Sociedade Brasileira de Educação Matemática

RIPEM

Teaching arithmetic mean from the perspective of statistical literacy in the final years: Reflections with teachers

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2238-0345 ISSN

10.37001/ripem.v14i3.3702 doi

Received • 17/12/2023
Approved • 01/03/2024
Published • 20/08/2024

Editor • Gilberto Januario

Abstract: The article discusses the results of a master's research that aimed to analyse reflections with Mathematics teachers of the final years of Elementary School on possibilities of teaching the arithmetic mean from the perspective of Statistical Literacy. Data were obtained from two virtual Focus Group meetings in which some types of errors about arithmetic mean were problematized, theorized about Iddo Gal's Statistical Literacy model and teaching proposals about arithmetic mean were elaborated. Four Mathematics teachers from the final years who teach in schools in different municipalities in the state of Pernambuco participated in the group. The approaches developed by the professors involved possibilities of mobilizing dispositional elements in articulation with elements of knowledge about the arithmetic mean. The study emphasizes the importance of introducing instructional approaches regarding the arithmetic mean within the framework of Statistical Literacy, with an emphasis on addressing critical issues.

Keywords: Arithmetic Mean. Statistical Literacy. Fundamental School Final Years. Teaching Proposal. Critical Issues.

La enseñanza de la media aritmética desde la perspectiva de la alfabetización estadística en el liceo: Reflexiones con profesores

Resumen: El artículo discute los resultados de una investigación que tiene como objetivo analizar las reflexiones con profesores de Matemática de los últimos años de la Enseñanza Fundamental sobre las posibilidades de la enseñanza de la media aritmética en la perspectiva de la Alfabetización Estadística. Los datos fueron obtenidos de dos reuniones virtuales de Focus Group en las que se problematizaron algunos tipos de errores sobre la media aritmética, se teorizó sobre el modelo de Alfabetización Estadística de Iddo Gal y se elaboraron propuestas didácticas sobre la media aritmética. Participaron cuatro profesores de Matemática que enseñan en escuelas de diferentes municipios del estado de Pernambuco. Los abordajes desarrollados por los profesores involucraron posibilidades de movilizar elementos disposicionales en articulación con elementos de conocimiento sobre la media aritmética. Resaltamos la importancia de implementar propuestas didácticas sobre el promedio desde la perspectiva de la Alfabetización Estadística con enfoque en temas críticos.

***Palabras Clave*:** Media Aritmética. Alfabetización Estadística. Liceo. Propuesta Didáctica. Cuestiones Críticas.

Ensino da média aritmética na perspectiva do letramento estatístico nos anos finais: Reflexões com professores

Resumo: O artigo discute resultados de uma pesquisa de mestrado que objetivou analisar reflexões com professores de matemática dos anos finais do ensino fundamental sobre possibilidades de ensino da média aritmética na perspectiva do letramento estatístico. Os dados foram obtidos a partir de dois encontros virtuais de grupo focal nos quais se problematizou alguns tipos de erros sobre média aritmética, teorizou-se sobre o modelo de letramento estatístico de Iddo Gal e elaborou-se propostas de ensino sobre a média aritmética. Participaram do grupo quatro professores de matemática dos anos finais que ensinam em escolas de diferentes municípios do estado de Pernambuco. As abordagens pedagógicas desenvolvidas pelos docentes participantes envolveram e evidenciaram possibilidades de mobilização de elementos disposicionais em articulação com elementos do conhecimento sobre a média aritmética. Destacamos a importância da implementação de propostas de ensino sobre a média na perspectiva do letramento estatístico com o foco em questões críticas.

***Palavras-chave*:** Média Aritmética. Letramento Estatístico. Ensino Fundamental: Anos Finais. Proposta de Ensino. Questões Críticas.

1 Introduction

The simple arithmetic mean is a measure of central tendency, present in the most varied contexts, used to represent the behavior and characteristics of a data set. Although not intentionally, people tend to intuitively use the average through estimates and inferences about everyday situations, such as the average annual grades in mathematics, the average number of children per couple, and the average number of goals (Cazorla, 2004).

The National Common Curriculum Base (BNCC) recommends teaching averages throughout the final years of fundamental school, precisely in the thematic unit of Probability and Statistics (Ministry of Education, 2018). According to these guidelines, students are expected to collect and organize data, representing them in Frames and various graphs, including measures of central tendency, interpret and analyze data present in everyday-life problem situations in different contexts, make inferences and judgments, and make informed and appropriate decisions. Despite curriculum recommendations, teachers usually refer to the arithmetic mean only based on technical aspects with an emphasis on calculation (Alves *et al*., 2019), failing to consider its properties (Silva Júnior, 2018; Strauss & Bichler, 1988) and their meanings (Batanero, 2000).

Some authors argue that exploring the concept of average, along with other statistical concepts, in issues associated with social contexts and practices is essential (Gal, 2002; Watson, 1997; Watson & Callingham, 2020). According to Costa Júnior *et al*. (2021), exploring the mean in multiple contexts encourages and motivates students to reflect on the problems presented, making it possible to understand the meanings of this notion.

Teaching the mean by exploring its varied contexts is at the heart of the research discussed here and refers to the perspective of statistical literacy. According to Gal (2002), in a literate society loaded with statistical data, citizens are expected to develop reading skills that consider the contexts of information. These skills are fundamental to the constitution of people's statistical literacy. Gal postulated a statistical literacy model that articulates the elements of knowledge (literacy skills, knowledge of mathematics, statistics, and context and

critical questions) and those of disposition (personal beliefs and attitudes, critical stances).

This statistical literacy perspective requires teaching processes that enhance the emergence of knowledge elements and dispositional factors, enabling connections between them. Considering the frequency of use of the arithmetic mean, we believe it is essential to analyze its approach from this statistical literacy perspective.

