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Global and ontosemiotic meanings of mathematical practices: an analysis of the didactic-mathematical skills and knowledge of students on Internships in High School

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Abstract: This article discusses aspects of the formation of didactic-mathematical skills and knowledge of students on a Mathematics Degree course based on the development and analysis of an activity that involved a question from the National High School Examination focusing on a linear function. The investigation, of which the activity is part, was inserted in a qualitative perspective and took as theoretical support the Ontosemiotic Approach with regard to the training of Mathematics teachers. Results lead to the understanding that it is important to have reflective and problematizing training, enabling future teachers to be teacher-researchers. With regard to the development of competence and didactic-mathematical knowledge, there is a commitment on the part of academics to develop resolutions based on didactic principles necessary for pedagogical mediation suitable for high school students.

Keywords: Didactic-Mathematical Skills and Knowledge. Ontosemiotic Focus in High School. Mathematics Teacher Training.

Significados globales y ontosemióticos de las prácticas matemáticas: un análisis de las habilidades y conocimientos didáctico-matemáticos de estudiantes en prácticas de Secundaria

Resumen: Este artículo analiza aspectos de la formación de habilidades y conocimientos didáctico-matemáticos de los estudiantes de la Licenciatura en Matemáticas a partir del desarrollo y análisis de una actividad que involucró una pregunta del Examen Nacional de Enseñanza Media centrada en una función lineal. La investigación, de la que forma parte la actividad, se insertó en una perspectiva cualitativa y tomó como sustento teórico el Enfoque Ontosemiótico en lo que respecta a la formación de profesores de Matemáticas. Los resultados llevan a comprender que es importante contar con una formación reflexiva y problematizadora, que permita a los futuros docentes ser docentes-investigadores. En lo que respecta al desarrollo de competencias y conocimientos didáctico-matemáticos, existe un compromiso por parte de los académicos de desarrollar resoluciones basadas en principios didácticos necesarios para la mediación pedagógica adecuada a los estudiantes de secundaria.

Palabras clave: Habilidades y Conocimientos Didáctico-Matemáticos. Escuela Secundaria con Enfoque Ontosemiótico. Formación de Profesores de Matemáticas.

Significados Globais e Ontossemióticos das Práticas Matemáticas: uma Análise das Competências e Conhecimentos Didático-Matemáticos de

Acadêmicos em Estágio no Ensino Médio

Resumo: O presente artigo discute aspectos da formação de competências e conhecimentos didático-matemáticos de acadêmicos de um curso de Licenciatura em Matemática a partir do desenvolvimento e análise de uma atividade que envolveu uma questão do Exame Nacional do Ensino Médio com foco em função linear. A investigação, da qual a atividade faz parte, se inseriu em uma perspectiva qualitativa e tomou como respaldo teórico o Enfoque Ontossemiótico no que se refere à formação de professores de Matemática. Resultados direcionam ao entendimento de que é importante que haja uma formação reflexiva e problematizadora, capacitando os futuros docentes a serem professores-pesquisadores. No que se refere ao desenvolvimento de competência e conhecimentos didático-matemáticos constata-se o empenho por parte dos acadêmicos em desenvolver resoluções a partir de princípios didáticos necessários para uma mediação pedagógica adequada a estudantes do Ensino Médio.

Palavras-chave: Competências e Conhecimentos Didático-Matemáticos. Enfoque Ontossemiótico no Ensino Médio. Formação de Professores de Matemática.

1 Introduction

Mathematics teachers' education is based on different perspectives and factors that impact their worldviews and understanding of how educational processes should occur. We understand that capturing how prospective teachers develop their knowledge and competencies is fundamental for better understanding their future practices and identifying the actions that formative courses must develop for their professional qualification.

Considering the context above, we developed qualitative-based research to investigate how academics from a mathematics teaching degree course mobilize didactic-mathematical competencies and knowledge in organizing, constituting, and implementing teaching practices for high school and formative activities based on the onto-semiotic approach. Linked to the doctoral study *The mobilization of didactic-mathematical competencies and knowledge of mathematics undergraduates for teaching in High School*[1], this research involved academics enrolled in the curriculum component Supervised Practicum in High School at a higher education institution in Metropolitan Porto Alegre/RS.

This article aims to discuss aspects of the formation of didactic-mathematical competencies and knowledge of academics in a mathematics teaching degree course about an activity that involved a question focused on linear functions extracted from the National High School Examination (ENEM). The analysis focused on the didactic-mathematical competencies and knowledge of the global and onto-semiotic meanings of practices, as well as different types of knowledge (epistemic, cognitive, interactional, affective, mediational, and ecological) mobilized during task-solving, as presented in Godino *et al*. (2017).

What follows presents the theoretical framework used in the analysis, which is at the basis of the research produced.

2 Theoretical Assumptions

Teaching competencies and knowledge reflect the attitude that teachers or prospective teachers develop professionally, their teaching practices, and their critical vision about how mathematics teaching should be conducted in the classroom. Those actions, as they relate to

[1] Thesis defended in the Postgraduate Program in Science and Mathematics Teaching at the Lutheran University of Brazil.

EOS2 (Godino *et al*., 2017), are developed within the scope of what the authors call didactic-mathematical competencies and knowledge and consider different assumptions about how teachers should perceive mathematics teaching and learning in the classroom. Supported by the onto-semiotic approach, what the authors affirm as necessary for teachers to consider in the classroom is highlighted, which refers to:

- the mathematical references used in learning processes, considering the actors involved (teacher, student, and objects of knowledge), and how they are used (epistemic view) in problem situations, rules, arguments, languages, and intra-mathematical and extra-mathematical relationships;
- the learning and meanings developed by students, considering their previous knowledge, those in development and future ones (cognitive vision);
- the necessary and existing interactions so that meanings are shared between those involved (students and teacher), as well as the necessary adjustments so that pedagogical obstacles are overcome (interactional vision);
- the socio-affective relationships necessary to guide the interest of the involved throughout the process, as well as their engagement and collaboration in educational practices (socio-affective vision);
- the objects, means, and time needed for the learning processes to come to fruition (mediational vision);
- the learning objects prescribed in the curriculum and adjustments, taking as a reference the connections within mathematics itself, with other areas, and with the school context in which it is developed (ecological vision).

