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Professional vision of Mathematics teachers from rural schools regarding aspects of teaching practice

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Abstract: This study aims to understand aspects of the professional vision of mathematics teachers from rural schools regarding their teaching practice mobilized in the context of continuing education. This qualitative research is empirical, in which the data emerge from discussions held with teachers about the relationships between tasks and mathematics teaching, particularly about a mathematical task proposed in a class from an inquiry-based teaching perspective. The results show evidence of the teachers' professional vision not only on important characteristics of the tasks but also on specific aspects of the dynamics of their classes. Inquiry-based teaching as a possible lens for analyzing materials from the classroom offered alternative references to direct teaching and favored reframing aspects of teaching practice such as proposed tasks, classroom communication, and curriculum management.

Keywords: Continuing Education. Rural Education. Mathematical Tasks. Inquiry-Based Teaching. Teacher's Noticing.

Visión profesional de los docentes de Matemáticas de escuelas rurales sobre aspectos de la práctica docente

Resumen: Este estudio tiene como objetivo comprender aspectos de la visión profesional de los profesores de matemáticas de escuelas rurales sobre su práctica docente, movilizados en un contexto de educación continua. Se trata de una investigación cualitativa de carácter empírico, en la que los datos surgen de discusiones sostenidas con profesores sobre las relaciones entre las tareas y la enseñanza de las matemáticas, en particular, sobre una tarea matemática propuesta en una clase desde la perspectiva de la Enseñanza Exploratoria. Los resultados muestran evidencia de la visión profesional de los docentes no sólo sobre características importantes de las tareas, sino también sobre aspectos particulares de la dinámica de sus propias clases. La perspectiva de la Enseñanza Exploratoria como posible lente para el análisis de materiales del aula ofreció referencias alternativas a la Enseñanza Directa y favoreció el replanteamiento de aspectos de la práctica docente como las tareas propuestas, la comunicación en el aula y la gestión curricular.

Palabras clave: Formación Continua. Educación Rural. Tareas Matemáticas. Enseñanza Exploratoria. Percepción Profesional.

Visão profissional de professores de Matemática de escolas do campo sobre aspectos da prática letiva

Resumo: Este estudo tem como objetivo compreender aspectos da visão profissional de

professores de matemática de escolas do campo sobre a sua prática letiva, mobilizados em um contexto de formação continuada. Trata-se de uma pesquisa qualitativa, de natureza empírica, na qual os dados emergem de discussões realizadas com os professores sobre as relações entre as tarefas e o ensino de matemática, em especial no que se refere a uma tarefa matemática proposta em uma aula na perspectiva do ensino exploratório. Os resultados evidenciam indícios da visão profissional dos professores não somente acerca de características importantes das tarefas, mas também sobre aspectos particulares da dinâmica de suas próprias aulas. A perspectiva do ensino exploratório, como uma possível lente de análise de materiais oriundos da sala de aula, ofereceu referências alternativas ao ensino direto e favoreceu a ressignificação de aspectos da prática letiva, tais como as tarefas propostas, a comunicação em sala de aula e a gestão curricular.

Palavras-chave: Formação Continuada. Educação do Campo. Tarefas Matemáticas. Ensino Exploratório. *Noticing* do Professor.

1 Introduction

Teachers' professional practice has various and peculiar characteristics that "are certainly one of the factors that most influence the quality of teaching and student learning" (Ponte & Serrazina, 2004, p. 2). Due to its complexity, "to describe and understand teachers' practice, it is essential to identify their actions, intentions, and motivations" (Oliveira, Menezes & Canavarro, 2013, p. 30).

Some studies on initial teacher education (Rodrigues, Cyrino & Oliveira, 2018; Rodrigues, Oliveira & Cyrino, 2022) and teacher continuing education (Sherin & van Es, 2009) have been based on the concept of professional vision (Goodwin, 1994) to investigate not only teaching actions but also the lenses through which they are seen and understood by the group of professionals who develop them. "Members of a profession use discursive practices to shape events in the domains to which their professional analyzes are subject" (Goodwin, 1994, p. 606, our translation); that is, they refer to what can be learned through of meanings or experience about specific situations in a group.

According to Sherin and van Es (2009), Goodwin (1994) introduced the term *professional vision* to describe the ability that members of a professional group share to interpret phenomena central to their work. In this sense, studying aspects of teachers' professional vision can highlight relevant processes of teacher professional development, which are directly linked to professional practice (Ponte & Serrazina, 2004). The primary aim of this research is to reveal aspects of the professional practice of mathematics teachers linked to rural schools.

According to Decree N. 7.352/2010 (Art. 1º, § 1º item II), rural schools are those located in rural areas, as defined by the Brazilian Institute of Geography and Statistics Foundation (IBGE), or in suburban neighborhoods with rural characteristics, and are designed to attend the people who live in those areas. Besides location, the rural school is aimed at rural people because "only schools built politically and pedagogically by rural subjects manage to have the way of being of the rural areas and incorporate in this way of being the rural people's forms of organization and work" (Caldart, 2003, p. 66).

