

Mathematics textbooks as producers of subjectivities based on female figures

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Abstract: Research literature has been discussing for some time the social representations produced through the use of textbooks by students in Basic Education, as well as the power relations behind the creation of these materials. To investigate how mathematics books can act as producers of subjectivities based on the female figures present in their pages, this article aims to answer the question: What can we discuss from the mathematics textbook about what “is” to be woman”? To this end, we developed a qualitative study with a post-structuralist bias and a feminist perspective, which analyzes emerging discourses in the speeches of mathematics undergraduates and teachers in basic education when perceiving gender representations arranged in books. mathematics teachings and evoke other possibilities for female representation that are more assertive and promote women's access to the most diverse social spaces. The research collaborators were participants in a workshop developed to produce data for a dissertation. The data analyzed in this text focuses on the possibilities of creative insubordination in the face of what is currently presented in textbooks, with an analysis of the collaborators' production as options that explore other ways of being a woman, different from the hegemonic one.

Keywords: Gender. Mathematics Textbook. Women.

Los libros de texto de Matemáticas como productores de subjetividades a partir de figuras femeninas

Resumen: La literatura de investigación viene discutiendo desde hace algún tiempo las representaciones sociales producidas a través del uso de libros de texto por parte de estudiantes de Educación Básica, así como las relaciones de poder detrás de la creación de estos materiales. Con el fin de investigar cómo los libros de matemáticas pueden actuar como productores de subjetividades a partir de las figuras femeninas presentes en sus páginas, este artículo tiene como objetivo responder a la pregunta: ¿Qué podemos discutir desde el libro de texto de matemáticas sobre qué “es” ser mujer? Para ello, desarrollamos un estudio cualitativo con sesgo postestructuralista y perspectiva feminista, que analiza los discursos emergentes en los discursos de estudiantes y docentes de matemáticas de educación básica al percibir representaciones de género dispuestas en los libros de enseñanza de las matemáticas y evocar otras posibilidades de representaciones femeninas que sean más asertivas y promuevan el acceso de las mujeres a los más diversos espacios sociales. Los colaboradores de la investigación participaron en un taller desarrollado para producir datos para una tesis. Los datos analizados en este texto se centran en las posibilidades de insubordinación creativa frente a lo que actualmente se presenta en los libros de texto, con un análisis de la producción de las

colaboradoras como opciones que exploran otras formas de ser mujer, distintas a la hegemónica.

Palabras clave: Género. Libro Didáctico de Matemáticas. Mujeres.

Livros didáticos de Matemática como produtores de subjetividades a partir de figuras femininas

Resumo: Há algum tempo, a influência dos livros didáticos na (re)produção de subjetividades de estudantes na Educação Básica, bem como as relações de poder que precedem sua elaboração, é tema de estudos na Educação Matemática. Com o intuito de investigar como livros de matemática podem atuar como produtores de subjetividades a partir das figuras femininas presentes em suas páginas, objetivamos responder ao questionamento: O que podemos discutir a partir do livro didático de matemática sobre o que é “ser mulher”? Este estudo qualitativo, pós-estruturalista e feminista, analisa discursos emergentes nas falas de licenciandas e professoras de matemática da educação básica ao perceberem as representações de gênero dispostas em livros didáticos de matemática. No contexto de uma dissertação, foi ofertada uma oficina cujos dados focam nas possibilidades de insubordinação criativa diante do que é, atualmente, apresentado em livros didáticos. Os resultados exploram outras formas de representar mulheres, em suas múltiplas e não-hegemônicas maneiras de existir. Eles destacam a necessidade de expandir as discussões interseccionais sobre gênero na Educação Matemática e de evocar outras possibilidades para representações femininas que sejam mais assertivas e promovam acesso das mulheres aos mais diversos espaços sociais.

Palavras-chave: Estudos de Gênero. Livro Didático de Matemática. Mulheres.

1 Introduction

Gender persists as a criterion of social differentiation that establishes hierarchies and marginalizes people in Brazilian society. It is common for us to witness different types of violations of women's rights, violence against their bodies and attacks on their lives. Defamation, murder, mutilation, beatings, and trafficking are just some of the dehumanization that haunt them. Swain¹ (2009) argues that sexual difference is established through indifference. With this affirmation, dehumanization is (re)produced and naturalized. It does not matter what happens to these women. After all, "these lives have no value. They serve a system based on patriarchy and the device of sexuality" (Swain, 2009, p. 389).

In this context, women are objects of appropriation and constructed as "others. From an early age, they face subordination and devaluation, are subjected to daily violence, and are defined by their absence from the subject that the norm recognizes: men, who accumulate privileges and "through repeated and daily observed recommendations, (...) serve as a reference for all" (Louro, 2008, p. 22). Those who do not fit into these parameters are destined to be censured, forbidden and, above all, excluded.

Women's space for self-constitution is limited by mechanisms that confine them to behaviors, ambitions, and occupations that are socially acceptable for them. The cultural messages that perpetuate these restrictions are everywhere, competing in the order of discourse, transforming their bodies into marketable objects or reducing them to the image of mother, wife, and domestic. According to Walshaw (2001, p. 481), these messages are not "static and personal mental frameworks, but public ones that agree on ways of doing things whose meanings change over time. Thus, the “ideal woman” is constantly being claimed, influencing

¹ We have chosen, the first time a person is quoted directly, to introduce them with their full name, in order to make their gender visible (knowing the limitations of assuming gender from their name) and their ancestry, with their full surname.

both the structure of institutions and the constitution of people in their thoughts, feelings, and actions.

Among these institutions is the school, which provides access to knowledge, not only scientific, but also political, cultural, and social, organized by hierarchies of power that compete in the construction of these students' subjectivities. According to Louro (2000, p. 60), "all schooling processes have always been and continue to be concerned with monitoring, controlling, shaping, correcting, and building the bodies of boys and girls. Therefore, school knowledge, whether explicit or covert, circulates, examines, and pedagogues bodies according to desirable notions of morality, civility, and normality (Louro, 2000).

In other words, our practices have political implications and are in tension with the relationship between knowledge and sources of social control. An illustrative example of this phenomenon can be seen in relation to mathematical knowledge. Mathematics is perceived by many as a noble and rigorous science in which errors are intolerable, deviations are punished, and its truth is placed above all other statements (Giraldo, 2018). In addition, the mastery of mathematics is sometimes valued above other forms of intelligence, and those who do not meet its expectations are relegated to failure.

The practice of school mathematics, influenced by such epistemological attitudes, tends to produce devastating effects of oppression and exclusion. Among the marginalized groups, we highlight the position of girls who, immersed in cultural patterns of subordination, are exposed to unequal and inferior experiences (Walkerdine, 1998). Finally, we are immersed in a socio-cultural context in which "masculinity functions as a deeply rooted metaphor for reason" (Hottinger, 2010, p. 56) and is considered an imperative and necessary transcendence in relation to the feminine.

The math textbook, in turn, reinforces such experiences. It is used "as an instrument of (trans)formation and conduction of the behavior of the subject and society" (Santos, 2019, p. 12). Whether through the reproduction of stereotypes or the unequal and devalued representation compared to men, these books have been identified as a limiting obstacle for girls, restricting future projections that deviate from the norm. This is because the pages they use so much do not provide them with enough representation to see multiple possible ways of "being a woman." (Blumberg, 2015).

In the article "Gender as problem in math textbooks: a practical handbook on how to be a girl/woman", Neto and Silva (2021) use the examples studied in the doctoral theses of Neto (2019) and Souza² to argue that math textbooks also serve as a kind of behavioral guide for girls. Through these examples, the author highlights the teachings on what they should understand by "femininity", as well as what their domestic and social skills and duties should be. The characteristics emphasized are:

- (i) girls should be careful; (ii) girls should be selfless; (iii) girls should be delicate;
- (iv) women should take care of time, (vi) girls should be organized and efficient and, finally, (vii) girls should know how to cook. (Neto & Silva, 2021, p. 215)

In addition to this article, other productions, such as those by Godoy, Lima and Musha (2019), Souza e Silva (2017 and 2018), Neto and Guida (2020), among others, denounce the extent to which mathematics textbooks are used to inscribe certain bodies into certain discursive practices. Regarding women's bodies in particular, they argue that relations of knowledge and

² Not completed due to the author's death.

power are often intertwined with mathematical knowledge. When teaching the content, be it basic mathematical operations, measurement or counting, textbooks use the apparent neutrality of mathematics and its overvalued status to perpetuate the norms targeted at girls. In the denunciations of these studies, it is evident "how much the mathematics curriculum is articulated with a language of difference for the production of social inequalities" (Souza & Silva, 2017, p. 379).

We therefore agree with Foucault (2014, p. 41) when he emphasizes: "every educational system is a political way of maintaining or modifying the appropriation of discourses, with the knowledge and power they entail". We argue here, however, that the school as we know it seems to remain in the realm of "maintaining". It and all its didactic-pedagogical apparatus are maintained, compromising "the guarantee of the right to quality education and entailing the exercise of a mutilated citizenship" (Junqueira, 2013, p. 481). In mathematics classes, our students are constantly underestimated and discouraged from excelling, which alienates them or does not allow them to pursue professional opportunities in careers that require aptitude in the exact sciences (Hottinger, 2010). Textbooks, especially mathematics textbooks, continue to (re)produce an ideal of a woman to be imitated (Neto & Silva, 2021; Souza & Silva, 2018), communicate ideologies of domination (Blumberg, 2015), and mobilize multiple erasures (Louro, 1997).

This begs the question: when, then, will we be able to make the transition to "change"? How can we transform our pedagogical practices into instruments of emancipation, instead of simply perpetuating and preserving inequalities and exclusions? How long will the school maintain gender as a determining parameter for restricted spaces of speech, legitimized silences, crushing invisibilities and the isolation of "undesirables"?

Faced with this chaotic scenario of deep-rooted prejudices and suspension of rights, we will reflect on the question: What can we discuss from the mathematics textbook about what it is to "be a woman"? Therefore, this qualitative study with a post-structuralist bias and a feminist outlook aims to reflect on female representations in mathematics textbooks, analyzing the discourses that emerge in the speeches of undergraduates and basic education teachers when they perceive the gender representations set out in mathematics textbooks and evoke other possibilities for female representation that are more assertive and promote women's access to the most diverse social spaces.