These elements are at the basis of the so-called didactic suitability, a theoretical construct presented within the scope of the onto-semiotic approach (Godino *et al*., 2017), which, together with four other theoretical groups, currently make up the OSA: Systems of Practices, Configurations of Objects and Mathematical Processes, Didactic Configurations, and Normative Dimension, with each of these groups presenting distinct competencies for mathematics teachers.

In the OSA, didactic suitability, as a theoretical group, seeks to establish criteria for effective intervention in the classroom, considering the epistemic, cognitive, interactional, emotional, mediational, and ecological components and empirical indicators that allow evaluating or conducting teaching practices. At the same time, as an object of investigation and qualification of teaching practices, it allows a critical interpretation of educational processes, taking into account a detailed look at ways of adapting, structuring, and, potentially, innovating those practices.

Understanding the education of mathematics teachers in Godino *et al*. (2017), these first elements (presented by didactic suitability) constitute didactic-mathematical knowledge, which teachers should master but can take on different meanings when mobilized. This

[2] The onto-semiotic approach to mathematics knowledge and instruction is a theoretical approach that articulates different perceptions and models of education and mathematics education based on anthropological, semiotic, and sociological assumptions about mathematics, mathematics teaching, and use for research. Its structure is based on the ontology of objects, which comes from the triad: socially shared problem-solving activity, symbolic language, and logically organized conceptual system. It is currently organized into five theoretical subgroups: a system of practices, the configuration of objects and mathematical processes, didactic configurations, normative dimension, and didactic suitability.

knowledge is presented in Table 1, considering its components and indicators.

Table 1: Didactic-Mathematical Knowledge

Components	Indicators
Epistemic	It refers to didactic-mathematical knowledge about the content itself and the way the teacher understands and knows mathematics. It includes common content knowledge (knowledge of mathematics shared with students) and expanded content knowledge (knowledge of mathematics at later levels). The relationship between this knowledge is understood as mathematical knowledge *per se.*
Cognitive	It refers to teachers' knowledge of how students learn, reason, and understand mathematics in different learning progress processes.
Affective	This is the teacher's knowledge of the affective and emotional aspects, and attitudes and beliefs regarding mathematical objects in the teaching and learning process.
Interactional	It concerns the teacher's knowledge of mathematics teaching in relation to the organization of mathematical tasks, the resolution of students' difficulties, and interactions that can be established in the classroom.
Mediating	It is the teacher's knowledge of technological, material, and temporal resources that are appropriate for their students' learning.
Ecological	Teachers' knowledge of mathematical content regarding other areas of knowledge and curriculum components, as well as socio-professional, political, and economic aspects that drive mathematics teaching and learning processes.

Source: Adapted from Godino *et al.* (2017)

The didactic-mathematical knowledge that teachers develop in their education and the constitution of their professional profile accumulate and transform as they develop new experiences and meanings throughout their work, considering the social, cultural, and historical aspects existing inside and outside school environments. In this context, teachers are expected to develop competencies to deal with didactic challenges in teaching and learning, taking into account the understanding of what is being put into play and how mathematical and didactic objects intervene in mathematical practices. In this context, Godino *et al.* (2017) add to the model of didactic-mathematical knowledge the notion of didactic-mathematical competencies, having as a background the five groups that structure the OSA, generating the following competencies: analysis of global meanings, onto-semiotic analysis of practices, analysis and management of didactic configurations, normative analysis, and analysis and evaluation through didactic suitability.

Theoretically, only the first two skills mentioned will be highlighted: competence in analyzing global meanings and competence in the on-semiotic analysis of mathematical practices. This article presents the analysis of the mobilization of these two skills by a group of prospective teachers and the analysis of the didactic-mathematical knowledge involved.

Regarding the analysis of global meanings, in the first phase, the meanings are understood pragmatically so that their understandings and representations can be read, written, and appreciated within the system of practices needed to solve mathematical or didactic problems. According to Godino *et al.* (2017), the analysis of these global meanings assumes that teachers have the competence to characterize both the institutional practices (different institutional meanings of the object), considering the different contexts in which they deal with problems and meanings that arise from them, and the personal practices (meanings that students are expected to develop). Thus, according to the authors, it may be an understanding that teachers mobilize the competence of onto-semiotic analysis of mathematical practices when they respond satisfactorily to the following questions:

- What are the meanings of the mathematical objects involved in the study?
- How do they interact with each other in the context of mathematical practice?

Thus, taking Godino *et al*. (2017) and Napar (2022) as a reference, we believe that the analysis of global meanings aims to analyze how teachers

> conceive the different meanings that objects have in other situations, knowing how to relate and articulate them in contexts involving mathematical practices. With this, potential teachers could show situations that must be considered when learning an object, knowing when to connect it with other mathematical themes and presenting to their students when these different meanings are shown in new or different situations (Napar, 2022, p. 273).

Concerning the competence of the onto-semiotic analysis of mathematical practices, we consider that prospective teachers can evaluate their own practices and understandings in teaching and learning, in the negotiation of meanings, in their implications for students' learning, and in how the goals of doing, representing, and sharing mathematics come to fruition. In this scenario, the teacher must "recognize and reflect on the configurations and processes that intervene in mathematical practices, especially those that imply how students are expected to solve mathematical tasks and problems." (Napar, 2022, p. 275-276).

In teaching and learning, the format and ways mathematics teachers can assess their students' progress are complex, especially when seeking to understand the different meanings of mathematical objects for them [the students]. In this sense, teachers must be prepared to analyze how students learn and how they represent and expose their knowledge in relation to mathematical practices (Godino *et al*., 2017). With this, "it is understood that they [the teacher] can evaluate problem resolutions, conflicts, and obstacles, in addition to [the] objects that circulate the teaching and learning process [...]" (Napar, 2022, p. 277), analyzing students' possible difficulties and ways of conducting a process that is more attentive to individualities and the classroom context.

