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A brief historical overview of Probability Theory and the connections with its teaching

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Abstract: In this article we present a brief historical frame of Probability Theory, cover the period from antiquity to the mid-twentieth century, in order to understand which theoretical constructs need to be privileged in its teaching. Using a Timeline organized into four periods (prehistory of probability, emergence of the concept, development of Classical Probability and modern period), we highlight the contributions of the main mathematicians to the Probability Calculus. We note that probability theory has been shaped by diverse contributions from brilliant minds from different eras and contexts, including mathematicians, philosophers, statisticians, and scientists. This conceptual and epistemological evolution has had a significant impact on the development of theory over the centuries and has influenced teaching and learning processes.

Keywords: Uncertainty. Probability Teaching. Historical and Epistemological Development.

Un breve panorama histórico sobre la teoría de la probabilidad y sus vínculos con su enseñanza

Resumen: En este artículo presentamos un breve recido histórico de la Teoría de la Probabilidad, que embraca desde la antigüedad hasta mediados del siglo XX, con el objetivo de comprender qué construos teóricos son importantes privilegiar en su enseñanza. Using a time line organized in four periods (la prehistoria de la probabilidad, el surgimiento del concepto, el desarrollo de la Probabilidad Clásica y el período moderna), we highlight the contributions of the mathematical principales to the Calculus of Probabilities. We note that la teoría de la probabilidad has been shaped by several contributions of brilliant minds from different eras and contexts, including mathematicians, philosophers, statisticians and scientists. Esta evolución conceptual y epistemológica ha tenido un impacto significa en el desarrollo de la teoría a lo largo de los siglos y ha influenciado los procesos de enseñanza y aprendizaje.

Palabras clave: Incertidumbre. Enseñanza de la Probabilidad. Desarrollo Histórico y Epistemológico.

Um breve panorama histórico sobre a Teoria da Probabilidade e as articulações com seu ensino[1]

Resumo: Neste artigo apresentamos um breve recorte histórico da Teoria da Probabilidade, abrangendo desde a antiguidade até meados do século XX, com o objetivo de compreender

[1] This article is an excerpt from a master's thesis defended in the Graduate Program in Scientific and Technological Education of the Federal University of Santa Catarina, written by the first author and guided by the second author.

quais constructos teóricos necessitam ser privilegiados em seu ensino. Utilizando uma Linha do Tempo organizada em quatro períodos (pré-história da probabilidade, surgimento do conceito, desenvolvimento da Probabilidade Clássica e período moderno), destacamos as contribuições dos principais matemáticos para o Cálculo de Probabilidade. Observamos que a teoria da probabilidade foi moldada por diversas contribuições de mentes brilhantes de diferentes épocas e contextos, incluindo matemáticos, filósofos, estatísticos e cientistas. Essa evolução conceitual e epistemológica teve um impacto significativo no desenvolvimento da teoria ao longo dos séculos e influenciou os processos de ensino e de aprendizagem.

Palavras-chave: Incerteza. Ensino de Probabilidade. Desenvolvimento Histórico e Epistemológico.

1 Introduction

The history of Mathematics has a long tradition of research and documentation. However, the history of Probability Theory is less studied and documented. In part, this is due to the lack of material available in Portuguese and of interest among experts in documenting its development. As a result, the history of this theory is often approached in a secondary way, in texts and documents that deal with Statistics, an field closely related to it. This can make it difficult to distinguish between developments in probability and statistics, as well as lead to an incomplete understanding of the history of probability as a separate discipline (Calabria & Cavalari, 2013; Viali, 2008). As we noted in *Google Scholar*, many of the historical documents on this theory are only available in languages that are less accessible to most students and researchers, such as Latin, French and German. So, the access to these materials is not so easy, which prevents researchers and teachers, like us, from using them to draw a historical line and a comprehensive view of this movement.