2 Methodology

Between denunciations and announcements, the methodological paths of this study emerge: qualitative research with a post-structuralist bias and a feminist stance. According to Creswel (2007, p. 26), qualitative research has a flexible structure with "a focus on individual meaning and the importance of interpreting the complexity of a situation. In this way, all investigative and analytical procedures incorporate the particularities of the environment, the people involved in the research, and the researcher. We therefore explore the "world of meanings of human actions and relationships, a side that is not perceptible and cannot be captured in equations, averages, and statistics" (Minayo, 1994, p. 22).

We also emphasize the post-structuralist bias, which, according to Peters (2000, p. 29), is "a complex network of thought - embodying different forms of critical practice. An approach to thinking and writing that does not assume the existence of a static, unchanging world. The emphasis is on the ability to "understand historically how truth effects are produced in discourses that are neither intrinsically true nor false" (Foucault, 2021b, p. 44).

Finally, the feminist perspective is aligned with the values and goals of the feminist

struggle. It brings an orientation that is fundamentally different from those that do not incorporate it, even though the methods used may seem similar (Crotty, 1998). Feminist research can offer new interpretations of life and identity, paving the way for unprecedented readings that contrast with the masculinized approach to science, precisely because it is a tireless search for equality and liberation from patriarchal domination. In short, as Crotty (1998, p. 201) has said, "feminist research is always a struggle to at least reduce, if not eliminate, the injustices and lack of freedom experienced by women."

The data for this research was collected in the context of a Cycle of Workshops called "Maria goes with which others? The construction of subjectivities based on the female figure in mathematics textbooks", as an extension action of the Research Group "MatematiQueer: Gender and Sexuality Studies in Mathematics Education". The choice of this name was a way of questioning who are the "others" presented in math textbooks to students, paraphrasing the popular expression "Maria goes with the others". However, the objective goes beyond that. The proposal aims to stimulate reflection and create interventions that challenge the hegemony of these representations. In other words, it seeks to instigate questions such as:

What "others" could the student who reads the math textbook go with? Only with housewives and mothers? Aren't there other girls/women who are brilliant in other social and professional areas, who can give these students more powerful identifications? Representations such as these that form students towards gender equality? (Durval, 2023, p. 83)

In fact, in some situations, this expression is used with a negative connotation, suggesting that the person follows the behavior of others without reflecting on his or her identity. However, inspired by the song "Maria vai com as outras" (Maria goes with the rest), by Midian Nascimento, a young artist from the Northeast of Brazil, we took a different approach to this phrase: we put it back under the lens of visibility and representativeness. We believe that highlighting the multiple and infinite combinations that make up "being a woman" "can serve as a model for [students] in the process of constructing their identities" (Godoy *et al.*, 2020, p. 985), and not just for girls. The participation of all people, regardless of their gender identity, is essential if reflections on gender structures in society are to be translated into concrete changes.

The structure of the workshops was designed to encourage reflection on the following questions, "What representations of women are allowed in math textbooks? How can we creatively resist what is imposed on us?" (Durval, 2023, p. 85). In addition, they sought to provide possible answers to these reflections, thinking: "How can we undermine the reinforcement of stereotypes by proposing tensions in what is presented in mathematics textbooks?" (Durval, 2023, p. 85). To this end, we organized the meetings into three four-week modules called: Representation, Disobedience, and Resistance, corresponding to each of the above questions.

The activities were organized in four virtual and synchronous sessions of three hours each on Saturday mornings in May 2022, complemented by six hours of asynchronous activities. The choice of the remote format was aimed at making participation more flexible in terms of mobility and location, while also considering the remnants of social distancing measures due to the Covid-19 pandemic, the effects of which were only recently felt.

At the end of the workshops, each collaborator had to create an educational resource with an approach and utility similar to that of a textbook. That is, it should use elements such as images, contextualized exercises, drawings, explanations, characters, curiosities, texts,

photographs, theories, examples, among others, to support the teaching and learning process. The emphasis of this material, which we call *Roteiros Didáticos Insubordinados* (Insubordinate Teaching Guides), is to ensure the prominence of the female figure in its presentation or to address social issues faced by women. In other words, it was essential to adapt to the guidelines of the National Textbook Plan (PNLD), which we consider to be the most complete. These are:

- 1.1.1. positively promotes the image of women, considering their participation in different jobs, professions, and spaces of power, reinforcing their visibility and social protagonism;
- 1.1.2. addresses gender issues with a view to building a non-sexist, fair and equal society, including about combating homophobia and transphobia;
- 1.1.3. provide a debate on contemporary commitments to overcoming all forms of violence, with special attention to the educational commitment to the agenda of non-violence against women; (Brasil, 2015, p. 31, emphasis added)

To get to know the people who collaborated in this part of the research, I present an identification chart. In order to protect their identities, each person is represented by a codename chosen in honor of a woman who has had a significant impact on their lives. In addition to personal information and social markers, the last column contains a brief excerpt from the texts written by the collaborators in tribute to these women: some "Marias" who carry with them the mark of life and a peculiar faith in the world.

Table: Identification of participants

Codename	Information	Tribute
Liliane	Cis woman, heterosexual, 43 years old, white, spiritist, Master in Mathematics Education, works in the final years and high school of the state and municipal schools in Rio de Janeiro.	"(...) she was and still is my foundation. I learned values from her that I'll treasure all my life. (...) Gratitude for this warrior woman. (...) Her greatest legacy is helping others!"
Helena	Cis man ³ , heterosexual, 28 years old, does not declare race, Protestant Christian, graduated in Mathematics, works in the final years of elementary school in the private and state schools of Rio de Janeiro.	"(...) for being a single mother and having fought hard to give me a whole structure so that I could dream without worrying about other demands. She is an example of strength and struggle for me."
Noemi	Cis woman, heterosexual, 31 years old, black, Christian, master's student in math teaching, works in high school in the private school system in Rio de Janeiro.	"With her love for teaching and her students, she never let me give up. Today I'm a graduate and a master's student, trying to be for my students what she was for us: a light."
Tereza	Cis woman, heterosexual, 24 years old, brown, non-practicing Protestant Christian, degree in mathematics, works as a private teacher in Pernambuco	"(...) a great worker and my example of honesty. As well as a strong woman, who was and is a mother who does everything for her children."

³ Since the codenames were chosen by the contributors in homage to women, even though we have a cis male contributor, we are adopting a generic feminine plural for this group of people, since the reader is already informed that Helena is a boy.

Source: Adapted from (Durval, 2023, p. 91-93).

It is important to note that the collaborators are immersed in diverse socio-economic contexts, places of residence, family structures, ethnicities and religions, elements that shape their experiences and perspectives. A detailed intersectional analysis would be necessary, although it is beyond the scope of this article. However, this observation urges us not to consider them just as numbers or statistics, but as real people with socio-culturally situated experiences. So, without losing sight of their humanities and respecting their social experiences, we revisited the recordings of the workshop meetings, the transcripts of their speeches and the materials prepared for discussion. Given the time constraints, we will highlight the reflections from the Insubordination Module.

3 Data Analysis

According to Walshaw (2001, p. 481), we believe that "the construction of actual truth is inextricably linked to the discursive practices and political struggles that involve the personal, psychic, and emotional investments that the girl [woman] makes in these practices and struggles. In other words, the truths of the collaborators in this study are manifested in discursive practices, and it is crucial to analyze how they make sense of the world through permitted and assumed meanings.

However, in addition to this, we aim to identify aspects in the speeches and interventions they propose that could contribute to a healthier and more powerful construction of girls' subjectivities in relation to mathematics. Aspects that have not yet been incorporated into textbooks, but that could be examples of more assertive approaches that promote girls' empowering representations. Therefore, the intention of this analysis is not to judge the productions, but rather to present observations and theorizations that highlight their relevance in the context in which they were conceived. Furthermore, we don't see them as finished works, but rather as evolving projects that can be adapted to different contexts, audiences, and goals.

Furthermore, it is worth pointing out that the changes we seek are not associated with an alternative, fanciful, or disconnected dimension of reality. Insubordination is possible in the real world, competing for space with the truths produced there. This is precisely because shifts in the order of discourse are not stable and absolute (Foucault, 2014). It is precisely because of the uncertainty of these displacements that the possibilities of confrontation and mobilization arise, the opportunity to contest space and resist. For this reason, we individually and collectively sought alternatives to the concerns we generated throughout the workshop, in order to "exercise the profession in a dignified and responsible manner, committed to improving human life" (D'Ambrosio & Lopes, 2015, p. 5), especially about women.

Let's start with Tereza, who analyzed the book "Mathematics and Reality" written by Iezzi, Dolce and Machado (2018) for the 7th grade of primary school. It reports:

I was looking to see how sport was being dealt with in the book. I noticed that there were very few questions, so I decided to look specifically at soccer and futsal because these are sports that I truly enjoy. (...) And then I brought three questions to demonstrate my annoyance at the lack of female representation on football-related issues. (Tereza)

Figure 1: Fans watching the soccer pre-match.

< Absolute value

The TV news

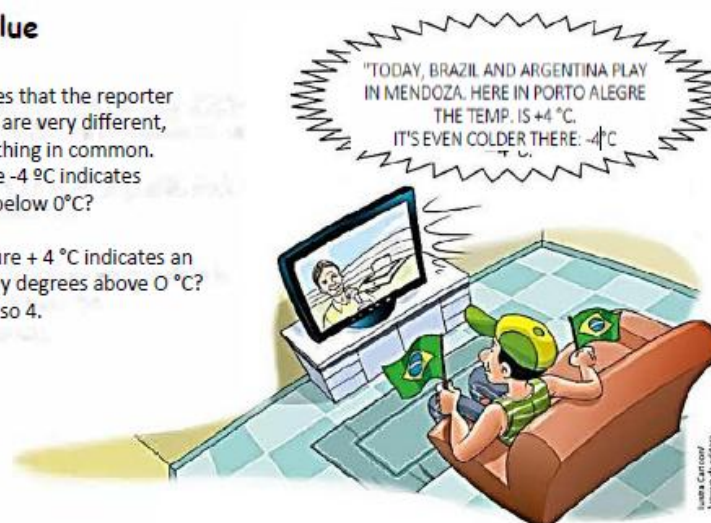
The temperatures that the reporter is announcing on TV are very different, but they have something in common.