Furthermore, some previous studies highlight the need for discussions and explorations about statistical literacy in initial and continuing teacher education (Ferreira *et al*., 2019; Macedo *et al*., 2016). However, more discussions and explorations on this topic in initial and continuing teacher education are necessary to connect the components of knowledge and the dispositional aspect of statistical literacy.

In this article, we discuss the results of a survey that sought to analyze reflections with mathematics teachers in the final years of fundamental school about possibilities for teaching the arithmetic mean from the perspective of statistical literacy. The research is part of a broader academic master's study carried out by the first author with the guidance and co-supervision of the second and third authors.

In addition to this introduction, we explore the properties and meanings of the arithmetic mean and, subsequently, Gal's (2002) statistical literacy model. We also present the methodological procedures, results, and discussion. Finally, we discuss the final considerations of the study.

2 Simple Arithmetic Mean

The arithmetic mean is the most important centrality measure used in everyday life. It describes the behavior and characteristics of a data set, expressing it around a single number (Duquia & Basto, 2006; Cazorla *et al*., 2019). The mean is defined as the "balance point of the deviations of the distribution values" or the "value that is equivalent to the center of mass of a data set" (Novaes & Coutinho, 2009, p. 80).

To obtain the simple arithmetic mean algorithm for an ungrouped dataset, simply divide the sum of the values of all data in that set by the number of elements (Cazorla *et al*., 2019). These authors emphasize that, despite the simplicity of obtaining the mean algorithm, appropriating and understanding this concept involve a complexity of properties and meanings, so their approach should not focus solely on the calculation procedure.

Strauss and Bichler (1988) list seven properties for understanding the mean: I. It is between the highest and lowest values, i.e., between extreme values unless all operationalized values are equal; II. The sum of the deviations of the variable values regarding the mean results in zero; III. It is influenced by values different from it, i.e., for all operationalized values; IV. It does not necessarily coincide with the operationalized values; V. It may not be based on real data, i.e., it can be a fraction, a decimal number; VI. When calculating the mean, all values are considered, including null and negative ones; VII. The mean value represents and approximates the calculated data, i.e., the mean represents the operationalized data.

Regarding the meanings of measures of central tendency, such as the simple arithmetic mean, Batanero (2000) shows that these meanings involve a complex character acquired throughout primary and secondary school. This author lists four meanings of the mean: I. The mean in estimating an unknown quantity in the presence of measurement errors, i.e., as the best estimate in repeated measurements; II. The mean as a fair quantity to distribute to achieve a uniform distribution, i.e., an equiframe distribution; III. The mean represents a dataset whose distribution is approximately symmetric, i.e., the mean seen as a representative element; IV.

The mean as a need to know the value obtained with the greatest probability using a random element from a population.

We understand that media knowledge is complex as it involves different properties and meanings, requiring explorations from different teaching situations. These can involve several methods and contexts, such as analyzing real data sets and solving real-world problems, enabling a deeper understanding of how to apply the mean. However, not only cognitive aspects (elements of knowledge) are involved in understanding the mean. In teaching the arithmetic mean from the perspective of statistical literacy, we must also consider the affective aspects (dispositional elements) that encompass critical stances and beliefs.

3 Statistical Literacy

Statistical literacy is a fundamental skill of statistical education. It can contribute to the lives of people in society in many ways, such as enabling the ability to access, understand, critically interpret, and evaluate statistical information and data presented in the media. Thus, it allows the construction of arguments in different contexts and helps subjects develop critical awareness, providing them with essential subsidies to position themselves in decision-making about trends and phenomena of social importance and in their daily choices (Gal, 2002). Gal (2002) proposes a statistical literacy model (Frame 1) that underpins this research.

Frame 1: *Iddo Gal's Statistical Literacy Model*

Knowledge elements	**Dispositional elements**
▪ Literacy Skills ▪ Statistical Knowlwsgw ▪ Mathematical Knowledge ▪ Context Knowledge ▪ Critical Questions	▪ Beliefs and Attitudes ▪ Critical Stance
Statistical Literacy	

Fonte: Adapted from Gal (2002)

According to Frame 1, statistical literacy derives from the connection between elements of knowledge (cognition) and those of disposition (affective or attitudinal). Gal (2002) highlights two interrelated skills underlying this model: I. the ability of people to interpret and critically evaluate statistical information related to data or stochastic phenomena (statistics, probability, and combinatory) they may encounter in different contexts; II. the ability to discuss or communicate their actions in the face of statistical information, understand the meaning of the information and express opinions about the implications of this information, or concerns regarding the acceptability of conclusions given. These skills must be taught, but to do so, teachers who teach statistics in basic education must master knowledge of statistical and mathematical content (specific knowledge) and know how to teach them (didactic knowledge).

Statistics is a science that permeates other areas of knowledge, although it has been included in the curriculum as a constituent part of mathematics. Gal (2019) and Lopes (2012, 2013) highlight that statistics, unlike mathematics, does not study data as just numbers but as numbers within a context to be explored. Numbers within contexts are loaded with intentions and meanings; therefore, teaching approaches from the statistical literacy perspective must consider the specificities of the context. For example, Gal's (2002) statistical literacy model has contributed to reflections on approaches within the scope of financial literacy in the context of financial education (Giordano, Lima, & Silva, 2021; Coutinho, 2021; Coutinho & Teixeira,

2015).

Financial education consists of a set of basic information that helps people make choices and decisions in the areas of personal finance in a critical way (Coutinho & Teixeira, 2015). The context of the information involves aspects related, for example, to personal and family monitoring and budgeting, investments, purchases, and saving. The link between financial education and financial literacy partly arises from the idea of increasing skills and a critical stance in choices and decisions in the personal finance areas (Coutinho, 2021).

