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Reading and writing the world with Mathematics in the initial training of Mathematics teachers

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Abstract: Rethinking initial teacher training considering the current dilemmas and needs of Education is a great challenge. Committed to this task, the objective of the research presented in this article was to analyze aspects that contribute to the critical and reflective training of future teachers in an investigative activity on Reading and Writing the World with Mathematics. This work was developed in a subject called Geometry for Basic Education of a Mathematics Degree course, in which a mapping of the quality of life in traffic in different neighborhoods of the city of Rio de Janeiro/RJ was proposed. After analyzing the activity developed, we highlight that there are four main formative aspects of the practice of Reading and Writing the World with Mathematics that must be considered in the degree: unpredictability, dialogue, dynamicity and transferability.

Keywords: Critical Mathematics Education. Teacher Training. Social Justice.

Leer y escribir el mundo con Matemáticas en la formación inicial de profesores de Matemáticas

Resumen: Repensar la formación inicial docente a la luz de los dilemas y necesidades actuales de la Educación es un gran desafío. Comprometidos con esta tarea, el objetivo de la investigación presentada en este artículo fue analizar aspectos que contribuyan a la formación crítica y reflexiva de los futuros docentes en una actividad investigativa sobre Leer y Escribir el Mundo con Matemáticas. Este trabajo se desarrolló en la asignatura Geometría para la Educación Básica de la Licenciatura en Matemáticas, en la que se propuso un mapeo de la calidad de vida en el tránsito en diferentes barrios de la ciudad de Rio de Janeiro/RJ. Tras analizar la actividad desarrollada, destacamos que existen cuatro aspectos formativos principales de la práctica de Leer y Escribir el Mundo con Matemáticas que deben ser considerados en la carrera: imprevisibilidad, diálogo, dinamismo y transferibilidad.

Palabras clave: Educación Matemática Crítica. Formación de Profesores. Justicia Social.

Leitura e Escrita de Mundo com a Matemática na formação inicial de professores de Matemática

Resumo: Repensar a formação inicial de professores diante dos dilemas e necessidades atuais da Educação é um grande desafio. Compromissados com essa tarefa, o objetivo da pesquisa apresentada neste artigo foi analisar aspectos que colaborem com a formação crítica e reflexiva de futuros professores em uma atividade investigativa de Leitura e Escrita de Mundo com a Matemática. Este trabalho foi desenvolvido em uma disciplina denominada Geometria para

Educação Básica em um curso de Licenciatura em Matemática, na qual foi proposta um mapeamento da qualidade de vida de populações de diferentes bairros da cidade do Rio de Janeiro/RJ. Após análise da atividade desenvolvida, ressaltamos que existem quatro principais aspectos formativos da prática da Leitura e Escrita de Mundo com a Matemática que devem ser consideradas na licenciatura: a imprevisibilidade, o diálogo, a dinamicidade e a transferibilidade.

Palavras-chave: Educação Matemática Crítica. Formação de Professores. Justiça Social.

1 Initial training of mathematics teachers for Social Justice

Forming critical and reflective citizens has been an important task undertaken by educators concerned with an Education that goes beyond the fulfillment of curriculum and focused on the exercise of citizenship.

In this process, we highlight the role of Mathematics as a science that can contribute to students' awareness and to foster transformative practices in the construction of a fairer society. This view has been more explored in recent decades, since, traditionally, Mathematics and its teaching were seen in a more technical way and based on a false sense of neutrality.

> Is the role of mathematics education to preserve mistaken views of social and political order, which are deeply rooted in society? Have we lost our way as educators? Or is it that Mathematics Education has always been guided by the interests of the job market and we, mathematics educators, have difficulty recognizing this? (Skovsmose, 2014, p.16)

If we understand the school space and, more specifically, Mathematics classrooms as spaces of power, we must question whose interests are being served when we opt for a certain practice or address certain topics in our classes. The way we encourage our students to think about different types of knowledge also influences the worldview that is created, bringing them closer to or further away from the area we teach.

It is worth highlighting that teaching practices reflect a multiplicity of knowledge and experiences, that is, their constitutions are the result of teachers' intentionality, but based on their training in a broad sense. According to Tardif (2002), we train as educators throughout our lives. However, the author highlights that initial training is the privileged and systematized moment to become professionals. In addition to curricular knowledge, we learned a lot from the examples and practices of teachers at the university. And sometimes, we reproduce practices that we experience, but we don't reflect on properly. Gutstein (2007) highlights that it is a challenge to develop an educator's stance towards social justice based on the experiences of other professionals, as most traditional teacher training courses still do not contribute satisfactorily to this aspect.

Therefore, we believe it is important to provide Mathematics degree students with moments in their initial training in which they have contact with pedagogical practices that present other possibilities for the Mathematics teacher to act in favor of learning that dialogues with the student's reality and make them reflect about the unequal structures to which they are subjected, aiming for a society in which social justice prevails. This type of practice promotes an educational culture that values differences and pays close attention to individual and collective demands.