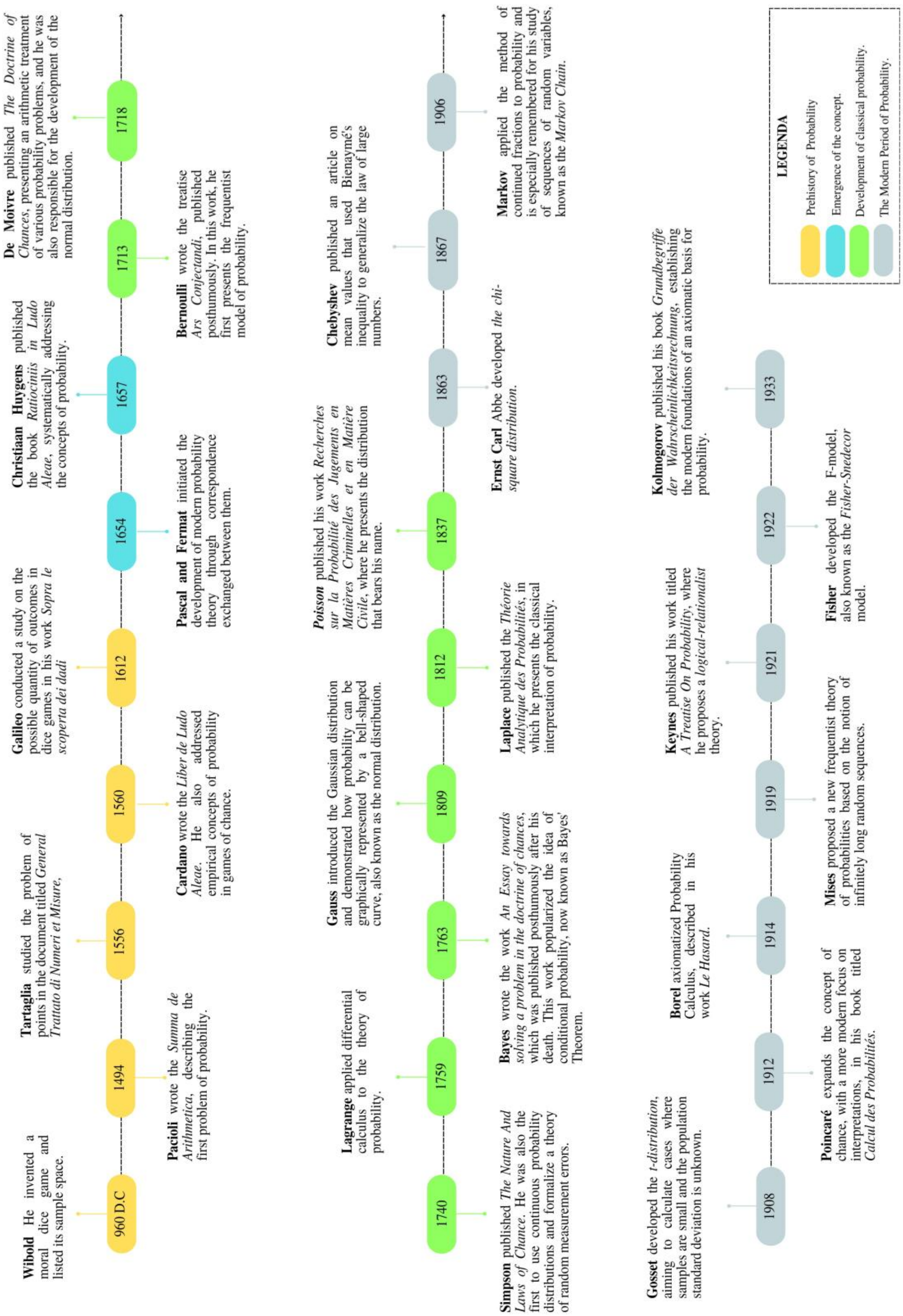
The historical line is important for understanding the development of Probability Theory, since it allows us to see that different contributions, ideas and contexts have influenced its evolution over time, and that it was built by many hands, with contributions from mathematicians, philosophers, statisticians and other scientists from different eras and contexts. Furthermore, the historical line helps us to comprehend how the conceptual and epistemological understanding of probability has changed over time and how this has impacted the development of the theory, as well as helps us to understand which theoretical constructs need to be privileged in its teaching. These contributions are the result of an evolutionary process that goes from the first attempts to understand uncertainty and chance, to modern applications of probability in areas such as Artificial Intelligence and Data Analysis.