Giordano, Lima, and Silva (2021) argue about the need to develop approaches and pedagogical propositions for developing students' financial literacy. Then, they propose a model of financial literacy based on the definitions of statistical literacy (Gal, 2002) and probabilistic literacy (Gal, 2005), highlighting as knowledge elements: basic ideas, calculation of the value of money over time, knowledge of language and context, and critical questions. Besides the elements of knowledge, there are elements of disposition, including the latter emotional components related to feelings about uncertainty and risk. Like those proposed by Gal, the financial literacy model involves the notion that literacy is more than reading and writing, implying a critical and conscious interaction with social and economic aspects.

According to Gal (2019): “The context is the driver for the questions that inform statistics instruction and the answers we generate based on provided data” (p. 4, our translation). In this way, the context and the formulation of critical questions are crucial in teaching processes that seek to enhance statistical literacy. We highlighted these aspects in our focus group meetings with teachers, as shown in the methodology below.

4 Methodology

Using a qualitative approach, the research analyzes our considerations with mathematics teachers teaching the final years of fundamental school about possibilities for teaching the arithmetic mean from the perspective of statistical literacy. The participants were four mathematics teachers - three women and one man- teaching the final years of primary public schools in different municipalities in Pernambuco. The women are between 25 and 30 years old, and the man is over 55. The three women have been teaching at this level of education for more than three years while the man has been teaching for more than 20 years.

The four teachers have degrees in mathematics and took at least one Statistics subject during their undergraduate studies; however, in their initial education, they did not study the properties and meanings of the arithmetic mean or statistical literacy. Only one participant reported that he took a subject as a special student in a stricto sensu postgraduate course, where measures of central tendency and dispersion were briefly discussed. In addition, he also participated in a workshop and a course on statistical literacy.

This data was produced in the master's research, which had two stages. The first was the application of a questionnaire, shared together with the free and informed consent form via WhatsApp and Facebook, answered voluntarily by teachers. The second stage involved a focus group discussion with those teachers who responded to the initial questionnaire and decided to continue collaborating with the research.

We conducted focus group (FG) work with the participants to explore the arithmetic mean from the perspective of statistical literacy. According to Gatti (2005), the FG is an important instrument in social research with qualitative approaches. This author also highlights that this instrument allows researchers to understand convergences and divergences in the participants' speeches on a given topic, being, therefore, a relevant procedure to develop not

only studies with groups that have some characteristics in common but also an experience with the topic under discussion. In our research, we have a group of teachers who teach mathematics in the final years of fundamental school and have experience in teaching arithmetic mean.

The FG work was conducted remotely in two synchronous meetings, each lasting two hours, held at night via Google Meet. The teachers allowed researchers to video and audio record them.

We created a WhatsApp group named Statistical Literacy. In the group, we first shared the texts to be discussed at the meetings (Frame 2), helped participants with doubts, and monitored their teaching proposals. They could solve doubts via WhatsApp and share the proposals before the second meeting. Frame 2 presents the organization of the meetings.

Frame 2: Meetings and actions proposed for the focus group interview

Meetings Synchronous	**Actions corresponding to learning meetings**	
1st Meeting 09/27/2022	*Problematization*	Analysis of types of errors in arithmetic mean problems. Discussion about assessing errors in mathematics based on Pessim and Leite (2020).
	Theorization	Discussion on statistical literacy based on Gal (2002) from slides and Carvalho *et al.*'s (2021) work.
	Teaching proposal	Teachers organized in pairs to prepare a teaching proposal on the arithmetic mean from the perspective of statistical literacy.
2nd Meeting 10/21/2022	*Socialization and discussion of proposals*	Presentation of teaching proposals. Discussion focusing on articulating knowledge elements and dispositional factors of statistical literacy, according to Gal's model (2002).

Fonte: Rocha (2022)

We previously sent the texts discussed in the synchronous meetings to the teachers via WhatsApp, and encouraged them to read them and raise questions.

The Frame 3 presents the problems and erroneous resolutions of the pairs the teachers analyzed during the first meeting. The first problem involves analyzing and interpreting the mean in information conveyed by television media, while the second requires calculating it.

As mentioned, the participating teachers had already answered the problems posed in Frame 3 in a virtual questionnaire initially carried out in our master's study. Therefore, it was not the first time they had encountered these issues; for this reason, we separated the student responses from Eugênio's survey.

Fictitious names will identify the teachers participating in the research in the analyses presented in the following section. The teachers were called Joaquim, Ana, Laura, and Maria. Joaquim and Ana formed Pair 1, and Laura and Maria formed Pair 2.

Frame 3: Problematization of errors made by students in the ninth grade of fundamental school

Problems	**Responses**

Problem 1 You are watching television and hear the following news: "On average, Brazilian families have 2.3 children." a) What can you say about this news? b) Why is the average 2.3 and not a number like 1, 2, 3, or 4?	**Pair 1**: (a) "That they must be two or three children." (b) "Because there are fewer families today than in the past." **Pair 2**: (a) "That families have two or three children, I think." (b) "Because they have to do the math, and that's what happened." **Pair 3**: (a) "We think they calculated and got this approximation." (b) "We think that people nowadays have fewer children; it must be a lot of work." **Pair 4**: (a) "That in Brazil that is the average number of children. That's it." (b) "It must be due to some crazy calculation or mathematical formula."
Problem 2 Flávio worked in an appliance store. On the first day of work, he delivered four refrigerators; in the second, five; in the third, seven; and in the fourth, six. What was the average number of refrigerators delivered on these days?	All pairs answered, "23 household appliances."

Fonte: Based on Eugênio (2013)

5 Results and discussion

The reflections from the FG enabled the organization of the obtained data into two categories: Teachers' analysis of types of errors in arithmetic mean problems and teaching proposals on the arithmetic mean.