According to Gutstein (2016), teaching from the perspective of Mathematics Education for Social Justice means "teaching students to learn and use mathematics to develop a sociopo-

litical awareness of the roots of injustice in their lives and in society at large, so that can eventually act to change the things they believe are wrong" (Gutstein, 2016, p. 64). This way, students will have the opportunity to experience important discussions for society based on mathematical knowledge.

The implementation of Public Policies aimed at valuing and encouraging initial and continuing teacher training contributes strongly to this perspective. It is worth highlighting actions such as the Institutional Teaching Initiation Scholarship Program (PIBID) and the Pedagogical Residency Program (RP) which, by inserting graduates into the school environment at different moments in their academic life, allow them to have contact with the reality experienced by educators and begin to design alternatives to the learning demands that arise from contact with basic education students. Furthermore, this moment presents itself as an excellent opportunity for articulation between academic theory and teaching practice.

The inclusion of the Mathematics teacher in initial training in everyday life, experiencing the school environment, allows the student to have access to a real classroom, an environment in which many factors, especially those external to the educational context, interfere with the development of classes. Skovsmose (2014) denounces this situation when he defines the concept of a stereotypical classroom, that is, an environment in which everything occurs in the best way possible, without problems, and the students' difficulties are restricted only to learning mathematical concepts.

However, teacher trainers concerned with education for social justice can introduce practices into their classes, so that students can experience teaching practices that will inspire their teaching actions in Basic Education in the future. Therefore, in this article we will reflect and analyze the process of developing and applying a teaching project that followed the principles of Mathematics Education for Social Justice in a Geometry for Basic Education subject of a Mathematics Degree course. The objective of the research was to analyze aspects that contribute to the critical and reflective training of future teachers in an investigative activity on Reading and Writing the World with Mathematics.

2 Reading and writing the world with Mathematics

The process of understanding the world/reality is quite complex and challenging. Every person is inserted in a social context and lives with others, being able to share similar experiences, however everyone perceives the world in a unique way. Attitudes in everyday decisions reflect lived experiences and the way they interpret the factors that make up the structure of which they are part. Taking a critical stance in the face of lived reality becomes feasible through a process of raising the individual's awareness of the factors that influence their relationship with people and the world.

When entering the school environment, students bring with them knowledge and cultural background obtained within the family, which will be important in the process of training these individuals. In this sense, the question arises about the teaching role in civic education and in the acquisition of scientific knowledge.

> The educator's mission is not to use their condition to profess or teach a discipline, to proselytize, that is, to convert the student to their doctrine, idea or discipline, but rather to use the disciplines to fulfill the greater objectives of education. In other words, the educator subordinates the subjects, in particular, the contents, to the objectives of education and does not subordinate education to the objectives, transmission and advancements of the subjects. The student must be, as an individual, the determinant of the knowledge that is transmitted to him (D'Ambrosio, 2016, p.32).

Assuming the same commitment to an Education that goes beyond disciplines and content, concerned with the critical and reflective formation of students, Freire and Macedo (2022) discuss in the book "Literacy: reading the world, reading the word" the literacy process of individuals, which is not restricted to the process of mechanical reading of words, but literacy that provides people with an understanding of their reality to the point of awakening the desire for a transformation of their social condition.

> Literacy, within this broader perspective, not only empowers people through a combination of pedagogical and critical analysis skills, but also becomes a vehicle for studying how cultural definitions of gender, race, class and subjectivity constitute themselves as constructs. both historical and social. Furthermore, literacy, in this case, becomes the fundamental pedagogical and political mechanism through which to establish the ideological conditions and social practices necessary for the development of social movements that recognize the imperatives of radical democracy and fight for them (Freire & Macedo, 2022, p. 43).

In this sense, the authors point out that teachers have an important contribution to this process, they argue that educators are promoters, through their pedagogical practice, of training that leads students to their emancipation. In other words, through the content taught in the classroom, a feeling of overcoming the inequalities that permeate social relations is created. "In other words, for radical literacy to come to fruition, the pedagogical must become more political and the political, more pedagogical." (Freire & Macedo, 2022, p. 42)

Eric Gutstein (2003;2006) was based on Paulo Freire's productions about reading the world and moves such reflections to the field of Mathematics Education so that this area also has the aim of a liberating education that is practiced by all people, including those who are not in a situation of social disadvantage, but who are important in the fight for a fairer society for all. This critical look, during mathematics classes, at the injustices present in everyday life and the desire for a change in conditions are defined by him as Reading and Writing the World with Mathematics.

Reading the world with Mathematics represents "using mathematics to understand power relations, resource inequalities, differences in opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language and other differences" (Gutstein, 2003, p. 45). In other words, students build their awareness process encouraged by teachers.

However, it is not enough to just identify problems through reading the world, it is necessary to act, create strategies to promote changes using the knowledge learned. This idea can be defined as the process of writing the world with Mathematics. This stage is considered the most difficult to execute, as writing the world through Mathematics requires preparation so that students feel capable of transforming society (Gutstein, 2006).

