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Teaching planning in Geometry from an inclusive perspective: contributions of Universal Design for Learning

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Abstract: This research seeks to emphasize the contributions of Universal Design for Learning (UDL) in the teaching of Geometry from an inclusive perspective. To achieve this, a qualitative approach was adopted, involving pedagogical intervention with 28 students, aged between 7 and 10 years, from the 3rd grade of Elementary School I in a municipal school in Araucária, Paraná, Brasil. Over the course of 19 sessions, 16 activities related to Geometry were implemented. In the analysis, we considered the dialogues and observations perceived during interactions with the students, the aspects of UDL incorporated into the planning by the teacher-researcher, and considerations regarding Inclusive Mathematics Education. Thus, it is possible to indicate that UDL promotes inclusive education by fostering accessibility, interaction, and understanding of mathematical/geometric concepts, providing a solid foundation for the promotion of inclusion in Mathematics Education.

Keywords: Accessibility. Elementary School. Mathematics. Inclusion. Pedagogical Practices.

Planificación docente en Geometría desde una perspectiva inclusiva: contribuciones del Diseño Universal para el Aprendizaje

Resumen: Esta investigación tiene como objetivo destacar las contribuciones del Diseño Universal para el Aprendizaje (DUA) en la enseñanza de la Geometría desde una perspectiva inclusiva. Para lograrlo, se adoptó un enfoque cualitativo que implicó intervención pedagógica con 28 estudiantes de entre 7 y 10 años, pertenecientes al tercer grado de la Educación Primaria en una escuela municipal en Araucária, Paraná, Brasil. A lo largo de 19 encuentros, se llevaron a cabo 16 actividades relacionadas con la Geometría. En el análisis, se tuvieron en cuenta los diálogos y observaciones percibidas durante el contacto con los estudiantes, los aspectos del DUA incorporados en la planificación de la práctica por parte de la profesora-investigadora y las consideraciones sobre la Educación Matemática Inclusiva. De este modo, es posible indicar que el DUA favorece una educación inclusiva, fomentando la accesibilidad, interacción y comprensión de los conceptos matemáticos/geométricos, proporcionando una base sólida para la promoción de la inclusión en la Educación Matemática.

Palabras clave: Accesibilidad. Educación Primaria. Matemáticas. Inclusión. Prácticas Pedagógicas.

O planejamento docente em Geometria na perspectiva inclusiva: contribuições do Desenho Universal para Aprendizagem

Resumo: Esta pesquisa tem como objetivo evidenciar as contribuições do Desenho Universal para Aprendizagem (DUA) no ensino de Geometria sob uma perspectiva inclusiva. Para isso, foi adotada uma abordagem qualitativa, utilizando intervenção pedagógica com 28 estudantes, com idade entre 7 e 10 anos, do 3º ano do Ensino Fundamental I de uma escola municipal de Araucária, Paraná. Ao longo de 19 encontros, foram aplicadas 16 atividades relacionadas à Geometria. Na análise, consideramos os diálogos e observações feitas durante o contato com os estudantes, os aspectos do DUA incorporados ao planejamento da prática da professora pesquisadora e as considerações sobre educação matemática inclusiva. Dessa forma, foi possível indicar que o DUA favorece uma educação inclusiva, promovendo acessibilidade, interação e compreensão dos conceitos matemáticos/geométricos, proporcionando uma base sólida para a promoção da inclusão na educação matemática.

Palavras-chave: Acessibilidade. Ensino Fundamental. Matemática. Inclusão. Práticas Pedagógicas.

1 Introduction

The recognition of Mathematics as a discipline intrinsically linked to human experience requires its development to be dynamic and impactful across various aspects, such as the articulation with social, political, historical-cultural, methodological, pedagogical, philosophical, and epistemological issues, including the professional sphere. Developing skills in logical thinking, investigation, and argumentation based on mathematical knowledge enables a deeper understanding and intervention in the world around us. In this sense, it is necessary to comprehend the interconnections between mathematical concepts and their applicability in different domains, fostering students' self-confidence and perseverance in constructing and utilizing mathematical knowledge.

Among the strategies used in the classroom are the development and discussion of thematic projects that address social issues relevant to the students' environment, guided by ethical, democratic, sustainable, and solidarity principles. These strategies value differences from an inclusive perspective, embracing the diversity of cultures, individuals, and opinions, with a focus on equity. As established in the National Common Curricular Base (BNCC),

> a planning focused on equity also requires a clear commitment to reversing the historical exclusion that marginalizes groups—such as the indigenous peoples and the populations of quilombola communities and other Afro-descendants—and individuals who have not been able to study or complete their education at the appropriate age. Equally, it demands a commitment to students with disabilities, recognizing the need for inclusive pedagogical practices and curricular differentiation (Brasil, 2017, p. 16).

Educational planning should not only aim for equal opportunities but also strive to overcome the historical and social inequalities that affect specific groups by adopting inclusive pedagogical practices. This underscores the importance of an equitable educational system that addresses different challenges, promoting inclusion and social justice, allowing students to express responses and derive lessons through various forms of expression (Brasil, 2017). This approach provides opportunities for different records and demonstrations of acquired knowledge, for example, through graphic representations, written texts, digital media, among others. This, combined with systematic observations of both numerical and qualitative aspects,

should be present in the classroom, providing challenges in various contexts, including imaginary scenarios.

By overcoming various barriers in Mathematics, we enter the field of inclusive mathematics education, ensuring that each student has the opportunity to develop their mathematical potential and fully participate in society (Viana & Manrique, 2019). The school environment should be a "space of learning and inclusive democracy, [where] it must strengthen the coercive practice of non-discrimination, non-prejudice, and respect for differences and diversities" (Brasil, 2017, p. 14). This is the true meaning of inclusion, providing affection, inclusion, opportunities, and acceptance; "in this movement, Mathematics Education will be constituted from an inclusive perspective, being redefined to promote a school centered on ethics, respect for individuality, and diversity" (Viana & Manrique, 2019, p. 664).

Teaching Mathematics from an inclusive perspective requires overcoming daily barriers, developing a sensitive approach to the diversity of individuals so that the appropriation of content occurs naturally. Silva and Diáz-Urdaneta (2021, p. 5) suggest reflecting on how to equip our students to provide "opportunities in the social practice and dynamics of society to seek, inquire, organize, represent, and publicize this information with critical, reflective, and mathematical approaches."