5.1 Teachers' analysis of types of errors in arithmetic mean problems

When presenting the protocol with the *types of errors* of the four pairs of students in Eugênio (2013) regarding *problem 1* (Frame 3), the teachers highlighted the pair(s) that they considered to be closer or further away from the expected answer. We emphasize that this strategy was adopted by the teachers when analyzing the responses, as shown in the speech excerpts:

> Ana: [pairs] *1 and 2*, in a way, they had an interpretation. Now, maybe not as an *approximation*, perhaps more in the sense of, yes, this data is there, but, in practice, it means that perhaps, it was in that sense.
>
> Maria: I agree! *They follow the reasoning that each family would have two or three children.* I believe they thought that. Maybe they still have difficulty showing their thoughts. Let's just say that rewriting what they think is difficult.
>
> Laura: *The one that comes closest to logic would be Pair 3. We think they did the math and got to this approximation.* Because they put approximation, right, because it is important, that they did the calculations. But, even though, yes, some things are not so exact, right? Regarding their opinion, they had a logic as to why, right? *Pair 4 said, yes, that was it, and that was it, right?* They really agreed with the news, without even thinking about it mathematically, right, that we

were talking about mathematics, right? So, we needed to go after mathematical knowledge, and they didn't even think about that; they just agreed, and that was it. *As for the others, Pair 1 and Pair 2, they said: "Yeah, it can't be 2.3; either it will be one thing or it will be another."* They thought that either it would be 2, a whole number, or it would be 3, another whole number.

Teachers Ana and Maria consider that pairs 1 and 2 made somehow an interpretation of the issue when they said that Brazilian families should have two or three children. However, we observed that these pairs indicated two and three, respectively, as the minimum and maximum points, considering that families have exactly two and three children. Despite the error, this answer is close to the first property of the mean (the mean is between the highest and lowest values) of Strauss and Bichler (1988).

Laura considered Pair 3's reasoning interesting as it indicated the performance of a mathematical procedure (calculation) and pointed out the idea of approximation. On the other hand, unlike Ana and Maria, she did not agree with the answers presented by Pairs 1 and 2, pointing out that the students did not observe that 2.3 is the result of an arithmetic mean and does not necessarily need to be an integer. Furthermore, she considered that the fourth pair merely agreed with the news without alluding to mathematical logic.

Therefore, in her answer, Laura refers to the fifth property of the mean by Strauss and Bichler (1988) –the mean can be a fraction, a decimal number. However, it seems that she did not analyze this mean from a critical stance and from the perspective of other properties. For example, the first (the mean is between the highest and lowest value), the fourth (the mean does not necessarily coincide with the operationalized values), the sixth (in calculating the mean, all values are considered, including null and negative ones) and the seventh (the mean value represents and approximates the calculated data).

In *item b* of *problem 1* (The Frame3), although the four teachers considered that the four pairs' answers were very creative, they claimed that the students did not understand what was requested and did not consider the concept worked on and the mathematical aspects, as shown in their answers (with our highlights):

Laura: *And they tried, didn't they, but they didn't take it to the mathematics side*. They didn't take it to the mathematics side; they took it more to the family side, and they didn't exactly answer the question. Then, they wanted to answer considering the family and not the mathematical question, not that there is logical reasoning there, with a rational number, but in a way that was for the family; they were much more regarding the family, you know? [...] I believe that was it, that, look, because the families, this is Pair 1, right, because there are fewer families today than in the past. Because they must do the math and they got this [Pair 2]. And I thought this 4 was funny, crazy calculation, a crazy formula [laughter]. So, I found the part where they said that funny; but even so, they didn't arrive at what we needed, right, which would be more logical reasoning, right, in relation to mathematics, and in relation to an arithmetic mean, right?

Maria: I confess that the answers were funny**,** *but they didn't really understand what the question asked.* But they were very creative.

Ana: Show us, yes, if we were to make a bridge with item a, *it's like they didn't understand anything*, apparently, because there they say: "ah, families must have two to three children." *Oh, it was an approximation.* But, when you get here, you ask like..., at least when we as teachers read it, we understood, right; then maybe, yeah, *they didn't understand what the question was saying or they actually didn't understand and didn't actually realize that it was an arithmetic mean, right?* [...]. So, if we were to relate these responses to the first ones, it is as if they had responded just for the sake of responding, but without actually leading to the concept of what mean is.

Joaquim: What did the student think that led him to this answer? So, empty answers. I was surprised when I saw the one about the number of children, and we usually have somebody who hits the answer, *but no one could verify that 2.3 children was not possible.*

Laura indicates that the students did not use mathematical logic and tried to interpret based on a comparison between the number of children in families in the past and those they found around them. Ana again highlights the idea of approximation and indicates that the students did not realize that it refers to an arithmetic mean and answered without reflecting. Maria points out that the four pairs, in fact, did not understand the issue; however, she does not analyze those students' responses.

Joaquim, in turn, claims that the students presented empty answers and could not identify that families cannot have 2.3 children. He claims that there are no families with 2.3 children. We consider it important to emphasize that this value in the problem is a mean, a value that represents a dataset. Furthermore, regarding the properties of the mean, we found that the value does not correspond to the physical reality, as evidenced by the fifth property discussed by Strauss and Bichler (1988).

When checking the types of errors in *problem 2*, teachers Ana, Laura, and Maria emphasized that the students only added, i.e., they did not calculate the simple arithmetic mean. Joaquim did not analyze this issue. Below, we those teachers' speeches with our remarks:

Laura: *They only gathered them, they didn't do an arithmetic mean, right, because they didn't divide it or anything.* They just gathered the numbers that appear there and just added them up, right? Just that. As a sequence, without reading. *They still have a huge gap in interpretation.* Yeah, I always ask my students to read it the first time, read it a second time if they don't understand, and read it the third time, right? I believe it is exactly that. *The difficulty they have in interpreting.* I believe it was like that. Maybe they were learning something similar and thought it would be a sequence. But they didn't interpret it correctly, especially that part there, hey, what was the mean, right? So I believe they did it.