We reiterate that the process of recognizing and overcoming social inequalities, power structures or situations involving oppression against minorities, through Mathematics, for example, are not accomplished naturally. It is necessary that everyone involved in the process, educators and students, is engaged and aware of a larger context in which they are inserted. And from this, they understand themselves as subjects prepared to contribute to social transformation.

Gutstein (2007) also highlights that mathematics teachers who wish to develop this practice in their classes have to work on the three C's with their students, which are: community knowledge, critical knowledge and classical knowledge. Community knowledge highlights that

the teacher needs to get involved and understand the community of which students are part, their customs, power relations, mobilized knowledge that is not learned in the school environment, in addition to the culture they experience. As for critical knowledge, it refers to the contribution to the process of critical understanding of society and the ability to question, ask questions and doubt the absolute truths often proffered, including at school. Finally, in classical knowledge, he highlights that it is knowledge disseminated in academic environments and that everyone must have access, especially those who occupy more vulnerable positions in society, as it will be possible to ascend socially from the acquisition of this knowledge that is valued in society.

We believe that for future Mathematics teachers, being in contact with Mathematics Education for Social Justice and Reading and Writing the World with Mathematics, from initial training, will strongly contribute to future actions when they take over their classes.

Therefore, we will present, next, an activity designed and executed following the presented precepts of Reading and Writing the World with Mathematics. We did not find activities of this nature in initial teacher training in the literature consulted, therefore, this work makes such a shift by proposing an exercise in a discipline that aimed to look at Geometry with a focus on Basic Education, as detailed below.

3 Reality investigation

The activity entitled "Mapping access to leisure, culture and sport in the city of Rio de Janeiro" was developed with 20 students enrolled in the subject "Geometry for Basic Education" offered for the Mathematics Degree course at a public university in Rio de Janeiro. January/RJ.

The subject is offered in conjunction with three other subjects: Algebra for Basic Education, Arithmetic for Basic Education and Statistics for Basic Education. The main objective is to think about how these curricular components can be addressed in the Final Years of Elementary School and High School, presenting possibilities that go beyond so-called traditional practices. In every semester, the subjects seek to work on a common thematic axis that culminates in the development of work that is presented to the four classes in a joint action to conclude that academic period. In the first half of 2023, the proposed theme was "How can mathematics be put into practice to understand the world in its multiple dimensions?".

After theoretical studies carried out on the search for peace as the responsibility of mathematicians (D'Ambrósio, 2011), Scenarios for Investigation (Skovsmose, 2000) and Reading and Writing the World with Mathematics (Gutstein, 2006; Moura & Faustino, 2017), the students were guided by the report made by Rico Gutstein of the "South Central" Project and, motivated by the professor/researcher (first author of the work), they designed a study on the city of Rio de Janeiro/RJ.

The South-Central Project, as Gutstein reports in an interview with Moura and Faustino (2017), was developed within the scope of continuing teacher training. As the researcher reports, South Central is a community located in Los Angeles, California/USA. The activity was motivated by the story of Rodney King, a black man who was brutally beaten by the police in a police raid, which led to several urban rebellions against such a dehumanizing act. Gutstein reports that in 2002, about two years after the incident, when listening to a radio program that mentioned the case, he heard from the announcer that at the time, within a radius of 5 km from the epicenter of the riots, there were 640 alcoholic beverage stores and no alcoholic beverage centers. community or cinema (Moura & Faustino, 2017). Concerns about this fact led Gutstein to propose a project with practicing professors to investigate, in a medium-sized US city, what should be a reasonable proportion of cinemas, community centers and alcohol stores.

When discussing this fact with students studying Geometry for Basic Education, comments arose, such as that some regions in the city of Rio de Janeiro would probably present a panorama similar today to that of the South-Central region in the 1990s. confirm (or not) this fact, we jointly designed an investigation, understood as a scenario for investigation with references to reality (Skovsmose, 2000).

3.1 Analyzing different contexts in the city of Rio de Janeiro

The first step was to organize the 21 students into five groups and carry out a random draw of five among the city's 158 neighborhoods. Each group was responsible for investigating one of the neighborhoods listed below: Group 1 – Cocotá (Ilha do Governador[1]); Group 2 – Pavuna (Pavuna); Group 3 – Água Santa (Meier); Group 4 – Vargem Grande (Barra da Tijuca) e Group 5 – Paciência (Santa Cruz)[2].

After that, each group chose a central point to study within a certain radius in the selected neighborhood. The choice of the point should consider a reference location for the neighborhood based on an initial investigation into it. For example, group 3, when investigating the history of the Água Santa neighborhood, determined that the central point of the studied area would be the Santa Cruz water company, since the neighborhood emerged from the discovery of the city's second hydromineral source there.