In this context, according to Godino *et al*. (2017), teachers are expected to master this competence when they can answer questions such as:

- What configurations of mathematical objects and processes imply the constitution of practices and meanings involved in the intended learning objects (epistemic configurations)?
- What configurations of objects and processes can students bring into play when solving problems (cognitive configurations)?

We understand that the theoretical aspects mentioned so far lead to a more pragmatic perception (which materializes or can materialize objectively in action) of how one can understand, assess, and analyze mathematics teaching and learning, paying special attention to the components of mathematics teacher education and how the teachers perceive the practices. Godino *et al.* (2017) lead us to understand that analyzing the didactic-mathematical competencies and knowledge of teachers or prospective teachers constitutes a task of a semiotic analysis of objects through the meanings and ways of interacting in mathematical practices on the one hand, and on the other hand, anthropological, social, and cultural interactions and their implications for sharing knowledge, which requires attention to understand and seek to qualify the initial or continuing mathematics teachers' education.

In the context of the research produced — and which gave rise to the analysis presented in this work — we consider that the support brought by the onto-semiotic approach provided, and still provides, the constitution of the instruments that were used to analyze the mobilization of the didactic-mathematical competencies and knowledge, as well as the possibility of establishing a formative process in which prospective teachers can reflect on and analyze their actions and professional development.

Having presented the theoretical basis of this work, the following section shows aspects of the methodology.

3 Methodological Aspects

As highlighted, this article discusses and analyzes aspects of the formation of didactic-mathematical competencies and knowledge of students attending a mathematics teaching degree course regarding an activity that involved a question focused on linear functions from the National High School Examination. In this sense, we study here the competencies of analysis of global meanings and onto-semiotic analysis of mathematical practices, as well as the epistemic, cognitive, interactional, affective/socio-emotional, mediational, and ecological didactic-mathematical knowledge mobilized by mathematics teachers attending a formative course.

Methodologically, the investigation that gave rise to what is presented and analyzed here was inserted in a qualitative and analytical-descriptive perspective, taking as a theoretical construct for analysis and reflections the onto-semiotic approach to knowledge and mathematics instruction (Godino; Batanero; Fonte, 2008; Godino *et al*., 2017). The participants were academics from a mathematics teaching degree course attending the curriculum component Supervised Practicum in Mathematics in High School, offered in the second semester of 2020 during the Covid-19 pandemic. The actions carried out had the consent of the institution, academics, and others involved. They met the criteria of the Research Ethics Committee of the university to which the researchers were bonded and were approved by the Certificate of Presentation of Ethical Appreciation number 15443019.0.0000.5349. For this specific article, we used materials from three research participants represented by the letters A, B, and C.

The activity that allowed data collection was organized into two distinct moments. In the first, we presented a question from the 2011 National High School Exam and asked participants to solve the problem, the emerging meanings and their representations, their relationships, possible understandings, and obstacles that high school students might have when solving it, the curriculum objects that can be related to those that students are learning and with the didactic-mathematical objectives designed to be achieved in mathematical practices. In the second, we presented one student's resolution that was extracted from Filho (2017), where inquiries involved the student's resolution and its solution, the potential objects and meanings attributed by him, the possible mathematical obstacles and the languages that could be identified, possible relationships and conditions for the student to perform similar tasks, and the assessment that could be carried out in light of the activity conducted. Regarding this work, we bring only the analysis results from the first moment.

The entire analysis was done considering two categories based on didactic-mathematical competencies: analysis of global meanings and onto-semiotic analysis of mathematical practices. Each category is analyzed with a focus on the results obtained from questions about the activity proposed to the undergraduates, discussing the mobilization of the two didactic-mathematical competencies, and paying attention to the didactic-mathematical knowledge that emerged in the discussion arising from the, at the time, prospective teachers.

For data analysis, we created a protocol considering the contributions of the onto-semiotic approach, as presented in Table 2, whose first column presents the competence or didactic-mathematical knowledge being referenced, and the second, the empirical indicator that, we understand, will allow identifying the mobilization of the mentioned competencies.

Table 2: Instrument for analyzing didactic-mathematical skills and knowledge

Competence	Indicators
Analysis of Global Meanings	Identifies, understands, mobilizes, and links the meanings of the mathematical objects involved, their relationships with the context of the problem situation, and their implications for mathematics teaching and learning.
Onto-semiotic analysis of mathematical practices	(a) Mobilizes understanding about which configurations of mathematical objects and processes intervene in the development of practices and their meanings in what the student is expected to develop in learning (epistemic configurations). (b) Understands which configurations of objects and processes students may bring into play during problem solving and their learning development (cognitive configurations).
Knowledge	**Indicators**
Epistemic	Presents his didactic-mathematical knowledge, shared (with the students) and expanded (from mathematics in other uses and advanced levels), about the objects discussed in mathematical practices, conducting arguments, proofs, justifications, discussions, and resolutions appropriate to the knowledge institutional expectations.
Cognitive	Provides adequate knowledge of how students develop their learning, reasoning, and understanding in mathematical practices.
Affective	Displays affective and emotional understandings and attitudes and beliefs involved or necessary in mathematics teaching and learning.
Interactional	Demonstrates understanding of the organization of mathematical tasks and the interactions necessary, or that occur, in the teacher-student-object relationship, as well as overcoming learning obstacles.
Mediator	Exposes knowledge of technological, material, and temporal resources suitable for teaching and learning processes.
Ecological	(a) Expresses knowledge of mathematical objects in the curriculum and how they are articulated in mathematics and other areas, considering political, economic, social, and cultural aspects in the environments where mathematical practices occur. (b) Presents knowledge of the curriculum and environmental adjustments necessary for the students' formative level.