Thus, field education is conceptualized as a field of research and social struggles in which the "bringing together rural social movements and government bodies was essential to highlight in official documents the emphasis on preserving the identities of this unique reality" (Silva & Silva Júnior, 2011, p. 323). Thus, it is necessary to consider the concrete conditions of the production and social reproduction of life in the countryside and "defend the right of a

population to think about the world from the place where they live" (Fernando, 2012, p. 97). In addition to a dialogue with pedagogical theory, field education is based on the particular reality of rural people to promote the education of the rural working population and, more broadly, their human formation (Caldart, 2004, p. 3). Therefore, the professional practice of the teacher who teaches mathematics in rural schools presents specific characteristics related to their identity.

Based on these assumptions, research has highlighted the importance of continuing teacher education that promotes dialogue between formal knowledge (academic/school mathematics) and social knowledge (practice or mathematical thinking for social needs) to give new meaning to the professional actions of the teacher who teaches mathematics (Moré & Rodrigues, 2023).

From this perspective, the investigation presented in this work was developed in the context of continuing education of mathematics teachers who work in rural schools. The research was based on the analysis of materials originating from teaching practice, linked to the tasks proposed in the classroom and their relationship with the mathematics teaching approaches that the teacher intends to develop. In particular, those that oppose direct education stand out (Bridge, 2005), such as inquiry-based teaching (IBT).

The inquiry-based education is rooted in an *inquiry-based teaching* perspective (Oliveira & Cyrino, 2013). It involves developing challenging situations in which students are asked to analyze and question phenomena and explain what they noticed (Hattie, 2009). However, when teachers set out to develop an exploratory practice, they encounter many challenges, for example, task selection, the identification of aspects of reasoning to value, and the type of communication to be developed in the classroom (Ponte, Mata-Pereira, Henriques & Quaresma, 2013; Stein, Engle, Smith & Hughes, 2008).

Another complex practice involves promoting mathematical discussions with the class to consolidate mathematical knowledge and contemplate students' personal and collective meanings (Stein *et al*., 2008). Therefore, we consider that IBT can be linked to field education through tasks that enable the execution of mathematical activities in which students feel challenged and able to develop autonomy based on the knowledge of those who live in and near rural areas.

In this formative context, this study discusses aspects of the mathematics teachers' professional vision in rural schools, focusing on their teaching practices. Considering that most of the work involving (prospective) teachers' professional vision refers to mathematics teaching in urban schools, this study becomes relevant when we intend to highlight teachers' professional vision regarding specific practice issues in the context of field education. In this scenario, the IBE perspective is adopted as a way to enable the development of classes that value belonging to the field, taking into account students' ideas and their needs (Arroyo, Caldart & Molina, 2004; Caldart, 2004, 2009; Teixeira Júnior, 2020).

To this end, this research is empirical in nature, and the data produced comes from the transcription of the recording of formative discussions on two different materials. The first is a text by Ponte (2005), which deals with curriculum management in the classroom, addressing, in an interconnected way, the establishment of teaching objectives, the methodological strategies adopted, and the tasks selected. The second material is the task "The Necklaces," chosen because it covers crucial characteristics for a class from the IBT perspective (Jesus, 2016) and mainly because it was implemented by the teacher educator (first author of this work) in a rural school classroom.

2 Teacher's professional practice and vision

Training practice encompasses initial and continuing education and professional development, as "education throughout one's career is part of one's professional life" (Ponte & Serrazina, 2004, p. 26). According to Baldini and Cyrino (2016), teachers' learning during education constitutes a key element for their professional development. Following Jesus (2011), in this work, we consider professional development from the perspective of lifelong construction, even before initial education, in a continuous process of analyzing the teacher's professional practice.

Figure 1 presents the description of teachers' professional practices and their main aspects, according to Ponte and Serrazina (2004).

Figure1: Description of teachers' professional practices

Professional practices of Teachers	Essential aspects	Description of aspects
(i) Teaching practices	- Proposed tasks - Materials utilized - Communication in the classroom - Curriculum management - Assessment	They occur in the classroom and are more closely oriented towards students' mathematical learning.
(ii) Professional practices in the institution	- Collaboration	It deals with interactions with other professionals within the school institution.
(iii) Training practices	- Education	These are aspects of the search for teaching knowledge and include initial and continuing education and, more recently, the need to highlight the importance of thinking equally about the teacher's professional development.

Source: Prepared by the authors based on Ponte and Serrazina (2004).

According to Ponte and Serrazina (2004), there are two central stimuli for teachers to participate in formative actions: (i) the need to acquire essential knowledge to implement new aspects of the programs and (ii) a general desire to update, based on the curriculum documents guidelines and methodologies that involve knowledge frequently associated with the use of technologies in teaching.

However, the lack of connection between what is proposed in training and the teacher's practice suggests a minimal impact on changes in classroom actions. Assemany, Costa, and Machiavelo (2020, p. 25) suggest that continuing education considers "the human (material) that is there, promoting moments of dialogue, unforeseen events, collaborative work, and changes in the planning made by the teacher educator." Furthermore, Mocrosky, Orlowski, Campanucci, and Pereira (2021) emphasize that formative actions that disregard the teacher's professional practice are still rooted in a colonizing formative perspective. For changes to occur, "it is essential that the teacher is engaged in the process so that he or she can develop a different vision of themselves and the teaching and learning processes" (Jesus, 2011, p. 18).

