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Resources of curriculum materials that induce teachers' professional knowledge of the additive conceptual field

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Abstract: Assuming that curriculum materials reverberate the knowledge of teachers, we sought to *identify and discuss resources of the materials, related to the additive field, which induce professional knowledge in Mathematics*. Through a documentary analysis of three 2nd, 3rd and 4th grade Mathematics Teacher's Manuals, the addition and subtraction problems were mapped and analyzed; the texts presenting the material and the units corresponding to the mapped problems were analyzed, as well as the specific guidelines for working with these problems. Considering the Conceptual Fields Theory and the Knowledge of Knowledge of Curriculum Embedded Mathematics as references, we identified a variety of problem classes, as well as guidelines on the different meanings of operations. We observed weaknesses in the approach taken in the 2nd grade Manual corresponding to the addition and subtraction approach, to the detriment of the other Manuals. We noted the absence of explicitly stated information in the three manuals, which may have an impact on the way teachers read and interpret the guidelines, and evaluate the problems, implying the teaching knowledge mobilized or constructed.

Keywords: Curriculum Materials. Additive Field. Professional Teaching Knowledge.

Recursos de materiales curriculares que inducen al conocimiento profesional docente sobre el campo conceptual aditivo

Resumen: Asumiendo que los materiales curriculares repercuten en el conocimiento de los docentes, buscamos *identificar y discutir recursos de los materiales, relacionadas con el campo aditivo, que inducen el conocimiento profesional docente en Matemáticas*. A través de un análisis documental de tres Manuales del Profesor de Matemáticas de 2°, 3° y 4° año, se mapearon y analizaron problemas de suma y resta; Se analizaron los textos que presentan el material y unidades correspondientes a los problemas mapeados, así como las orientaciones específicas para trabajar con estos problemas. Tomando como referencia la Teoría de los Campos Conceptuales y el Conocimiento Matemático Incorporado al Currículo, identificamos una variedad de clases de problemas, así como orientaciones sobre los diferentes significados de las operaciones. Observamos debilidades en el enfoque adoptado en el Manual de 2º año correspondiente al enfoque de suma y resta en detrimento de otros Manuales. Destacamos la ausencia de información explícita en los tres Manuales, lo que puede impactar la forma en que

los docentes leen e interpretan las orientaciones y evalúan los problemas, implicando conocimiento docente movilizado o construido.

Palabras clave: Materiales Curriculares. Campo Aditivo. Conocimiento Profesional Docente.

Recursos dos materiais curriculares que induzem o conhecimento profissional docente sobre o campo conceitual aditivo

Resumo: Assumindo que materiais curriculares reverberam o conhecimento de professores, buscou-se *identificar e discutir recursos dos materiais, relativos ao campo aditivo, que induzem o conhecimento profissional docente em Matemática*. Por meio de uma análise documental em três Manuais do Professor, de Matemática do 2°, 3° e 4° anos, foram mapeados e analisados os problemas de adição e subtração; analisados os textos de apresentação do material e das unidades correspondentes aos problemas mapeados, e as orientações específicas ao trabalho com esses problemas. Considerando a Teoria dos Campos Conceituais e o Conhecimento da Matemática Incorporada ao Currículo como referenciais, identificamos uma variedade de classes dos problemas, bem como orientações sobre os diferentes significados das operações. Observamos fragilidade na abordagem assumida no Manual do 2° ano correspondente à abordagem de adição e subtração em detrimento aos outros Manuais. Evidenciamos a ausência de informações explicitamente declaradas nos três Manuais, o que pode impactar sobre o modo como professoras leem e interpretam as orientações, e avaliam os problemas, implicando o conhecimento docente mobilizado ou construído.

Palavras-chave: Materiais curriculares. Campo Aditivo. Conhecimento Profissional Docente.

1 Curriculum Materials as a Source of Professional Teaching Knowledge

According to S. Lima (2014), curriculum materials are produced with the purpose of serving as essential tools in the educational space, and these materials reflect interpretations of the prescribed curriculum, helping to disseminate political, social and educational issues. In order for the materials to disseminate the educational objectives proposed in the prescriptions, teachers develop the proposals present in them and create opportunities for students to construct learning. The curriculum presented to teachers in the curriculum materials is realized through teaching practices and the use of these materials (Lima, S., 2014).

According to Brown (2009), curriculum materials, especially textbooks, have been an important tool that teachers have used in their teaching practice in order to achieve the desired objectives in terms of teaching and learning mathematics. In this sense, teachers use curriculum materials to read and interpret the content and tasks proposed in them. By reading and interpreting the curricular proposals contained in them, these professionals select tasks in order to enhance or create learning situations for their students.

When using curriculum materials as a tool to support their math teaching practices, teachers establish a relationship with these materials; based on this relationship, each teacher perceives and uses the materials in different ways (Brown, 2009). Thus, the materials help the teachers in their teaching objectives, but the potential and restrictions present in them are perceived as knowledge related to mathematics and its teaching is mobilized.

When reading, interpreting, evaluating and selecting these materials, or part of them, teachers need to mobilize some knowledge so that the prescribed guidelines can be incorporated into the planning and implementation of lessons. In this sense, the beliefs and conceptions that these professionals have about mathematics and the resources and procedures needed to teach it can serve as a beacon for the teacher-curriculum materials relationship, and these beliefs and

conceptions mediate the formation and construction of mathematical concepts by their students (Januario, Lima & Perovano, 2021).

When dealing with the teacher-curriculum materials relationship from the perspective of teacher professional knowledge, Remillard and Kim (2017) understand that when reading, interpreting, evaluating and selecting the proposals in the materials, teachers need to mobilize or construct knowledge so that there can be a dynamic and interactive relationship. In this way, for this relationship to occur, it is necessary to perceive *affordances*, in other words, to identify possibilities for action that can support classroom practice.

Commonly, in public schools, the materials used by teachers are those evaluated and distributed by the National Textbook and Didactic Material Program [Programa Nacional do Livro e do Material Didático — PNLD]. According to S. Lima (2014), these materials are a way of disseminating the curriculum and presenting it to teachers. The preparation of these materials is driven by the prescriptions and guidelines present in curriculum materials, for example, the National Common Curriculum Base [Base Nacional Comum Curricular — BNCC] (Brasil, 2017).

One of the guidelines in this document is that the teaching and learning of mathematics, in terms of working with addition and subtraction, should focus on the different meanings associated with these operations. According to the theories proposed by Vergnaud (2009), this is an approach based on the Conceptual Fields Theory, specifically the additive field or additive classes.