Source: Own elaboration

The analysis will be conducted taking as a reference the first moment of the activity proposed to the students, indicating which questions were used to investigate the didactic-mathematical skills in the two categories already mentioned: analysis of global meanings and the onto-semiotic analysis of mathematical practices. As each category is presented, the didactic-mathematical knowledge that is identified from the academics' responses is discussed, considering the mobilization of this knowledge as an object for mastering the category of competence to which it refers.

What follows presents the data analysis and the emerging discussion of its content.

4 Data Analysis and Discussion

In the first moment of the activity carried out with the students, we asked them to carefully read a question from the National Secondary Education Examination (ENEM) year

2011, presented in Table 3[3].

Table 3: Wording of the ENEM question

(ENEM 2011) The balance of hiring in the formal market in the retail sector in metropolitan São Paulo registered an increase. Comparing hiring in this sector in February with January of this year, there was an increase of 4,300 vacancies, totaling 880,605 workers with a formal contract.
Source of the subject available at: http://www.folha.uol.com.br. Accessed on: April 26th. 2010
Suppose that the increase in workers in the retail sector is always the same in the first six months of the year. Considering that y and x represent, respectively, the quantities of workers in the retail sector and the months, January being the first, February the second, and so on, the algebraic expression that relates these quantities in these months is:
a) y = 4.300x
b) y = 884.905x
c) y = 872.005 + 4.300x
d) y = 876.305 + 4.300x
e) y = 880.605 + 4.300x

Source: Napar (2022)

Based on the problem situation, we conducted the questions according to the categories of didactic-mathematical competencies, as described in Table 4.

Table 4: Competence-based questions about the first moment of the activity

Competence Category	Questions
Analysis of Global Meanings	(1) Solve the question, describing the path used to find the solution so that it can be explained to high school students; (3) What types of mathematical objects (concepts, definitions, propositions, theorems, etc.) must a high school student have for them to be able to solve this task? Present, if you consider it relevant, specific and/or generic examples to justify, as you prefer; (4) How do the mathematical objects you mentioned in item 3 relate to each other? For example, there is a relationship between the content of functions and algebraic equations since one can have f(x) = 0 to find the root of the function. Indicate the mathematical connections that you consider to be present; (5) What types of mathematical languages can be used to solve this task? Indicate sections of the resolution made in item 1 where these appear or could appear. (6) Could the mathematical objects you highlighted in item 4, as well as the task itself, be addressed in other areas of knowledge? If so, which ones? Give examples.
Onto-Semiotic Analysis of Mathematical Practices	(2) If you were to use this question in the classroom, in high school, with what type of pedagogical/didactic objective could you use it? (7) What competencies and abilities would be related or could be developed when exploring this activity in the classroom? (8) What types of mathematical difficulties or conflicts might high school students face when trying to solve this task?

Source: Research data

As explained above, didactic-mathematical knowledge that emerged from academics' resolution of questions will also be analyzed for each competence. In what follows, we present the discussion on the competence of analysis of global meanings.

4.1 Analysis of the Problem Situation: Competence of Analysis of Global Meanings

For the first topic (1) Solve the question, describing the path used to find the solution, so that this solution can be explained to high school students, we took as an analytical parameter that academics resolved the question appropriately, recognizing that the number of vacancies

[3] All the figures that represent a solution used by the academics had to be translated into English.

(880 605) indicated referred to the second month of the year (February) and that, therefore, two increment units should be subtracted from the number of vacancies (4 300 multiplied by 2 which is equivalent to 8 600) to go back to December base. Thus, considering the value of December, the given increment and the number of months would find the correct alternative (alternative C), as we have that 880 605 – 8 600 = 872 005, forming the model y = 872 005 + 4 300x. Furthermore, a high school student should be able to understand the presented solution.

All participants adequately expressed the path to solving the problem. However, we must consider that teachers in initial education were asked to carry out a resolution suitable in theoretical and didactic terms for high school students.

Considering the mentioned context, academic A presented a solution that, although it led to the correct answer, did not contain elements that explained it didactically since it did not present justifications or arguments that would lead secondary school students to understand the process involved in the explanation, such as highlighted in Table 5. We understand that even the organization of the explanation and the notation did not allow a high school student to understand what was done.

Table 5: Part of the resolution of topic 1 academic A

Data: a + 4,300 = 880,605 contracts signed without the month of February a = 876,305 – 4,300 are the contracts signed before the month of February a = 872,005 4,300 contracts signed in February. Y = number of workers in the sector X = months

Source: Research data

On the other hand, Academic B used a short answer in which he directly explained why the increase should be deducted twice. However, this answer does not provide enough elements for a high school student to understand how to organize the initial number of job vacancies (872 005) and the monthly job change rate (4 300) in a first-grade functional model, as presented in Table 6.

Table 6: Academic B's resolution

Taking as a starting point the representation of a linear equation y = ax + b, we can write it as: 872,005 + 4,300x (with x varying from 1 to 6, 1st half of the year) Replacing 2 with x, we get: February (2) = 872,005 + 4,300 (2) = 82,005 +8,600 = 880,605 (value given by the exercise) Therefore, the algebraic representation will be given by y = 872.005+4.300x, letter C.

Source: Research data

Prospective teacher C described in detail the process of deducing the increment and searching for the model that provides the solution to the problem, using a resolution organized by an algorithm (e.g., do a, then b, then c, etc.) with representation made through a chart, as shown in Table 7.

Although he did not explain the function and did not use the terms “function” or “algebraic expression,” as in the wording (he used the term “equation”), we understand that when presenting the months in the chart, going back to the month before the first month of the year considered, the academic established better conditions for understanding the solution presented.