Ana: Until getting this answer. *Reading, they did read; but they didn't interpret it, they didn't reflect on it. A very common concern in our reality, right? It's something that we are, you know, really perplexed, because, any student, they would add up the numbers that are there, yeah, written. (. . .) They still added up wrong, right?* The account is thus messed up. (. . .) *We realize that they have no knowledge of the mean*. They don't have this knowledge of mean, of how it is calculated, at least like this, until then, right? [...] *They did not demonstrate an understanding of the subject.*

Maria: I was surprised by the answers, as they were ninth graders. *We can see the difficulty they still feel in the four basic operations,* and interpretation.

Laura: That's the basics, right? This is the basic. But, you know what? It's because they didn't read it properly. *They still have a huge gap in interpretation.* Yeah, I always ask my students to read it the first time, read it a second time if they don't understand, and read it the third time, right? I believe it is exactly that. What? (. . .) *They have no understanding of what mean is, what a statistics account is, because this is basic statistics, the mean, the weighted mean, right?*

The three teachers (Ana, Maria, and Laura) reinforce the importance of students interpreting the question and point out that, in addition to making mistakes in the mathematical procedure, the students performed the wrong sum; that is, instead of 22, they got 23. Thus, their statements indicated that the four pairs did not understand the concept of mean and how it is calculated.

After the teachers analyzed the types of student errors, we talked to them about the

statistical literacy perspective and how, in this approach, the elaboration of critical questions based on the context can help students construct meanings. For this purpose, we used slides emphasizing the model by Gal (2002). Furthermore, we highlighted some excerpts from the text by Carvalho *et al*. (2021), previously sent to them via WhatsApp, in which the authors discuss the importance of the context in the interpretation of statistical data, using COVID-19 pandemic situations as an example.

An aspect highlighted on the occasion consisted of reinforcing for teachers how the elaboration of critical questions can be the link to intersect knowledge and dispositional elements in statistical education from the perspective of statistical literacy. At the end of the first meeting, we requested and guided the teachers, in pairs, to develop teaching proposals about the arithmetic mean from the perspective of statistical literacy and we maintained contact with them over the weeks leading up to the last meeting, in which they shared the teaching proposals they created.

5.2 Teaching proposals on the arithmetic mean

To explore the concept of mean from the perspective of statistical literacy, we asked the four teachers, in pairs (Pairs 1 and 2), to articulate teaching proposals, at least one from each pair, focusing on critical issues. Therefore, Pair 1 (Laura and Maria) and Pair 2 (Joaquim and Ana) presented two teaching possibilities each, totaling four proposals.

However, we will not consider one of Pair 1's activities in this study because another author developed it for educational purposes, with copyright and for-profit intentions. To access the material, one must pay a fee; it cannot be shared with others. Furthermore, this material focuses only on the technical aspects of the mean and other measures of central tendency that is, calculating the mean from sets of numbers. Therefore, three teaching proposals will be presented throughout this topic, one from Pair 1 and two from Pair 2.

Pair 1 (Laura and Maria) suggested the development of research based on themes that are part of the students' reality (context), as shown in Frame 4:

Frame 4: Pair 1's (Laura and Maria) teaching proposition

Grades: *8th or 9th year*
Initially, the teacher must survey topics that are part of the students' reality, such as games, films, series, etc., that is, survey topics that students like. Next, students should be asked to conduct research according to the teacher's initial survey of the topic. This research can involve the entire school or just the classroom.
A few exam 1. How many students have a specific game installed on their mobiles? 2. At school, how many people have this game downloaded on their mobiles? 3. Which game do students like most? 4. What is their favorite movie? 5. What *anime* do students follow? Which do they like best? 6. Who likes hamburgers? 7. Who likes eating pizza?
Then, teachers could ask students questions about these topics to gather data and explore the concept of mean. Furthermore, they could ask students critical questions about the means obtained to find out what they consider about them.

Fonte: Organized by Rocha (2022)

Laura and Maria (Pair 1) highlight the importance of working with students on pedagogical proposals that enable the construction of mathematical and statistical knowledge based on practical situations that mobilize students' criticality. In other words, they have situations that involve the exploration of collected data (data processing) and calculated means, noting that they belong to real situations, close to the students' context.

This duo also highlights that each class and school has its own reality. This must be considered when organizing lessons because, depending on such reality, teachers may or may not be able to apply certain proposals in mathematics classes.

The first proposal from Pair 2 (Ana and Joaquim), on the other hand, proposes a pedagogical work on the arithmetic mean on the topic of bullying based on teamwork, as illustrated in Frame 5:

Frame 5: The first teaching proposal from Pair 2 (Ana and Joaquim)

Theme: *Bullying*/ Grade: 7th year
Organization of teams Initially, the teacher reflects on the topic through a video. Then, students are divided into teams, each receiving a specific activity. *Team 1 (Communication team):* responsible for defining bullying to the classes and its consequences in the victims' lives. *Team 2, 3, 4, and 5 (Research teams):* Responsible for carrying out research in the classes.
Data collection Data collection is based on four questions: 1. Have you ever suffered bullying? 2. Do you have a friend who has suffered bullying? 3. Do you think this practice harms the learning of those who suffer from it? 4. Have you ever helped someone in a bullying situation?
When addressing the classes, each team asks its respective question and notes the number of students who raise their hands, in addition to the total number of students present in the room. After collection, students must organize the data in a Frame. Next, some types of graphs and their usefulness are explained. After the explanation, the teams must meet and, with the teacher, choose the graphs according to the answers obtained, which can be column graphs. Some data can be grouped when they refer to different classes. When choosing graphics, students can use a cardboard to construct and present them. When presenting and analyzing the data, the teacher proposes that students calculate the mean number of students who answered each of the questions. From this, the idea of mean and its implication in the researched reality will be explored, for example: how many students suffer bullying on average?