Goodwin's research (1994, p. 607) uses the term *professional vision* to describe the

perspectives shared by professional group members in interpreting events highlighted as necessary to their work, which "is a socially situated activity carried out through the implementation of several historically constituted discursive practices." More broadly, *professional vision* is defined as "socially organized ways of seeing and understanding events that are responsible before the distinct interests of a given social group" (p. 606). Particularly, the ability to pay attention to certain events and attribute them meaning based on the analysis of classroom interactions is a critical aspect in the constitution of the teacher's specific professional vision. This process occurs through two distinct subprocesses: selective attention and knowledge-based reasoning (Sherin, 2007).

For Sherin and van Es (2009), in the classroom, many things happen simultaneously, and teachers focus on what they consider most significant; in other words, they direct their attention to what they believe to be relevant. This choice about what to pay attention to highlights some events as relevant in classroom interactions, whose understanding depends on the knowledge mobilized about them. This process is described by Sherin (2007) as knowledge-based reasoning. Sherin and van Es (2009) exemplify that teachers can reason about a certain event based on their knowledge of the subject, the curriculum, or students' previous comments.

Therefore, developing teachers' professional vision also involves learning processes (Rodrigues, 2017). According to Baldini and Cyrino (2016, p. 186), only "courses, seminars, workshops" are not enough for learning and changes to occur, "it is necessary to encourage them to investigate their own practice, to solve the problems of this practice, to develop collaborative work so that they can manage and make a commitment to their education."

These are fundamental elements in the education promoted within the scope of this research. We sought to articulate the teaching practices of participating teachers with representative elements of practices, such as the case of the ET. This is accomplished through sharing resolutions and analyzing a task used in a class in the rural school context, given that *highlighting* significant aspects of the analyzed situation is not a practice that develops in isolation or individually but that "connects relevant characteristics of a scenario with the activity that is being carried out in that scenario" (Goodwin, 1994, p. 628).

3 Exploratory teaching and the tasks proposed in the classroom

Exploratory education is based on inquiry-based teaching (Maaß & Artigue, 2013; Cyrino & Oliveira, 2016). This approach foresees the development of students' mathematical learning based on their investigative, questioning position, and connected to the construction of meanings. This is achieved by engaging in tasks that are challenging for them. We assume here the concept of tasks, as defined by Bridge (2014), that differentiates them from activity:

> A task may or may not have potential in terms of mathematical concepts and processes that it can help mobilize. It can give rise to different activities, depending on how it was proposed, how students work, the learning environment, and their own capacity and previous experience. For its part, an activity corresponds to one or more tasks carried out within the framework of a specific situation (Ponte, 2014, p. 16).

Based on Stein and Smith (2009), tasks that require only memorization and procedures without connection to meanings are not seen as cognitive challenges for students. Those that require the development of procedures linked to meanings or that involve the practice of doing mathematics are considered cognitively challenging.

To ensure an adequate level of cognitive demand for students (Stein & Smith, 2009), it

is recommended that the task is by the specificities of the class. Furthermore, it must be structured to lead to analysis and reflection, enabling a gradual construction of concepts (Jesus, Cyrino & Oliveira, 2018). Furthermore, for students to be able to develop cognitively challenging tasks, it is necessary to offer them the conditions to develop different resolution strategies and representations. This favors student autonomy –but does not disregard the importance of teacher support– from the promotion of a dynamic of dialogical interactions (Wells, 2004), which, according to Rodrigues (2017, p. 30), involves "communication, between pairs or collectively and, which considers the other's ideas in the joint construction of knowledge."

According to Cyrino and Oliveira (2016), the dialogical perspective (Wells, 2004), on which ET is based, foresees the construction of knowledge in a collaborative way, highlighting the role of communication in interactions between teacher and student and between the students themselves. Therefore, the construction of knowledge is a process "that integrates action in cooperation with others and reflection on what was learned in this process" (Cyrino & Oliveira, 2016, p. 22). This dynamic is opposed to direct teaching, based on transmitting information and the technical resolution of exercises (Bridge, 2005). According to Jesus, Cyrino, and Oliveira (2020), societal changes require teachers to cast aside the idea of teaching as transmitting information and embrace the role of guiding student learning.

EE requires teachers to pay attention to aspects considered before class. This involves "a careful preparation of the class by the teacher, who allows flexibility in conducting the class, making decisions and changing the pre-established planning based on students' activity" (Oliveira, Menezes & Canavarro, 2013, p. 49). The selection or elaboration of the task is one of the primary aspects at this stage, as it involves mobilizing teachers' expectations and intentions regarding what they want the students to develop or understand, together with predicting how they intend to carry out the class dynamics, its actions, interactions, and decisions. Canavarro, Oliveira, and Menezes (2012, p. 264) show that "the overwhelming majority of the teacher's actions arise as a response to her interpretation of the needs of the class or some particular students given the mathematical purposes for the class."