Initially, the idea was to carry out the analysis of a radius of 5 km, as proposed in Gutstein's activity. However, when carrying out initial tests on some tools, for example, Calc Maps[3] e Maps Directions[4], it was found that the chosen area was not ideal, as it covered very large regions encompassing different neighborhoods and would not provide a specific portrait of a location. The suggestion to reduce the radius to 3 km was discussed with the entire class and accepted as the best option. Figure 1 shows the radius delimited by Group 3.

It is possible to see that the region goes beyond the Água Santa neighborhood, however, considering the purposes of the activity, this was not understood as a problem. The objective was to analyze the surroundings of a certain previously selected region to understand how residents in the center of the circle access certain services and spaces, which is why the analysis could be considered in this way.

After defining the area, students should estimate the population within the radius. Therefore, they carried out research on official population data in the neighborhoods present in the previously defined circle. After that, they started from the assumption of a uniform distribution in urban areas, discarded non-urban areas and used their knowledge of geometry to, from the region understood as inhabited, make a proportional calculation of the population.

After estimating the population, the whole class debated and defined which spaces should be accounted for and which would allow discussion about the quality of life of the residents. After a conversation with the entire class, it was established that the number of schools (public, private, language), daycare centers, hospitals and health centers, pharmacies, libraries, parks and leisure areas, universities (public and private), museums, police stations, prisons, subprefectures, theaters, cinemas, bars, shopping malls.

To do the counting, students used the Google Maps resource, carrying out searches when

1The administrative region to which the neighborhood belongs is indicated in parentheses.

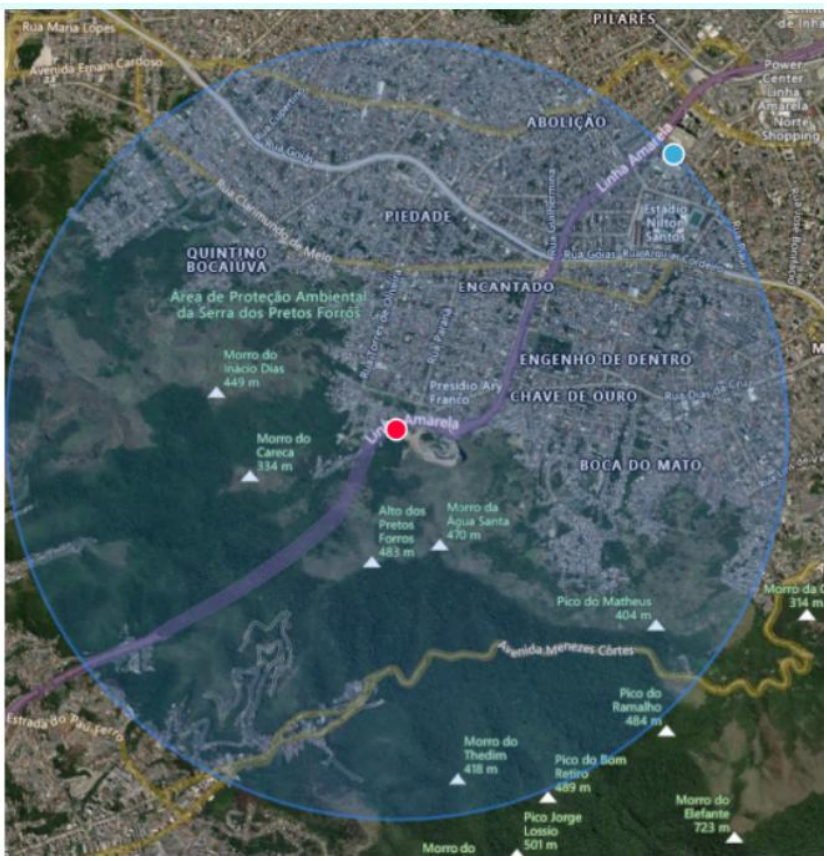
[2] The Data Rio website contains open data for free access on various aspects of Rio de Janeiro and was widely used by students in the activity. We suggest that readers explore it if they want to learn more about each of the neighborhoods. The website also has a map of the city of Rio de Janeiro divided into neighborhoods. Available at: https://www.data.rio/. Accessed on: May 28, 2024.

3Available at: https://www.calcmaps.com/pt/map-radius/. Accessed on November 29, 2023.

4Available at: https://www.mapsdirections.info/pt/mapa-circulo-raio/#google_vignette. Accessed on November 30, 2023.

the information was not evident. The data are recorded in Table 1.

Figura 1: Region delimited by Group 3 for investigation.



Source: Researchers' collection.

Table 1: Number of different spaces counted by students.