Presenting a variety of fundamental concepts and methods for problem-solving is recommended in the BNCC, especially when addressing the teaching of Geometry in Mathematics. It is necessary for students to explore concepts such as positioning and movement in space, as well as the shapes and relationships between elements of two-dimensional and three-dimensional figures, and the development of geometric thinking. This is an essential skill for investigating properties, formulating hypotheses, and constructing solid geometric arguments (Brasil, 2017).

Lorenzato (1995) asserts that the absence of study in Geometry can result in the lack of development of geometric thinking and visual reasoning, making the resolution of real-life situations a challenging task. Corroborating this author, Poi *et al*. (2019) reaffirm such evidence, but refer to the absence of study in Geometry in the initial training of Mathematics teachers. Additionally, Coelho and Góes (2021) find that the teaching of Geometry in basic education occurs through textbooks, involving repetitive practices and emphasizing the memorization of algebraic formulas, neglecting the development of understanding problem situations in the students' daily contexts. These facts imply that "without studying Geometry, people will not develop geometric thinking or visual calculation, and without these skills, they simply will not be able to solve the life situations that have been geometrized" (Lorenzato, 1995, p. 5). Teachers need to face the challenges, which include understanding planar and spatial figures, because it is "in school that we must show students this interface between daily mathematical knowledge and school knowledge" (Góes & Góes, 2023, p. 77).

In the quest to demonstrate approaches for addressing geometric concepts and content in practice from the perspective of inclusive mathematics education, we analyzed data from Stellfeld's (2023) research, highlighting the contributions of Universal Design for Learning (UDL) in instructional planning. UDL has emerged as a fundamental approach in promoting educational equity and accessibility. With its three principles (Engagement; Representation; Action and Expression) and guidelines, it provides a valuable guide for creating flexible and innovative teaching strategies, contributing to the reduction of physical and pedagogical barriers (CAST, 2018).

It is important to highlight that, as this research involves human subjects, it was approved by the Research Ethics Committee on Human Subjects of the Federal University of

Paraná, under Protocol CAAE No. 59276522.0.0000.0214 and No. 5.545.178, dated July 26, 2022.

2 Universal Design for Leaning

UDL has its origins in the ideas of Universal Design (UD), developed in the 1960s by American architects and designers, stemming from questions about designing spaces exclusively for people without disabilities, which sparked discussions about accessibility and the rights of people with disabilities, especially due to the high number of mutilated ex-combatants after World War II (Góes & Costa, 2022). During the 1980s and early 1990s, the concept of UD began to gain widespread accessibility, particularly with the proposal of architect Ronald Mace, a wheelchair user and artificial respirator user, highlighting the growing importance of accessibility among professionals in the construction field (Cassano *et al.*, 2022).

UD encompasses the "[...] design of products and environments that can be used by all people, to the greatest extent possible, without the need for adaptation or specialized design for individuals with disabilities" (Carletto & Cambiaghi, 2007, p. 10). According to Góes and Costa (2022), it is not about creating a science or style, but about enhancing the awareness when designing environments and products, making them usable by everyone.

It is evident that this conception focused on everyday societal projects would influence the school environment, as

> the school, as an integral part of society, cannot lag behind such discussions, not only regarding physical access, appropriate furniture, and equipment but also about the need to expand and promote accessibility for all students to learning (Cassano *et al.*, 2022, p. 5).

Thus, in the 1990s, the concept of UDL emerged from discussions by researchers at the Center for Applied Special Technology (CAST) to develop an accessible digital book for every user. These discussions led to the understanding that it is necessary to eliminate or reduce methodological barriers to learning in order to ensure an equitable approach, meaning it can be applied to each and every student, while also being flexible to meet specific educational needs. This approach provides a diverse set of possibilities that enhance and strengthen student learning, regardless of whether or not they have disabilities, promoting the construction of knowledge in a universalized manner (Góes & Costa, 2022).

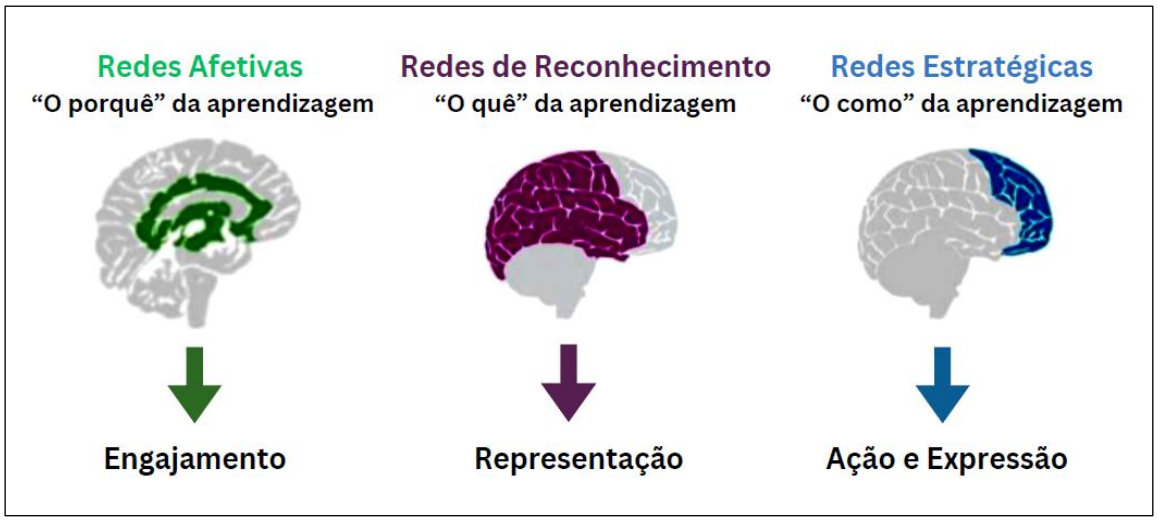
> When UDL is adopted in teaching, it becomes an extension of our shared values, reinforcing the commitment to equity and justice for all. This transforms the way participants in the teaching and learning process reflect on diversity, moving away from the idea that disability is synonymous with limitation and embracing the notion that diversity is the essence of the educational experience (Góes *et al.*, 2023a, p. 28).