As a result of the proposal to work with the different meanings attributed to the operations addition and subtraction, when using curriculum materials, teachers need to mobilize knowledge to read and interpret the tasks and didactic-methodological proposals, in order to create the conditions for their students to form concepts related to the additive field. By reading and interpreting the mathematics embedded in the curriculum materials, in order to evaluate and select teaching proposals, teachers not only mobilize knowledge about working with and forming concepts about the additive field, but they can also build new knowledge about the approach to be taken when teaching additive operations.

In this respect, curriculum materials can not only promote student learning, but also help teachers learn about mathematics and its teaching, because by reading and interpreting curriculum materials, especially the Teacher's Manual, they can (re)construct concepts about the additive field. From the perspective that these materials are important tools for teachers when planning and carrying out lessons, they can also be a resource for studying and improving pedagogical action, which characterizes the importance of researching which aspects are present and incorporated into the materials that contribute to teacher learning. To this end, the aim of this paper is *to identify and discuss resources of materials related to the additive field that induce professional knowledge in Mathematics.*

2 Additive Field and Teacher Professional Knowledge

It is at school that students' knowledge from their daily lives and social interaction is formalized and consolidated, favoring the conditions for obtaining the conceptual dimension of school and scientific knowledge. According to Pais (2002), however, it is not the excessive memorization of expressions and rules that will guarantee students' learning, because learning the meaning of a concept is not restricted to linguistic messages or expressions (formulas).

In this context, according to the Conceptual Fields Theory, the development of a concept will not occur through memorization or repetition of the same problem situation, but rather

through various situations, since multiple factors are related, interfere and influence the development of a concept (Vergnaud, 1986, 2009). The understanding of one concept is related to the understanding of others, just as a situation, however simple, involves numerous concepts, so that concepts are not learned in isolation, but in an integral way, because a task or problem situation will require several concepts to be involved (Magina *et al*., 2008; Vergnaud, 2009).

Mastery of additive classes requires that students are given various types of problem situations and that they are able to solve them, which goes beyond the mere ability to master or operate numerically, and these problems can be of the simplest or most complex type. The concepts or conceptions will emerge from the students' realization or interaction with the proposed situations, but the level of complexity present in the variety of each situation will require greater or different competence (*Magina et al*., 2008).

According to Magina *et al*. (2008), numerous situations need to be proposed to students, from the simplest to the most complex, and this variety is necessary for students to be able to master the additive field. In this sense, based on the studies and categorizations of Vergnaud (1986), these authors present and classify a set of problem situations (classes) belonging to the additive field, which are: composition, transformation, comparison, composition of transformations, transformation of composition, comparison with composition of transformations, transformation of relation and composition of relations.

As mentioned above, teachers need to develop knowledge about the classes that make up the additive field, as well as the concepts involved. It is therefore considered that teachers need to mobilize knowledge about mathematics and its teaching, knowledge that can be enhanced through the relationship they establish with curriculum materials (Collopy, 2003; Remillard & Kim, 2017). Collopy (2003) considers that teachers, during their professional development, need opportunities to build new knowledge about teaching mathematics in order to build their students' learning.

From this perspective, Collopy (2003) and Remillard and Kim (2017) highlight the importance of materials as a support for the teaching and learning of mathematical content and for professional teacher training. In addition to the support that these materials provide for teachers when planning, designing and carrying out their lessons, they are also seen as an important tool for developing new beliefs and knowledge on the part of teachers (Collopy, 2003).

In the quest to build new knowledge or expand it, curriculum materials are allies for teachers and can contribute to their professional knowledge by proposing situations and guidelines on the additive field and its classes.

3 Methodological design

When dealing with Mathematics and its teaching, and the possibilities of knowledge that research in Mathematics Education has promoted, contributing to the practice of teachers, we highlight studies on curriculum materials. Research has taken the perspective of materials as enhancers of teaching practices (Soares, 2020; Antunes, 2022; Machado, 2023).

Curriculum materials, specifically the Teacher's Manual, can provide resources to expand or give new meaning to the learning and improvement of teachers' pedagogical practice. These materials are tools used by teachers as a source of consultation, improvement and (re)construction of teaching practices (Januario & Lima, 2019). From this perspective, curriculum materials can serve to enhance the teaching practices of current teachers as well as those still in training.

When we return to the objective of the study — *to identify and discuss material resources relating to the additive field that induce professional knowledge in Mathematics* — the data collection procedure leads to a documentary analysis, the source of which are three volumes of Teacher's Manuals. According to what is proposed in the National Textbook and Didactic Material Program (PNLD), this type of material, the Teacher's Manual, has the purpose of presenting specific content guidelines, as well as the approaches and procedures to be adopted during the teaching process, in addition to reproducing the pages of the student material.

According to Sá-Silva, Almeida and Guindani (2009), documents can be used as sources of information and clarification for research. It is in this understanding that we consider curriculum materials to be documents, particularly the Teacher's Manual; it is a document designed to support teaching and which presents elements that broaden teachers' vision of the approach and treatment of mathematics, inducing teaching practices, such as planning and carrying out lessons, monitoring students and designing and developing assessment resources.

Based on Lüdke and André (1986), there are reasons that guide the choice of materials to be analyzed, and this choice is guided by purposes. The study portrayed here is part of a larger study involving three teachers who work in the 2nd, 3rd and 4th grade of the Elementary School. That's why we chose the Teacher's Manual for these school years, as these materials are used in all three classes. The three volumes are part of the *Ápis Mais* collection, published by Ática, organized by Luiz Roberto Dante and Fernando Viana, published in 2021, and distributed to schools in 2022 as part of the PNLD selection process for the 2023-2025 cycle.

Unlike the materials to be used by students, the Teacher's Manuals have an introductory chapter with specific guidelines for teachers, followed by pages from the Student's Book with a presentation and specific guidelines for each chapter and answers to the tasks.

The introductory part of the Manuals analyzed emphasizes the process of preparing the collection and its alignment with the Common National Curriculum Base. In this part, the guidance texts refer to the general principles of Mathematics Education; theoretical foundations — BNCC and some methodological guidelines for teaching Mathematics —; assessment processes; information on how to use the collection and its possibilities; references for further study by the teacher and a specific part with skills and guidelines for the year of schooling corresponding to the volume.