The temperature -4°C indicates how many degrees below 0°C ?

The answer is 4.

The temperature $+4^{\circ}\text{C}$ indicates an amount of how many degrees above 0°C ?

The answer is also 4.



o The balance of the game

The result of a soccer match between Brazil and Argentina was:

Brazil 3-1 Argentina

What was Brazil's goal difference?

$$3 - 1 = +2$$

And Argentina's?

$$1 - 3 = -2$$



Brazil and Argentina played a friendly match, held at the Arena das Dunas stadium, Natal (RN), November 2014.

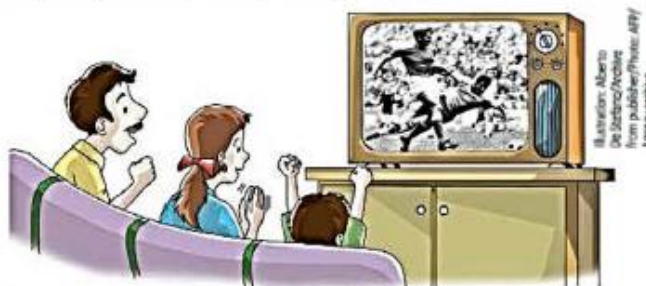
Source: Iezzi, 2018, p. 22

Figure 2: Cheering while watching soccer match

36 What was the score of the Brazil-Italy soccer match in the 1970 World Cup final? Find out by calculating the expressions:

a) Brazil: $(-4)^3 : (-2)^5 + 2 : (-10)^0$ 4

b) Italy: $8^2 : [3^2 - (1 - 2^3)] + (-3)^1$ 1



Source: Iezzi, 2018, p. 64

Figure 3: Football championship at school

Calculation of the difference

Recalling the goal difference

In the table opposite, we calculate difference for each team in the student championship.

When the team has more goals for than against, the balance is positive; when it has more goals against than for, the balance is negative.

If the team has scored as many goals as it has conceded, the balance is zero.

To calculate the balance, we perform a subtraction operation. Subtraction doesn't always work with natural numbers; it does with integers.

For example:

$16 - 20$ - does not occur in all natural numbers.

$16 - 20 = -4$ - takes place in the set of whole numbers;

$22 - 12 = 10$ - occurs in the set of natural numbers and in the set of integers.

In the set of natural numbers, we cannot subtract from a number another number greater than it. In the set of integers, yes; the result is a negative integer.

In the set of natural numbers, we can only subtract from a number another number smaller than it; the result is a natural number. In the set of integers, too, the result is a positive whole number

Team	Goals for	Goals against	Goal balance,
Lions	22	12	$22 - 12 = 10$
Tigers	16	20	$16 - 20 = -4$
Bulls	12	18	$12 - 18 = -6$
Bears	14	14	$14 - 14 = 0$



Futsal football match.

Source: Iezzi, 2018, p. 64

Using these three images as an example, Tereza illustrated that the representations of spectators, players, and commentators were predominantly male. Even when depicting a scenario of inter-class school games, the players were depicted as boys, excluding girls from these spaces. The only time a woman was depicted was as a spectator at the men's World Cup, in a family context - with her partner and son, both men, on the living room sofa - and her posture stood out from the rest. Tereza describes: "What bothered me was that the two men were more exuberant in their celebration than the woman, who was just clapping her hands," highlighting a representation of female docility in relation to sport.

In addition, when examining the general context of the book, Tereza made the following observation: "Whenever people discuss games, especially soccer, they use the words: Brazilian national team. A generalization to refer to Brazil. As if only the men's national team existed, ignoring the participation of the women's national team". These generalizations reflect a social imaginary that associates soccer with a predominantly masculine and masculinizing practice, conceived as an activity done by men and for men. The competitiveness, aggressiveness, and freedom present in this sport that is so symbolic for Brazil are used as elements that legitimize masculinity. This legitimacy applies not only to the players, but also to the fans who follow the men's soccer championships all year round, with ample support from free-to-air television.

Regarding the mathematics curriculum practiced in the textbooks, Souza and Silva (2018) have already shown the attempt to fix and standardize the subject of soccer as a masculinized identity, while teaching that the site of feminized identity is primarily playing with dolls and stuffed animals. The authors argue that the discursive constructions around games and toys serve a cultural pedagogy of behavior that relegates girls to ideals of passivity and restricted environments, in contrast to the behavioral projections of boys. Although not coercive, these constructions suggest and limit their personal projections and cultural interests, evoking the "subject of the desirable feminine, who is not excluded from playing, but communicates in the images her absence in the objective ability to play soccer" (Souza & Silva,


2018, p. 154).

"I was very annoyed to realize that women are not mentioned throughout the book. Neither the famous players like Marta and Formiga, nor the women's national team," says Tereza. Faced with this discomfort, she decided to intervene. After all, in this game of forces and relationships, "we are never trapped by power: we can always modify its dominance, under certain conditions and according to a precise strategy" (Foucault, 2001, p. 267). The strategy she adopted was to give visibility to women in football, who are constantly hidden in the textbook she analyzed. The following images show the Insubordinate Didactic Script created by Tereza.

This script uses graphic resources to contextualize important information for students about women's participation and challenges in women's soccer. This includes the historical ban on women playing soccer, the prejudice faced after the ban was lifted, and the remarkable achievements of Marta and the women's national soccer team. The material provides an in-depth look at this information, with links available throughout the document.

In addition, the collaborator assumes that the 2nd grade students who will carry out the activity have already learned the necessary content for this purpose. Thus, the script addresses historical and cultural challenges legitimized by biological difference, mobilizes relevant mathematical knowledge for decision-making, and analyzes information based on real data.

Figure 4: Insubordinate Didactic Script produced by Tereza



MATHEMATICS
Combinatorial Analysis | Percentage

THE HISTORICAL PREJUDICE AGAINST WOMEN'S
For a long time, various sports were classified as "not a woman's thing". Who hasn't heard the words "Women's soccer is bad" or "Women soccer players earn less because they play badly"? These words show the prejudice against this sport, which for years was forbidden to women, who played without being landmarks, only as circus acts, as if they were putting on a show, subjected to such performances since they couldn't compete. It was in this sense that soccer players tried for years to gain a foothold in society. It is worth noting that in this same censorship of prejudice, in 1941 in Brazil, under the government of Getúlio Vargas, a law was passed that institutionalized prejudice and curtailed women's rights in various sports, including soccer. Law nº 3.199 in its art. 54, says: "Women shall not be allowed to practise sports incompatible with the conditions of their nature, and to this end the National Sports Council shall issue the necessary instructions to the country's sporting organizations".

40 YEARS OF BANNING
It wasn't until 1979 that the law banning women from playing soccer in Brazil was repealed, but still with a lot of prejudice and tension. Even so, the end of the 1970s marked a new journey for the sport around women.

Warriors with soccer boots
The first official FIFA Women's World Cup took place in 1991 in China. Participation in the Olympic Games began in 1996, at the games held in Atlanta, USA. Another major championship for the women's national team is the Pan American Games, in which the sport's participation began in 1996, in Winnipeg, Canada. Even with little investment in the athletes and the sport, the Brazilian women's national soccer team has taken part in all three of these events. It is worth highlighting their great historical achievements: they were silver medalists in 2004 and 2008 at the Olympic Games; and they have accumulated 3 golds (2003, 2007 and 2015) and 1 silver medal in 2011 at the Pan American Games.

Queen Marta
Considered the best and greatest player in the world, Marta was voted the best female footballer on the planet 5 times in a row, from 2006 to 2010, and won another title in 2018, becoming the first female or male footballer to win 6 titles. Marta is synonymous with talent, skill and endurance. At the age of 36, she continues to dribble past everyone.

1st. Exercise
Observe the following situation
The players called up for the Brazilian women's soccer team are distributed in 5 positions (as shown in the picture and table). Take a look at the 23 players and the arrangement (4-3-3 tactical formation) set up on the pitch by coach Pia Sundhage. a) Taking into account the players' positions, how many different formations can be set up?

Forwards
Debinha
Marta
Giovana
Gabi Nunes
Kerolin
Geise

Midfielders
Julia Bianchi
Duda
Formiga
Adriana
Ivana Fuso
Ana Vitória
Ary Borges
Angelina

Full-backs
Bruninha
Tamires
Katrine
Yasmim

Defenders
Dailane
Antônia
Talmar
Lauren

Goalkeepers
Lorena
Letícia Roldão

b) If Marta, the team captain, is a fixed player among the three forwards, how many team formations can be set up under this condition?

MEN'S WORLD CUP 2018

A budget of 400 MILLION dollars distributed to 32 NATIONAL TEAMS.

The French team, champions, took home the prize of 32 MILLION dollars each.

The United States team, eliminated in the group stage, took home the prize of 8 MILLION dollars.

WOMEN'S WORLD CUP 2019

A budget of 30 MILLION dollars distributed to 24 NATIONAL TEAMS.

The United States team, champions, took home the prize of 4 MILLION dollars each.

No details of the other teams' prizes have been made available.

2nd exercise:

THE GENDER GAP IN FOOTBALL is glaring. If we look at the 2018 and 2019 World Cups, for men and women respectively, we can see the gulf in investment in the national teams and athletes, as well as in the two FIFA competitions. Look at the data and make comparisons between the two competitions, calculating percentages and describing what you see:

3rd exercise:

Continuing on the subject of gender inequality in soccer, we can see a gap of at least 40 years between the two disciplines, in which women were banned from playing and when they returned to the field they didn't have the necessary investment in the sport. From this point of view, salaries reflect this inequality between female and male athletes. Take a look at the data available on the Politize website:

AMONG THE SALARIES

TOP 5 IN MEN'S BUDGET

Player	Salary
Cristiano Ronaldo	400,000,000
Lionel Messi	360,000,000
Antony	140,000,000
Paulinho	140,000,000
Lucas	140,000,000

TOP 5 IN WOMEN'S BUDGET

Player	Salary
Christine Sinclair	400,000
Lucy Bronze	360,000
Fran Kirby	140,000
Vivianne Miedema	140,000
Wendie Renard	140,000

a) The percentage difference between the sum of the 5 players' salaries and the Brazilian player's salary is?