UDL is a framework grounded in cognitive neuroscience, developed to design learning experiences that ensure each student attains the status of a "expert learner," that is, one who possesses extensive knowledge, significant skills, continuous learning, and intrinsic motivation to seek more knowledge (CAST, 2018). The essence of this distribution and the structure of organization in neuroscience can be observed in **Figure 1.**

The three distinct brain networks are each related to one of the UDL principles (CAST, 2018), which seek to provide different ways for students to engage in the learning situation (Engagement principle), how they acquire information (Representation principle), and how

they express and use it (Action and Expression principle). Thus, it constitutes a comprehensive approach that addresses the various forms of student learning, as it is not rigid, inflexible, or closed, but rather guiding (CAST, 2018).

Figure 1: Principles Based on Affective, Recognition, and Strategic Networks



Source: CAST (2018), adapted and translated by the authors (2024).

In addition to the three principles, the UDL framework unfolds into nine guidelines and 31 checkpoints, which assist teachers in planning. We emphasize that the organization of the principles, guidelines, and checkpoints is presented on the CAST website and in the texts by Sebastián Heredero (2020) and Góes *et al.* (2023a), as shown in **Figure 2.**

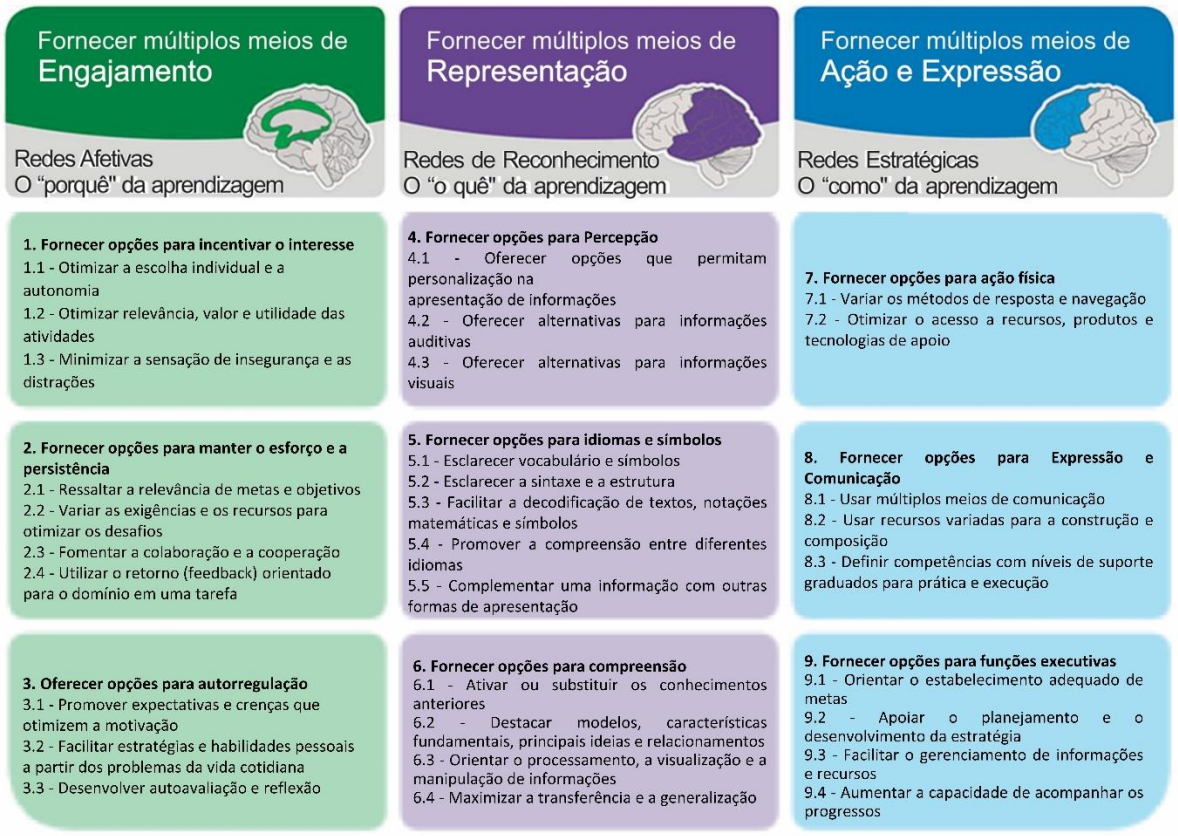
The search for an effective approach in organizing and developing educational methodologies and practices begins with an important reflection on the teaching methods that best align with the nuances of each brain network's functioning. When questioning how the UDL framework can be employed, the need arises to understand how materials play a fundamental role in providing the necessary support to enable this practical approach (CAST, 2018). In this context, the Engagement principle stands out as an essential foundation for meaningful learning. By exploring "the why" of learning, this principle aims not only to identify but also to cultivate individual student interests. Through this intrinsic connection with personal interests, it seeks to both motivate and promote deeper engagement in the educational process. This implies the need for instructional strategies that transcend barriers, spark interest, and encourage active student participation (CAST, 2018).

As highlighted by CAST (2018), Engagement is considered the fundamental element because, without active student participation, other forms of representation or expression become less effective. Furthermore, it emphasizes the importance of feedback and awareness of progress as contributing elements to stimulate intrinsic motivation and ensure continuous student engagement. Thus, the interconnection between the personalization of educational methods, the principle of Engagement, and the emphasis on feedback reveals itself as a pathway in the pursuit of more effective and meaningful education.

The principle of Representation addresses "the what" of learning, seeking to provide students with various ways of perceiving the information presented and converting it into practical knowledge applicable to their personal experiences. In this context, it is important to ask: what is the most essential content or skill we want our students to acquire? Do the multiple representations align with the content? Are they closely related to the learning objectives? It is important to emphasize the importance of evaluating all materials in light of the learning

objectives, the nature of the information, and the specific characteristics of the students (CAST, 2018). In the teaching of Mathematics, this may be where it is necessary to incorporate more representations, as it not only involves the use of symbolic expression of numbers but also the use of manipulatives, allowing students to perform operations and visualize concepts in a direct and perceptible manner (CAST, 2018). As students differ in the way they perceive and understand the information presented to them, there is no one means of representation that works for everyone.