Fonte: Organized by Rocha (2022)

According to Ana and Joaquim, this proposal arose from situations they had experienced at school since the beginning of the school year. They noticed that many students experienced bullying, having witnessed several cases at school. According to the pair, the theme is in the scope of socio-emotional education, being recommended by BNCC.

The pair also highlight that these themes can make students reflect on situations experienced outside the school. Regarding critical issues, Ana underlines some questions that can be asked based on the means obtained, according to her speech with our highlights:

Ana: *What is going on? How are we behaving? What can we do, considering this data to have it reduced, right, so that we can mitigate all of this?* That's why one of the questions is: "Have you ever helped someone, right?" (. . .) A student even said: "Look, teacher, how do we want to talk about this?" [In an activity] I realized that this student experiences a lot of bullying. I realized he was without a group. Then I went to look at the groups, the group was incomplete to fit him. Then we got a group for him. So, I was very happy just hearing her say that, right? So, it was already a positive result, which (...) it already brings us a perspective of improvement, right?

Ana presents a teaching experience she had developing an activity in a Mathematics class, in which one of the students was without a group. Given this observation, with the students' help, she managed to fit him into a team and identify the situation that was occurring in her own classroom.

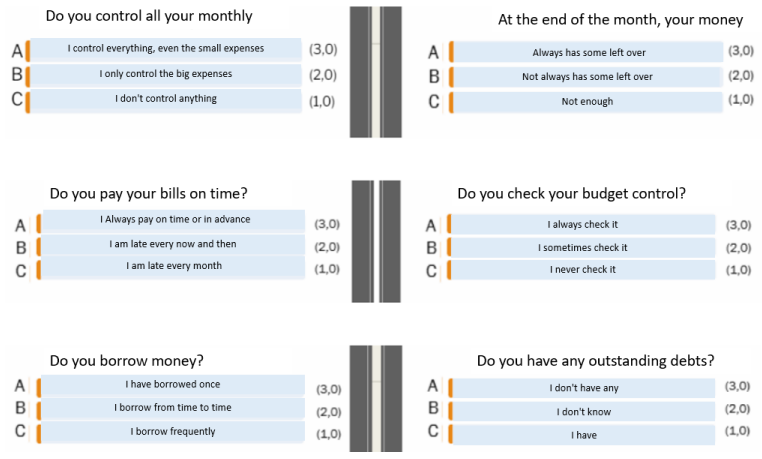
Regarding the critical questions she suggests asking students when promoting debates, it seems they focus on personal reflections on actions and attitudes at school, in the classroom, in everyday life, and in interpersonal relationships. From this, we must seek solutions to promote inclusion and dispel acts of bullying, as occurred in the case of the student without a group. Not always cases of bullying are visible. Although the proposal is based on a problem that can be experienced in everyday school life, a lag was observed about the concept of simple arithmetic mean.

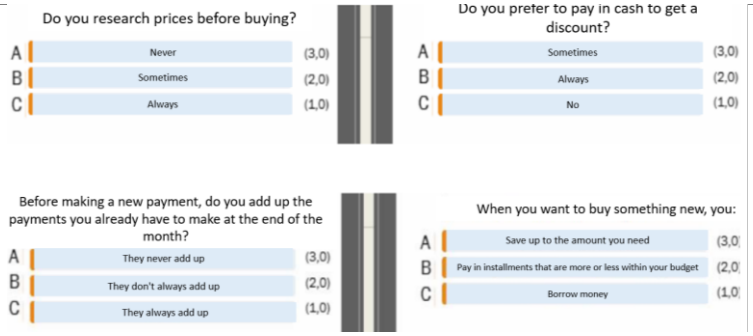
We understand that these classroom discussions and reflections are important for developing critical attitudes among students and teachers and, therefore, for constructing procedural and dynamic statistical literacy. There is no point in just discussing bullying in class. We must develop actions and attitudes that minimize cases in the classroom, encouraging students to have conscious and critical postures and attitudes toward the research results and situations that may arise in the school context.

In Frame 6, we present the second proposal from Pair 2, composed of Ana and Joaquim:

Frame 6: The second teaching proposal from Pair 2 (Ana and Joaquim)

Theme: Financial Profile/Grades: 8th or 9th year
Initially, we ask students to conduct research with 10 to 15 people about their financial profile, using the following questions:





After each interview, students must inform each interviewee's score and read the message according to the sum of points, as shown below:

Caution!

Score from 0 to 10! You should evaluate your financial life and monitor your income and expenses more closely.

Congratulations!

Score from 21 to 30! You are on the right track. Remember that simply monitoring your budget is not enough. Planning and investment are necessary for your dreams to become reality.

Attention!

Score from 11 to 20! You have little knowledge about your finances and those of your family. Start controlling your budget right now

Then, with the data collected and organized in Frames according to each financial profile (care, congratulations, and attention), students build sector graphs. For each profile, there is a pie chart.

Using the graphs and data from each profile, teachers work and problematize the arithmetic mean concept.

Fonte: Organized by Rocha (2022)

The theme of this proposal (Frame 6) arose from an elective curriculum component Ana taught in high school, focusing on financial education topics. Therefore, the pair discusses financial education for the final years of fundamental school and designs an activity that eighth and ninth graders can develop. According to them, this topic is important, as the BNCC recommends it and allows for teaching arithmetic mean from a meaningful context for students.

The proposed approach suggests carrying out field research, aiming to identify people's financial profiles, working on statistical representations, such as graphs and Frames, carrying out calculations, and exploring the concept of arithmetic mean. For each financial profile, we suggest producing a pie chart. Furthermore, the pair highlights that this proposal can explore other mathematical concepts, such as the idea of degree and circumference due to the type of graph proposed, and the concept of division.