However, due to the multifaceted history of probability and its complexity, it is difficult to establish a clear, linear timeline of important events and developments. Moreover, the contributions and ideas of the various scientists who worked in the area did not occur in a precise chronological order and there were often mutual interactions and influences between these ideas. Even so, learning from the past and evaluating the progress of Probability Theory over time is fundamental to understanding its evolution and application.

Although we can establish an approximate timeline of the history of probability, it is not linear and it should be seen as a useful simplification to understand the evolution of the theory. In addition, this history is made up of ideas and concepts that have evolved over time. It is not just a history of isolated facts that occurred in a given period. We need to analyze the context and influences that led to the development of these ideas and concepts in order to understand the complexity and non-linearity of it.

The facts do not unfold linearly, but it is possible to present a historical description of the probability in a linear fashion, following an approximate chronology of important events and developments. Our presentation (Figure 1) is linear, but the history of probability is full of interconnections and mutual influences between different developments and contexts.

Figure 1: Timeline of the historical development of Probability Theory (950 AD – 1933)



Source: Authors (2023).

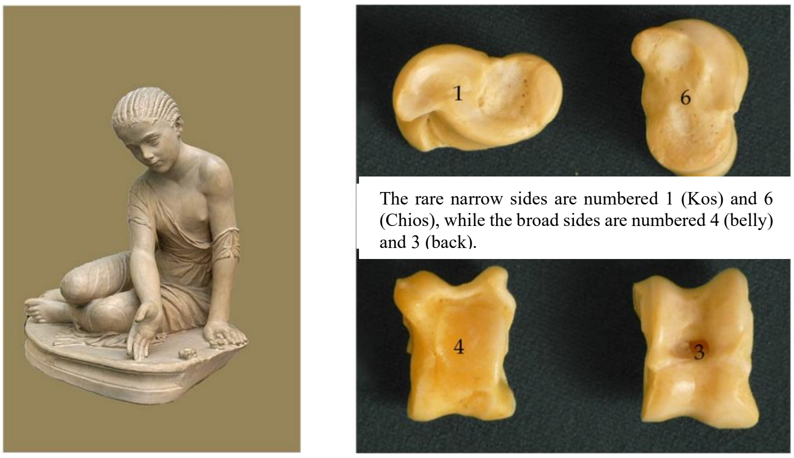
In order to understand the evolution of probability over the centuries, we have divided the historical line into four distinct periods. The **first** covers the Prehistory of Probability, from ancient times to Galileo's studies, characterized by gambling games and the quest to understand chance and uncertainty in different contexts. The **second** period, entitled The Emergence of the Concept of Probability, begins with the exchange of letters between Fermat and Pascal, and extends to the works of Huygens. During this period, the first ideas about probability began to emerge, laying the foundation for its future development. The **third** period, called The Development of Classical Probability, marks the moment when probability was formalized as a mathematical discipline, from Bernoulli's contributions to Poisson's works, consolidating it in this field. Finally, The Modern Period of Probability (**fourth** period), which goes from Ernst Carl Abbe to Kolmogorov, is characterized by the establishment of the modern foundations of axiomatic probability and the development of practical applications in areas such as Statistics, Physics and Engineering.

This division into periods, although useful for study and understanding, does not fully encompass the complexity and interconnectedness of the history of probability, since many developments occurred simultaneously in different parts of the world and in various areas of knowledge.

2 Prehistory of Probability: from ancient times to Galileo's studies

According to David (1962) and Viali (2008), the first records involving probability came about through gambling practices. Primitive forms of dice were discovered in archaeological sites in Mesopotamia and the Nile Valley region of Egypt during the Paleolithic period. These rudimentary dice were bones extracted from the paws of some animals namely sheep, horses, rams, deer, and were called *astragalus* or *talus* (Figure 2). Because they were cube-shaped, they were widely used in gambling as a kind of ancestor of modern dice during Antiquity, especially in Mesopotamia and Greece.

Figure2: Configuration of possible scores on the dice roll



Source: Adapted from http://www.roemisches-rheinland.com/Spiele

According to Vega-Amaya (2002), the peoples of Mesopotamia, such as the Sumerians and Assyrians, carved the *astragalus* bone to use it in games and entertainment activities. This random device had four possible positions when played on flat surfaces and, thus, allowed the creation of games with simple rules and a certain degree of randomness. But, apart from entertainment, it is unclear whether the carvings on the *astragalus* were used for other purposes, such as religious practices or symbols of deities. Paintings found in Egyptian tombs show the presence of *astragalus* and of record tablets of gods, suggesting that their use was not only limited to entertainment, but could also have religious and symbolic implications.