Figure 2: Principles, Guidelines, and Checkpoints of UDL



Source: CAST (2018), adapted and translated by Stellfeld (2023, p. 59).

The principle of Action and Expression, "the how" of learning, aims to allow students to express themselves in different ways regarding the knowledge they have acquired. Furthermore, it is necessary to continuously and openly evaluate, validate the responses, provide feedback to the students, and intervene when necessary to ensure equity (CAST, 2018).

Thus, the guidelines of UDL seek to address a significant problem in education: the presence of fixed, inflexible, and restrictive curricula designed to meet the needs of some students but not each one. These three principles provide unique pathways to a common goal, reflecting in the curriculum by observing its four main components and designing them in a universal manner.

The first component addresses the objectives that must be clearly defined and communicated to individualize the learning path. The second component is the materials used in the classroom, which must have flexibility so that individualization can occur and each student can utilize them. The third component involves considering the methods, analyzing how the teacher teaches: what is their teaching practice? How do they participate and provide collaborative groups? Do they conduct discussion circles? Do they offer real-world

experiences? Finally, it is essential to consider the method of assessment, reflecting on how we can be sure that learning is truly occurring, which is intrinsically related to the learning objective rather than the teaching objective. As Stellfeld (2023, p. 64) mentions, "in addition to considering the content that students are learning, it is important to evaluate how they are assimilating this knowledge and in which situations they perform best." These considerations are fundamental to verifying whether the curriculum is hindering or limiting each student's learning.

By adopting UDL and creating flexible proposals from the beginning of planning, customizable options are offered that allow each student to progress from their individual starting point (Sebastián-Heredero, 2020). In this sense, Meyer, Rose, and Gordon (2014) indicate scientific evidence that underpins UDL practices to ensure inclusive education, addressing the diverse needs of students, and contributing to the creation of more equitable and responsive learning environments. The four stages that underpin UDL practices can be observed in **Table 1.**

Table 1: Scientific Evidence Supporting UDL Practices

Área de investigação	Evidências científicas
Foundations of UDL	Research across various disciplines, including neuroscience, education, and cognitive psychology, consolidated in the concepts of the zone of proximal development, constructivist stages, and the works of Piaget, Vygotsky, Bruner, and Bloom.
Principles of UDL	Based on modern neuroscience: the principles are built on the knowledge that our brain is formed by three different networks used in the learning process: recognition, affective, and strategic.
Promising UDL Practices	Studies over the years have proven effective and align with the UDL approach, following its principles and guidelines. Thus, didactic processes within this approach are necessary to confirm its results as efficient.
Implementation of UDL	Related to research and specific applications of UDL in learning environments, "including the necessary conditions for implementation, the most common barriers, and the contributions of this practice" (Sebastián-Heredero, 2020, p. 743).

Source: Sebastián-Heredero (2020), adapted by Stellfeld (2023, p. 56).

The UDL approach can strengthen inclusion in schools by eliminating attitudinal and pedagogical barriers and providing a flexible curriculum to meet the diversity of students. In this context, assessment should be designed considering the understanding and performance of all students, in addition to being an opportunity for them to reflect on their own learning.

> UDL is much more than an approach for redesigning practices, instructional materials, and assessments for students with disabilities; it is a worldview in which diversity is celebrated, equal opportunities are a reality, and inclusion is the foundation of education. It shapes a future in which each student has the opportunity to reach their maximum potential. It emphasizes creativity, explores new ways of learning, stimulates forms of expression, and encourages each individual to freely pursue their educational journey. (Góes *et al*., 2023a, p. 29).

In summary, UDL is an innovative approach that confronts biases, breaks down barriers, recognizes diversity, and seeks to challenge paradigms in the field of education. "When barriers are removed, equity in opportunities becomes a reality, allowing each student to reach their

maximum potential" (Góes *et al.,* 2023a, p. 29). By adopting it, each student will have equitable access to education, as demonstrated in the research whose methodological procedures we describe below.

3 Methodology

This study adopts a qualitative approach, involving "direct and prolonged contact of the teacher-researcher with the environment and the situation being investigated, typically through intensive fieldwork" (Lüdke & André, 2018, p. 12). It is a type of pedagogical intervention, where, through the design and execution of interventions (changes, innovations), the aim is to generate progress and improvements in the participants' learning processes, followed by the assessment of the impacts of these interventions (Damiani *et al*., 2013). This research model meets the researcher's need to be engaged with their research object, generating reflections and memories, respecting the research subjects with their values, interests, and principles, thereby broadening the construction of knowledge (Lüdke & André, 2018).

As previously mentioned, the methodological processes analyzed in this text were produced in the research of Stellfeld (2023), developed and applied in a 3rd-grade class of Elementary School, consisting of 28 participants, over 19 sessions through 16 practical activities, in a municipal school in Araucária, Paraná. It is worth noting that we focused our analysis on activities related to Geometry, as Góes and Góes (2023, p. 21) state that, "regarding Geometry, teaching and formal demonstrations need to be preceded by activities involving experimentation and construction."

Among the research participants were students with specific characteristics, such as Arthur, an 8-year-old diagnosed with Autism Spectrum Disorder (ASD), and Pedro, a 9-year-old undergoing investigation for indications of Intellectual Disability (ID). It is important to note that, to preserve the anonymity of the participants, fictitious names were used. Additionally, more than half of the children involved in the research were repeating the grade, facing learning challenges. In this context, the age range of the participants was between 7 and 10 years old.

During the didactic process in Mathematics, it was possible to observe the integration of Geometry concepts with other fields, as indicated in the official documents of the municipality where the research was conducted:

> Understanding the relationships between concepts and procedures from different fields of Mathematics (Arithmetic, Algebra, Geometry, Statistics, and Probability) and other areas of knowledge, while feeling confident in one's own ability to construct and apply mathematical knowledge, thus developing self-esteem and perseverance in the search for solutions. (Araucária, 2019, p. 18).

The activities developed were grounded in the principles and guidelines of UDL, aiming to promote inclusive education, encouraging students to reflect on an inclusive society, and developing their self-esteem. In addition to addressing mathematical concepts, the initiative explored ways to promote an inclusive society.