In organizing this activity, Ana and Joaquim used a virtual questionnaire found on the blog Cresol (https://educacaofinanceira.coop.br/teste-de-perfil-financeiro/). This questionnaire consists of ten questions that outline the respondents' financial profiles. This questionnaire was

adapted for students to conduct the research (interview) in person; thus, alternatives A, B, and C of the questions received, respectively, scores 1, 2, and 3, as illustrated in Frame 6.

To awaken critical questions in students, i.e., mobilize statistical literacy skills, teachers highlight that the very context of this proposal is part of students' social reality, considering that the discussion about financial education is quite important in schools. Furthermore, before producing the research data, Ana and Joaquim pointed out that teachers can ask students about the results they expect from the respondents' financial profiles and what they conclude based on the mean of each profile. They also highlighted what students consider can be done with those who are not at the desired level, i.e., who need to be careful and attentive with their financial planning.

Pair 2 also presents some questions and actions that can be provided based on this proposal, as shown in their statements, which follow with our highlights https://educacaofinanceira.coop.br/teste-de-perfil-financeiro/:

> Ana: *On average, how many people were in this profile, for example, right?* If most people, for example, were on an unwanted profile, which is from zero to ten, that is very critical, isn't it?
>
> It means that people don't know how to deal with their money, they owe a lot, they're late on their bills, and then bringing this reflection based on the data, right?
>
> Joaquim: When you bring up the question of evaluating the points one can achieve to know what type of financial education it is, the situation one is in, one can act, take measures, try to correct it, for example, when one does not achieve the desirable. So, doing these reflections with students, what could be done? What could they try to do? What are the perspectives of financial education, together with the perspective of statistical literacy, so that they can reflect on and try to incorporate them into their reality?

Therefore, Ana and Joaquim propose that students know what type of financial profile the interviewees are in and propose reflections on what actions they need to take to improve when they are not in the desirable profile. The objective would be to provide students with reflections on the data and averages obtained to incorporate this discussion into their realities. These teachers also highlighted the mean as a portrait of the reality researched. However, we emphasize that the average does not always reflect the majority it depends on the dataset and the absence of discrepant data.

While discussing the proposals, Ana establishes an association with Carvalho *et al.*'s (2021) article, read during the theorization, highlighting that students can observe the reliability of the information by analyzing how the data are processed. At another point, she highlights that she could problematize graphs with incorrect and manipulated information based on this text so that students can critically analyze the data they collect and organize. The discussion of that text may have induced this aspect.

This teachers' proposal demonstrates possibilities for combining statistical literacy (Gal, 2002) and financial literacy (Giordano, Lima, & Silva, 2021), making it possible to mobilize skills from the components of both literacy models. On the other hand, we observe that we must consider some aspects, such as the variables produced after applying the proposal, and we must make some modifications to calculate the mean.

The GF discussion allowed us to observe that teachers demonstrate difficulties in understanding the mean, its properties, and meanings; that is, they focus more on defining the simple arithmetic mean. The three teaching approaches proposed, one by Pair 1 and two by Pair

2, present relevant contexts for students. They suggest carrying out research in the school context and/or outside it to process the collected data subsequently.

However, based on the analysis of student errors in problems involving the arithmetic mean (Frame 3), teachers reinforced that they do not know this concept in terms of its properties and meanings, nor do they know how to identify the variables that can generate the mean, only the sum and count. Pair 1's proposal and Pair 2's first proposal suggest research that generates empirical variables (qualitative variables). However, in treating the collected data, the proposals were structured so that only counting and summing –not calculating the mean– were possible. Therefore, one proposition would be to include other variables, such as grade and biological sex, as quantitative variables.

Pair 2's second proposition deals with conceptual variables. However, the ideal would be not to classify the possible research results into *careful*, *congratulations*, and *attention* categories (Frame 6). Therefore, it is impossible to calculate the mean, as these are qualitative variables, those that do not have numerical values and can be organized into categories. They can be unordered (ordinal variables), such as biological sex, eye color, hair color, etc., or ordered, such as the month of observation (January, February, ..., December). Therefore, in formative processes, we consider the need to work on the nature of variables as a key concept for teachers to understand that only from quantitative variables can we calculate the mean of a dataset to, thus, develop proposals for this concept from the perspective of statistical literacy.

One possibility of teaching about the nature of variables is to present the propositions developed by the participants of this research, to observe whether teachers realize that it is impossible to determine the mean in the way they were structured and encourage them to develop suggestions so that the work with this concept is possible. One way to teach about the nature of variables is to present the propositions developed by the participants of this research, observing whether teachers realize that determining the mean in the way they were structured is impossible. This can encourage them to develop suggestions so that working with this concept becomes feasible. After that, we could discuss the types of variables and explain that only quantitative ones allow us to calculate the mean.

6 Final considerations

The article analyzed reflections with mathematics teachers working in the final years of fundamental school on possibilities for teaching the arithmetic mean from the perspective of statistical literacy. We observed that when analyzing types of errors, teachers focused more on the technical aspects of the mean (procedure), observing whether students knew how to calculate (definition of simple mean), and on the structure of the problems, pointing out that they should contain other data to assist students in solving these problems. Furthermore, we found that, in their speeches, the teachers made errors like those presented by the students in Eugênio's research (2013). They demonstrated difficulty mainly interpreting the mean as a representative value of a dataset, given that this concept does not always represent the majority and that the value found approximates the operationalized data.

This work presented three teaching propositions prepared by the four teachers, one by Pair 1 and two by Pair 2. They suggest carrying out statistical research on topics that are part of students' reality and the educational context, such as financial education, bullying, films, series, anime, food, and games. According to the pairs, those studies can be carried out with students in class and involve the entire school or the public outside the school context. However, these proposals did not indicate whether teachers know the concept and use of the simple mean.