Batanero and Romero (1995) describe that the *talus* game was used for various purposes, including predicting the future, decision-making and entertainment activities. The authors point out that by using the *talus*, it was avoided to give advantage to any of the interested parties, as it was believed that the randomness could not be humanly controlled.

The *astragalus*, with its six faces, presented an unequal distribution of probabilities. Faces 1 and 6 had a probability of approximately 12%, while faces 3 and 4 had a probabilities of 37% and 39% respectively. This disparity allowed it to be used in gambling and therefore increased the unpredictability and challenge as the outcome could not be controlled. This made *astragalus* ideal for betting, where players could choose which face would be revealed, based on individual guesses and odds (Viali, 2008).

In the 10th century, the Belgian Bishop Wibold[2] (~960 AD) carried out a mathematical study of a game and paved the way for a more rational and strategic approach to games. Although gaming was considered sinful by the Church at the time, Wibold saw an opportunity to use such an activity as a way of promoting and teaching a virtuous life. He developed *Ludus Regularis Seu Clericalis,* a board game suitable for the clergy, encouraging priests to practice it as a formative activity that contributed to their personal and spiritual development. This initiative was a milestone in the history of games, since in addition to providing a more rational and strategic approach, it promoted the idea that games could have educational and moral benefits for religious players.

The *Ludus Regularis Seu Clericalis* became popular among the clergy. It used three dice to generate 56 possible combinations, each corresponding to a virtue, with a score ranging from 3 to 18, and aimed to accumulate as many virtues as possible, some being easier to obtain than others. The game emphasized the connection between the virtues and allowed complementary combinations that demonstrated their relationship. The numerical combinations of the dice represented specific virtues, with numerical values related to religious and moral principles, which provided a didactic approach during the game. For example, the combination of Charity (3 points) and Humility (18 points) resulted in a total of 21 points, thus demonstrating the connection between these two virtues. Likewise, the combination of Faith (4 points) and Continence (17 points) resulted in 21 points, as did the combination of Justice (6 points) and Hilarity (15 points).

Later, around 1220 and 1250, Richard de Fornival[3](1201-1260) wrote the poem *De Vetula*, which describes the calculation of the number of possible outcomes for the roll of three dice, including the permutations. This poem (Figure 3) is considered the oldest manuscript on the subject and established the relationship between the frequencies observed in the throws, as well as the enumeration of possible combinations. Fornival presents the results of rolling a single dice and explains the calculation of the combinations for three dice, showing the sixteen possible sums of the faces of the dice and how the permutations are calculated for each sum. The poem emphasizes the importance of permutations and makes it possible to calculate the total number of possible combinations for the throw of three dice. This work contributed significantly to the Probability Theory and influenced later researchers, as it presented in a concise and didactic way the enumeration of possible combinations in the launches obtained (Batanero, Henry & Parzysz, 2005).

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[2](c. 900-966, Northern France) – Archdeacon of the church in Noyon, France, responsible for the supervision of the Clergy of the Diocese, in 965.

[3]Born in Amiens, Northern France. He had extensive knowledge in several areas, such as Philosophy, Poetry, Music and Medicine, in addition to being a scientist.

Figure 3: Extract from a medieval manuscript describing probabilistic calculations

Perhaps, however, you will say that certain numbers are better
Than others which players use, for the reason that,
Since a die has six sides and six single numbers,
On three dice there eighteen,
Of which only can be on top of the dice.
These vary in different ways and from them,
Sixteen compound numbers are produced. They are not, however,
Of equal value, since the larger and the smaller of them
Come rarely and the middle ones frequently,
And the rest, the closer they are to the middle ones,
The better they are and more frequently they come.
These, when they occur, have only onde configuration of pips on the dice,
Those have six, and the remaining ones have configurations midway between the two,
Such that there are two larger numbers and just as many smaller ones,
And these have one configuration. The two which follow,
The one larger, the other smaller, have two configurations of pips on the dice apiece.
Again, after them they have three apiece, then four apiece.
And five apiece, as they follow them in succession approaching
The four middle numbers which have six configurations of pips on the dice apiece

Source: Adapted from Belhouse (2000, p. 134)

In the late 15th and early 16th centuries, Italian mathematicians such as Luca Pacioli (1445-1517), Niccolo Fontana (1499-1557) and Girolamo Cardano (1501-1576) began to explore probabilistic calculations when solving concrete gambling problems. Although they did not develop concepts or theorems, they advanced beyond the mere enumeration of possibilities by investigating how the odds of winning or losing were influenced by combinations of dice.