Inspirations

Athletes from the 2019 Brazilian Women's National Football Team - Illustrator Eneo Lage

04 Mathematics

Source: (Durval, 2023, p. 224)

Still in the context of sport, Noemi directed her analysis to the book from the Ápis collection by Dante (2017c), intended for the 5th grade. She also expressed her discomfort with the predominantly male representations in the material, and shared her intention to address this identified gap.

In this week's assignment, I was annoyed by a book in which they only put sports, predominantly male sports. They put them on podiums, in short. There were very few exercises that I saw that named women in the Olympics. That's why I was considering bringing in this book. Because I thought it was absurd.

Note that Noemi's statements are very similar to Tereza's observations. Both analyzed teaching materials for the final years of elementary school, from different publishers and written by different authors. However, the absence of female representations in high-profile sports emerges as a common denominator, not only establishing hierarchies between sports considered more or less acceptable for women, but also reflecting their visibility in performance awards. This pattern is evident in Figure 5, which corresponds to the opening of Unit 7 of the book investigated by Noemi.

Figure 5: Men's competition podium



Source: Dante, 2017c, p. 164 e 165

Given this scenario, Noemi's intervention proposal was a little different from the others. First, because her script was produced for immediate use in a first-year-high school class and, because of the specific needs of this class, it was the only one organized in slide format. It was also created in response to a reflection on a lesson that had already been taught. See what Noemi says in the presentation of her production:

The funny thing is that I had already given this class, with a different proposal. And after I had the lessons with you, I was like: "Guys, I'm doing everything wrong". It's something we end up doing, you see: this lesson I had given last year, I put in all-male sports. I put in basketball, men's soccer... I redid the whole lesson. I even reworked the questions because everything was in a male context. I put in all-male sports and all-male exercises. This becomes a danger. And in the book it's just: 'in basketball, a player...', 'in a competition, the swimmer...'. Guys, it can't be like that!

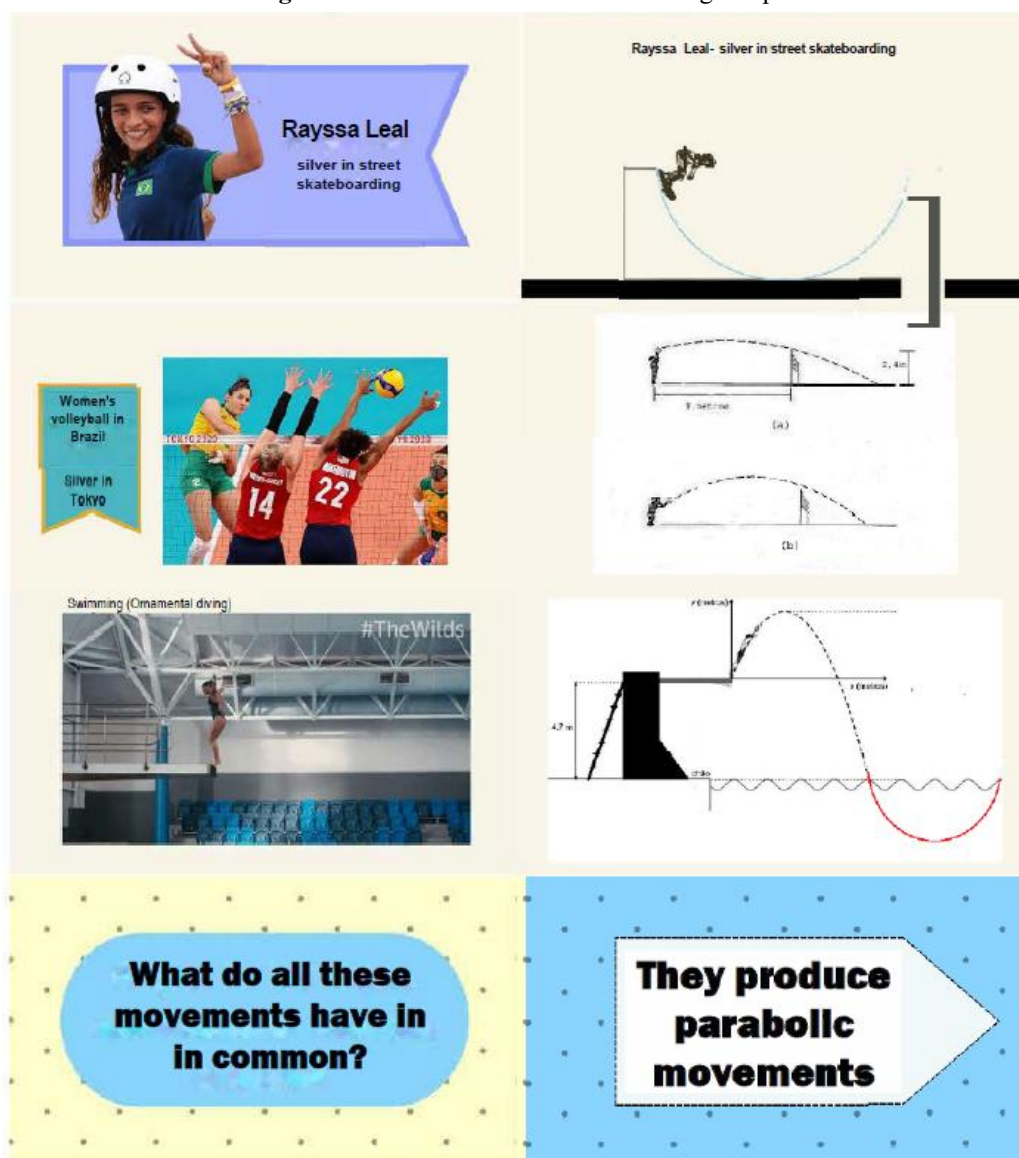
In Figure 6, there are some pages from that document.

The exposure to the provocations raised in the workshops and the apparent underrepresentation of women in mathematics teaching materials, combined with memories of her own professional career, led Noemi to realize that, in addition to teaching mathematical knowledge, her classes also perpetuated normative gender stereotypes. These reproductions are internalized in our sociocultural experiences, in the reference resources we use, and in the very act of teaching mathematics. According to Louro (1997, p. 64, author's emphasis), "routine and common practices, banalized gestures and words must be the object of renewed attention, questioning and, above all, suspicion. In this way, "perhaps the most urgent task is precisely this: to be suspicious of what is taken for granted" (Louro, 1997, p. 64).

In the first version of the lesson (in the context of men's sports), a basketball player identified with an example given. "He was thrilled to play and to see himself in what he was doing," said Noemi. Note how personal identification and affection connected the student to the mathematical knowledge being developed. Noemi points out that the girls didn't show the same behavior, which may be due to the lack of representation and the lack of the necessary tools to

see themselves in these contexts. In addition, their sensory and bodily experiences are regulated from an early age in the name of behavioral norms and body aesthetics.

Figure 6: Noemi's Insubordinate Teaching Script



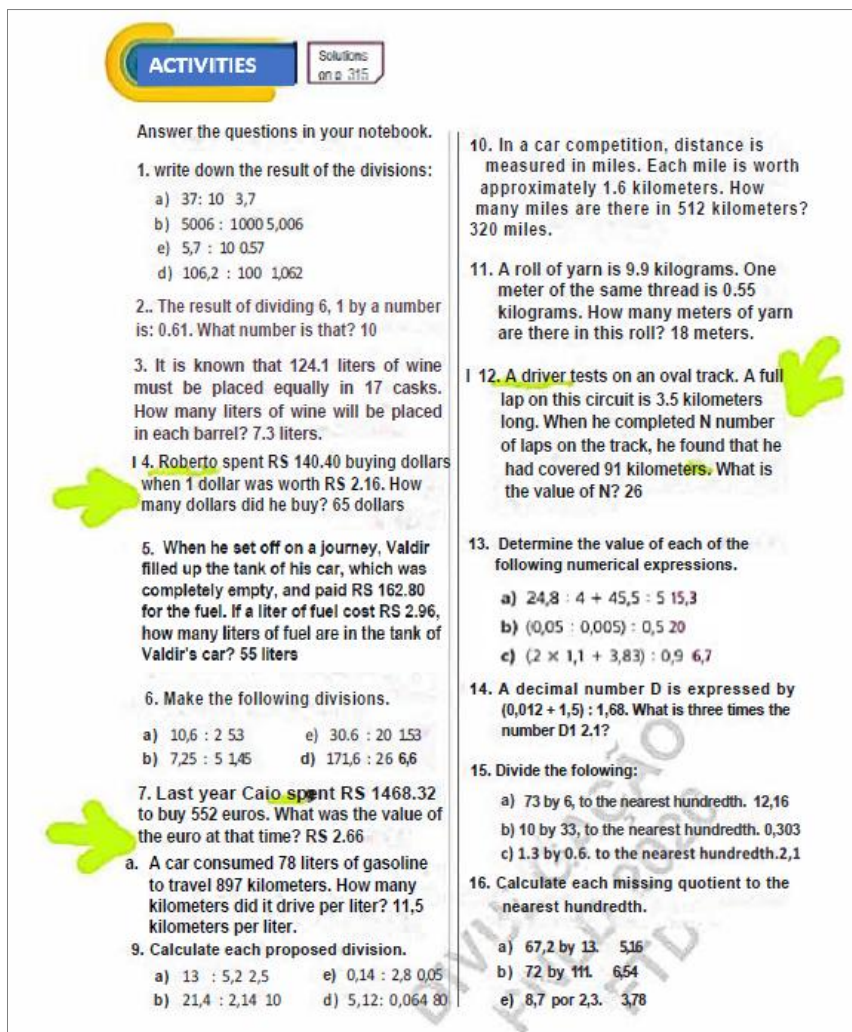
Source: Research data.