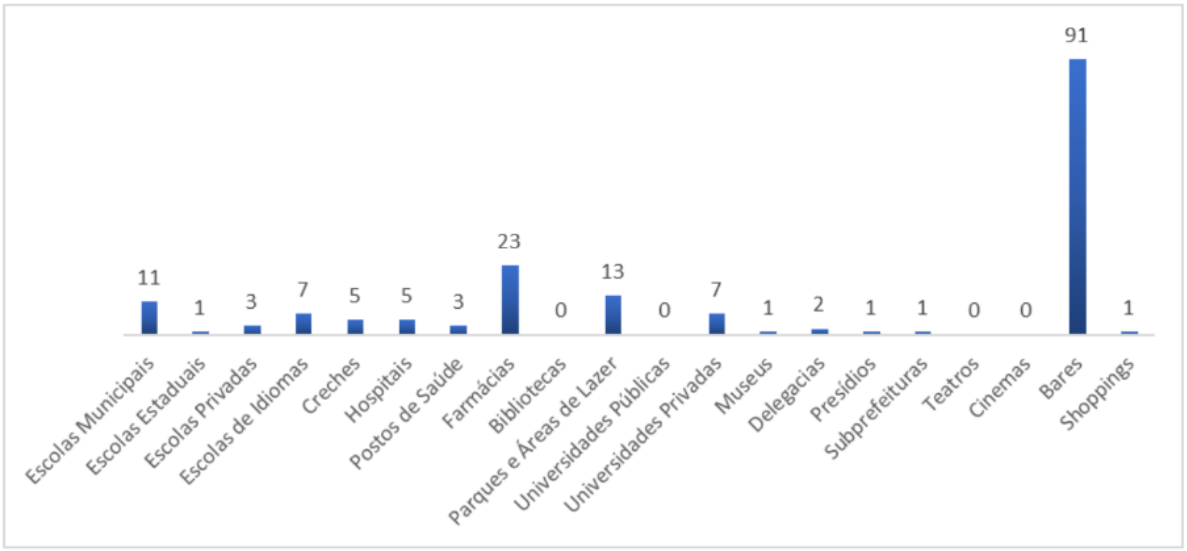
Groups	1	2	3	4	5
Neighborhoods	Cocotá	Pavuna	Água Santa	Vargem Grande	Paciência
Public schools and daycare centers	23	18	15	7	19
Private schools and daycare centers	13	16	12	9	18
Hospitals and health centers	16	15	8	6	5
Pharmacies	36	29	23	18	36
Libraries	1	0	0	0	0
Parks and leisure areas	19	13	13	6	10
Public universities	0	0	0	0	0
Private Universities	6	9	7	1	6
Museums	0	4	1	0	0
Police stations	1	4	2	1	1
Prisons	0	0	1	0	0
Subprefectures	1	0	1	0	0

Theaters	0	1	0	0	0
Cinemas	1	4	0	2	0
Bars	98	22	91	18	34
Shopping malls	0	1	1	2	0

Source: Prepared by the authors.

Each group used a different strategy for registration and presenting the data collected. For example, we present Graph 1 produced by Group 3.

Graph 1: Data collected about the area defined by Group 3 – Água Santa.



Source: Researchers' collection.

It is worth noting that the data presents an estimate of the location's portrait, as not all spaces are registered on Google Maps and locations that may have closed recently may still be registered on the website. However, approximate data was sufficient to achieve the objectives of the activity. The lack of supply of cultural and leisure spaces is evident in all regions investigated, in contrast to the high number of bars. This was a reality similar to that perceived by Gutstein (2017) and which was highlighted by all groups.

Next, another aspect investigated by the students was the quality of the local streets and sidewalks. The Street View feature on Google Maps was used to observe some parts of the area. Group 3, for example, brought notes from some different locations within the traced radius. One of them was the region of the "Nilton Santos" Olympic Stadium, also known as Engenhão. Near the stadium, the streets were free of potholes and had wide sidewalks suitable for physical activity. On the other hand, students perceived a different situation for less busy streets whose sidewalks were not accessible for people with reduced mobility, as depicted in Figure 2.

With this analysis, the different groups reflected on the issue of mobility, also understood as a basic aspect of the quality of life of the community in each location.

Figure 2: Record of one of the streets in the analyzed region made by Group 3.

Source: Researchers' collection.

3.2 Investigating distances and access with Taxi Geometry

With the first analysis process completed, the class was encouraged to think about distances. After the quantitative analysis of the establishments, the students learned about Taxi Geometry. According to Barbaresco and Morgado (2013), it is not a "new geometry", but another metric for Euclidean Geometry that considers the plane as a map of a city and, therefore, the shortest distance between two points should take considering that movement can only be done through the streets.

> Taxi Geometry is a geometry capable of modeling the trajectories, "by broken lines", of citizens and vehicles moving between blocks, along the axes of streets and avenues. Distance is not measured like the flight of a bird, but like the journey of a taxi in a city, whose streets extend vertically and horizontally in a block or urban grid, which can conveniently be associated with the Euclidean plane (Barbaresco and Morgado, 2013, p.4).

To learn more about Taxi Geometry, students were introduced to semi-reality problems, in which an ideal world is considered in which streets are represented by horizontal and vertical lines in a checkerboard grid on an orthogonal Cartesian plane.