The analysis considered the introductory part common to all three volumes of the Manuals, and the guidelines that accompany the problems on the pages that reproduce the Student's Book, as well as the problems to be proposed for the students to solve.

Reading the guidelines, in the initial part of the Manuals, we tried to identify an implicit or explicit approach that mentioned didactic, methodological and conceptual aspects of the additive field. With regard to the mapping of additive problems, they were initially read and identified, and classified according to the classes of problems in the additive field and the operation used as a result of the position of the unknown in the situation presented. Once the problems had been mapped, the accompanying guidelines were read in detail.

We will now analyze the three volumes of the Teacher's Manual considering the discussions on Conceptual Fields Theory, particularly the additive conceptual field, and on professional teaching knowledge as referential theorizations.

4 Additive Conceptual Field and its approach in curriculum materials

Based on the substantial idea that the curriculum materials distributed by the PNLD

follow the prescriptions and guidelines contained in the BNCC (Brasil, 2017), we read this document in order to identify the skills related to the thematic unit *Numbers* for the Elementary School. Table 2 shows the skills that refer to the meaning of the operations addition and subtraction.

Table 2: Skills prescribed in the BNCC regarding the different meanings of the operations addition and subtraction

Grade	Skill
1st	(EF01MA08) Solving and working out addition and subtraction problems involving numbers up to two digits, with the meanings of **joining, adding, separating and taking away**, with the support of images and/or manipulatives, using personal strategies and forms of recording. (EF01MA03) **Estimating and comparing** quantities of objects from two sets (around 20 elements), by estimation and/or by correspondence (one to one, two to two) to indicate "there is more", "there is less" or "there is the same amount".
2nd	(EF02MA06) Solve and work out addition and subtraction problems involving numbers of up to three orders, with the meanings of **joining, adding, separating and taking away**, using personal strategies.
3rd	(EF03MA06) Solving and working out addition and subtraction problems with the meanings of **joining, adding, separating, taking away, comparing and completing quantities**, using different exact or approximate calculation strategies, including mental calculation.
4th	—
5th	—

Source: BNCC (Brasil, 2017, p. 281, 285, 289; emphasis added)

The desire to read the skills for the five years of schooling is justified by the relevance of knowing the prescriptions present in this document for working with the different meanings of the operations addition and subtraction, since the guidelines reverberate in the development of curriculum materials, so that they may or may not provide teachers with learning situations to be developed with students.

As can be seen in Table 2, there are no skills mentioning addition and subtraction from their different meanings for the 4th and 5th grades. Despite not exploring their different meanings, these operations are prescribed for these school years, as can be seen in the following skills:

> (EF04MA03) Solve and work out problems with natural numbers involving addition and subtraction, using different strategies such as calculation, mental calculation and algorithms, as well as estimating the result. (Brasil, 2017, p. 293)
>
> (EF05MA07) Solve and work out addition and subtraction problems with natural numbers and rational numbers whose decimal representation is finite, using different strategies such as estimation, mental calculation and algorithms. (Brasil, 2017, p. 297)

The lack of reference to skills that guide work with the different meanings of these operations in these two years of schooling indicates an inconsistency with the literature on the additive conceptual field. Magina *et al.* (2008) point out that the formation of concepts based on working with problems involving the operations addition and subtraction is not consolidated in a given year of schooling, but is a perennial process.

We understand that the absence of skills that suggest working with the meanings of addition and subtraction implies teaching practices, since the materials used by the teachers follow the prescriptions of official documents, in this case the BNCC. Even though there are no skills that specifically deal with the classes belonging to the additive field, it is possible to

identify, after analyzing the 4th grade material, that problems involving these operations are incorporated into the Teacher's Manual of the *Ápis Mais* material.

By examining the terms used in the BNCC and comparing them with the meanings attributed to additive operations, according to Vergnaud's studies (1986, 2009), it is possible to conclude that only three classes are identified in the BNCC, with regard to work in the Elementary School: composition, transformation and comparison.

According to the studies of Vergnaud (1986), discussed by Magina et al. (2008), *composition* involves putting two quantities together to determine the whole, or even subtracting one part from the whole to obtain the other part; the *transformation* class is classified as situations in which the idea of time is involved, in which there is an initial state that is modified or transformed, either by loss/gain, increase/decrease; and the *comparison* class is made up of situations in which two quantities (values) are compared, one called referent and the other referred to.

When analyzing the terms used in the skills described in the BNCC, as well as in the Manuals, we identified *joining, separating, removing and completing quantities* to deal with problems involving the *composition* class; in relation to the *transformation* class, we identified the expressions *add, remove and separate* and the terms, *compare and estimate* to designate problems involving the *comparison* class. Based on the analysis and comparison of the terms used in the BNCC and in the curriculum materials manuals, in relation to the theoretical framework studied, we understand that there is a consonance between the two.

As announced in the previous section, the research described here is based on an analysis of the copies of the Teacher's Manual for the 2nd, 3rd and 4th grades. Once we had the three manuals, we read the problems to identify the classes related to the additive field, as shown in Tables 1 to 3, which we will discuss below.

Table 1: Number of problems in the composition class

Grade	Total	By operation		Location of the unknown	
				Parts	All
2nd	46	Addition	37	4	33
		Subtraction	9	9	—
3rd	33	Addition	27	1	26
		Subtraction	6	6	—
4th	23	Addition	17	—	17
		Subtraction	6	6	—

Source: Survey data, 2023

Table 2: Number of problems in the transformation class

Grade	Total	By Operation		Location of the unknown			
				Initial Status	Positive Transformation	Negative Transformation	Final Status
2nd	33	Addition	14	3	1	—	10
		Subtraction	19	1	3	—	15
3rd	22	Addition	10	—	1	—	9

		Subtraction	12	—	—	2	10
4th	24	Addition	12	5	—	—	7
		Subtraction	12	5	1	—	6

Source: Survey data, 2023

Table 3: Number of problems in the comparison class

Grade	Total	By Operation		Location of the unknown			
				Referent	Positive Relationship	Negative Relationship	Referred
2nd	35	Addition	4	—	—	—	4
		Subtraction	31	—	18	11	2
3rd	19	Addition	1	—	—	—	1
		Subtraction	18	—	10	6	2
4th	15	Addition	1	—	—	1	—
		Subtraction	14	—	8	4	2

Source: Survey data, 2023

In accordance with the mapping carried out, it is possible to highlight that the three Manuals analyzed show a predominance of composition (102), transformation (79) and comparison (69) classes in their approaches, which according to Magina *et al*. (2008) and Vergnaud (2009), are the three basic groups of problems that make up the additive field.