After the workshops, Noemi reformulated her entire lesson, shifting the focus to highlighting the presence of women in sports such as skateboarding, basketball, diving, Olympic gymnastics, and volleyball, using images of female role models. Rather than simply identifying the problem, she creatively defied it, challenging discriminatory and/or exclusionary discourses. She chose to listen to "her students, her subject, her colleagues, rather than the guidelines set by the institutions [and reflected in the textbook]". (D'Ambrosio & Lopes, 2015, p. 13, emphasis added). Noemi reframed her experiences, questioned what seemed natural, and concluded: "So it is possible to make women visible and women in sport!".

The realization that it is possible to create material that challenges some of the ideas that are repeated, propagated and validated in math textbooks is very welcome. While the teaching profession often faces "a deep bureaucratic and technocratic control that limits and conditions pedagogical and investigative action" (D'Ambrosio & Lopes, 2015, p. 6), the collaborators of

this research did not shy away. In the proposed textbook analysis, Liliane studied the book "A Conquista da Matemática" by Giovanni Jr (2018a), for the 6th grade of primary school. She highlighted the page of the book shown in Figure 6 with her italics and notes.

Figure 7: Page with no female figure



ACTIVITIES Solutions on p. 315

Answer the questions in your notebook.

- Write down the result of the divisions:
 - $37 : 10$ 3,7
 - $5006 : 1000$ 5,006
 - $5,7 : 10$ 0,57
 - $106,2 : 100$ 1,062
- The result of dividing 6,1 by a number is: 0,61. What number is that? 10
- It is known that 124,1 liters of wine must be placed equally in 17 casks. How many liters of wine will be placed in each barrel? 7,3 liters.
- Roberto spent R\$ 140,40 buying dollars when 1 dollar was worth R\$ 2,16. How many dollars did he buy? 65 dollars
- When he set off on a journey, Valdir filled up the tank of his car, which was completely empty, and paid R\$ 162,80 for the fuel. If a liter of fuel cost R\$ 2,96, how many liters of fuel are in the tank of Valdir's car? 55 liters
- Make the following divisions.
 - $10,6 : 2$ 5,3
 - $30,6 : 20$ 1,53
 - $7,25 : 5$ 1,45
 - $171,6 : 26$ 6,6
- Last year Caio spent R\$ 1468,32 to buy 552 euros. What was the value of the euro at that time? R\$ 2,66
- A car consumed 78 liters of gasoline to travel 897 kilometers. How many kilometers did it drive per liter? 11,5 kilometers per liter.
- Calculate each proposed division.
 - $13 : 5,2$ 2,5
 - $0,14 : 2,8$ 0,05
 - $21,4 : 2,14$ 10
 - $5,12 : 0,064$ 80
- In a car competition, distance is measured in miles. Each mile is worth approximately 1,6 kilometers. How many miles are there in 512 kilometers? 320 miles.
- A roll of yarn is 9,9 kilograms. One meter of the same thread is 0,55 kilograms. How many meters of yarn are there in this roll? 18 meters.
- A driver tests on an oval track. A full lap on this circuit is 3,5 kilometers long. When he completed N number of laps on the track, he found that he had covered 91 kilometers. What is the value of N? 26
- Determine the value of each of the following numerical expressions.
 - $24,8 : 4 + 45,5 : 5$ 15,3
 - $(0,05 : 0,005) : 0,5$ 20
 - $(2 \times 1,1 + 3,83) : 0,9$ 6,7
- A decimal number D is expressed by $(0,012 + 1,5) : 1,68$. What is three times the number D? 2,1
- Divide the following:
 - 73 by 6, to the nearest hundredth. 12,16
 - 10 by 33, to the nearest hundredth. 0,303
 - 1,3 by 0,6, to the nearest hundredth. 2,1
- Calculate each missing quotient to the nearest hundredth.
 - 67,2 by 13. 5,16
 - 72 by 111. 6,54
 - 8,7 por 2,3. 3,78

Source: Giovanni Jr., 2018a, p. 189.

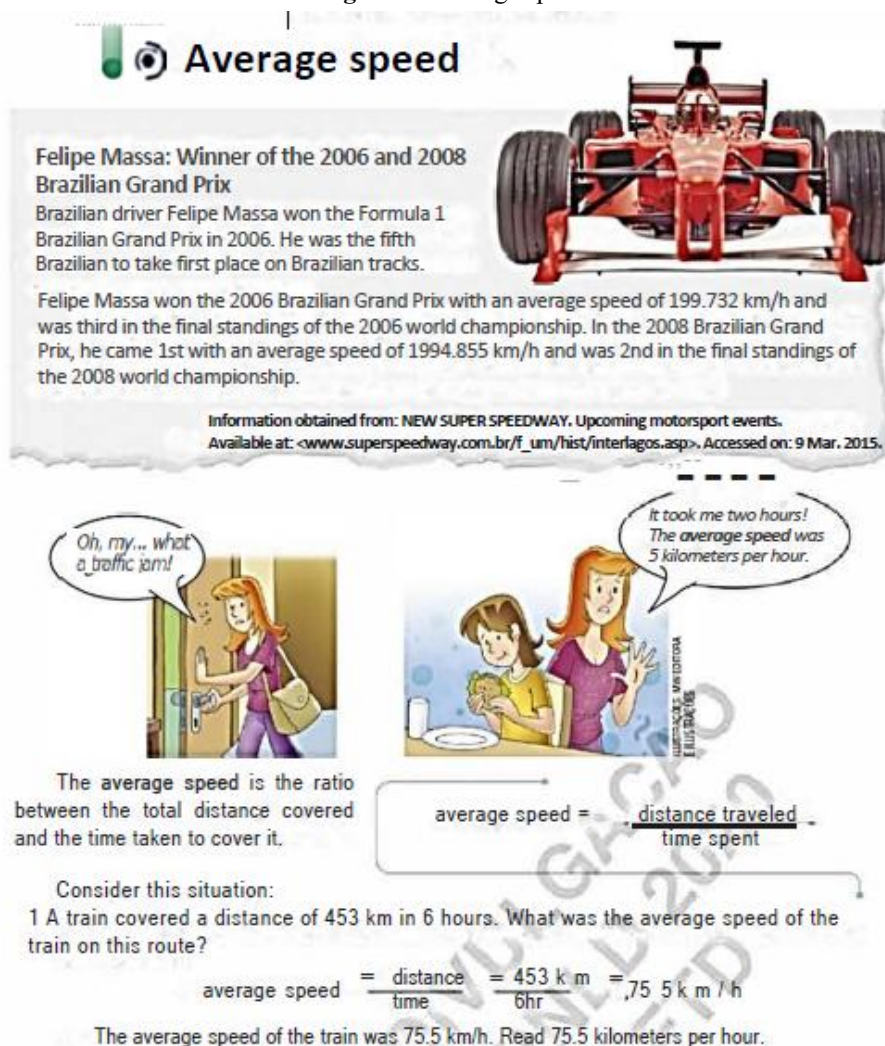
Liliane explains the highlights made in the texts in Figure 7 and comments on her impressions:

I was struck by the absence of the female figure in the suggested questions throughout the page. These questions are about buying dollars, euros, filling up one's car, actions that require refined cognitive skills, such as testing an oval track. Actions that are often considered "masculine." (Liliane)

In her view, this example illustrates part of the movement portrayed in the book when it comes to activities that are socially coded as masculine. She continues: "Women are taken out of the picture, and once again the idea of inferiority, the prejudice against them, and the stereotypes surrounding the female figure are reinforced." Especially when they involve large sums of money or require motor skills suited to these challenging external contexts. If the woman is not removed from the scene, she is framed in a behavior that is diametrically opposed to that of the man. This dynamic was observed in Helena's analysis, based on the 8th grade book

in the same collection.

Figure 8: Average speed



Source: Giovanni Jr., 2018b, p. 255.

Helena makes a few remarks about Figure 8:

When I looked at her [the image], I immediately thought of the sexism that exists in relation to driving. The first text shows how hard the athlete ran, emphasizing his performance. Then, to illustrate another average speed - only slower - it shows a mother who has been stuck in traffic and who, by the look on her face, is tired and worried. What struck me most was the woman's face. I think there could have been a comic where she complained about the traffic, but not with this maternal weight. (...) It's okay, she's stuck in traffic. Everybody gets stuck in traffic. But it indicates that she's late. There's obviously stress about feeding her daughter and everything. For me, it brings up the idea of responsibility. That got my attention. I didn't need the second image, the composition with her daughter eating a sandwich quickly. All I needed was for her to say that she'd been stuck in traffic.

While the male image is associated with high speed, professional success, fun, and personal satisfaction, the female image contrasts in all these aspects. She is shown arriving home, justifying her tardiness, and taking on the domestic or family responsibilities that await her. We don't even know if she was driving in the traffic. Finally, despite the reference to traffic

jams, the choice shows the woman confined to the walls of her home.

Helena's discomfort is therefore more than understandable and underlines the contrast between the two characters and their behavior. Contexts that weren't even explored to further the book's explanations. It is noticeable that the text briefly introduces a third problem situation, indicating that the construction of the first two, apparently intended to contextualize the production of mathematical knowledge, is hardly used for this purpose. On the other hand, it serves efficiently to encode binary behavior.


Ana's experience was also marked by the identification of very limiting stereotypes. When analyzing math textbooks, she focused on the third grade book written by Dante (2017b), although she also examined other books from the same collection. Ana noticed a clear connection between femininity and the appreciation of flowers. Some examples she highlighted are shown in the figures below.

Figure 9: Subtraction idea with flowers

5 SEPARATE ONE QUANTITY FROM ANOTHER

a) Look at the sequence of scenes and complete it accordingly.

The images are not represented in proportion



- In the 1st scene, there were 7 flowers on the table.
- In the 2nd scene, Maria picked 3 flowers to put in the vase.
- In the 3rd scene, there were 4 flowers left on the table, out of the vase.

b) Now, indicate the corresponding subtraction. $7 - 3 = 4$


Source: Dante, 2017b, p. 77.