The practical activities were divided into two stages. The first occurred over seven sessions, encompassing four activities related to presenting the research to the school community, including administrators, teachers, staff, and guardians. The second stage, focused on pedagogical intervention in the classroom, consisted of 12 sessions and 12 activities. In brief, these activities explored the analysis of the shapes of packaging; recognition of geometric

figures, associating them with real-world phenomena; representation of two-dimensional and three-dimensional objects with observations from the front, side, and top views; identification of the number of faces, vertices, and edges of packaging in the form of spatial geometry; plane geometry through the unfolding of packaging; association of packaging with spatial geometric figures, such as cubes, rectangular prisms, spheres, and cylinders; and primitive notions of geometry, such as plane and space.

Table 2 presents the objectives of the activities planned to address geometric concepts and content, relating them to the principles and guidelines of UDL.

Table 2: Organization of Activities

Objectives	**UDL Principles and Guidelines**
Teaching Geometry beyond concepts and content	
Present a video of the Paralympics, five-a-side football (blind futsal), highlighting that in this modality the players are blind or visually impaired. Compare the measurements of official courts used by people with and without disabilities. Apply plane geometry in the analysis of futsal courts. Associate geometric shapes with forms presented in the courts and objects used in the games, such as the ball.	**Engagement Principle** **Guideline 1:** Provide options for recruiting interest. **Guideline 2:** Provide options for sustaining effort and persistence.. **Representation Principle** **Guideline 4:** Provide options for perception.
Discovering the geometric universe by exploring three-dimensional packaging	
Explore, analyze, and record information related to recyclable packaging. Manipulate three-dimensional shapes, such as cubes (hexahedrons), rectangular blocks (parallelepipeds, prisms), pyramids, cones, cylinders, and spheres, identifying elements of geometric shapes.	**Engagement Principle** **Guideline 3:** Provide options for self-regulation. **Representation Principle** **Guideline 5:** Provide options for language and symbols.
Exploring shapes in everyday life, puzzles, and geometric challenges	
Solve spatial geometry problems through puzzles and investigative challenges. Identify the presence of plane and spatial figures. Identify plane and spatial figures in objects from the school and home environment. Visualize and represent two-dimensional and three-dimensional objects in various positions (top, front, and side views).	**Representation Principle** **Guideline 4:** Provide options for perception. **Guideline 6:** Provide options for comprehension. **Action and Expression Principle** **Guideline 7:** Provide options for physical action. **Guideline 8:** Provide options for expression and communication.
Learning different languages	
Analyze geometric shapes through instructions provided regarding accessibility, format, textures, colors, measures of mass, and volume. Interpret numerical information and identify values and forms of spatial recognition. Recognize accessibility present in the packaging, noting that many products have Braille writing on	**Representation Principle** **Guideline 4:** Provide options for perception. **Guideline 5:** Provide options for language and symbols.

their packaging or a QR code that directs to videos with translation in Brazilian Sign Language (Libras).	
Exploring and redefining spaces in accessible and non-accessible markets	
Discuss the conception of an ideal market, accessible to everyone. Develop concepts of ordering, categorization, spatial orientation, proportion, representation, and spatial visualization.	**Action and Expression Principle** **Guideline 9:** Provide options for executive functions.

Source: Stellfeld (2023), adapted by the authors (2024).

By analyzing the data from observations and audio and video recordings conducted by the teacher-researcher, as well as materials created by the students, we evidenced the contributions of UDL that provided a mathematics education from the perspective of inclusive education.

4 Results e analysis

In this section, we present moments of the research application, highlighting the geometric concepts and content addressed and the remarks or observations of the teacher-researcher. We also emphasize how the aspects of UDL (principles, guidelines, and checkpoints), considered in the planning of activities, contributed to the practice. Each of the principles, even though the foundations of the approach do not indicate that such principles need to be applied separately or that all of them must be present in practice (Góes, Costa & Góes, 2023), is expressed. It should be clarified that, for the identification of the UDL guidelines and checkpoints, we used Góes *et al*. (2023) as a reference.

a) *Teaching Geometry Beyond Concepts and Content*

Considering the principle of Engagement, the use of videos was planned to connect students to the experiences of Paralympic athletes. The goal was to address significant challenges to comprehension when information does not receive proper attention and does not involve cognitive activity (Sebastián-Heredero, 2020). In this context, UDL Guideline 1, which seeks to provide options for recruiting interest, played an important role by offering options that stimulate students' interest, motivating the exploration of knowledge. This aligns with Checkpoint 1.2, optimizing the relevance, value, and utility of educational activities (Góes *et al.*, 2023a).

During the activities, there was a demonstration of enthusiasm and admiration for the achievements of athletes with physical disabilities. This sparked curiosity, as illustrated by Arthur, an 8-year-old student, who asked, "*How can he play ball if he can't see? I find it difficult even though I can see.*" This observation allows us to associate curiosity with his daily life, highlighting the principle of Representation in UDL. The student used prior knowledge of Geometry by mentioning the word "ball," associating it with the spatial geometric figure, the sphere. According to Sebastián-Heredero (2020, p. 736), "[...] learning and the transfer of learning occur when multiple representations are used, as this allows students to make internal connections as well as connections between concepts."

The teacher-researcher seized this moment to introduce five-a-side football (blind futsal) as an example, highlighting the uniqueness of this sport and the use of bells inside the ball, which allow players to locate it by sound. She emphasized that the goalkeeper does not have a visual impairment and that the court is smaller in size compared to traditional football. By presenting this lesser-known reality to the students, she fostered curiosity and connected it to mathematics education.

In this practice by the teacher-researcher, we observe Sebastián-Heredero's (2020, p. 760) remark that "[...] a significant part of teachers' activities is dedicated to capturing students' attention and involvement, given that they differ significantly in what attracts their attention and motivates their interest." This is related to Guideline 2, which offers alternatives for sustaining effort and persistence in motivating students towards inclusive education, providing options to maintain their motivation (Góes *et al.*, 2023a). Checkpoint 2.2 addresses the variability of resources to optimize challenges, encouraging students in their activities. (Góes *et al.,* 2023a).

The comparison between futsal courts for people with and without disabilities enabled the analysis of elements of plane geometry, emphasizing the importance of Mathematics in daily life through UDL Guideline 4, which proposes providing options for perception to facilitate the understanding and interpretation of content (Góes *et al.*, 2023a). This aligns with Checkpoint 4.1, which suggests personalization in the presentation of information, enhancing perceptual clarity, and highlighting essential information to meet the needs of a wide diversity of students.