Luca Pacioli[4] (1445-1517), an Italian Franciscan monk, made significant contributions to mathematics during the Renaissance. Even though he did not publish any original work, his book *Summa de Arithmetica, Geometria*, *Proportione et Proportionalitá*[5], printed in Venice in 1494, served as a comprehensive summary of the mathematical knowledge of the time and incorporated works of Leonardo Pisano (1170-1250), better known as Fibonacci.[6] Pacioli's book played a crucial role in the development of Probability Theory as it introduced the study of a famous problem known as “the problem of points” or “the division of bets.” This problem involved a gambling game between two players with equal chances of winning each round (Viali, 2008). Each player contributed equally to the prize and they agreed that the first player to reach 6 points would get the entire prize. However, if the game was interrupted when one player had 5 points and the other had 3 points, how would they divide the prize fairly? Pacioli discussed this problem in his work, although his solution was incorrect. He suggested that the first player should receive 5/8 of the total prize, while the second should receive 3/8. Despite the mistake, Pacioli's solution inspired other mathematicians to discuss the problem, including Tartaglia and Cardano, which eventually led to the development of modern probability theory in the 17th century.

Niccolò Fontana[7] (1499-1577), better known as Tartaglia, was an important Italian mathematician of the 16th century, known for his contributions to Algebra and the Theory of Equations. In his treatise *General Trattato di Numeri et Misurre*[8] (General Treatise on Numbers and Measures), he discussed the problem of points, presented by Pacioli, but disagreed with his solution. Nevertheless, Tartaglia independently arrived at the same result and concluded that

[4]Luca Bartolomeo de Pacioli (Sansepolcro, 1445 – Sansepolcro, 1517) was a Franciscan friar and famous Italian mathematician.
[5]https://archive.org/details/summa-de-arithmetica-geometria-proportioni-et-proportionalita
[6]Italian mathematician, considered the first European mathematician of the Middle Ages.
[7]Tartaglia, born in Brescia, Italy, was a mathematics teacher in Venice who rose to prominence as a mathematician due to his contributions, including the Tartaglia Triangle and the solution of the third degree equation.
[8]https://echo.mpiwg-berlin.mpg.de/ECHOdocuView?url=/permanent/library/H5BAMGAN/

the first player should receive 5/8 of the total bet amount and the second should receive 3/8. This solution was based on the ratio of the points. Subsequently, mathematicians such as Pierre de Fermat and Blaise Pascal correctly solved the problem, using the binomial formula to calculate the probability of each player winning in a specific number of rounds. Then, these probabilities were used to determine the fair division of the prize (David, 1962; Viali, 2008).

Another very important figure is Girolamo Cardano[9] (1501-1576), a renowned Italian mathematician, physicist and physician who played a significant role in the development of probability calculus. His numerous works cover several areas and his main work, *Ars Magna*[10], was the first Latin treatise dedicated exclusively to Algebra, in which methods of solving third and fourth degree equations were presented. In addition, Cardano pioneered the study of Probability Theory by exploring the throwing of dice to demonstrate the influence of chance on gambling. In his treatise *Liber de Ludo Aleae*[11] (Book of Gambling), published in 1560, he addressed problems of combinatorial analysis, revisited issues raised by Pacioli and offered advice on how to avoid fraud, aiming to assist in making better decisions regarding the games that existed at that time (Silva, 2020; Morales, 1985).

Although published posthumously in 1663, the *Liber de Ludo Aleae* is considered a remarkable manual for players of that time as it presents several aspects of the games, including descriptions and strategies against opponents. A specific chapter presents the number of favorable outcomes for two- and three-dice rolls. Although his analyses are restricted to gambling, especially to dice, this treatise by Girolamo Cardano is recognized as the first systematic work to develop statistical principles of probability. Despite not being perfect and containing some mistakes, the work represents a milestone in the human understanding of uncertainty, and its method for dealing with randomness is remarkable for its effectiveness and simplicity (Gadelha, 2004; Viali, 2008).