In addition to the three classes of problem found, we identified situations relating to the classes *composition of transformations and transformation of composition*. Due to the small number, we have chosen to describe this data. The class *composition of transformations* was identified in all three volumes of the Manuals with a total of 9 situations. Of these, 2 are in the 2nd grade Manual and involve both transformations, i.e. positive and negative transformations; 3 situations are in the 3rd grade Manual and also involve both transformations; and 4 situations are in the 4th grade Manual, of which three involve positive transformations, and 1 both transformations. Of the 9 problems found, 1 in the 3rd grade Manual has its unknown in the transformations, and in the remaining 8 problems the unknown is located in the composition.

With regard to the *composition transformation* class, 4 problems were found, 3 of which are in the 2nd grade Manual and 1 in the 4th grade Manual. In one of the problems found in the 2nd grade Manual, the unknown is in both the transformation and the composition, while in the other two, the unknown is in the transformations. The problem found in the 4th grade Manual has its unknown in the composition.

With a smaller number, the 2nd and 4th grade Manual present problems with the *composition transformation* class (4), while in the 3rd grade no problems involving this class were found. With regard to the class *composition of transformations*, 9 problems were found in the three Teacher's Manual. One point to consider about the *classes transformation of compositions and composition of transformations* is that, even though they were identified in the Manuals, no skills were found that included them in the BNCC.

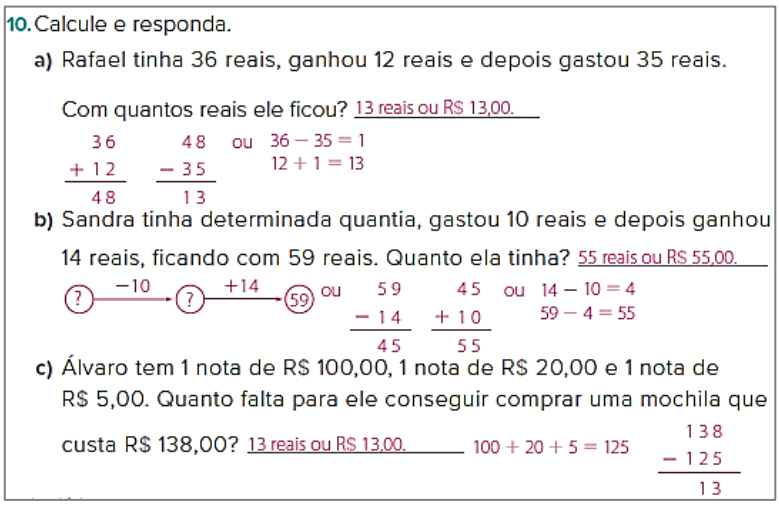
Problems involving the classes *comparison with composition of transformation, transformation of relation and composition of relations* were not identified in any of the Teacher's Manual analyzed. The lack of approach to these classes is in line with Magina et al. (2008) and Vergnaud (2009), who say that students should be presented with a variety of

problems in order to form concepts relating to the additive field. The plurality of problems, as well as their complexity, ensures that students have access to a diversity of situations from which concepts emerge and are formed.

With regard to the approach to problem classes throughout the Teacher's Manual analyzed — in the part that reproduces the pages of the Student's Book — they vary from one another. When dealing with the operations addition and subtraction, they are dealt with in the same unit/section, but first the meanings of *joining and adding* are dealt with as addition, and then the meanings of subtraction: *separating, removing, completing and comparing*. After dealing with the meanings of addition and subtraction, the relationship established as inverse operations is presented. In contrast to the 3rd and 4th grade Manuals, the 2nd grade Manual has separate units for working with these operations, so work with inverse operations is done in the *Subtraction* unit.

The 2nd grade manual presents a first unit called *Numbers up to 199*, in which the operations *addition and subtraction* are alternated, but two other units called *Addition and Subtraction* are presented later, with the addition unit preceding the subtraction unit. In the unit entitled *Addition*, there are 26 problems involving addition, 3 of which can be solved using subtraction (in the comparison class). In the unit entitled *Subtraction*, 57 problems were identified in which operations involving subtraction predominate. Of these, the remaining 37 involve addition and subtraction simultaneously, as exemplified in the problems in Figure 1.

Figure 1: Problems in which addition and subtraction are used simultaneously[1]



Source: Ápis Mais, 2nd grade (2021, p. 174)

In contrast to the theoretical framework adopted here, the approach used in the 2nd grade manual - in which the operations addition and subtraction are worked on in isolation - diverges from the understanding that understanding a concept involves associating it with other concepts; thus, the concepts belonging to the additive field are interwoven between addition and subtraction operations (Magina *et al.*, 2008; Vergnaud, 2009). As a result of the initial approach in the 2nd grade manual, i.e. the lack of simultaneity in the approach to operations, students may not experience situations in which the different concepts necessary for the formation of additive concepts are involved.

[1] 10. Calculate and answer. a) Rafael had 36 Reals, earned 12 Reals and then spent 35 Reals. How many reais did he get? b) Sandra had a certain amount of money, spent 10 reais and then won 14 reais, leaving her with 59 reais. How much did she have? c) Álvaro has 1 banknote worth R$100.00, 1 banknote worth R$20.00 and 1 banknote worth R$5.00. How much money does he need to buy a backpack that costs R$ 138.00?

When reproducing the pages of the Student's Book, the 3rd and 4th grade Teacher's Manual, unlike the 2nd grade Manual, present the operations addition and subtraction in the same section. The 2nd grade Manual, as we've discussed, presents an approach that diverges from what the theory suggests; however, the 3rd and 4th grade Manuals, despite addressing addition and subtraction in the same unit/section, address the meanings separately according to the operation.

It can be seen in the three Teacher's Manual analyzed that the problems involving these operations are not restricted to the *Numbers* thematic unit; on the contrary, the additive field classes are also dealt with in the *Greatness and Measures* thematic unit, as occurs in the 2nd grade material (greatness and its measures); in the 3rd grade Manual (length, mass and capacity); and in the 4th grade Manual (mass, capacity, time interval and temperature).

The analysis showed that the 2nd grade Manual has a higher number of problems in the additive field than the 3rd and 4th grade Manual (114). According to Tables 1, 2 and 3, it was also possible to see that, in the three Manuals, there is an approximate number of addition and subtraction operations, totaling 123 problems involving addition and 127 involving subtraction.