Figure 10: Multiplication idea with flowers

2 Gina made flower arrangements in vases for her birthday party. Observe and complete

The images are not represented in proportion

- There are 3 vases.
- There are 5 flowers in each vase.
- There are 15 flowers in total.
- Corresponding addition:
 $5 + 5 + 5 = 15$
- Corresponding multiplication:
 $3 \times 5 = 15$



Flower pots for Gina's party.

Source: Dante, 2017b, p.121.

Figure 11: Division idea with flowers

2 Camila wants to divide these flowers equally between the 2 vases.

a) Help her by drawing the flowers in each vase,



b) As there is 1 flower left, we indicate this division as follows:

$$\underline{7} \div \underline{2} = \underline{3} \text{ and remainder } \underline{1}.$$

Source: Dante, 2017b, p. 150.

The pictures illustrate exercises in which the women have to decide how many flowers are in each vase. Problems like these recur in almost every section of the book, from the introduction to operations between natural numbers, except for addition. However, this absence is compensated for in the division operation, where the opening pages of the chapter and the preliminary discussion of the content involve a girl and flowers, as shown in Figures 12 and 13.

Figure 12: Cover of the Division Unit



Source: Dante, 2017b, p. 146 e 147.

Figure 13: Solving problems with flowers

TO START

Look at how Carla has arranged the flowers in the vases. Situations like this, in which a quantity is divided into equal parts, are solved using the division operation.

In this unit we'll start studying this operation.

- Analyze the scene on the opening pages of this Unit. Talk to your colleagues and answer the following questions.

How many flowers was Carla holding in the first picture?
12 flowers

And how many vases were in the second comic?
3 vases

How many flowers did she put in each vase in the third picture?
4 flowers

How many flowers would Carla put in each vase if distributed equally over 4 vases?
3 flowers
 $12 = 3 + 3 + 3 + 3$ or $12 = 4 \times 3$

What would happen if Carla wanted to divide 11 flowers equally in 3 vases?
She would put 3 flowers in each vase, but there would be 2 flowers left over.
 $3 + 3 + 3 = 9$ and $9 + 2 = 11$ or $3 \times 3 = 9$ and $9 + 2 = 11$

The images are not represented in proportion

- Talk to your colleagues about these further questions.

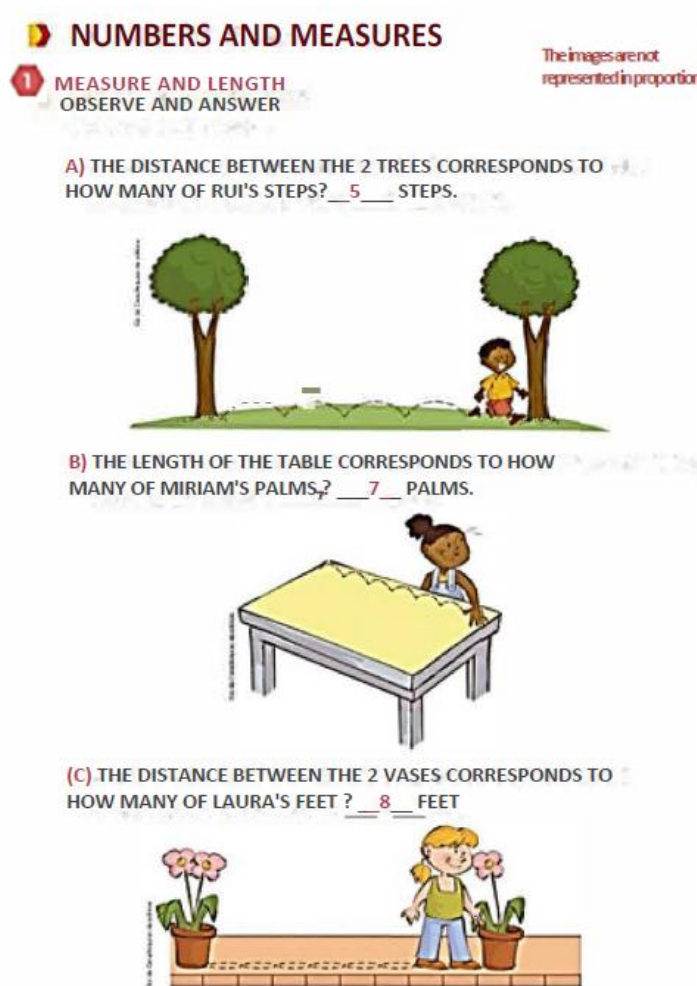
Source: Dante, 2017b, p. 148.

This task, which marks the beginning of the exploration of division, poses several questions that will help students develop strategies for making decisions. It not only encourages individual reflection, but also suggests a collective approach. Faced with this insistent association, the collaborator warns:

I was shocked that in the same book, of all the examples you can give about women, almost all the examples about surgery were about flowers. I was very irritated by this, how they simplify and repeat the nature of women. (...) So we have to be careful and aware of the way women are simplified into repetitive and domestic tasks. They constantly try to make us docile, cute, pretty, wear pink. I may like pink, but to always be associated with pink is very complicated. Always being associated with the flower...

Ana's indignation at the simplification of "being a woman" is an observation that is part of much of the discussion in this article. These stereotypes reduce the image of women to the cliché of the "weaker sex", constantly associated with delicacy, passivity, and beauty - except when it comes to protecting their children. This recurrence shows more than a behavioral appeal. They are strategies for producing knowledge and power that ensure that aesthetics are overvalued, especially the aesthetics of women's bodies and postures. They act as universalizers, delineating the limits of what is considered feminine. These constructions, far from being innocent, become less subtle when we examine the following image, taken from another book in the same collection, intended for the 1st year of elementary school.

Figure 14: Boys in the field, girls at home



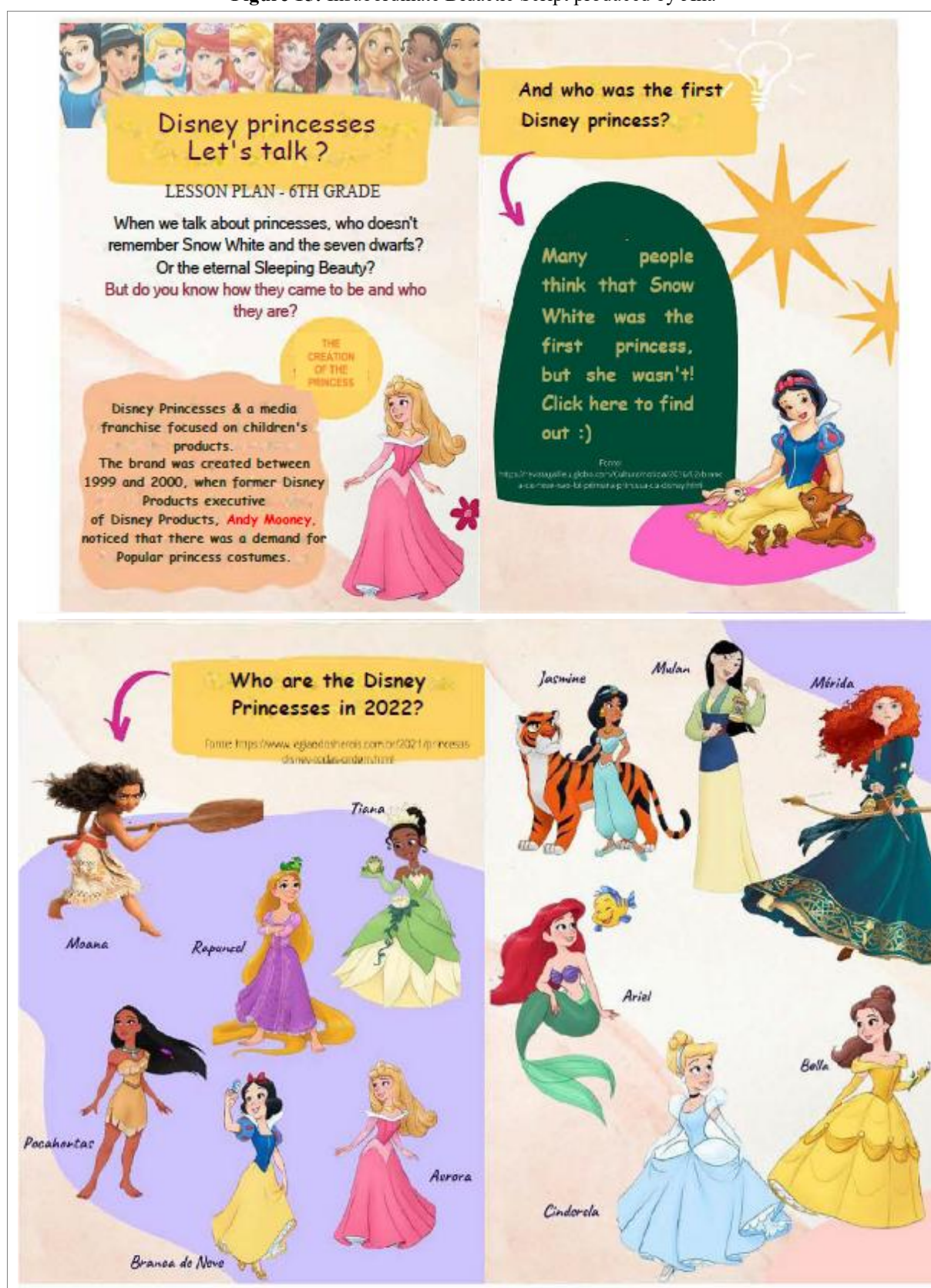
Source: Dante, 2017a, p. 56.

Ana expressed her dissatisfaction with this exercise:

In this question, they are working out a problem by measuring the distance from the girl to the flower pots. That's absurd! You have so many other examples to give, and you want to measure how many feet Laura is from the other flower pot? Absurd! Absurd because it's so, so, false. It's a lack of sense of reality. Then there's the confinement within the environment itself. The boy is always represented in a worldly, external environment, and the girl is confined.