Table 7: Excerpt from the resolution presented by academic C

880,605 + 4,300 | 884,905 + 4,300 | 889,205 + 4,300 | 893,505 + 4,300

Start	January	**February**	March	April	May	June
872,005	876,305	**880,605**	884,905	889,205	893,505	897,805

876,305 − 4,300 | 880,605 − 4,300

But there is a simpler way to determine the number of employees in any given month, and this is through a first degree equation. To put together this equation, we will highlight:
a - Angular coefficient = 4,300b - linear coefficient = (to be determined)
x = 2 (February) – the number of months
y = 880,605 (Workers hired in February)

Source: Research data

Through the academics' solutions, we can infer that the prospective teachers have adequate epistemic knowledge to solve the task since they used mathematical knowledge to solve the activity. However, it is essential to consider that participants were asked to come up with a resolution that could be used as an explanation so that potential students could take ownership of the topic in question, which only participant C showed signs of achieving. From a didactic-mathematical perspective, it was expected that teachers would be concerned with the cognitive (learning development) and interactional (learning obstacles) -besides the epistemic- impacts the resolution would have on the studies of their potential students (Godino *et al.*, 2017). Evaluating the points mentioned is important to understand the need for materials prepared by teachers to be didactically self-explanatory, providing students with adequate guidance for their studies so that they can conduct them autonomously (Napar, 2022). In this context, the common knowledge of the content (that object that both the student and the teacher make use of as the object of functions in this situation), when made explicit in the teaching and learning process, needs to consider how the students will receive it, valuing clear and pedagogically coherent communication, because it is the primary means for attributing meanings and structuring mathematical knowledge (Godino, 2014) and the path to be followed for generalization.

Regarding mathematical objects, the question mentions: (3) What types of mathematical objects (concepts, definitions, propositions, theorems, etc.) should a high school student have so that they can solve this task? Present, if you consider it relevant, specific and/or generic examples to justify it, as you prefer. In the answer taken as a parameter, we perceived the need to mention the following: polynomial function of the first degree, dependence relationship between variables, generalization of a model of a problem situation through a polynomial function of the first degree and algebraic equations, considering the hypothesis that the student can apply for x the number of months in the function to check the number of vacancies/months ratio.

Academics A and B highlighted objects coherent to the situation, such as dependence relationships, algebraic expressions, a polynomial function of the first degree, and algebraic equations. Prospective teachers mention that without these well-structured concepts, students could not solve the task. This highlights epistemic didactic-mathematical knowledge, as they highlight the objects present in the situation and others linked to them. It also highlights cognitive knowledge, as they mention the prior knowledge students need to solve the task. Particularly, teacher C adds elementary operations and highlights his concern with ensuring that students are prepared to face problem situations of this nature, resuming knowledge that makes them feel familiar with solving the task. Thus, although prospective teachers expressed themselves appropriately on this topic, they did not use the elements mentioned to answer the previous topic (topic 1). To illustrate the elements mentioned here, Table 8 presents the academics' answers.

Table 8: Answers presented by academics

Teacher	Pointed objects
A	First degree equation, algebraic expressions, and construction of polynomial models using 1st and 2nd degree functions.
B	Concept of first degree equations, dependence and first degree equations.
C	Dependence relationships between quantities, affine function and linear function, and elementary operations.

Source: Research data

From a didactic-mathematical perspective, prospective teachers are expected to be prepared to understand students' potential difficulties and act as possible facilitators of this learning (Godino, 2014). Although these facilitating actions do not constitute primary elements for learning, the teacher's stance when revealing this concern can allow the student to feel more comfortable in carrying out the activities, bringing them security and confidence in their ability. (Moreira, 2002), as expected from affective didactic-mathematical knowledge (Godino, 2014).

In topics (4) and (6), the question was: (4) How do the mathematical objects you mentioned in item 3 relate to each other? For example, there is a relationship between the content of functions and algebraic equations since one can have f(x) = 0 to find the root of the function. Indicate the mathematical connections that you consider to be present, and (6) Could the mathematical objects highlighted by you in item 4, as well as the task itself, be addressed in other areas of knowledge? If so, which ones? Give examples. The resolution taken as a reference has as a parameter the relationships that can be perceived when developing, in practice, procedures with the mathematical objects involved, of which the following can be considered:

- polynomial function of the first degree is related to the dependence relationship between quantities since the primordial concept of function is based on this idea;
- a first-degree polynomial function is involved with the generalization of models, as a problem situation may require the student to perform this action, as requested in the problem;
- polynomial function of the first degree and algebraic equations when the subject values the function;
- generalization of a model of a problem with a dependency relationship between
- quantities because, when formulating a functional model, the subject must establish the relationships between variables.

In topic (6), the understanding that the concepts covered in item 4 can be worked on in physics was taken as a response parameter since this area uses mathematical concepts and languages to represent and work with certain notions, such as Hooke's law and momentum of a force, for example. Furthermore, we understand that the notion of a polynomial function can be used in biology to calculate the linear regression of the metabolic activity of a specific bacterium, for example. Also, it is possible to say that the notion of polynomial functions and the relationship between quantities is addressed in geography when working on linear regression of graphs that present earthquake data. Finally, we can say that first-degree polynomial functions can be used in economics to represent the cost growth of a given product due to inflation in each period in which the growth exhibited such behavior.

In this question, academics A and B only considered the existing potential relationship, in their understanding, between polynomial functions of the first degree and algebraic expressions. Furthermore, they highlighted an argument that the problem above could not occur in other areas of knowledge. We believe that the participants' arguments were fragile and did not reveal mastery of the issue, as they could have pointed out other relationships between the objects that they themselves had already mentioned, such as the dependence relationship that can be expressed from a polynomial function. Furthermore, we understand that the prospective teachers did not envisage the possibility that the problem-situation model could be generalized and worked on in other areas, such as those already mentioned. In this context, we felt that the required elements lacked connection, which leads to the understanding that teachers in initial education are still beginning to establish and improve their primary didactic-mathematical knowledge with an epistemic and ecological basis, needing to expand the meanings they attribute to mathematical practices. Through a continuous and problematizing process of reflection based on necessary tools and theoretical foundations, it would be possible to provide an environment in which the conceptions of prospective teachers would be constituted by the negotiation of meanings of mathematical objects (Godino *et al.*, 2017) and development of a teacher-researcher profile capable of enabling self-reflection on practices and conceptions of school contexts (Schön, 1992; Napar, 2022).