For Sullivan, Clarke, and Clarke (2013), teachers' actions when selecting or preparing the task are influenced by their previous considerations about students' difficulties and, possibly, by their confidence in their own mathematical knowledge. These authors highlight the importance of teachers working on the task before class to maximize its mathematical and pedagogical possibilities. Solving the task is an essential step in anticipation. It is crucial that teachers "know the task they will propose to the students, solving it in different ways, anticipating different strategies and representations, as this will make it possible to outline students' potential difficulties" (Jesus, Cyrino & Oliveira, 2020, p. 116).

By anticipating possible situations, teachers can predict responses that allow them to guide students without threatening their autonomy. Furthermore, its objectives regarding mathematical learning aim to organize a dynamic to improve class management. In the practice of anticipating possible task resolutions, teachers are allowed to predict different strategies to be developed by students during the four phases of the class (Canavarro, Oliveira & Menezes, 2012; Cyrino & Oliveira, 2016), in which the IBT is structured:

1st phase—Presentation and proposal of the task: This phase consists of presenting the task to the students, checking how they interpret the statement, and encouraging students to take ownership of the task and carry it out.

2nd phase—Task development: Involves the teacher's actions to ensure students

develop the task independently, without reducing cognitive demand (Stein & Smith, 2009). At this stage, there is a need for the teacher's perception (Rodrigues, Cyrino & Oliveira, 2019) of the "work carried out by the different groups in order to choose and sequence the resolutions to be presented in a large group" (Cyrino & Oliveira, 2016, p. 24).

3rd phase—Collective discussion: This phase consists of organizing appropriate conditions for verbal and written presentation of the task's resolutions. This is when the procedures and strategies students use to solve the task are communicated. At this stage, the teacher's actions require managing interactions to improve the mathematical quality of the presentation.

4th phase—Systematization: In this stage, the teacher seeks to articulate the main concepts, ideas, or mathematical procedures, aligning them with what the students have constructed and their objectives for mathematical learning.

The class structure in ET implies two main and specific actions on the part of the teacher: 1) students' mathematical learning, and 2) class management, organizing, and managing interactions (Oliveira, Menezes & Canavarro, 2013). When addressed in formative actions, these aspects tend to promote reflections that provoke thinking in anticipation as a way of preparing for decision-making during the action.

> To describe and understand the teacher's teaching practice, it is essential to identify not only their actions but also the intentions that are incorporated in those actions, the reasons that justify their behaving in a certain way, namely those that derive from their teaching context (Canavarro, Oliveira & Menezes, 2012, p. 257).

In the next section, focused on the teachers' professional vision, we will situate the research and describe how data was produced based on the analyses proposed to teachers in the context of continuing education.

4 Context and methodological procedures

This study brings data from the activities developed in an extension project for analysis. The project focused on a continuing education course for mathematics teachers who teach rural schools and included the participation of 12 professionals from different locations. It is, therefore, qualitative research based on interpretative analysis (Erickson, 1986).

Erickson (1986, p. 122) says that the central questions of interpretive research "concern questions of human choice and meaning and, in this sense, they concern questions of improvement in educational practice." This training aimed to promote a space where teachers felt part of something, encouraging them to recognize and value the actions in the rural school and understanding them through dialogue and reflection (Cyrino & Jesus, 2014; Nagy & Cyrino, 2014).

We opted for online Google Meet weekly meetings because rural schools are located far from each other and the city. These meetings began on May 10th and, after a break during the month of July, ended on August 16th, 2022, totaling eight meetings lasting two hours on average.

Some activities were carried out asynchronously and noted in the *field diary* to contribute to discussions in subsequent meetings. All 12 participating mathematics teachers work in rural schools. Ten were from Mato Grosso do Sul, and two were from São Paulo. The objectives of each meeting were based on establishing situations to promote interactions to

understand the specificities of the profession and its actions in mathematics teaching in rural schools. This article will deal with situations experienced during the 3rd and 4th meetings. However, to highlight elements of the formative context, we will also describe how the 1st and 2nd meetings were developed, as we understand that the entire process influences the aspects identified in the data.

1st meeting: Teachers were asked about their expectations for training and what they expected as a contribution to their profession. We proposed some agreements and drew attention to the importance of individual and collective participation, the responsibility assumed, and even developing research, considering the advances already achieved in field education.

Collaboration, which promoted interactions throughout the training, consisted of verbal participation during discussions in the eight synchronous meetings, as well as written participation, both in the Google Meet chat and in the notes in the field diary. This instrument used in training is a Word document that participants complete during the week and hand out to trigger discussions at the meeting. For the following meeting, we asked each participating teacher to record in their respective diaries a task used to teach mathematical concepts in the classroom at the rural school to share with their peers.

2nd meeting: We initially focused on the presentation of tasks selected or prepared by the participating teachers to reflect on the aspects that influenced this process. Furthermore, there were discussions about *tasks* and how the term differs from *activities.* Finally, we discussed the concepts of rural schools from the perspective of these professionals, aiming to raise discussions related to the context and how this is linked to the decisions made in selecting or preparing the task. For the next meeting, we handed out an article (Ponte, 2005) and asked them to answer some questions on the text, which should be returned in the field diaries.

