way without belittling the work done by their students**.

Source: Survey data (2023)

The two categories relating to the teacher — *Encouraging the teacher's knowledge of problem-solving strategies* (PT3, PT8, PT9, and PT10) and *Introducing students to problem-solving strategies* (PT11) — show that these prospective teachers emphasized to a greater degree the need for the teacher to know about problem-solving strategies. This is important for the training of prospective teachers and is in line with what is advocated by authors such as Krulik and Rudnik (1982), Vale and Pimentel (2004), and Proença (2018) to know and understand problem-solving strategies for teaching mathematics.

PT9 highlights this issue by saying, "*Knowing all the strategies behind a problem helps the teacher be prepared*." The studies by Lockwood and Gibson (2016) and Oliveira and Proença (2022) provided this preparation for prospective teachers, and they were able to reflect on the strategies addressed, moving to the level of understanding the strategies and their differences, limits, and possibilities. In addition, the two categories relating to teachers reveal the need for training in mathematics undergraduate courses and continuing teacher training. This leads to knowing the strategies for solving combinatorial problems, as shown in the study by Gomes and Viseu (2017) on prospective teachers' difficulties in solving problems and diversifying strategies. It also leads to understanding problem-solving strategies for other contents (Proença, 2012; Mendes & Proença, 2020).

The two categories relating to students — *Valuing students' thoughts* (PT4, PT5, and PT7) and *Valuing students' strategies* (PT1, PT2, and PT6) — show that the prospective teachers were more concerned about students' learning. The answers may show attention to the students' thinking in the use of strategies, according to Vale and Pimentel (2004), to the psychological processes of thought schemes for the use of strategies, according to Pozo and Angón (1998), and the students' strategic preferences, according to Sternberg (2010).

This concern for students and their strategies was revealed in the study by Pantziara, Gagatsis, and Elia (2009), involving 194 sixth-grade students. Since not all of them had the cognitive aspects aligned to a certain type of diagram when solving combinatorial problems, they concluded there was a need to value the students' preferences. By recognizing the students' thinking and strategies, the study participants show that this is important for their training as prospective teachers, as it constitutes pedagogical and mathematical knowledge for teaching (Shulman, 1986).

6 Conclusions

This article aimed to analyze and highlight the reflections of prospective mathematics teachers on their strategies when solving a simple combination problem. To do this, 11 prospective teachers solved the proposed problem and, after discussions in the course, motivated by the fact that they said the strategies were the same, they answered an online questionnaire.

The categories of analysis emerged from the participants' answers. They showed that the prospective teachers' understanding, after solving the problem, indicated that, for them, there was no clear difference between the diagram and table strategies used. Considering the discussions and reflections in class, the categories of analysis showed that the prospective teachers understood the difference between the strategies that used the diagram and the table. In this way, the categories of analysis of their reflections on problem-solving strategies in the classroom showed that the prospective teachers valued the importance of knowing the problem-solving strategies and, above all, recognizing the students' strategies and ways of thinking.

However, the prospective teachers' reflections in the classes led them to understand that the strategies used differ. They showed that they have different representations and organizations even with the same principle (combinatorics). These reflections allow us to conclude that these prospective teachers have understood the importance of valuing the students' strategies and thoughts when using them in classroom teaching.

Although this study naturally focused on the diagram and table strategies, there was also a differentiation between the other strategies covered in the lessons. This is because the participants realized that the simple combination formula used, the polygon diagonal formula, and the representation of the data by drawing a hexagon were different from each other and the others. One limitation of this work may have been that it did not attempt to obtain results on the students' reflections on the three strategies so that they could identify similarities and differences with the strategies analyzed. Also, we did not try to explore whether there were more strategies that, in some way, used other mathematical content or forms of representation.

Finally, this study contributes to the training of prospective teachers in the field of problem solving by dealing with these resolution strategies, as it enables an understanding of the differences for use in the classroom. From the point of view of scientific research, this study contributes by advancing studies that have already been carried out. It sought out and showed the reflection of prospective teachers on their strategies, unlike most studies that have focused on revealing the types of problem-solving strategies of the participants. Therefore, future research could investigate the differences, potentialities, and limits of using various strategies for the same problem. Future research could even deal with problems in different contexts to broaden teachers' and prospective teachers' understanding of these strategies.

Acknowledgments

The authors would like to thank the Academic Writing Center (Centro de Escrita Acadêmica, CEA) of the State University of Maringá (UEM) for assistance with the English language and developmental editing.

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