The application of UDL by the teacher-researcher, considering the importance of emotions and affectivity in learning, emphasized the need to provoke and motivate students in diverse ways. This approach aligns with the aspect highlighted by Vargas (2022, p. 59), stating that "affectivity is present in all stages of human development, thus observing the fundamental role of emotions and affection in the formation of psychic life." It is about understanding the true meaning of inclusion, providing affection, inclusion, opportunities, and acceptance.

b) *Discovering the Geometric Universe by Exploring Three-Dimensional Packaging*

In a subsequent stage of the activity, which involved the principle of Representation through the use of packaging, the students explored concepts of spatial geometry by manipulating three-dimensional shapes, such as cubes (hexahedrons), rectangular blocks (parallelepipeds, prisms), pyramids, cones, cylinders, and spheres, with reference to UDL Guideline 5, aimed at providing options for language and symbols. During the demonstration of identifying and associating these shapes with everyday objects, the students mentioned examples such as the cube represented by a coffee packaging box; the parallelepiped, related to the school desk and locker; the cone, to the birthday hat, ice cream, and funnel; the cylinder, to the milk can, chocolate powder can, yogurt pot, PET bottle, and fan blade; and the sphere, to the ball and globe. **Figure 3** illustrates the students analyzing some of the packaging.

In addition to associating the geometric nomenclature with the shape of each package, the students highlighted the similarities and differences between them. This process revealed distinct perceptions in the interpretation and understanding of the presented information (CAST, 2018), stimulating the students' interest in mathematics education, focusing on Geometry concepts. As Sebastián-Heredero (2020, p. 760) points out, "alternative ways to promote interest and strategies that correspond to the intra/interindividual differences among students for their engagement" are fundamental. This action emphasized UDL Guideline 3, which proposes providing options for self-regulation to students, associating learning with everyday reality, offering "[...] opportunities for students to explore and understand their abilities and experiences, which is essential as it promotes the belief in each one's potential" (Góes *et al.*, 2023a, p. 37). This approach also aligned with the application of Checkpoint 3.2, by facilitating personal strategies and skills from real-life problems, encompassing the promotion of positive emotions through the presentation of real and inspiring situations that help overcome daily challenges (Góes *et al.*, 2023a).

Figure 3: Exploration of Recyclable Packaging

Source: Stellfeld (2023, p. 176).

In a subsequent activity, it became evident the ability to relate the shapes of packaging to real objects, demonstrating mastery of mathematical language, as when Raquel, aged 10, said, "*Look, teacher, that die over there looks like a cube, that box looks like a rectangular prism, that one looks like a sphere, and this can here looks like a cylinder because it rolls*," and Tatiana, aged 9, commented, "*This coffee box also looks like a rectangular prism. That name is really long."* UDL Guideline 5, aimed at providing options for language and symbols, was also present at this moment through Checkpoints 5.1, related to clarifying vocabulary and symbols, which "aim to facilitate understanding and access to content for a broader range of audiences" (Góes *et al.*, 2023a, p. 38); 5.2, concerning clarifying syntax and structure, as mathematical equations and languages of geometric concepts are methods that can facilitate student understanding; and 5.4, linked to understanding different languages, assisting in the comprehension of specific Geometry terms, promoting broader understanding, fostering inclusion, and accessibility in the educational environment. In this context, mathematics education assumed an inclusive perspective, "being resized to promote a school centered on ethics, respect for individualities, and the valorization of diversity," as reported by Viana and Manrique (2019, p. 664).

c) *Exploring shapes in daily life, puzzles, and geometric challenges*

In the stage involving puzzles and investigative challenges, guided by the Representation principle of UDL, students solved spatial geometry problems by visualizing and representing two-dimensional and three-dimensional objects in various positions (orthographic views), planned with Directive 4, in the sense of providing options for perception. João, aged 8, expressed his enthusiasm by stating, "*That's cool, now I see a geometric shape in everything I look at!".*

Within the scope of the Action and Expression principle, a notable moment occurred when students solved geometric figure problems through a puzzle, relating two-dimensional and three-dimensional representations. This practice aligned with Directive 7 of UDL, which aims to provide options for physical action, specifically Checkpoint 7.1, concerning varying methods of response and navigation, offering alternatives that accommodate the pace, speed, and extent of movements required for interaction (Góes *et al*., 2023a).

Some observations perceived align with UDL Guideline 8, which aims to provide options for expression and communication, promoting collaboration in their everyday experiences (Góes *et al.*, 2023a), through resources like manipulative math materials (Checkpoint 8.2), represented in this instance by packaging associated with spatial geometric figures.

In accordance with the assumptions of the BNCC, which emphasize the study of Geometry in understanding a broad set of concepts and procedures necessary to solve problems in the physical world and different areas of knowledge (Brasil, 2017), the students were tasked with determining the number of faces, vertices, and edges by unfolding some packaging. In this process, they recognized that the faces of the three-dimensional figures form into two-dimensional shapes; regarding this, Camila, aged 8, shared her experience: "*This coffee box here, when we open it, looks like a bunch of rectangles".*

Another example was the excitement of Felipe, aged 8, expressing: "*Look! My wallet, the classroom door, and the teacher's cupboard are rectangles.*" His statement demonstrates prior knowledge and associates a flat geometric shape with objects from his daily life, revealing evidence of the Action and Expression principle of UDL, differing in the ways "they seek knowledge and express what they know" (Sebastián-Heredero, 2020, p. 736), which is often easier to grasp in Geometry when shapes are all around us (Oliveira *et al.*, 2022).