After this exploration, we returned to the investigation of the delimited region and the students were encouraged to calculate the shortest distance that a resident who hypothetically resided in the center of the established circle would have to move to access each of the spaces listed above. The shortest distances should be found even for places that exceed a 3 km radius, if none were available within the delimited area. The data indicated by students in their work are compiled in Table 2.

Table 2: Shortest distance from the center of the circle for each of the locations investigated.

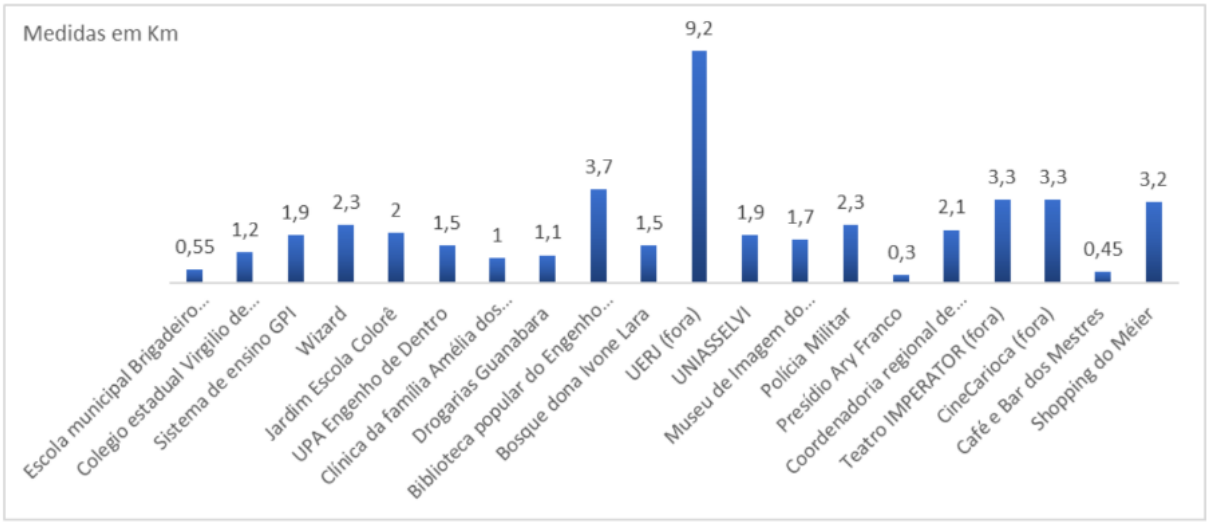
Grupos	Grupo 1	Grupo 2	Grupo 3	Grupo 4	Grupo 5
Bairros	Cocotá	Pavuna	Água Santa	Vargem Grande	Paciência
Escolas e creches públicas	0,8 km	0,6 km	0,55 km	0,7 km	0,25 km
Escolas e creches privadas	0,8 km	0,3 km	1,9 km	0,7 km	0,19 km
Hospitais e postos de saúde	0,5 km	2,1 km	1,5 km	2,2 km	1,2 km

Farmácias	1 km	0,05 km	1,1 km	1 km	0,14 km
Bibliotecas	0,05 km	5 km	3,7 km	2,3 km	4 km
Parques e áreas de lazer	0,6 km	1,5 km	1,5 km	0,5 km	0,11 km
Universidades públicas	12 km	19 km	9,2 km	32,4 km	6,4 km
Universidades privadas	1,2 km	2,3 km	1,9 km	4,6 km	0,15 km
Museus	24 km	1,5 km	1,7 km	20,5 km	7 km
Delegacias	2,5 km	2 km	2,3 km	5,2 km	3,8 km
Presídios	20 km	19 km	0,3 km	29,7 km	24 km
Subprefeituras	3 km	6 km	2,1 km	19,2 km	4 km
Teatros	23 km	2,5 km	3,3 km	5,4 km	7 km
Cinemas	12 km	2,8 km	3,3 km	5,3 km	11 km
Bares	0,05 km	0,02 km	0,45 km	0,3 km	0,01 km
Shoppings	12 km	2,8 km	3,2 km	0,8 km	11 km

Source: Prepared by the authors.

As with the quantitative representation of spaces, each group used its own form of registration. To illustrate, we present again the graphic record made by Group 3, now to indicate the distances and which will be presented in Graph 2.

Graph 2: Shortest distance between the center of the circle and the environments investigated by group 3.



Source: Researchers' collection.

As expected, the most frequent spaces had a shorter distance, and scarcer spaces required greater travel by residents to be able to access them.

We consider that the process described promoted a Reading of the World with Mathematics for students (Gutstein, 2007). Future teachers were able to reflect on the reality of a given group and aspects that are related to their quality of life. It is worth highlighting that even if it is not decisive, the context experienced influences people's future perspectives. Therefore, we emphasize the importance of offering and promoting access to the most different goods and services for everyone, in different areas, such as education, health, security, leisure, among others.

To conclude the process of reflection on reality, students were asked the question: Does proximity guarantee access? Table 3 presents excerpts from the groups' responses.