3rd meeting: It was initially based on the post-reading discussion of the text by Ponte (2005), received during the previous meeting. The text addresses, in an interconnected manner, curriculum management, teaching objectives, tasks proposed in class, and the methodological strategies adopted. This reading aimed to investigate the most frequent mathematical tasks in their classes and, mainly, introduce task analysis according to the possibilities of adopting them from the ET perspective. For the following week, we agreed that they would solve the "The Necklaces" task in their field diaries and, later, answer some questions to analyze the task, illustrated in Figures 2 and 3.

The chosen task, considered cognitively challenging, involves patterns and regularities and can potentially develop students' algebraic thinking based on different types of representations and a variety of possible solutions (Jesus, 2016). The field diary also included some questions related to an analysis of the implementation of this task in the classroom.

4th meeting: The participating teachers solved the task "The Necklaces" and forwarded their productions in advance to the teacher educator, who organized the resolutions into photo slides and returned them to ensure sharing. During this synchronous meeting, when the resolutions were presented and discussed, the focus was on the feasibility of implementing this task in the classroom, specifically in the rural schools where the teachers work.

The analysis materials come from audio and video recordings of synchronous encounters, represented by the acronym SE, followed by the number corresponding to the meeting. In this article, we will specifically focus on the discussions of the 3rd and 4th meetings, which, with express authorization in the Free and Informed Consent Form (TCLE), were recorded and transcribed, with minimal correction of errors in the spoken language to preserve

the original context. We used fictitious names to guarantee participants' anonymity (except for Camila, the teacher educator).

Figure 2: "The Necklaces"

Inês made three necklaces with black and white beads, as shown in figures 1, 2, and 3.

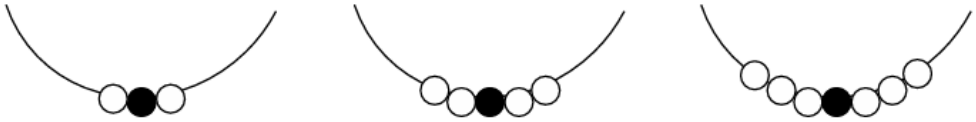

Fig. 1 **Fig. 2** **Fig. 3**

Number of necklace beads

1. Indicate the total number of beads for each figure.
2. Continuing this sequence of necklaces, how many beads would the necklace corresponding to the following figure have?
3. How many beads would the necklace corresponding to Figure 8 have?
4. Without drawing, find out how many beads the necklace corresponding to Figure 19 would have.
5. Is there a necklace with 55 beads in the sequence? Explain your reasoning in detail.
6. Describe a rule that allows you to determine the total number of beads in any figure in the sequence.

Source: Adapted from Pedro (2013).

Figure 3: Questions for analyzing "The Necklaces"

Consider the mathematical task "The Necklaces."

1. Solve the task and present your solution.
2. What mathematical ideas can emerge from solving this task?
3. What strategies and representations can students use to solve this task?
4. What difficulties might students have when solving this task?
5. What is the potential of this task for students' mathematical learning?
6. At what level of education can this task be worked on?
7. Would necklaces be an idea for an attractive task, does it refer to something in your students' context?
8. If the answer was no: Would you be able to adapt this task to something at your school? If the answer was yes: Can you describe what the class dynamics would be like by developing this task?

Source: Prepared by the authors.

Due to the number of participants and questions, the criteria for choosing excerpts from the discussions in the formative meetings aimed at accomplishing the research objective outlined in the introduction of this work.

5 Data description and analysis

Next, the data will be described and analyzed in three subsections, each characterizing aspects of the professional vision of mathematics teachers from rural schools in relation to their teaching practice, which were mobilized in the context of continuing education.

5.1 Initial considerations about teaching practice based on the IBT perspective – 3rd meeting

At this meeting, we used the presentation tool of Google Meet to address points highlighted in the text, which led us to discuss the IBT:

(A1) Camila: *Regarding the text (Ponte, 2005), what did you identify in the text that connects with your practice? [...]*

(A2) Suellen: *[...] This investigative work is a lot of work; the teacher has to have time to prepare it. [...] As we must cover the curriculum, we talk, we go ahead and explain, give exercises and don't leave space for all this investigation, this creation on the part of the student [...].*

(A3) Fabiola: *Besides being hard work, I see the following, sometimes you prepare a plan and when the time comes you need to go elsewhere, due to the pandemic we are now with recomposition of learning, you will work on content, the teacher, but which way does it go? You have to go back all over again, explain where that came from, the beginning, so you can start imparting that content. So it's a little complicated because of that too, the recomposition of learning thing. We have to teach not only content that should be covered in a specific class, that grade's content, but bring what they haven't seen before, that is, "seen" in quotation marks, right, because they haven't really seen it.*

(A4) Lorena: *It's like that, right? I think everyone is, I was feeling really behind this year with the content with my students because I have to backtrack all the time. I went to talk to other teachers from other schools and everyone is in the same situation, we always have to go back to previous content because they are really struggling.*

(A5) Pearl: *Camila, I also use Fabíola's words, I was, I mean, we, I'm still from the last century, we are trained to convey concepts; as I read the text, I didn't do the activities for various reasons this week, but I read it and I feel this difficulty. Wow, is that a problem? Am I doing the investigation? Am I in the exploratory field? Because I didn't learn that way. [...] So, in this investigative way, you encourage the student to think. So this is very good, not only in the field but also as a professional, because those who are now leaving university already carry this vision, while we from the last century have more difficulty.*