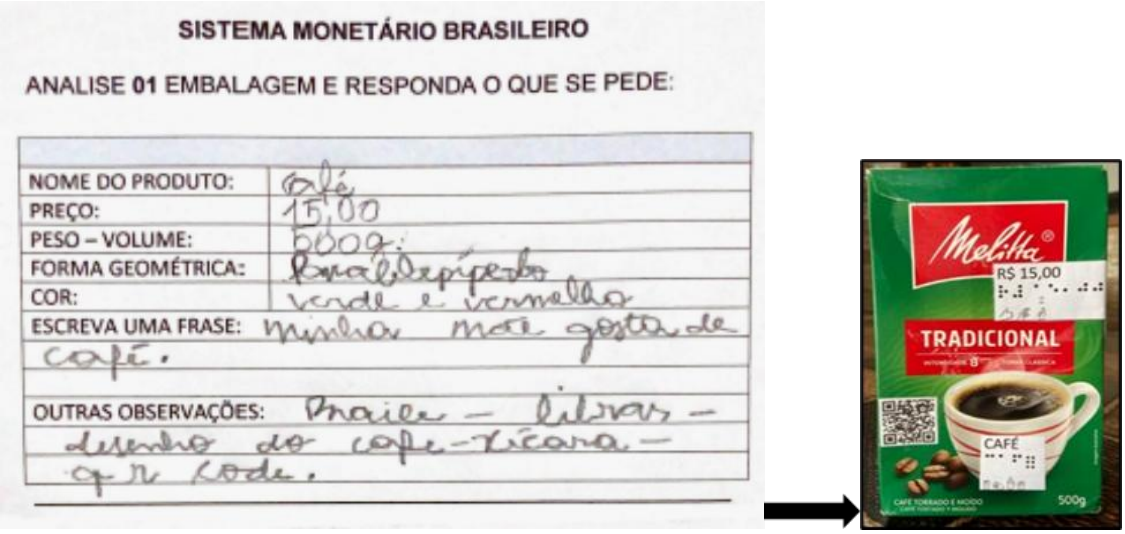
The enthusiastic expressions of the students when identifying the presence of flat and spatial figures, such as rectangles on the classroom desks, parallelepipeds on the cupboard and door, cylinders on the fan, milk cans, and yogurt pot, and spheres on the ball and globe, highlight their ability to apply mathematical concepts to visualize objects around them. This underscores the relevance of including in planning the UDL principle 6, which suggests providing options for comprehension, facilitating interpretation, and ensuring effective knowledge acquisition tailored to individual needs (Góes *et al*., 2023a). In this context, point of verification 6.1 stood out, concerning activating or substituting prior knowledge, which proposes "the application of resources that activate students' prior knowledge, aiming to eliminate barriers and disparities in the assimilation of concepts" (Góes *et al.*, 2023a, p. 39). Through the guidelines of the Representation principle, the aim was to cultivate an informed and creative student, empowering them in the active construction of their knowledge. These principles "should be considered when planning inclusive activities, therefore, we consider it necessary to adopt them in proposals for Geometry teaching activities," as mentioned by Lima *et al.* (2020).

d) *Learning of different languages*

In a subsequent moment, record sheets were distributed to each group member, associating specific questions with the packaging chosen by the researching teacher. This practice was planned based on guideline 4 of UDL, which focuses on providing options for perception, aligning with checkpoint 4.2 by offering alternatives for auditory information, and 4.3 by offering alternatives for visual information. This allowed students to enhance their reading and writing skills, interpret numerical information, identify values, and recognize spatial forms, demonstrating that Mathematics transcends its boundaries, contributing to the completeness of human formation.

Another example of this life preparation is the development of knowledge about accessibility present in packaging, indicating that many products already feature Braille writing on their packaging or QR codes that link to videos with Brazilian Sign Language (Libras) translation, practices not mandated in the curriculum. Consequently, these languages have been incorporated into packaging, making them accessible to diverse individuals (**Figure 4**).

Figure 4: Activity of analysis and registration of recyclable packaging



Source: Stellfeld (2023, p. 176).

This approach reflects the importance of employing "a variety of diverse resources and materials [being] essential to make learning" (Góes *et al.*, 2023a, p. 39) meaningful, accessible, and redesigned to meet the diverse needs of individuals, even if the classroom in question does not have blind or deaf students. Redesigning, as justified by Góes, Costa, and Góes (2023, p. 24), "[...] expands didactic practices and resources to incorporate the new student into the group without excluding them. This understanding is the foundation for defining the learning objectives of each person, who has unique abilities, interests, and ways of learning," serving as an example of a product that would be ready if the classroom were to receive a blind or deaf student. As emphasized by Góes *et al*. (2023b, p. 18), "when addressing the education of blind people, it is crucial to use tactile materials that complement oral explanations of concepts." In this context, it is crucial to consider the ergonomics of these materials, ensuring that their dimensions and characteristics are planned to optimize their effectiveness in communicating the concepts addressed.

Also, the UDL was considered in the planning with the objective consistent with guideline 5, aimed at providing options for language and symbols, allowing for shared understanding and a more comprehensive interpretation of languages and symbols (Góes *et al*., 2023a). This approach aligns with checkpoint 5.2, clarifying syntax and structure, and checkpoint 5.4, promoting understanding across different languages. In the context of geometry teaching, this supported student learning, including notations of geometric concepts such as vertices, faces, edges, and names of geometric figures, both flat and spatial. These elements can be considered mathematical languages, a different language in the student's vocabulary, which connects with checkpoint 5.1 mentioned earlier. This approach also aligned with the curriculum guidelines of the municipality of Araucária, which emphasizes the importance of describing the characteristics of spatial geometric figures, classifying them, comparing them, and recognizing the number of faces, vertices, and edges of a spatial geometric figure. The ability to carry out plannings, understanding that the faces of three-dimensional figures form two-dimensional figures, is integrated into this educational perspective. (Araucária, 2019).

e) *Exploring and redefining spaces in accessible and non-accessible markets.*

Throughout the activities based on the principle of Action and Expression, an effort was made to encourage reflection on the conception of an ideal market, accessible to everyone. The teacher-researcher posed questions to the students, addressing topics such as appropriate product categorization, the ideal design for aisles, and the proper width of spaces. Additionally, issues related to product packaging and identification methods were discussed, emphasizing the importance of organizing the environment to ensure an efficient flow of people and goods. This approach aligned with Guideline 9, which seeks to provide options for executive functions, developing strategies that optimize knowledge assimilation (Góes *et al.,* 2023a). This highlights the importance of activities that address perception and visualization from the early grades, as emphasized by Oliveira *et al*. (2022).