Participant C saw more possibilities for relationships compared to his colleagues. He highlighted the relationship between algebraic expressions and polynomial functions of the first degree, between the dependence relationship and polynomial functions and basic operations with all other objects. Despite not explaining or exemplifying the relationships, we can infer that the prospective teacher's mentions are adequate and demonstrate an expanded condition of his epistemic didactic-mathematical knowledge and the meanings he attributes to mathematical objects. Regarding the relationships between the objects that the teacher mentioned and other areas, he cites possibilities for their use in areas such as physics, chemistry, biology, and, within mathematics, modeling, as suggested by Table 9.

Table 9: Academic C's answer on relationships between objects

6. Could the mathematical objects you highlighted in item 4, as well as the task itself, be addressed in other areas of knowledge? If so, which ones? Cite examples.
Solution:
Yes. For example, quadratic equations with one or two variables and modeling problems are often used in physics, chemistry, biology, computer science, engineering, etc. Furthermore, the four fundamental operations have a countless range of applications.

Source: Research data

Although it is not possible to determine the academic's level of understanding of the connections indicated, it is possible to infer that there is an attribution of meaning to the aspects mentioned that would lead him to indicate these relationships, even if he does not know exactly when these are executed in action. These elements can come from contact with teachers' learning practices or actions and experiential knowledge generated in their historical-social and cultural context (Tardif, 2002), which allow the prospective teacher to have triggers of possibilities or creativity regarding the global meanings of the objects that they are expected to have developed.

The last topic to be presented is (5): What types of mathematical languages can be used to solve this task? The minimally suitable languages for this use are algebraic, numerical, natural, and graphical. Although graphic language is not necessary to resolve the issue, it can be used with a pedagogical bias to illustrate to high school students an alternative possibility when numerical-algebraic or natural language understanding is insufficient to solve the task.

Academics A and C mentioned that algebraic and natural languages were used, emphasizing them from the perspective of solving the problem. On the other hand, Academic B did not demonstrate that he adequately grasped the term "languages," expressing in his answer his understanding of the steps he used to solve the problem, as presented in Table 10.

Table 10: Academic B's answer about the mathematical languages used

5. What types of mathematical languages can be used to solve this task? Indicate sections of the resolution made in item 1 where these appear or could appear.
Answer:
a) The mathematical languages that can be used are:
b) There was an increase of 4,300 vacancies each month, that is, there was a constant growth in the number of vacancies, that is, 4,300 vacancies multiplied by x months;
In the month of January there were 876,305, as there were 4,300 fewer vacancies than in the month of February.

Source: Research data

We infer that academics A and C proposed suitable mathematical languages to solve the problem. However, we can say that they could also cite numerical language to express symbols of quantities such as the number of worker vacancies. In this context, they demonstrate that they have control over the means of sharing and expressing mathematical knowledge (epistemic didactic-mathematical knowledge) and particular meanings about how they communicated mathematics in resolving the issue. However, we must emphasize that prospective teacher B did not present a satisfactory answer, and it is not possible to determine whether he did not understand the question or did not recognize the term "mathematical language" as the target of the answer. It is noteworthy that teachers' knowledge and recognition of different mathematical languages allow them to establish ways of representing mathematical objects that are important in mathematical learning and in establishing meanings and new meanings that are developed in the classroom (Godino, 2014). However, we must note that a graphical representation was not used to explain the question, which is very common when working with functions, especially considering the request that the explanation be suitable for high school students.

From the analysis, we infer that those academics mobilized didactic-mathematical knowledge of an epistemic, cognitive, interactional, and ecological nature. We think that teacher C stood out in relation to his colleagues. He provided more details about his view on each question, enabling a better understanding of how he is situated pedagogically and mathematically in a system of practices. However, we believe that the other academics presented answers that made it possible to infer that the differentiation of the academics' answers may be due to more personal factors, such as the degree of availability for the task, or formative ones, such as the diversity of locations and sources of formation for teachers in initial education. Despite these differences, prospective teachers have a central place for structuring knowledge, the teaching degree course, time, and space in which they should have had the opportunity to develop and negotiate the global meanings they presented throughout the activity.

What follows presents the analysis from the perspective of the second competence.

4.2 Analysis of the Problem Situation: Competence of Onto-Semiotic Analysis of

Mathematical Practices

In this competence category, the question was, firstly, whether the prospective teachers used that question in their classroom and with what objective it should be implemented. As a response parameter, we considered that a coherent pedagogical objective would seek to develop knowledge, competencies, and skills related to solving problems involving first-degree polynomial functions, also presenting pedagogical arguments. From this perspective, we could analyze academics' reflections and understanding.

Academic A established a context in which the pedagogical objective would be to show possibilities for contextualizing school mathematics and connecting it with reality, as we infer from what is presented in Table 11.

Table 11: Academic A's answer about the pedagogical objective

2) Using this question in the classroom, we can demonstrate to students how mathematics can be applied in various ways; in this specific example, we can see that a 1st degree polynomial function can be used to calculate worker increase monthly and that we can also demonstrate that this teaching can take on several other types and concepts that we can use in our daily lives.

Source: Research data

This prospective teacher's perception highlights his epistemic didactic-mathematical knowledge (by thinking about the contextualization of objects) and ecological (application in the social context), elements to be considered necessary when seeking to establish a practice attentive to the needs of students' community (Godino, 2014). However, we think academics should focus on the curriculum and didactic objectives of the mathematical object of the question, considering the teachers must establish proposals combined with the content and area they are teaching (Godino *et al.*, 2017).

Teacher B considers that the activity can be used as a game or an alternative methodology for the classroom. In this context, we disagree with the academic, as we understand that problem situations like this must be present in teaching and learning since working with situations from different contexts, realities, or applications motivates students to develop multiple meanings about how mathematical objects are inserted in educational processes (Godino, 2014). Therefore, prospective teachers must understand that these problems arise in nature from the social, cultural, and historical uses of mathematics, constituting an essential element in teaching practice (Cury, 2001; Godino, 2014; Napar, 2022).