(A6) Lucas: *I just wanted to make a speech, amending a little what my colleagues said and what you brought in the presentation about the reports, which we have to do and, today, students don't want to write, few want to read and write [...] in the New High School, the subject I worked on we were requested to solve the task, and the students had to record the methods for solving as if it were an investigative activity, but with the pandemic this didn't work [...] and now, in mathematics modeling in the field school I am working, I ask the students to report haw they had solve [the problem], but few do it The students say: "Teacher, I like doing it." You know, they like to see the result, but they don't like to explain how they got there, or they want to explain it by saying that it's easier than writing it down, they don't like writing it down. [...] I speak, write, put it on paper.*

(A7) Karla: *The State is content-oriented. One must impart the content, and that's it. It doesn't matter whether the student learned or managed to get there, but we have always tried to try other ways. We participate in study groups because our students don't learn the way we learned before.*

(ES, 3)

In A2, Suellen seems to recognize the importance of promoting students' development of autonomy and the construction of knowledge, avoiding providing direct answers. She emphasizes that, to this end, the teacher must anticipate the class (Canavarro, Oliveira & Menezes, 2012; Cyrino & Oliveira, 2016). However, she understands that such aspects of an inquiry-based education practice require a lot of work, given the professional problems linked to the lack of time to prepare the class and the requirement to comply with the curriculum.

This vision is confirmed and contextualized by Fabíola in A3 when she shares the

curriculum requirements proposed by the Mato Grosso do Sul Department of Education (SED/MS), called *recomposition of learning*. Those measures were implemented to address the educational impacts generated by the COVID-19 pandemic during 2020 and 2021.

Next, in A4, Lorena also directs her attention to this situation that worries her. She interprets that she is always late with the content foreseen in the curriculum, as she needs to revisit previous content. The perception of these aspects, recognized in the speech of other teachers who experience the same professional reality (Goodwin, 1994), highlights evidence that "students' mathematical learning is strongly influenced by teaching practices, in the context in which they occur" (Cyrino & Oliveira, 2016, p. 23). In addition to highlighting the difficulties students presented after the pandemic regarding curriculum demands, Perla (in A5) associates teachers' difficulties in rethinking their mathematics classes with the strong influence of the direct teaching practice experienced in school and initial education.

Lucas's focus in A6 is on the challenges inherent to mathematical communication in the classroom, typical of the dialogical perspective (Rodrigues, 2017; Wells, 2004). According to the text, he understands that, when developing the task, students should register how they developed the mathematical strategy used to solve it. Furthermore, in the collective discussion, it is necessary to explain it to the whole class. However, from the inquiry-based education (IBE) perspective, these forms of interaction expected from students in class phases do not match the usual communication method in mathematics classes, where they are usually just listeners.

In A7, teacher Karla relates the constant demand to comply with the State-approved curriculum (SED/SP) to a frustration generated by realizing that this requirement prevails over students' learning. However, she explains that she has sought other references in study groups to promote student learning.

Teachers highlighted challenges in implementing mathematics classes from the IBE perspective, including the transformation from the way one learns to how one teaches, academic education influence, the discrepancy between the amount of content in the curriculum and the time available in mathematics classes, and the predominant forms of mathematical communication in the classroom. Such perceptions of actions of class management and anticipation seem to constitute a discourse of the profession that influences these teachers' views on the IBT practice (Goodwin, 1994).

5.2 Considerations on the teachers' resolutions of the task "The Necklaces" and their resulting didactic intentions – 4th meeting

Teachers' task resolutions were presented for the analysis of their characteristics, sparking discussions about their implementation in the classroom.

At the beginning of the presentation, we noticed less interaction, possibly due to participants' lack of confidence in presenting their resolutions. Some participants reported this feeling in informal conversations carried out individually on WhatsApp. As the educator progressed with the presentations, the teachers began to recognize the mathematical validity of their strategies and interact more.

When presenting the resolutions to Question 5 of the task, the educator asked a question that triggered the following discussion:

(A8) Camila: *Did you imagine that you would have different resolutions like this?*

(A9) Karla: *Look, I think like this: the same way we do this one (question 5) using division, 54 divided by 2, which gives 1 more, but the student with difficulty with division will go to the side they know better. So,*

they will do both parts; they have understood that there is the same amount on both sides. If the figure has 19 [...]

(A10) Camila: *Yes, you did it based on the idea of the sum of both sides; do you think the students would reason this way?*

(A11) Karla: *Yes, they don't choose division; if they have difficulty with division, they choose the algorithm that they know; I think that most of my (students) would go by this one (solution using the sum).*

(A12) Vanessa: *So, I found it interesting. I was working on percentages with my students, I taught them various methods, I had their methods too, you know, and then each one did it differently. Then, they looked at the others' and said: theirs is wrong, they didn't do what I did. [...] So, I found it very interesting that everyone did it in their own way and now here, like the teachers too, everyone did the task in their own way.*

(A13) Camila: *That's it. The interesting thing is to notice how the other person thought. They think that mathematics only has one correct way, one process, and when you bring this type of task up, you validate that there are several ways.*

(ES, 4)

In A9, teacher Karla highlights her knowledge of students' difficulties with division and predicts an alternative way of solving this issue through addition. Thus, she seems to understand that the task allows students to rely on their learning experiences, an aspect highlighted by Wells (2004). In A11, she concludes that students use strategies based on their own knowledge.