The pedagogical process developed for the FG interview meetings proved satisfactory

regarding the organization of its elements and time. The teachers' teaching approaches involved appropriate contexts for the final years of fundamental school that could interest the students.

However, the propositions reinforced that teachers do not know the concept of mean in depth, its properties and meanings, or how to identify the variables that can generate the mean. Given this, we consider it necessary to work on the nature of variables as a key concept for teachers to understand when they can calculate the mean of a dataset and to develop proposals for this notion from the perspective of statistical literacy.

Therefore, we emphasize the importance of problematizing this discussion with mathematics teachers so that they reflect on their teaching practice. We highlight, then, the importance of developing and carrying out teaching approaches on the mean from the perspective of statistical literacy, focusing on the context and critical issues and, above all, the elaboration of a formative proposal that constitutes a model for working with the simple arithmetic mean with teachers working in the final years.

Aknowledgments

This work was carried out with the support of the Coordination for the Improvement of Higher Education Personnel - Brazil (CAPES) - Financing Code 001.

References

Alves, T. A. S.; Silva, A. F. G.; Amorim, M. E. & Pietropaolo, R. C. (2019). *Professores explicitam seus saberes acerca das propriedades da média ao início de uma formação continuada*. In: XIII Encontro Nacional de Educação Matemática, Cuiabá, Mato Grosso, Brasil.

Batanero, C. (2000). Significado y comprensión de las medidas de posición central. UNO: *Revista de didática de las matemáticas, 25*, 41-58.

Carvalho, L. M. T. L.; Carvalho, C. F. & Carvalho, R. N. (2021). Dados Estatísticos e Pandemia Covid-19: Reflexões Sobre Dimensões do Letramento Estatístico. In C. E. F. Monteiro & L. M. T. L. Carvalho (Org.). *Temas emergentes em Letramento Estatístico.* Editora Universitária.

Cazorla, I. M. (2004). Estatística ao alcance de todos. *Encontro Nacional de Educação Matemática*, 8, 1-11.

Cazorla, I. M.; Santana, E. R. S. & Utsumi, M. C. (2019). O campo conceitual da média aritmética: uma primeira aproximação. *Revemat*, 14, 1-21.

Costa Júnior, J. R.; Monteiro, C. E. F. & Cavalcante, N. I. S. (2021). Letramento estatístico: explorando dimensões críticas com licenciados em matemática. *EDUFCG*.

Coutinho, C. (2021). Letramento estatístico e letramento financeiro: uma reflexão sobre suas possíveis articulações. *Revista Chilena de Educación Matemática*, *13*(1), 30-41.

Coutinho, S. & Teixeira, J. (2015). Letramento Financeiro: um diagnóstico de saberes docentes. *Revista Eletrônica de Educação Matemática*, *10*(2).

Duquia, R. P. & Basto, J. L. D. (2006). Medidas de tendência central: onde a maior parte dos indivíduos se encontra? *Scientia Medica*, *16*(4).

Eugênio, R. S. (2013). *Explorações sobre a média no software TinkerPlots 2.0 por estudantes do Ensino Fundamental*. [Dissertação Mestrado]. Universidade Federal de Pernambuco.

Ferreira, R. S.; Sarges, A. B. & Lacerda, A. G. (2019). O Ensino de Conceitos Estatísticos: uma proposta com o uso de planilha eletrônica a partir da análise do Índice de Massa Corporal. *Revista de Ciência e Tecnologia, 5*(9).

Gal, I. (2002). Adult statistical literacy: Meanings, components, responsibilities. *International Statistical Review, 1*(70), 1-25.

Gal, I. (2005). Exploring probability in school: Challenges for teaching and learning (pp. 39-63).

Gal, I. (2019). Understanding statistical literacy: About knowledge of contexts and models. In J. M. Contreras, M. M. Gea, M. M. López-Martín & E. Molina-Portillo (Eds.), *Actas del Tercer Congreso Internacional Virtual de Educación Estadística*. Universidad de Granada (UGR).

Gatti, B. A. (2005). *Grupo Focal em Ciências Sociais e Humanas*. Líber Livro Editora.

Giordano, C.; Lima, R. F. & Silva, A. W. (2021). Literacia estatística, probabilística e financeira: caminhos que se cruzam. *Revista de Ensino de Ciências e Matemática, 12*(6), 1-26.

Lopes, C. E. (2012). A Educação Estocástica na Infância. *Revista Eletrônica de Educação, 6*(1), artigos.

Lopes, C. E. (2013). Educação Estatística no Curso de Licenciatura em Matemática. *Bolema, 27*(47), 901-915. Rio Claro - SP.

Macedo, R. C., Pietropaolo, R. C. & Carvalho, J. I. F. (2016). Um estudo sobre estatística na formação continuada de professores: experiência com a curva normal. In: *XII Encontro Nacional de Educação Matemática*, São Paulo, São Paulo, Brasil.

Ministério da Educação. (2018). *Base Nacional Comum Curricular (BNCC)*. MEC.

Novaes, D. V. & Coutinho, C. Q. S. (2009). *Estatística para Educação Profissional*. Atlas.

Silva Júnior, A. V. (2018). *Efeitos do ciclo investigativo PPDAC e das transformações de representações semióticas no desenvolvimento de conceitos estatísticos no ensino fundamental*. [Dissertação Mestrado]. Universidade Estadual de Santa Cruz.

Strauss, S. & Bichler, E. (1988). The development of children's concepts of the arithmetic average. *Journal for Research in Mathematics Education*, 19, 64-80.

Watson, J. (1997). Assessing statistical thinking using the media. In J. Garfield & I. Gal (Org.), *The assessment challenge in statistics education* (pp. 107-121). IOS Press.

Watson, J. & Callingham, R. (2020). Covid-19 and the need for statistical literacy. *Australian Mathematics Education Journal, 2*(2), 16-21.