Academic C sought to establish three essential objectives from different perspectives: the pedagogical objective with the mathematical object, skills that the student could develop, and a pedagogical/curriculum objective combined with government guidelines, as follows in Table 12.

Table 12: Objectives for working on the activity, according to academic C

1. Working on related functions, first degree equations with two variables by solving problem situations, leading the student to realize that the study of equations has practical and theoretical applications and is fundamental to solving everyday problems 2. Developing logical reasoning and the ability to relate everyday problems with the concepts of equations previously studied. 3. Addressing one of the transversal themes proposed by the PCNs, namely, work and consumption. And through this, lead the student to develop fully by understanding their reality and becoming aware that they can improve it.

Source: Research data

We infer that the prospective teacher was concerned with establishing objectives that would be part of his pedagogical organization when working with mathematical objects to be developed with high school students. In this sense, we highlight the prospective teacher's understanding in establishing proposals for activities that are attentive to students' educational and social needs. Godino *et al.* (2017) state that mathematical content knowledge is insufficient for teachers to conduct educational processes. Therefore, we must create a formative profile in which teachers understand the importance of looking at teaching and learning in different contexts, considering the objects that should be taught, the mode, the pedagogical practices, and the curricular background surrounding these actions (Godino *et al.*, 2017; Napar, 2022). Based on this context, we infer that the teacher has an expanded perspective of institutional onto-semiotic practices (epistemic perspective), demonstrating his epistemic didactic-mathematical knowledge (knowledge of objectives related to the mathematical objective), cognitive (of the concern for learning that develops the student's reasoning) and ecological (by paying attention to how prescribed guidelines consider a possible didactic teaching path).

From an onto-semiotic perspective of institutional mathematical practices, academics were asked about competencies and skills that could be explored using the situation above. As a solution, we thought it appropriate for them to mention that they took into account the National Common Curriculum Base — High School (BNCC - Ensino Médio) (Brasil, 2018, p. 67), specific competence 3 in the area of mathematics: "Use mathematical strategies, concepts, and procedures, in their fields — arithmetic, algebra, quantities and measures, geometry, probability, and statistics — , to interpret, build models, and solve problems in different contexts, analyzing the plausibility of the results and the adequacy of the proposed solutions to build a consistent argument." As for skills, they considered: "(EM13MAT302) - Solve and elaborate problems whose models are 1st and 2nd-degree polynomial functions, in different contexts, including or not digital technologies" (Brasil, 2018, p. 67). From a more personal, experiential perspective, the following competence is considered: Demonstrate knowledge of modeling first-degree polynomial functions, using mathematical instruments to solve problem situations.

Here, the academics demonstrated their ecological didactic-mathematical knowledge by describing and using competencies and skills prescribed in the National Common Curriculum Base. The competencies cited by academics coherently highlight the use of mathematical knowledge to interpret daily and socioeconomic issues, understanding the construction of mathematical knowledge and its use in social practices, and the expansion of reasoning in structuring new mental schemes. Regarding skills, they mentioned interpreting mathematical data and application in situations of first-degree polynomial functions, solving mathematical problems with everyday contextualization, and creating models using knowledge of first-degree polynomial functions. All academics presented similar competencies and skills, and we infer that their ecological knowledge of the theme was shared, probably during their formative path along their degree.

The academics adequately interpreted the problem situation to determine how competencies and skills could be developed. In this context, we highlight the importance of these teachers' formation in providing the necessary knowledge to identify how their proposals should be linked to their intended objectives. We also emphasize how the teaching degree course advises on the decisions and didactic-mathematical methodologies used and designed in relation to educational processes involved in solving problem situations (Godino, 2014; Napar, 2022).

The last question sought to analyze the competence of onto-semiotic analysis of student learning practices during problem solving (cognitive perspective), asking: (8) What types of mathematical difficulties or conflicts could high school students face when trying to solve this task? As a viable solution, we thought that students could have conflicts when interpreting the task and the data that arise from it. For example, the student could not consider reducing the increment by one unit necessary, leading him to define an inadequate model for the activity. Also, reducing the number of months makes it possible to build a model that needs to start in month 0, a restriction that, if used, does not satisfy the problem.

The academics' answers coincide in that one of the students' difficulties would be interpreting the mathematical problem, but they highlight different sources of information for this statement.

Prospective teacher A told us about his experience developed throughout the supervised practicum, mentioning that students would demonstrate difficulty interpreting the problem and relating the resolution to the problem solution, as presented in Table 13.

Table 13: Academic A's answer about students' potential difficulties

Nowadays, I can use my practicum as a reference; students have a lot of difficulty interpreting mathematical problems; in this specific case, I believe that some of the students would have a lot of difficulty interpreting that they would have to find the value of b before putting together the correct function.

Source: Research data

Academic A's indication highlights how the knowledge developed in teaching practices and the knowledge learned in contact with the practicum context enabled him to develop a cognitive and interactional perception that expands his didactic-mathematical knowledge. According to Fiorentini (2005), the practicum is the locus so prospective teachers can improve their knowledge through *praxis* and realize how the school context unfolds daily. This makes it possible for interactions between students, teachers, and objects to bring a context of immersive and collaborative practices (Godino *et al.*, 2017), enabling reflection on teaching practice and how these professionals can develop, implement, and self-reflect on how to conduct a process that minimizes the difficulties that can be -or are- expressed by students. This understanding returns to the importance of the Supervised Practicum and how this moment of the teaching degree plays a crucial role in the education of mathematics teachers.

Teacher B's answer is based on his understanding of the false idea that reading is not necessary to study mathematics, as presented in Table 14.

Table 14: Academic B's answer about students' potential difficulties

One of the main difficulties faced by students is that they lack the ability to interpret when solving a problem and that they have the false idea that reading is not necessary to study mathematics.