Based on what Karla brought in A10 and A11, Vanessa, in A12, points to the students' belief that mathematics is a ready-made science and that there is only one correct way of solving it. According to Sullivan, Clarke, and Clarke (2013), when the task can be solved in different ways, it can engage students in an intellectual activity that develops autonomy and self-confidence. Furthermore, it seems to recognize the importance of students presenting resolutions based on their formative experience. In the context of IBE, this is the collective discussion phase of the task (Canavarro, Oliveira & Menezes, 2012).

5.3 Projections on teaching practice with the task "The Necklaces" in the context of the rural school – 4th meeting

When the discussion started, the teachers were more confident and participative while presenting the resolutions. Thus, they focused on the intentions and possible challenges of teaching practice based on the "The Necklaces" task in the rural school context.

(A14) Camila: *I brought here clippings of your answers, but I would like you to verbally express your impressions, whether this was what you expected from a task in exploratory education, whether this was what you remembered, those who had already studied it, and then we will discuss the question of context. Solving the task is an important part of analyzing this task [...] I found it interesting about the content; some suggested the idea of doubling because it would be possible to think of another way [...]*

(A15) Karla: *So, I put this double part there because they'll think twice. It's on one side, and it's on the other. It's double, and it reminded me of this content because there's a question down there that asks where you can use this, in which grades, I think from the 6th grade onwards, have the learning recuperation program here, just like you said. The activity is initially very simple; then, you can get students to start counting and drawing. Depending on the direction you give the activity, the questions become complex and reach other audiences.*

(ES, 4)

In A15, teacher Karla analyzes the knowledge of her rural school students and reports

the possible paths they can take. She also analyzes the task, addressing one of the important characteristics of a cognitively challenging task (Stein & Smith, 2009): the progressive increase in difficulty, which starts from a simple question to offer support and confidence for the student to build more complex reasoning for challenging questions (Ponte, 2005).

On the one hand, this characteristic Karla noticed makes her describe the task as possibly assisting students with or without difficulties in the same class. Wells (2004) highlights this aspect, addressing the importance of offering opportunities for the students to act following what they know while moving toward what is yet to be learned. On the other hand, Karla seems to interpret that only some students can solve the task. It highlights the role of teacher guidance in the task development phase (Cyrino & Oliveira, 2016). At this point, Karla's perception diverges from everyday observation. In particular, she recognizes that there needs to be flexibility in the direction given to students as they develop at different rates and have different strengths (Louie, 2018).

Next, the participants' statements address Question 4, read again by the educator, to spark discussion:

(A16) Camila*: What difficulties might students have when solving this task?*

(A17) Elza*: I think they could answer the number of beads in any necklace, but create an expression, that formation law, right, a little expression to calculate the number of beads in a given necklace, I think it's difficult for them.*

(A18) Fabiola: *So, what Elza said, I saw it today in a 1st-grade class, the teacher explained the numerical sequence and asked them to formulate, the logic, the law of formation of that sequence, and it was like Elza said, they knew the result, but they had great difficulty formulating the law of formation of this sequence.*

(A19) Camila: *Yes, and this is not something simple. But I really believe in developing, because they have this capacity, in a way they are used to not needing it, but they have the capacity.*

(A20) Pearl: *They receive it and that is it, they are technically used to it, you just throw them the formula and they go to the technical side there. [...]I thought that with the drawings, we would start working from the 5th and 6th grades onwards. I found it very interesting, and as I am also a support teacher, I am a support teacher in the first years of elementary school; in the 3rd grade, I worked on a way of teaching even and odd numbers. Children in the 3rd grade, they have it like this, if you put + 1, it's an odd number, if you put + 2, it's an even number [...] and here they also found the formula for odd numbers, so when I saw this task I thought that it would be possible to work on it, of course, without the creation of a formation law, but a way for you to show it to the student.*

(ES, 4)

In this meeting, we identify signs of a search to understand how to act from the EE perspective when faced with students' difficulties with the task. The excerpt represents a significant movement in how teachers see and understand these difficulties, i.e., in their professional vision. Building on the discussions held at the third meeting, the challenges are interpreted differently here.

According to the IBT, teachers seem to understand how to create strategies to support students' advancement. They see IBT as a mathematics teaching perspective that differs from direct teaching. In this approach, responding appropriately to the student's thinking is based on the perception of mathematical understanding and does not mean providing the right answer immediately but rather enabling the student to act in the process of solving the task (Nagy & Cyrino, 2014; Sherin & van Es, 2009).