In the market activity and as a proposal to cultivate empathy, the students assumed the roles of people belonging to priority groups, such as blind individuals, people with physical disabilities, autistic individuals, pregnant women, mothers with babies in strollers, and the elderly. During the simulation of an inclusive market (**Figure 5**), a non-accessible market was also designed, where the students faced obstacles such as narrow aisles, high shelves, and a lack of signage, experiencing difficulties in moving around and shopping. Additionally, they got lost in the spaces, requiring the assistance of their peers to reach the checkout.

Figure 5: a) Accessible market (left) and non-accessible market (right); b) Highlight of the accessible market

a) 

b) 

Source: Stellfeld (2023, p. 133).

During the construction of the inclusive market, the students committed themselves to achieving their objectives, verifying the design of aisles, shelves, and other strategies, strengthened by including Checkpoint 9.2 in the planning, which is related to "using goals, project planning, and task explanation to unify guidance and strategies" (Góes *et al*., 2023a, p. 42). This created a flexible school environment, allowing them to utilize resources that highlighted their abilities in solving the proposed activities.

The students' remarks revealed that, in practical activities such as the analysis of packaging and the simulation of an inclusive market, the integration with mathematical concepts manifested in various ways, including written, oral, and abstract expression. To highlight the achieved objectives, we share observations from students who reflected on their peers' understanding, emphasizing the importance of reference points to facilitate spatial orientation, allowing each "customer" to find products independently:

> *I think everything needs to be organized, teacher. Each product should be together: cleaning products with cleaning products, food with food, cookies with cookies. The prices also need to be big, because my grandmother can't see those tiny letters, you know!* (Fabiana, 8 anos).

> *There needs to be someone in each area to help the elderly and people who can't see. The aisles also need to be wide and the shelves low, teacher. Because neither my mother nor I can reach those products up high. It's so tall!* (Valentina, 9 anos).
>
> *It's important to have some benches for older people to sit and wait their turn because the checkout line takes a long time and people get tired of standing. There should also be people to help with paying for the purchases and packing them* (Patrícia, 10 anos).
>
> *You know that video we watched? The one with the different flooring where the blind person uses a cane to navigate? That should be in the accessible market too!* (Fábio, 9 anos).

These remarks reflect insights that demonstrate the importance of designing a plan based on the UDL approach, providing opportunities for learning about concepts of ordering and categorization. Specifically, regarding geometric concepts and content, issues related to spatial orientation, space proportion, and spatial representation and visualization were evident. It became clear that students perceive Mathematics as a tool that can facilitate inclusive and accessible markets, as it enables the planning of open spaces and shelves arranged at appropriate heights concepts that are intrinsically related to spatial perception and measurement comparison.

Regarding spatial relationships, the BNCC emphasizes that "when addressing the concept of space, the development of topological, projective, and Euclidean spatial relationships is stimulated, in addition to geographical reasoning, which are important for the process of cartographic literacy and learning with various languages" (Brasil, 2017, p. 364), as forms of representation and spatial thinking.

5 Considerations

Throughout this study, we observed the application of UDL integrated into mathematics education, focusing on Geometry content, for a 3rd-grade class of Elementary School. The practical and contextualized activities involving themes such as the Paralympics, sports adaptations for the visually impaired, packaging analysis, and market simulation proved effective in promoting students' Engagement, Representation, and Action and Expression.

Some UDL guidelines were observed through the activities carried out, even if they were not considered during the planning. A clear example is Guideline 3, which provides options for students' self-regulation. Additionally, Guideline 1 suggests options to recruit interest, and Guideline 8 aims to provide alternatives for expression and communication.

The interconnection between the guidelines reflects a comprehensive approach aimed at the holistic development of students, demonstrating that, even when not initially planned together, the UDL guidelines coexist synergistically. This intertwining enhances the effectiveness of instructional design, promoting a more inclusive educational environment tailored to the diverse needs of students (Góes, Costa & Góes, 2023).

Regarding the principle of Engagement, we observed that the use of videos about the Paralympics sparked students' interest in mathematics education, providing a connection between abstract content and their everyday reality. The association of geometric figures present in packaging with objects in the school environment demonstrated the students' ability to apply mathematical concepts appropriately. This was made possible by planning based on the UDL approach, which, being flexible, allowed for the exploration of different engagement methods, recognizing the diversity of learning styles.

In the context of the principle of Representation, the connection between abstract mathematical concepts and practical everyday situations of the students stood out. The activities of packaging analysis and market simulation provided multiple forms of representation, promoting an understanding and practical application of mathematical knowledge for teaching Geometry.

The guidelines of the principle of Action and Expression encouraged students to seek new mathematical knowledge and express their prior understandings. The integration of mathematical concepts into practical activities, such as the simulation of an inclusive market, revealed the applicability of Mathematics in planning accessible spaces. These aspects allowed students to express their knowledge in written, oral, and abstract forms, respecting their individual preferences and contributing to a rich educational environment.

The integration of UDL into mathematics education planning has proven to be an approach that promotes inclusion, critical thinking, and the development of students' geometric skills in everyday life, addressing the diversity of their learning styles and needs, and fostering inclusive mathematics education. Additionally, it raises awareness of diversity and the importance of accessibility, not only to academic/scientific concepts but also to the interests and experiences of the students.

Beyond the geometric concepts addressed, the presented activities can be used with students in subsequent years, providing deeper exploration into Geometry topics such as area and volume calculations, unit conversions, among others. In summary, these activities allowed students to experience and understand concepts within their daily lives, fostering inclusion, critical thinking, and the development of socio-environmental skills.

We can conclude by affirming that the contributions of UDL in the development of activities for students make them inclusive, as it is necessary to consider each student; flexible, as the teacher must be open to the “unexpected” and use it to guide their practice; and receptive to each student, who should find joy in engaging with what is being proposed. “Thus, it is necessary to welcome and value the uniqueness and multidimensionality of each individual, providing equity in education, and forming engaged citizens who are aware of their role in society” (Góes *et al.*, 2023a, p. 30), enhancing pedagogical practices for a rich educational environment that allows academic flourishing and the development of sensitive, ethical, and supportive citizens.

We suggest that future propositions address the extension and generalization of the presented activities, redesigning them for students in subsequent years and exploring new topics within Geometry, such as area calculations, volume, and unit conversions. These studies could deepen our understanding of how UDL can contribute to more inclusive Mathematics teaching, taking into account the diversity of learning styles and needs of the students.

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