Besides the contributions of Tartaglia and Cardano to the origin of Probability Theory, Galileo Galilei[12] (1564-1642), a renowned Italian mathematician, physicist, philosopher and astronomer, also played an important role in this subject. In his work *Sopra le scoperta dei dadi*[13] (On the game of dice), written around 1612, Galilei carried out a detailed study about the possible outcomes in dice games based on previous works, particularly those of Cardano. At the request of the Grand Duke of Tuscany, he investigated why the sum of 10 occurs more often than the sum of 9 when three common dice are rolled. (Calabria & Cavalari, 2013).

It's clear that to get a sum of 9 there are six different combinations, just as there are six combinations to get a sum of 10. However, according to Viana (2013), Galilei did not consider simple permutations and permutations with repetitive elements. For example, to obtain 9 points with the results 1, 2 and 6, we could have, in addition to the triple (1; 2; 6), the triples (1; 6; 2), (2; 1; 6), (2; 6; 1), (6; 1; 2) and (6; 2; 1). Thus, we would have a simple permutation of 3, i.e.,P_3 $3! = 6$.

Considering these permutations, Galilei identified that "there were more inversions or different ways of obtaining each of these combinations, noting that the total for sum 10 was 27, while for sum 9 there were 25 inversions" (Silva, 2020, p. 24). Therefore, he proved that events were not always equiprobable and that it was more likely to get a sum of 10 than a sum of 9.

[9]He was an Italian mathematician, physicist and physician, well known for his work Ars Magna – a treatise dedicated exclusively to Algebra and published in 1545.
[10]https://www.math.ksu.edu/~cjbalm/570s14/arsmagna.pdf
[11]https://books.google.at/books?id=4l9rySi8XbEC&pg=PA262#v=onepage&q&f=false
[12]He is an astronomer, physicist and mathematician born in Pisa, Italy. He is considered the father of observational astronomy.
[13]https://archive.org/details/agh6462.0008.001.umich.edu/

Thus, the works of Cardano and Galilei offer explanations and counting techniques that are in line with the concept of Classical Probability, and that address the equiprobability of events, as well as the possibility of winning in games. Moreover, the use of dice in these games has made it easier to study the "rules of chance", since the number of possible outcomes is known. Although there was no theoretical formulation of Probability Theory, the contributions of Pacioli, Cardano and Galilei laid the foundations for its later development, which would be driven by researchers such as Pascal, Bernoulli and Laplace (Sá, 2008).

3 From letters to calculations: The development of Probability Theory by Fermat, Pascal and Huygens

A century after the Italian contribution to the introduction of concepts of sample space and equiprobability, Pierre de Fermat[14] (1601-1665) and Blaise Pascal[15] (1623-1662), two renowned French mathematicians, continued the development of Probability Theory. After being introduced by a mutual friend, Pierre de Carcavi[16], Fermat and Pascal began to study the theory through correspondence. Motivated by questions proposed by Antoine Gombaud[17], known as the Knight of Méré, they realized the need to use more advanced mathematical tools to solve these problems. So, together they developed an advanced mathematics to calculate probabilities, which culminated in the creation of Pascal's famous triangle, described in Pascal's manuscript *Traité du triangle arithmétique*[18] (Caballero, 2001).

The correspondences of Pascal and Fermat, published in 1679 in Toulouse (France), are considered the origin of the development of the Mathematical Theory of Probability. These correspondences not only had significant content, but also the prestige of the two renowned mathematicians, which attracted other scientists of that era to the subject. Before this, the discussion of chance had not been approached in a systematic way, mainly because the idea of gambling and betting was not taken seriously and in earlier times its association with religious issues might have prevented its exploration. It was through the numerous letters exchanged between Pascal and Fermat that the concepts of probability and mathematical hope were formalized. But it is important to note that although they have made important contributions to Probability Theory, they did not write any specific books on the subject.