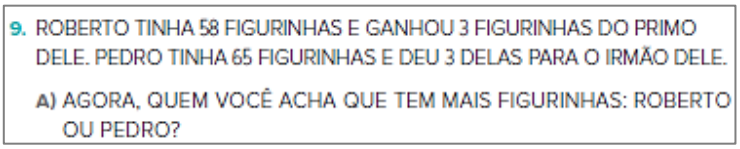
Based on the data above, we believe that the 2nd grade manual provides both teachers and students with more contact with problems in the additive field, which can help them mobilize more knowledge to work with addition and subtraction. From the perspective of student learning, they will be able to expand their knowledge based on the various problems proposed. With regard to the number of problems involving addition and subtraction identified in the three volumes, it can be inferred that students are exposed to a variety of problems in which both operations are tackled without privileging one over the other.

Based on the analysis, we found problems in the 2nd and 3rd grade Teacher's Manual in which more than one class is used to solve them. In this sense, we identified that *composition and transformation* are the initial classes involved in solving problems that cover the meanings of *comparison*; and the *composition* class is involved in situations that cover *transformation*. The following three problems — Figures 2, 3 and 4 — exemplify part of this approach.

Figure 2: Transformation with composition support[2]

Source: Ápis Mais, 3rd grade (2021, p. 109)

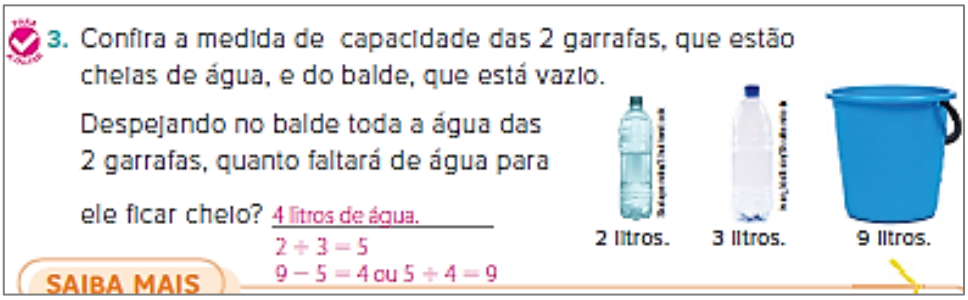
Figure 3: Comparison with transformation support[3]



Source: Ápis Mais, 2nd grade (2021, p. 73)

[2] 9. Márcia's father has R$1,000.00 in his bank account and is going to deposit these 4 notes. How many reais will his bank balance be?
[3] 9. Roberto had 58 stickers and got 3 stickers from his cousin. Pedro had 65 stickers and gave 3 of them to his brother. A) Now, who do you think has more stickers: Roberto or Pedro?

Figure 4: Comparison with composition support[4]



Source: Ápis Mais, 2nd grade (2021, p. 253)

Looking at Figures 3 and 4, we can see that as well as involving more than one class in the problems proposed, they also involve the operations addition and subtraction to solve them.

After analyzing the Teacher's Manuals for the 2nd, 3rd and 4th grades, and examining the presence and approach given to the additive field, we considered its implications for the professional knowledge of teachers who teach mathematics in these grades, an aspect that we will now analyze and discuss.

5 Professional teaching knowledge incorporated into the Teacher's Manuals

Based on the theoretical references that address both professional teaching knowledge and the teacher-curriculum materials relationship (Collopy, 2003; Remillard & Kim, 2017), we will analyze the three volumes of the Teacher's Manual, based on the resources underlying the materials that induce professional teaching knowledge when dealing with the additive field. We assume that, in possession of the Manuals, teachers can not only (re)think mathematical ideas and concepts, but they can also (re)mean the teaching of mathematics by expanding their knowledge of the meanings attributed to the operations addition and subtraction.

To begin the analysis, we started with the introductory part of the *Ápis Mais* material, which is common to the three volumes analyzed, entitled General Part (p. 5). It contains five sections of guidelines, including *Mathematics Education* (5-6), which highlights the importance of working with mathematics, based on the idea that students need to be encouraged to exercise their skills with creativity, responsibility and autonomy, using the knowledge built up in school to solve and resolve the problems of daily life. It is also pointed out, from excerpts taken from the BNCC, that working with mathematics should provide students with a source for solving problems, and that they will use concepts, procedures and results to interpret and solve problems according to their context. Thus, the teaching of mathematics will make sense if the situations or problems of daily life are solved using the knowledge that emerges from the school environment, so that learning in the classroom can become meaningful to students.

As we read through the introductory texts in the Manuals, we noticed that when dealing with some pedagogical practices for teaching mathematics and promoting learning situations, it is made clear that working with mathematical ideas and concepts needs to come before symbology.

> For example, before recording the sentence 1+ 3 = 4 on the blackboard, we recommend building the concept of number, exploring the ideas of addition (adding quantities or adding one quantity to another)

[4] 3. Check the capacity of 2 bottles, which are full of water, and the bucket, which is empty. If you pour all the water from the 2 bottles into the bucket, how much water will be left before it is full?

and the meaning of each symbol used, such as addition and equality (Ápis Mais, 2nd grade, 2021, p. 8).

Based on the excerpt, which is common in all volumes of the *Ápis Mais* material, the wording of the initial part in the Teacher's Manual emphasizes working with operations and recommends that the initial approach should focus on ideas and concepts and then introduce the symbologies involved. In this case, the meanings of addition (joining and adding), cited in the passage, serve as an example of an approach, without explaining what these meanings are.

The three volumes of the Teacher's Manual emphasize that the problems presented in the material follow an order and a connection between them and their different levels of cognitive demand, evolving from situations that require various types of reasoning, from the simplest to the most complex, as can be seen in the excerpt from the section *Some practical suggestions for working with problem solving*: "Start by working with simple problems and gradually introduce more complex ones. This strengthens each student's self-esteem and self-confidence" (Ápis Mais, 2nd grade, 2021, p. 14). Recognizing the potential of the information presented to the teachers, we understand that the information can contribute to expanding *horizontal knowledge of the content* (Ball, Thames & Phelps, 2008), as well as activating the way in which the teachers understand the *learning paths* present in the materials (Remillard & Kim, 2017).