This is evidenced when, in A20, Perla associates technicism with direct teaching

exercises but adds that it is possible to rethink this practice, as IBT contributes to significant learning (Cyrino & Oliveira, 2016). Therefore, she considers the possibility of developing classes with tasks that have such characteristics and implementing IBT, especially in the task development phase. At this stage, the teacher helps students solve the problem by developing procedures that have meanings connected to the situation explored in the task. There is an understanding that the changes are difficult not only for teachers but for students, too. These are, therefore, issues of classroom culture, which depend on continuous actions and time to be modified.

With this, after the discussion encouraged by the analysis of the task, we identified the creation of new meanings for aspects of the teachers' professional vision, which until then were mobilized in the characteristics of the tasks and the mathematics classes concerning the way the statement is constructed and arranged in the task; ways to enable student involvement, reasoning skills, and versatility in solving the task; and strategies for adapting to the curriculum, based on the connection of different mathematical ideas that can emerge from the resolution and discussion (Sullivan, Clarke & Clarke, 2013), concerning curriculum management, as pointed out by Ponte and Serrazina (2004).

6 Conclusions and some considerations

In this qualitative and empirical study, the data were produced in discussions that took place during the continuing education of mathematics teachers who teach in rural schools, based on the study of the relationships between the tasks proposed by the teacher in the classroom and the teaching experiences one wants to develop in this context. This process investigation allowed us to identify signs of the development of the participating teachers' professional visions, not only regarding important characteristics of the tasks for students' mathematical learning but also regarding aspects of the class dynamics specific to their practices, considering references and materials inherent to the inquiry-based teaching (IBT) perspective as a lens for analysis.

In particular, the discussions promoted in the 3rd and 4th meetings made it possible to analyze aspects of the professional practice of these teachers. This allowed the recognition of at least three aspects of their teaching practice, as presented by Ponte and Serrazina (2004): the proposed tasks, communication in the classroom, and curriculum management.

When considering these aspects mobilized in the IBT, we perceived a movement of resignification regarding i) the proposed tasks that can enable the production of mathematical meanings to the procedures created by the students, ii) the development of students' autonomy in solving these tasks; iii) consideration of students' previous knowledge, and iv) the importance of communication in different ways between teachers and students and between the students themselves.

Initially, these aspects were interpreted based on discourses related to direct education, which were strongly present in their teaching experiences and academic background. This is evident, especially when they recognize the need to promote classes where students engage with cognitively challenging tasks (Stein & Smith, 2009). However, at the same time, they consider that students will not be able to solve them or will only express themselves in writing, presenting the final answer. Likewise, when they discuss that students bring difficulties from previous years resulting from the pandemic period, they consider the transmission of information as a way of not delaying the content.

In the 4th meeting, after the presentation and discussion of their resolutions for "The

Necklaces" task, we identified aspects that are recognized as important for student learning, inherent to the IBT class dynamics (Canavarro, Oliveira & Menezes, 2012; Cyrino & Oliveira, 2016): a type of support offered by the teacher to students, and which does not harm their autonomy. This is a prominent aspect in the 2nd phase of the IBT and in the 3rd phase of the class, in the learning possibilities involved in the collective discussion of the task resolutions, as occurred in continuing education.

The collective study of Ponte's text (2005) raised discussions on specific issues in mathematics teaching. In particular, the teachers showed signs of understanding important aspects from the IBT perspective, such as the anticipation of the class and the forms of communication in the classroom, both between the teacher and the students, so as not to provide them with direct answers that would harm their autonomy, as well as that of the students with the teacher and with other students, when recording and explaining their mathematical ideas. However, when sharing similar teaching experiences, teachers' interpretations of these aspects seem to have focused more on the discourses produced among members of the professional group (Goodwin, 1994), linked to the lack of time to prepare classes, curricular demands and the learning difficulties of students after the pandemic, as well as the influences of the direct teaching practice experienced in initial education, without envisaging alternatives to deal with these problems.

Goodwin (1994, p. 610) explains that these "characteristics can be difficult to see" when referring to aspects whose understanding is considered significantly relevant to the learning of a certain professional group. Therefore, professional lenses from professionally produced artifacts are provided to assist the perception of essential aspects and move towards reifying the object to which they are intended to give visibility. Thus, as teacher educators, we proposed discussing the text and professional learning situations based on the resolution and analysis of a mathematical task to discursively promote the perception of characteristics of the task and the class from the IBT perspective. We consider that this can expand the possibilities of thinking about mathematics teaching practice in rural schools beyond the discourses predominantly based on direct education and towards the principles of field education (Brasil, 2005).

Furthermore, the results denote recognition of the influence of classroom culture, especially those pertinent to formative practices already experienced by teachers. However, the way the materials relating to the practice of ET were arranged, in conjunction with the issues addressed and culminating in discussions, enabled teachers' involvement in the analysis of their individual practice with collaborative contributions in a group of professionals who share similar experiences (Baldini & Cyrino, 2016), with which they could identify and reason based on their knowledge about the curriculum, students, and context (Sherin & van Es, 2009).

However, the fact that the meetings were not in person may have influenced participation more eloquently, as it requires using tools such as a microphone and camera for interaction. Regarding the peculiarities of the schools where these teachers teach mathematics, the relevance of contextualized tasks (Sullivan, Clarke & Clarke, 2013) and other aspects that are the focus of general research must be addressed in other works due to their complexity.

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