Analyzing the three characters on the page, it becomes clear that the two girls are confined to confined spaces, while the boy is enjoying the outdoors, among trees. This representation not only reflects the limited vision of "being a woman," but also highlights the restrictive nature of this concept. In other words, rather than adhering to very specific archetypes of femininity, there is a deliberate transmission of generic messages associated with the content of the math curriculum. Faced with these concerns, Ana takes a stand by proposing an insubordinate teaching guide that uses female stereotypes in an intentional and emancipatory way, as shown in the figure below.

Figure 15: Insubordinate Didactic Script produced by Ana



PRINCESSES BEING "PEOPLE LIKE US"
Some curiosities

Some balloons contain more information. Find out what they are by **CLICKING**

Pocahontas really did exist and lived between 1594 and 11617. She was the daughter of the chief of an Indian tribe and lived in the area that is now known as the coast of the state of Virginia, in the USA. Want to know more?
[Click here.](#)

Mulan is based on the legend of the Chinese warrior Huo Mulan, whose story is told in the Chinese poem "The Ballad of Mulan".

Mulan is also the only princess who is not part of royalty either by birth or marriage. She earns the title for her great act of heroism.

Merida is the first princess not to marry Prince Charming.

Moana is the first Polynesian princess, with origins in the Pacific and the Maori culture. Find out more about Maori culture [HERE](#).

Tiona foi a primeiro príncipa africano criada Feio Disney. Seu ídolo é inspirado em um dublador. [Ano N. 1. 2012](#)

1. Research other princesses and choose one (it can be any princess ever created) that you identify with or see similarities in someone close to you. Then write it down in your notebook, justifying your choice.
Be creative!

2. Also think about princesses who don't exist yet. What do you think?

FORMALIZING THE IDEA CALCULATING PERCENTAGES INTRODUCTION

Calculate 20% of 400

Method 1: To do this, we can make a fractional representation of 20%, and then multiply this fraction by 400.

There are various applications of percentages, and for each there are different methods of solving them. To solve these percentage problems, it is quite common to use the simple rule of three or operations with fractions.

METHOD 2

In a class of 40 students, a survey was carried out which showed that 30 students like to play sports. What percentage of the students like sports?

$20\% \rightarrow \frac{20}{100}$
 $\frac{20}{100} \cdot 400 = \frac{8000}{100} = 80$

% Students	
100	40
x	30

$40x = 100 \cdot 30$
 $40x = 3000$
 $x = 3000/40$
 $x = 75\%$

Now it's up to you

Based on the text and everything you've seen, give examples of the situations below:

- From the princesses, calculate the percentage of black princesses.
- From the princesses presented, calculate the percentage of indigenous princesses.
- From the princesses, calculate the percentage of Chinese princesses.

Based on your answers, comment in class on the percentages you found, trying to understand why some princesses are more numerous than others. Then suggest on how this problem could be solved. Use your creativity in the final answer <3

Source: Research data.

Ana prepared a teaching guide using Disney princesses as a context. In conversation with the students, she proposes questions that begin with the creation of the first character and go on to include contemporary princesses. Ana asks the students to identify elements of everyday life that can bring them closer to these characters, beyond the limited vision of fantasy and "fairy tales". After all, some of these stories are based on real women and deal with social issues such as royalty, marriage, and ethnic/racial issues.

By moving between the affective imagination of the drawings and the knowledge gained

from the students' different perspectives on the world, Ana opens up discussions that can be explored on different levels. However, we can see the power of using percentages to reflect on these issues that are sometimes ignored in school mathematics. It is also important to highlight the subversive nature of the chosen context, which uses female stereotypes in favor of girls (Hottinger, 2010). The script aims to create an emotional connection that is sociocultural constructed between girls and princesses, while maintaining a necessary critical stance.

In a classroom that includes all gender identities, introducing contexts that are considered feminine to inform discussions is a revolutionary approach. This is because only contexts that are culturally understood as masculine are generally considered acceptable, universal, or neutral. Furthermore, it is more socially acceptable to privilege the bodily, sensory, and spatial experiences of (cis and heterosexual) boys. In this scenario, girls who deviate from this norm are marginalized and must conform or face exclusion. Ana's proposal opposes this social convention, not intending to restrict some harming others, but intending to broaden horizons and encourage more diverse and inclusive reflections.

Elisabete chose to analyze the book "Matemática Completa", by Bonjorno, Giovanni Jr and Sousa (2016). At the time of the workshops, this book had already given way to those approved by the PNLD 2021. However, the discussions raised by the collaborator when examining it were fundamental in shaping the direction of her intervention proposal. The collaborator begins by pointing out that the book chosen "doesn't present any women as mathematicians or philosophers, who are important in the construction of the content. That's the first thing you see, just by leafing through the book. There's no image, nothing.

Figure 16: Mathematics throughout the book



Source: Bonjorno; Giovanni Jr & Sousa, 2016, (1) p. 40, (2) p. 113, (3) p. 172, (4) p. 180, (5) p. 213.

Figure 16 shows some mathematicians (all men) who are highlighted throughout the

book. As new topics are introduced, the faces, stories, and work of these men who contributed to the development of a particular mathematical topic are highlighted. In addition, the book concludes with an infographic that organizes the mathematical production of the major topics covered, from prehistory to the present. This visual resource presents an attractive and well-structured timeline that aims to address concerns related to the erasure of the historical development of this science.

The authors criticize the notion of mathematics as detached from reality and highly abstract, arguing that societies would not have overcome obstacles, from the most complex to the most mundane, without the evolution of mathematical knowledge in interaction with human history. Still in this context, they point out that "it would not be an exaggeration to paraphrase the German philosopher Nietzsche (1844-1900) when he said that the mathematical field is 'human, all too human'" (Bonjorno; Giovanni Jr & Sousa, 2016, p. 249). However, the eight pages of the infographic seem to adopt a very specific notion of "human, all too human": men. Among more than 40 names, the contributions of women are not mentioned once, not even as co-participants.

The recognition of the erasure of women is not limited to the sphere of mathematical knowledge production. Elisabete identifies at least two forms of silencing, which she highlights as "late representation" and "assistance". The first is mentioned by the collaborator when she says

I've noticed that female representation is usually late in books, right? And one of them shocked me a little by presenting the first female representation on page 37. Only on page 37, people. There was no problem relating to a woman, nothing. Both the male language, until then, and the male figures too.

In fact, all the contexts, problem situations, or characters used male characters or referred to people in a generic way. In other words, throughout the first chapter, there was no space for women.

Elisabete's second concern was that the teaching materials she analyzed "(...) put women in a supporting role. As those who don't have knowledge or don't know how to solve a problem. Or men in a position of power. Given what she's seen, she suspects: "It seems like it's just there to say there's a woman. But really, it's not necessary to have that woman there, you know? [...] She was kind of thrown in just to do what was needed." These patterns of assistance or subordination are well illustrated in the example given by Elisabete in Figure 17.

In this problem situation, used to explore concepts of sets and introductory notions of functions, Elisabete observes: "The highest score comes from a boy, and the lowest scores come from girls. Women are in secondary positions and men are in positions where they seem to have more knowledge or ability [...]". These findings extend beyond this specific example. Although it may seem that such marginalizations occur through inadvertence, studies such as those by Hottinger (2010), Walkerdine (1998), Neto and Guida (2020), Souza and Silva (2018) and Neto and Silva (2021) show a logic of sociocultural perpetuation of these discourses in the management of life, especially when mathematical knowledge is in question.

Since the agenda is rationality and money, women only gain space when they are associated with the financial knowledge needed to manage the home or the needs of their daughters and daughters' daughters. In these cases, they are the protagonists, represented as having the most acute instincts of care and protection, as tireless givers of themselves (Neto & Guida, 2020). However, motivated by these observations and the collaborative atmosphere of

the workshops, Elisabete seeks alternatives, the first version of which is shown in the images of figure 18.

Figure 17: Example of a problem about sets

Exercícios resolvidos

1 The 1st year high school class at a school received their marks for the first History test of the year. To check the students' performance, the teacher made the following arrow diagram with the grades of the students who got less than 5.0, which were Ana, Caio, Tiago, Joana, Helena and Diego.

Based on this information, answer:

- The diagram shown relates which quantities?
- Did any of the students mentioned fail the History test?
- Were there any marks left without a student?

Source: Bonjorno; Giovanni Jr & Sousa, 2016, p. 40.

In her work, Elisabete focused on telling the story of women who played a significant role in the scientific progress of mathematics. The collaborator broadened the approach of the book under review, transcending the narrative of mathematics by men for men. In exploring analytic geometry, specifically elliptic conic sections, she introduced discussions contextualized with the study of planetary orbits, making visible the advances in astronomy achieved by women researchers such as Cecilia Helena Payne-Gaposchkin and Yeda Veiga Ferraz Pereira. It also highlighted the women behind the revolutionary ideas, usually attributed exclusively to the German astronomer Johannes Kepler.

While analyzing different textbooks, Elisabete noticed that some of them tried to represent women. She notes: "Sometimes a woman doctor or engineer appears. But if you look more closely, there's always a subtle male chauvinism". This subtlety underscores that to combat the sophisticated mechanism of producing female subjectivities that are incompatible with the exact sciences, it is necessary to do more than just put female figures in the background.

According to Foucault (2021a), the division between what is said and what is not said is not binary. Silences and erasures are more complex. Rather than simply identifying what is refused or forbidden to be said, it is crucial to explore the different ways of not saying, how authorized and unauthorized voices are distributed, and what kind of discourse is allowed or demanded of different groups. Therefore, it is necessary to analyze how discourse shapes or rejects behavior, how it distances or brings women closer to certain spaces. Finally, there is no absolute limbo or static antagonism. As highlighted in various parts of this analysis, there are different mechanisms of visibility and silencing that need to be considered within the whole discursive and power network. This is because "there is not just one, but many silences that are an integral part of the strategies that support and intersect discourses" (Foucault, 2021a, p. 31).

Figure 18: Insubordinate Didactic Script produced by Elisabete.

Analytic Geometry

Conic sections: Ellipse

In this section, we'll look at an important subject in geometry: the ellipse. This geometry can be found in many places, including in our solar system.

Astronomy

Astronomy is considered to be one of humanity's oldest sciences. The Sun and the Moon were the first stars to be noticed. After that, the stars began to be observed. And a series of mythological relationships and studies began to emerge.