At the end of the analysis of the introductory part of the Manuals, we highlight the lack of transparency or clarity in the guidelines for teachers. No explicitly stated guidelines were identified that could lead teachers to think about the meanings of addition and subtraction based on the problems to be worked on from 1st to 5th grade. In this way, we understand that teachers' professional knowledge based on the teacher-curriculum materials relationship may be affected, since there is insufficient information and teaching guidelines in these materials. This insufficiency could have repercussions on the expansion or (re)signification of the teachers' understanding of the additive field — *fundamental ideas* highlighted by Remillard and Kim (2017), referring to procedures that mathematically justify some resolution strategies, which are incorporated into the symbolic representation and invariants.

When we examine the section called *Unit Introduction*, which precedes the opening of each unit or chapter of the Student's Book, we notice that the information varies according to the year of schooling, and that explicit statements are made regarding the intent and objectives of each unit or chapter. In an analysis of the 2nd grade Manual, the *Introduction to the Unit* section (p. 135 and 153), which precedes the *Addition and Subtraction* units, states that "In this unit, we revisit the ideas associated with subtraction and introduce different algorithms for performing this operation" (Ápis Mais, 2nd grade, 2021, p. 153). It is understood that the 1st grade material presents the initial ideas about addition and subtraction, and that these ideas are linked to the meanings of the operations contained in the 2nd grade material.

Also in the *Introduction to the Unit* section of the 2nd grade Manual's, the objectives are explained, as well as guidelines on how to introduce and construct the concepts related to addition (adding and adding) and subtraction (taking away, completing, comparing and separating from subtraction). It also discusses the role of teachers in supporting students to develop their own strategies and to record them in different ways, including drawings.

In contrast to the 2nd grade Manual, the operations addition and subtraction in the 3rd and 4th grade Teacher's Manual are dealt with in a single unit or chapter. From our analysis, we can see that the *Unit Introduction* section in these two materials presents the objectives to be achieved, as well as signaling some pedagogical approaches. However, the 3rd grade manual, unlike the 2nd and 4th grade manuals, makes no mention of the meanings of addition and

subtraction.

In the *Unit Introduction* section of the 4th grade Manual, teachers are told that "In this Unit, the ideas associated with addition and subtraction are revisited and various challenges and problem situations are proposed using knowledge involving these operations and their algorithms" (Ápis Mais, 4th grade, 2021, p. 139), leading to the interpretation that the development of concepts and meanings is presented in a structured order, taking into account the guidelines presented in previous years' volumes.

We believe that the approach taken in the introductory part of the three volumes, as well as the lack of clarity in the guidelines on the meanings of addition and subtraction in the *Unit Introduction* section of the 3rd grade volume, indicate a weakness in the approach presented. It is through the explicitly stated approaches present in the materials that teachers, as they read and interpret the information contained in the Manuals, can (re)mean mathematics and its teaching, and can also expand their knowledge. We consider the lack of explicit information in the materials to be an obstacle to expanding teachers' knowledge of the additive field.

Based on the lack of clarity in the materials, we infer that the teachers will have to mobilize two of the dimensions pointed out by Remillard and Kim (2017), mobilization that would occur even if there were details of the meanings associated with the operations addition and subtraction. In order to understand and reason about these details in the Manuals, the teachers need to mobilize, or construct, knowledge related to *fundamental ideas* and *mathematical learning paths*. From the *fundamental ideas* dimension, teachers can recognize relationships and properties underlying the expected resolutions and articulate them to the teaching objectives; with regard to *learning paths*, teachers need to understand how the contents are related or interconnected over the years in order to make approaches that contribute to the formation of the expected concepts and skills relating to the operations addition and subtraction in later periods and years.

The approach to the meanings of addition and subtraction presented in the 2nd grade manual tends to explain each operation in isolation, separated by units or chapters. Despite this type of approach, we highlight the following introductory passage in the *Subtraction* unit: "Finally, in the Unit we highlight the relationship between addition and subtraction as inverse operations [...]" (Ápis Mais, 2nd grade, 2021, p. 153). We understand that this excerpt addresses what Magina *et al*. (2008) state about these operations being worked on concurrently as part of the additive field. However, the situations that address the meanings of the addition operation are presented before those of subtraction, which could lead to the teachers' understanding that problems involving addition need to be worked on initially, as a prerequisite or even in isolation. This approach in the Manual affects the teachers' understanding of learning paths.

When analyzing the pages that reproduce the Student's Book, we identified the use of images or scenes as resources to introduce the study of the units, scenes that refer to situations apparently familiar to the students. To contextualize the *Addition and Subtraction* units in the 2nd grade material, the scenes refer, respectively, to a shop window with products displayed and their respective values and to children playing a board game. In the 3rd grade material, the opening scene shows children playing with darts and in the 4th grade material, a bar graph is shown, with students and a teacher reading it.

The way in which the content is presented and introduced is in line with the categories of knowledge elaborated by Shulman (1987), in which the teacher needs to have *knowledge of the students and their characteristics and knowledge of the educational contexts* so that the teachers are able to explore the ideas underlying the scenes presented.

Thus, the problems involving addition and subtraction are accompanied by guidelines for the teachers, in order to explain the meanings involved in each situation presented; possible procedures to be adopted; the possibility of using manipulable materials; difficulties that may be presented by the students in solving the tasks; possibilities for solving using different strategies; and multiple representations that the student can use to portray the mathematical reasoning involved.

It is possible to highlight that, in addition to the meanings of addition and subtraction, the Manuals present, in their guidelines, possibilities for expanding the *teachers' knowledge of the content and the students* — the teachers, based on their knowledge of mathematics and their students, can anticipate the possible difficulties presented during the lessons; and *knowledge of the content and teaching* — the teachers choose the methodological strategies that best integrate with the specificities of their students (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008).

When starting the *Addition and Subtraction* units, we found that the 2nd grade manual presents problems involving the meanings of *joining and adding* for the addition operation and the meanings of *separating, taking away, comparing and completing* for the subtraction operation. The guidelines next to the tasks encourage teachers to go over the expressions associated with these operations with the students. In the 4th grade manual, at the beginning of the section *Reviewing the ideas of addition: joining and adding*, the teachers are told that "The ideas of joining and adding were worked on in the previous volumes of this collection" (Ápis Mais, 4th grade, 2021, p. 143). Also in the 4th grade volume, in the *Unit Introduction* section (p. 139), the ideas associated with addition and subtraction are taken up and various challenges or problems are proposed using knowledge involving these operations and their algorithms, emphasizing that the usual subtraction algorithm has already been presented as well as the ideas associated with the operation as they are taken up.