Table 3: Group responses to the question "does proximity guarantee access?"

Groups	Responses
1 - Cocotá	Ilha do Governador has a privileged structure of services, allowing the population to benefit from them, including UFRJ, where the distance from Cocotá to Ilha do Fundão is around 24 km. The question that remains for reflection is whether the available bus lines meet the demand of the population of this neighborhood and whether the population is encouraged to take advantage of the various outdoor leisure areas in the Ilha do Governador neighborhood. […] In an initial analysis, Cocotá has good access to the different regions of the city of Rio de Janeiro, but, the resident of Cocotá wanting to go to the city center, where there are several cultural and leisure options, on weekends and holidays, It is restricted to only the bus line, with alternative timetables, as the ferries, with easy access to Praça XV, do not operate.
2 - Pavuna	Proximity guarantees access to this resident only if he wishes to carry out activities within his neighborhood, because, as we have seen, if he wishes to go to public college or go to the beach, for example, he will have to travel for more than an hour on public transport. After all, even though the subway has made access easier for him, his neighborhood is still located in an area far from the center of Rio.
3 – Água Santa	Living in Água Santa means being close to some cultural and leisure options that the North Zone of Rio de Janeiro offers, it borders several well-known neighborhoods in the capital of Rio, such as: Jacarepaguá, Freguesia, Engenho de Dentro, Abolição, Méier, Quintino, Piedade and Encantado. In addition to having low access to infrastructure for everyday services, given that there is only access to buses; in other words, there is no access to trains and subways so close by. When it comes to access to public universities, there are none in the neighborhood or nearby. The number of municipal and state schools differs greatly, showing that there is a prioritization of the initial years and "leaving secondary education a little more abandoned", as well as language teaching by private institutions. The neighborhood has a considerable number of pharmacies nearby and a huge number of bars. As for culture, the neighborhood is not and should not be considered a highlight, as it leaves something to be desired in relation to many other neighborhoods in the city of Rio.
4 – Vargem Grande	In terms of access to the places mentioned, as we saw previously, most of the services mentioned in this report are present in the studied area, however, the number of these locations not present in the radius is also large. Therefore, using the public transport system present in the region, composed primarily of bus lines, a resident would have guaranteed access to all these services. Exceptions are found in access to culture, due to the distance between the resident and museums, cinemas and universities. Furthermore, we found it difficult to access the health system, as there is a low density of health posts in the region, with the only public health post being far from the region's urban centers.

5 - Paciência	It's a neighborhood with no nearby leisure areas, so the most immediate entertainment havens for residents seem to be bars. With no cinema, theaters or museums present in the neighborhood within the radius determined in the room, the cultural aspect appears to be precarious, citizens must leave the neighborhood in search of these areas. For basic education and language schools, the neighborhood does not present access problems, except the lack of federal institutions. Still in terms of education, we have several private colleges in the neighborhood, so private higher education is very present in the region, which does not apply to public colleges that are not in the researched area. Non-serious health cases have services such as pharmacies and UPAs nearby, but for serious cases, residents must go far away in search of treatment, in an emergency, this can complicate the chance of recovery.

Source: Prepared by the researchers.

We realized from the reports that many aspects of the reality of the regions could be grasped by the students who indicated the need for investment in urban mobility to access more distant locations and, also, local infrastructure in the provision of cultural and health spaces, in this case from group 4, for example.

After the process of Reading the World with Mathematics, students should put themselves in the role of public managers and develop intervention proposals in the region to strengthen possible identified weaknesses. This movement can be understood as Writing the World with Mathematics, in which knowledge is used to mobilize transformations aiming for a fair and more equitable society. Table 4 presents excerpts from the undergraduate students' proposals.

Table 4: Proposals for improving the regions.

Groups	Responses
1 - Cocotá	Given what was presented, we believe that the priority of the location is access, so the first act would be to improve transport lines through graph theory, as in many places the public transport travel time is the same as walking. Improve the paving of the site, avoiding accidents and inconvenience to the population. Encourage leisure activities at Parque Manoel Bandeira.
2 - Pavuna	To improve the neighborhood, it would be necessary to pave the streets, traffic control and a sub-prefecture on site. After all, with the streets in the states that residents face, flooding often makes it difficult to get around on rainy days. Without proper traffic control, cars are parked in inappropriate places such as sidewalks and interfere with pedestrian passage. Furthermore, the sub-prefecture is urgent because, without it, it is not possible to manage public services in specific areas of a city, meeting the unique needs and demands of each region and there is also a certain lag in street and road maintenance, public cleaning, supervision of local businesses, urban planning and community development. Furthermore, without them, more direct contact with local authorities is also difficult, resulting in a better quality of life for residents and a more balanced development of the city. After all, these are the areas that the subprefecture manages and the closest to the neighborhood is in Oswaldo Cruz, near Madureira.