Source: Research data

Flemming, Luz, and Mello (2005) confirm the prospective teacher's statement since historical aspects indicate that mathematics, in more traditional proposals, was worked out of context, in which the application of models and resolution of tasks used procedures that were not adequately constructed from a historical-social perspective. This part of the activity, from the mathematics education perspective, considered that it was necessary to insert a teaching tendency in mathematics for the reading and interpretation of mathematical textual genres (Flemming; Luz; Mello, 2005), encouraging students to develop a vision of mathematics as a science and culture and enabling them to create competencies in interpretation, argumentation, proof, or justification and mathematical writing.

Finally, prospective teacher C presented specific difficulties related to the activity, as shown in Table 15.

Table 15: Teacher C's answer to the question about learning obstacles

Solution: The biggest difficulty would be being able to relate the text with the content of equations and knowing how to work with the data provided to achieve a mathematical model displaying the correct number of employees each month. Calculating the result for the first months, although it helps in determining the general equation, is also not easy for those just starting this study. The calculations for the months after February may be easier than those for January, not the subtraction calculation, but the logical reasoning required.

Source: Research data

Concerning prospective teachers A and B, prospective teacher C presented in more detail his ability to conduct an argument focused on highlighting each obstacle that a student could experience in resolving the question, thus demonstrating greater didactic-mathematical mastery of an interactional (description of learning obstacles) and cognitive nature (indicates learning conflicts in resolution). We infer that this academic's answer also represents his ability to analyze conflicts between student and object, which can broaden his perception regarding how the resolution and the resources and methods used (mediational vision), from a didactic and pedagogical perspective, could be presented in a way that meets students' needs. In this sense, his ability to analyze practice from the onto-semiotic perspective of practices (cognitive vision; learning in problem solving), becomes more efficient, bringing him a vision that can make him establish connections with meanings and obstacles they generate, potentially providing more personalized learning for their students.

Within the scope of the onto-semiotic analysis of practices, the prospective teachers adequately mobilized their competence, considering their individualities of ecological, interactional, and cognitive didactic-mathematical experiences and knowledge. We can say that academic C stood out in the last question, presenting a deeper view of the learning process and the obstacles that his students would be experiencing. We infer that the competencies and skills, pedagogical objectives, and learning obstacles mentioned by the academics reflect a lot on how their undergraduate education was conducted, indicating how they organized, appropriated, and gave meaning to their studies, leading them, even if intuitively, to develop and mobilize the analyzed didactic-mathematical competencies and knowledge.

5 Final considerations

We understand that teacher education courses must be attentive to how their academics conceive, constitute, and give meaning to the different knowledge and experiences they acquire throughout their education. Thus, the need for theoretical proposals that seek to identify and analyze the skills and knowledge being developed and mobilized from the practices produced within the scope of the courses is indisputable.

In this context, the analyses indicated that teachers adequately mobilized the didactic-mathematical competencies of analysis of the global and onto-semiotic meanings of mathematical practices. They cited mathematical objects relevant to the problem, such as polynomial functions of the first degree and algebraic expressions, and related the objects to each other. Although the academics had established adequate relationships between mathematical objects, only teacher C considered relationships with other areas, such as physics and biology, even though he did not specify or exemplify to which situation he was referring. Teachers A and C mentioned coherent mathematical languages, such as the algebraic and the natural, but did not do so regarding numerical and graphical languages. All academics analyzed learning obstacles that the student could face when solving the task, such as textual interpretation, and the competencies and skills that would be developed with that activity, such as understanding mathematics as a resource for solving everyday problems.

As the prospective teachers answered the questions, they presented their didactic-mathematical knowledge, such as: epistemic, relating mathematical objects; cognitive, mentioning previous knowledge necessary to solve the task; interactional, in identifying learning obstacles; affective, about how the student could react to certain aspects of the activity; and ecological, about institutional objectives expected in the learning process. We consider that academics mobilized their knowledge interrelatedly, empirically presenting the objects of their education.

Given these considerations, we highlight that participants have adequate epistemic knowledge to solve the task. Still, improvement is needed concerning how they communicate and share this knowledge, especially considering the didactic-mathematical perspective. Furthermore, prospective teachers must be prepared to understand the mathematical difficulties and conflicts that high school students may face when solving problems of this type and develop competencies and skills related to mathematics teaching and learning. The analysis highlights the importance of promoting a formative environment that provides continuous and problematizing reflection using appropriate tools and theoretical foundations. We believe this will allow prospective teachers to expand the meanings attributed to mathematical practices, developing a teacher-researcher profile capable of promoting self-reflection on their actions and conceptions. Furthermore, it seems essential that prospective teachers recognize and master different mathematical languages to facilitate communication and establish meanings in the classroom with their students.

The analysis showed that teachers mobilize didactic-mathematical competencies and knowledge, which are important and necessary to evaluate their ability to analyze mathematical objects and processes, as they did throughout the proposed activity, and for conducting educational activities. Furthermore, we believe we demonstrated teachers' need to understand their pedagogical objectives when teaching a problem situation, indicating they must reflect on which competencies and skills they must stand to work on. A reflection was conducted on the learning obstacles that students may have when working with a problem situation, indicating the construction of a perspective that teaching and learning need to be personalized to the real needs of students and mathematical communities.

The questions designed to investigate the mobilization of competencies and knowledge made it possible to understand how prospective teachers demonstrate those objects in practice. In other words, a mathematical and educational activity that takes shape in the arguments and empirical meanings that come from academics' experiences and formation.

To conclude, we believe the action conducted in the investigation served not only as a proposal to analyze skills and knowledge but also as a reflection on the teacher's role in the teaching and learning process. We understand that this action can be included in teacher education processes when considering a course that focuses on the constitution of an investigative and critical profile in the professional context, enabling, through theoretical components such as those of the onto-semiotic approach, reflective development and natural competencies and didactic-mathematical knowledge.

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