Based on an analysis of the three Manuals, we can see that the 2nd grade volume contains a greater amount of information and guidance for teachers on the meanings of addition and subtraction, as can be seen in passages such as: "The activities allow students to have a first contact with the content that will be covered in the Unit, such as the ideas of addition and adding" (p. 138); "In item a, ask students if they remember any other expressions associated with the addition operation" (p. 138); "Work with the students on the first idea of addition we presented: adding quantities together" (p. 139); "Ask them which idea is associated with the situation in each item: adding quantities in item a and adding quantities in item b. Make them realize that, regardless of the values chosen to complete the situations, the ideas of addition remain the same" (p. 139).

Other suggestions and guidelines can be found in the Manual, such as: "Ask them to create problem situations with the ideas of addition, worked on in the topic *Ideas of addition*, using the words *join, complete or add*" (p. 140); "The activities allow students to have their first contact with the content that will be covered in the Unit, such as the ideas of taking away, comparing, completing and separating subtraction" (p. 156); "In this topic's activities, ask the students if they remember other words that can be associated with subtraction, such as taking away, completing and comparing, and explore them" (p. 156); "In item a, ask the students if they remember other words that can be associated with subtraction, such as *take away, complete and compare,* and explore them too" (p. 156); "The activities in this topic explore the ideas associated with the subtraction operation: taking one quantity away from another, completing a quantity ("How many are missing?"; "What's the difference?"), separating a quantity ("How much has been left over?") and comparing 2 quantities ("How many more?"; "How many less?")" (p. 157); "This activity works on the next subtraction idea: completing a quantity" (p.

158); “This activity works on the last idea of subtraction: separating one quantity from another” (p. 158); “[...] draw attention to the fact that the idea of taking one quantity from another is being used, related to subtraction [...]” (p. 160); "Read the problem with them and draw attention to the fact that the idea of comparing quantities is being used [...]” (p. 177).

To a lesser extent than the 2nd grade manual, the 3rd grade manual also provides guidance on the meanings of addition and subtraction, according to the following excerpts: “In the questions asked by the characters, the ideas of adding quantities (addition) and comparing quantities (subtraction) are addressed” (p. 122); “the activities in this topic explore the ideas of addition: adding quantities (activity 1) and adding one quantity to another that already exists (activity 2)” (p. 123); “[...] students elaborate the question using the idea of joining quantities and, for the others, using the idea of adding one quantity to another” (p. 123); “[...] students to work out the question using the idea of adding quantities together and, for the others, using the idea of adding one quantity to another" (p. 131); “Give the students some concrete situations that allow them to use the ideas of subtraction” (p. 132); “Group the students so that all the ideas of subtraction are used in working out at least one of the problems” (p. 133); and “In this activity, the ideas of adding (addition) and comparing (subtraction) are covered [...]” (p. 140).

In the 4th grade manual, we can highlight some guidelines such as: “The ideas of adding and adding have been worked on in previous volumes of this collection” (p. 143); “Explain to them that this activity uses the addition idea of adding one quantity to another” (p. 145); “The problem presented in this activity deals with the subtraction idea of taking one quantity from another” (p. 150); “The first three questions deal with the idea of comparing quantities [...] the fourth question deals with the idea of completing [...]” (p. 151); “This activity presents the idea of subtraction related to separating one quantity from another” (p. 152).

In the 2nd grade material, the *Addition* (p. 136-151) and *Subtraction* (p. 154-179) units present topics for each of the meanings involved. In the *Addition* unit, the topics *Join quantities* (p. 139) and *Add quantities* (p. 139) are presented; in the *Subtraction* unit, the topics are: *Taking one quantity from another* (p. 157); *Comparing quantities (how many more or less)* (p. 157); *Completing a quantity* (p. 158); and *Separating one quantity or amount from another* (p. 158). The 3rd grade material also presents the meanings of addition and subtraction through topics similar to the 2nd grade material, but with the addition of the topic *Compare*: *what's the difference* (p. 133). The 4th grade material presents the ideas of subtraction in just one heading, on page 150: *Subtraction with natural numbers: Reviewing the ideas of subtraction: taking away, comparing, completing and separating.*

Still on the 2nd grade material, in the Subtraction unit, the topic that deals with *Addition and Subtraction: inverse operations* (p. 165), problems are presented which can be solved using both operations. The Teacher's Manual also states that students may have difficulty with this type of problem, suggesting that teachers rewrite the problem in a straightforward manner, and that after solving it, they return to the initial statement.

> This activity involves solving problems using strategies that involve addition and subtraction as inverse operations. In general, students find it easier to solve direct problems. So if they find it difficult to understand the problems proposed, rewrite them in a straightforward way. Once they have solved these problems, go back to the original statements and compare them. Remembering the actions taken to solve the direct problem can help them understand the actions needed to solve the inverse problem. What's more, the next time they encounter this type of problem, they will have the background to interpret and solve it without having to resort to the corresponding direct problem. (Ápis Mais, 2nd grade, 2021, p. 167).

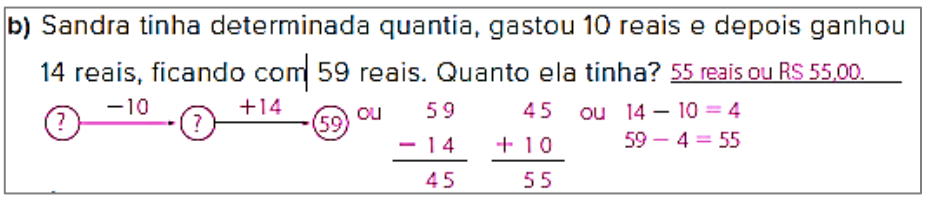
The way in which the guidelines are provided in the excerpt may have an impact on the knowledge to be mobilized by the teachers from reading and interpreting these guidelines. We believe that knowledge about *representations and their connections*, as well as the *complexity of the problem*, are some of the main knowledge developed from the interaction between the teachers and the Manuals. By reading and interpreting the guidelines, the teachers can reflect on the complexity of the problem proposed, taking into account the learning level of their students; they can also think of ways to make the problems more accessible by changing the way the situation is communicated.

When analyzing the other thematic units in the three volumes of the Teacher's Manual, we took into account the problems involving the operations addition and subtraction and their respective guidelines. We found that the explanations and guidelines inherent in the meanings of addition and subtraction are found less frequently than the specific units for working with addition and subtraction.