3 – Água Santa	The neighborhood has suffered for a long time with a constant lack of water, knowing that it is an essential asset, as mayors, we would solve these problems in the neighborhood and surrounding areas. We would try to accelerate the project established in June 2000 where Municipal Law No. 3,035/2000 was enacted by the Rio de Janeiro City Council, which provides for the creation of an Ecological Park in the neighborhood with: leisure areas, playground, sports courts, swimming pools, cycling and running areas, skateboarding rink, gym equipment, among others. Furthermore, promote the integration of the community, industry and commerce to promote tourism, the economy, culture and leisure in the locality. Finally, we would create a housing complex in the prison area, considering that the neighborhood suffers from intense rain in several nearby neighborhoods and there are still people living on social rent.
4 – Vargem Grande	As mayors of the city, we would propose the expansion of BRT routes to Vargem Grande, creating a new line that connects the Pontal terminal with the center of Vargem Grande. In this way, we will expand neighborhood residents' access to public transport, creating direct integration with subway, train and bus lines. Thus, increasing ease of access to other regions of the city. As a second proposal, we would improve the population's access to the public health system, promoting a biannual or annual event with the presence of professionals from different areas of health. In this way, we would increase the quality of life of residents.
5 - Paciência	As mayors of a neighborhood without a university and with few leisure areas, we would like to implement a project to build an educational and recreational complex. This complex would consist of a university and a technical school, offering courses in the areas of business, technology, engineering and social sciences, as well as mechanics, electronics, IT and health. This would provide young people in the community with options to specialize and qualify for the job market. Additionally, a recreational complex would be built, including green areas and open spaces for physical and sporting activities, such as football fields, basketball and volleyball courts, as well as play areas for children and families, a public swimming pool and a running track. . With this project, the neighborhood would become an educational and recreational center for the region, providing employment and development opportunities for the local community.

Fonte: Elaborado pelos pesquisadores.

The graduates put into practice the critical analysis of the location and the entire process of defining investment priorities in the regions. We highlight the role of Mathematics in the process, since the movement of estimating the population, quantifying spaces and analyzing distances were essential steps in decision-making when preparing proposals.

We highlight that the 3 C's presented previously (Gutstein, 2007) were mobilized. Students put scientific knowledge into practice through mathematical concepts involving quantities and measurements, statistics and geometry. Furthermore, they had to learn about the reality of the neighborhoods by studying maps, news and articles about the different regions, mobilizing the community's knowledge. Finally, critical knowledge was present in the analyzes to answer questions proposed by the teacher and which emerged organically in the research process.

4 Final Considerations

The activity "Mapping access to leisure, culture and sport in the city of Rio de Janeiro" allowed undergraduate students to experience the role of protagonists in their own learning. At the end of the activity in question, the teacher/researcher promoted reflections on how to put such actions into practice in Basic Education and the students pointed out that after experiencing the investigation, they were able to understand in practice how to conduct the dynamics of a class from the perspective of Mathematics Education for Social Justice. After analyzing the activity developed, we emphasize that there are four main formative aspects of the practice of Reading and Writing the World with Mathematics that must be considered in the degree: unpredictability, dialogue, dynamicity and transferability.

Investigative activities place teachers in a "risk zone", as discussed by Penteado and Skovsmose (2002), despite causing fear for many teachers, the risks bring with them possibilities. Unpredictability was seen as a power factor for the activity arising from its flexibility. Even though guidelines were previously drawn up, developments occurred beyond what was initially predicted. Students from different groups brought different solutions and observed the needs of each reality investigated. The objective of the activity and the way it was conducted allowed them, through critical knowledge, to mobilize scientific knowledge to expand what they knew about the community/municipality in which they live.

Dialogue was present in all meetings and not only based on the logic of communicating ideas, but a complex and politicized act, where active listening and the creation of a democratic environment are valued (Alrø; Skovsmose, 2006). It was possible to notice this during negotiations regarding general agreements on the activity, in work between groups and in decision-making.

In each of the meetings of this activity, students developed different tasks, such as: research, debates, sharing of ideas, construction of graphs and tables, statistical analyzes and investigations of distances. In this way, classes became more dynamic, as students took on the role of agents of their own learning, carrying out different tasks and even allowing different skills to be valued.

Finally, we highlight the power of the transferability of knowledge about the practice experienced. Future teachers who were able to experience Reading and Writing the World with Mathematics as students reported at the end of the activity that they felt more prepared to develop activities following the same precepts than when they had just read about the topic. In this way, we highlight the importance of experience, so that knowledge from initial training can be mobilized for future actions in schools.

Based on the aspects mentioned, we argue that activities from the perspective of Reading and Writing the World with Mathematics can and should be part of the practice of Higher Education teachers to encourage such moments to be carried out in the future in Basic Education, aiming for a new meaning of the role of school Mathematics in student training. We hope that future research involving Reading and Writing the World with Mathematics in initial teacher training will allow reflection on the aspects learned here, so that more and more conscious and intentional initiatives are developed with the aim of understanding such processes, in depth and allow educational spaces (school or university) to be open and encourage the development of autonomy and criticality.

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