She was the first person to show that the Sun is composed mainly of hydrogen. In 1925, contradicting the thinking of the time that the Sun had a composition similar to that of the Earth.

She was the first professional astronomer in Brazilian history. Among her research, she dedicated herself to studies on the Earth's rotation.

Movie **Alexandria** (2009): The story of the Greek polymath ahead of her time: the mathematician, philosopher and astronomer, Hypatia of Alexandria, who dared to teach at the Neoplatonic Academy at a time when women were not allowed.

Ellipse

Be a plan β , and a right circular cone. Consider that the plane bisects the conic surface and does not pass through the vertex O . If β is oblique to the axis e , then the conic section is an ellipse.

Elliptical orbits

The planets in the solar system move around the sun in elliptical orbits.

She had the mission of calculating on orbit light, which required the construction of a worldwide computer network. This made it possible to synchronize the lunar income of the Apollo project, which allowed astronauts to land on the surface of the Moon. In 2015, former US President Barack Obama presented the scientist with the Presidential Medal of Freedom.

A self-taught astronomer, her work became the basis for modern planetary orbit studies.

Definition of an ellipse

Given two distinct points F_1 and F_2 , belonging to a plane B . Let $2c$ be the distance between them. An ellipse is the set of points on B , whose sum of the distances to F_1 and F_2 is the constant $2a$, where $2a > 2c$.

$$|AF_1| + |AF_2| = |BF_1| + |BF_2| = \dots = |TF_1| + |TF_2| = |T| = 2a.$$

Elements

- Focus: the points F_1 and F_2
- Focal length: is the distance $2c$ between the foci
- Center: is the midpoint C of the F_1F_2 segment
- Major axis: the segment A_1A_2 of length $2a$
- Minor axis: is segment B_1B_2 of length $2b$
- Vertices: points A_1, A_2, B_1 and B_2

Eccentricity

- It is the number given by $e=c/a$
- Since $0 < a$ we have $0 < e < 1$
- If e is close to 0, then the ellipse is closer to a circle
- If e is close to 1, then the ellipse is flatter
- The further the foci are from the center, the flatter the ellipse is. However, if the foci are closer to the center, the closer the ellipse is to a circle

Eccentricity in space

The trajectories of the Earth around the Sun and the Moon around the Earth are ellipses with eccentricities equal to 0.016 and 0.054 respectively.


As these values are very close to zero, ellipses are often considered to be circles.

Considering the ellipse generated by the Moon's rotation around the Earth, Apogee is the furthest point from the Earth and Perigee is the closest point to the Earth. It is during this phase that the moon appears largest.

Kepler's Laws

Published between 1609 and 1618, Kepler's laws were the three basic proofs needed to explain the movements of celestial bodies around the Sun. These laws are still used today.


1. Planetary orbits are ellipses of which the Sun occupies one of the foci.
2. A planet moves faster when it is closer to the Sun and slower when it is further from it during its orbit.
3. There is a mathematical relationship between the time it takes a planet to complete an orbit around the Sun and the radius of the planet's orbit.



Because of her self-taught skills, she was asked to work at the Uraniborg observatory. She worked on a catalog detailing the position of the planets and the stellar background. This data was used by Johannes Kepler to develop the laws relating to orbits.

Sophie Germain
(1776-1831)

She was a translator of Johannes Kepler's works, improving the astronomical tables. With the work *Urania propitia* (Oels-Silesia, 1650), Cunitz gained a huge reputation in Europe, showing that she not only mastered advanced mathematics and astronomy, but was on a par with the best minds of her time in these fields.



Maria Cunitz
(1624-1694)

Equation of the ellipse with center at the origin

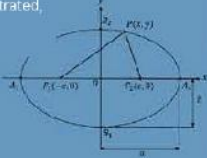
First case

- Let's consider the case in which the ellipse has its major axis positioned on the x-axis, with center C(0,0)
- Being P(x,y) be any point on the ellipse illustrated, whose foci are F1=(-c,0) and F2=(c,0).
- By definition:

$$|PF_1| + |PF_2| = d(P, F_1) + d(P, F_2) = 2a.$$
- Thus, we can arrive at the following expression:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Which is the reduced equation for this case.

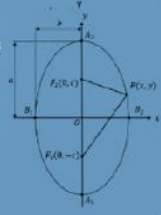


Second Case

- Now let's consider the case where the ellipse has its major axis positioned on the y-axis, with center C(0,0)
- P(x,y) is any point on the ellipse illustrated, and the foci are F1=(0,-c) and F2=(0,c).
- Similarly to the previous case, we find:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

Which is the reduced equation for this case.



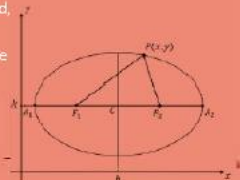
Equation of the Ellipse with center outside the

First case

- Let's consider the case in which the ellipse has its major axis parallel to the x-axis, with center C(h,k).
- Let P(x,y) be any point on the ellipse illustrated, whose foci are F1 and F2.
- We can arrive at the following equation for the ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

Que é a equação reduzida para este caso.

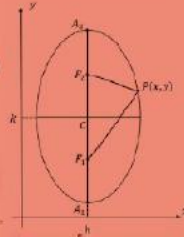


Second case


- Now let's consider the case in which the ellipse has its major axis parallel to the y-axis, with center C(h,k)
- Let P(x,y) be any point on the ellipse illustrated, whose foci are F1 and F2.
- We find the following equation for the ellipse:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1.$$

Which is the reduced equation for this case.



Exercising



Exercise 1: From the equation $44x^2 + y^2 - 16 = 0$ determine:

- (a) the measure of the axes;
- (b) A sketch of the graph;
- (c) The foci;
- (d) The eccentricity.

Exercise 2: Find the reduced equation of the ellipse with center at the origin and foci on the abscissa axes, where F1=(-3,0), with major axis equal to 8 units.

Exercise 3: An ellipse, whose major axis is parallel to the y-axis, has center C=(-4,-2), eccentricity $e=1/2$ and a minor axis of measure 6. What is the equation of this ellipse?

Source: Research data.

4 Conclusion

To conclude these reflections, I would like to highlight some observations of the

collaborators that revealed the changes of perspective and new visions that the workshops offered in relation to math textbooks. Even about other ways of teaching. They emphasized how the discussions sharpened their critical skills and made them aware of previously naturalized norms. In addition, they became more aware of their actions, more attentive to the ways in which these issues affect their students, and aware of the power that their professional practices can have when we adopt a stance of creative disobedience.

Before, I didn't think about it and I didn't question myself, right? So it's quite different now, having a more questioning look at how the ideas in the book are put together. (Helena)

I can say that the workshops of the "Maria vai com quais outras?" (Mary goes with which others?) project provided rich spaces for discussion, which led me to begin to perceive female identities in textbooks in a critical way. (Tereza)

"It's funny how there are things we don't pay attention to, until a certain moment when we start to open our minds. Before, I didn't even think about these textbook issues. It's amazing how certain things that would have passed me by two weeks ago. Now they bother me to the point of thinking: "How can that be?". That's interesting. (Josyane)

Look, I confess it was a new movement. Because - me, at least, right? - we analyze, but occasionally, we miss certain things that you don't even realize. This course has been great for me. (...) Because we have a different perspective. Now, for example, when I put an activity like this on the board, I change everything. That's really nice for us. It's been really enriching for me. Thank you very much. Because I didn't have that look, okay? I'm being honest, people. (Liliane)

(...) raised significant questions about women, looking at books, contexts, readings. This provocation makes us more critical of everything around us. I will incorporate your perspective into my reading and especially into my practice as a woman and future teacher (Ana).

As the collaborators in this research concluded, it is necessary to provide emotional, pedagogical and cultural memories that encourage girls to recognize themselves as capable of producing mathematics or occupying any desired space. As professionals, we must adopt a critical, active, and responsible stance towards the teaching materials used, ensuring that our teaching practices demonstrate "sensitivity to perceive and respect the process of intellectual and emotional development of students" (D'Ambrosio & Lopes, 2015, p. 5). In particular, so that our students can identify with others who have challenged pre-established standards and found unconventional ways to assert themselves as practitioners of mathematics.

So we return to the question: "Mary walks with what others? Who are the girls/women (and representations of them) that accompany the trajectories of our mathematics students? Are they limited to normative female models who are portrayed as domestic and submissive in classroom materials? Do they conform to (re)produced stereotypes of appearance, behavior, and professional success? Or do they have the opportunity to create their own narratives, to reinvent "happily ever after" according to their needs and desires?

We're not looking for fixed, finished answers to these questions. We are looking for the possibility of constant movement, alienation, and reflection. As such, our findings point to the importance of the field of mathematics education increasingly appropriating research and professional actions that make women's multiple existences visible, promote powerful representations for our students, and problematize urgent and under-discussed issues. Body and appearance stereotypes, self-esteem, fat phobia, homophobia and transphobia, ageism, domestic violence, harassment and abuse, ableism, issues of class, race, and ethnicity are

examples of issues that populate these girls' realities and shape their narratives. These are issues that must permeate our professional work, our research, and our educational materials so that we can move forward equitably, denounce norms, fight forced cisgenderism, and undermine gender oppression.

Furthermore, although not possible within the scope of this research, we would like to emphasize the need for gender studies to adopt an intersectional perspective to deepen discussions. After all, the people we are referring to identify themselves in multiple and even simultaneous categories that shape their subjectivities and experiences. Markers that carry with them not only that person's experience, but a whole historical, social, and cultural baggage built around identities such as race, class, geolocation, religion, and others. Finally, the change we seek is not just about adding the occasional success story to math textbooks. It's not even about superficial female inclusions that serve only as decoration or rigidly defined stereotypes. In fact, it's about welcoming diverse interpretations of the world that are far removed from the narratives perpetuated by patriarchy. It's about considering individual perspectives and recognizing the different ways in which women influence and are influenced by the spaces they occupy. As Rago (2013, pp. 93 and 94) points out, this applies not only to interpreting the past, but also to "building more liberating forms of coexistence in the present." In particular, embracing the marks, experiences, and infinite possibilities of the experience of becoming a woman.

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