In order to understand and infer aspects of the approaches and their relationship with ideas about the additive field, we highlight the guidelines that deal with the development and recording of strategies through images and drawings, i.e. symbolic representations according to the Conceptual Fields Theory (Vergnaud, 2009). The guidelines emphasize that before presenting the usual algorithm, "counting", teachers need to encourage the development of personal strategies and their recording. In our opinion, these guidelines are in line with what the theory says about relational calculation. In all three volumes, we can see that relational calculation is emphasized to the detriment of the premature presentation of the usual algorithm, i.e. numerical calculation.

In addition to the guidelines on relational calculus, we found a problem in the 2nd grade manual in which relational calculus is used as an example in the solving guidelines.

Figure 5: Presentation of relational calculus as a resolution strategy[5]



Source: Ápis Mais, 2nd grade (2021, p. 174)

By reading the guidelines described for developing and recording different solving strategies, teachers can articulate *specialized knowledge of the content* (Ball, Thames & Phelps, 2008), as well as articulate the form of *representations and their connections* present in the materials (Remillard & Kim, 2017). Thus, in contact with the guidelines, teachers can formulate and activate knowledge about the approaches to be used by their students, considering the strategies that best suit each problem.

Finally, we identified that greater emphasis is placed on the meanings of addition and subtraction in the introductory parts of the units and in the task guidelines — problems mapped and analyzed in the previous section — in contrast to the introductory part of the Manual (common to all grades), in which the meanings pertinent to the operations addition and subtraction and their relationship to the problems that will be proposed in the students' tasks are presented.

[5] b) Sandra had a certain amount of money, spent 10 reais and then won 14 reais, leaving her with 59 reais. How much did she have?

We should also point out that, as shown, the 2nd grade volume has a higher rate of problems mapped onto the additive conceptual field than the other volumes. The manual in question also incorporates a greater number of guidelines on the meanings of addition and subtraction.

6 Conclusion

Based on the assumption that curriculum materials, especially the Teacher's Manual, can contribute to the expansion of knowledge about the processes of teaching and learning mathematics, as teachers read and interpret the teaching guidelines, as well as the tasks proposed for students, the study portrayed in this paper sought *to identify and discuss resources in the materials, related to the additive field, that induce professional knowledge in Mathematics.*

In order to achieve the proposed objective, we carried out a documentary analysis of three Teacher's Manuals for the 2nd, 3rd and 4th grades. The analysis revealed that the three volumes explicitly present the meanings of the operations addition and subtraction in problems involving different meanings attributed to each of the operations, as well as guidance for teachers on working with these meanings. It was also possible to identify that the classes of problems in the additive field found with the greatest predominance in the three volumes are: composition, transformation and comparison; and with the least predominance, they are transformation of composition and composition of transformations.

When comparing the approach of the three Manuals analyzed, we found that the 2nd grade volume has different units or sections for working with addition and subtraction, in contrast to the approach taken in the 3rd and 4th grade volumes, in which these operations are presented in a single unit or section. We also found that in addition to the specific units or sections for working with addition and subtraction, problems involving these operations were found in the unit on Quantities and Measures.

In the introductory section, common to all three volumes analyzed, there is no evidence of guidance on the different meanings of addition and subtraction. However, in contrast to this lack of approach, when reproducing the students' tasks, the units or sections that deal specifically with these operations explicitly address the meanings of these operations, as well as guidance for teachers on the proposed work. In the sections or units entitled *Addition and Subtraction* (2nd grade manual), *Addition and Subtraction* (3rd grade manual) and *Addition and Subtraction with natural numbers* (4th grade manual), the meanings of addition and subtraction are elucidated through subheadings that express each of the meanings involved in the problems presented, as well as guidance for teachers on the meanings that are explored.

When we consider the lack of reference to the meanings of the operations addition and subtraction in the BNCC, with regard to working with the 4th and 5th grades, we point out that the lack of coherence in relation to the other years of schooling may imply the way in which the teachers read and interpret these skills. Depending on their beliefs, conceptions and knowledge about mathematics and its teaching, teachers may understand that working with the meanings of additive operations should only be privileged in the 1st, 2nd and 3rd grades. This interpretation can reinforce educational practices based on solving problems from a procedural perspective, i.e. focusing on operative techniques as opposed to the meanings underlying each problem situation.

Another aspect revealed by the research refers to the approach taken in the three Manuals analyzed, specifically the 2nd grade Manual. Presenting the operations addition and subtraction in isolation, or even approaching addition before subtraction, may have

repercussions on the teachers' interpretation that this is the correct approach, i.e. the operations should be worked on in isolation, and that subtraction should be presented after working with addition.

Based on the guidelines contained in the 2nd grade manual on the resumption of ideas related to subtraction, we infer that in the 1st grade volume the meanings of the operations are presented in such a way as to articulate them with the ideas presented in the 2nd grade volume. In order to identify the information contained in this volume, we read the introduction to the unit and observed that the ideas relating to the operations of addition and subtraction are privileged and emphasized. In this sense, teachers can refer to the 1st grade volume to appropriate the initial ideas presented, but when planning their lessons, 2nd grade teachers may or may not have access to this material.

In this way, the approaches found throughout the Manuals, as well as the lack of information explicitly stated in the common part of these materials, the teachers, as they read and interpret the proposed guidelines and tasks, can mobilize knowledge about mathematics and its teaching. We found that knowledge about *fundamental ideas and mathematical learning paths; knowledge of content and students; knowledge of content and teaching; specialized knowledge of content; representations and their connections; knowledge of students and their characteristics and knowledge of educational contexts* can be activated from the teacher-curriculum materials relationship as teachers read and interpret the guidelines and tasks underlying the Teacher's Manuals.

The research presented here does not exhaust all the possibilities of analysis on the subject presented, since we are starting from a specific collection of materials, which may reverberate the findings regarding the number of situations involving the additive field, as well as the guidelines and approaches adopted in the collection analyzed. We also stress the importance of the fact that the Manuals analyzed provide guidelines and conditions for expanding teachers' knowledge. Based on this idea, we believe that these teachers need to be made aware of the need to read and study the Manuals in order to expand their knowledge and have the opportunity to get closer to knowledge and concepts that are similar to continuing education.

Therefore, reading and interpreting these manuals can be done during the teachers' planning time, so that they can see the curriculum materials as allies in the teaching and learning of mathematics.

We would also point out that the information underlying these materials may or may not influence the teachers' practice as they read and interpret these materials, since knowledge needs to be mobilized, which can be unique to each professional in particular, depending on how they read and interpret this information.

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