The first book on the Probability Theory was written by Christiaan Huygens[19] (1629-1695) in 1657 and was entitled *Ratiociniis in Ludo Aleae*[20]. In it, the author solved problems related to gambling and introduced the rules of classical probability and the concept of mathematical hope. Huygens' book underwent revisions over time, became a famous introduction to Probability Theory and was used until the 18th century. The problems presented in it are essentially random, which indicates the author's penchant for the frequent approach.

Huygens' work had a significant impact on the field of probability, since through it the emergence of a new fundamental mathematical theory was perceived. His influence extended to contemporary mathematicians such as Jacques Bernoulli, who played a crucial role in establishing the foundations of Probability Theory. Additionally, these advances not only benefited the aforementioned theory, but also had repercussions in other disciplines, such as

[14]French lawyer and mathematician recognized for his work in number theory, in particular Fermat's Last Theorem.
[15]French mathematician, writer, physicist, philosopher and theologian. He laid the foundations for Probability Theory and formulated the principle of pascal pressure.
[16]Known as Carcavi, he was a French mathematician, secretary of the National Library of France.
[17]He lived in France in the 17th century (1607-1684). He was a mathematician and gambling addict.
[18]https://archive.org/details/bub_gb_UqgUAAAAQAAJ.
[19]Born in The Hague, Netherlands, he was a Dutch physicist, mathematician, astronomer and horologist, responsible for developing a mechanism that increases the capacity of telescopes.
[20]https://archive.org/details/ita-bnc-mag-00001383-001.

Physics and Astronomy, by demonstrating that random phenomena can be described and understood through precise mathematical laws.

4 Development of Classical Probability

The development of classical probability was driven by events and contributions from diverse mathematicians and philosophers over several centuries. However, some periods are particularly important for the development of Probability Theory. One of these remarkable mathematicians was Jacob (Jacques) Bernoulli[21] (1654-1705). Influenced by Fermat's combinatorial approaches and Huygens' book *De Ratiociniis in Ludo Aleae*, Bernoulli played a significant role in the systematization of Probability Theory. Although he left gambling aside in his theoretical approach, in 1685 he began his investigation into problems related to them and published several studies on the subject.

Bernoulli is best known for his posthumous treatise *Ars Conjectandi*[22] (The Art of Conjecture), published in 1713 by his nephew, Nicolaus I[23]. The book is structured in four parts: a reprint of Huygens' book with commentaries; the application of the theory of combinations to gambling games; the discussion on the Law of Large Numbers; and applications in civic, moral, and economic problems. The author used calculations with binomial coefficients to demonstrate the Law of Large Numbers, which relates probability to relative frequency. The book also explores mathematical hope and moral, as well as the meaning of probability as a measure of certainty. *Ars Conjectandi* traced the beginning of a more systematic Probability Theory, with important contributions (Gadelha, 2004).

In 1718, Abraham De Moivre[24] (1667-1754), a renowned French mathematician, published *The Doctrine of Chance*[25], with the aim of approaching problems arithmetically and making the subject more accessible. He proposed the reduction of probability problems to differential equations and the use of generative functions to solve them. In 1730, De Moivre published *Miscellanea Analytica*[26], where he made the first attempt to demonstrate the formula that is now known as Stirling's Formula. Three years later, in a supplement entitled *Approximatio ad Summam Terminorum Binomii* $(a + b)^n$ *in Seriem Expansi*[27], he used this formula to perform the first derivation of the normal distribution as an approximation of the binomial distribution (Silva, 2020, p. 39). In 1738, in the second edition of *The Doctrine of Chance*, he presented the normal distribution as an approximation of the binomial distribution, and, in 1756, in the third edition, he demonstrated a specific case of this approximation, which resulted in the Central Limit Theorem. In summary, De Moivre's contributions to Probability Theory include the definition of independence of events, the use of differential equations and generative functions, the derivation of the normal distribution as an approximation of the binomial distribution and the demonstration of the Central Limit Theorem.

[21]Jacob Bernoulli, also known as Jacques or Jacob I Bernoulli, was a pioneering mathematician who extended infinitesimal calculus beyond the proposals of Newton and Leibniz. However, it was mainly in the field of probability that he gained recognition.
[22]https://eprints.ucm.es/id/eprint/57575/1/ARS%20CONJECTANDI.pdf
[23]He was born and resided throughout his life in Basel, Switzerland, and was an important mathematician recognized for his correspondence with other mathematicians, including Euler and Leibniz.
[24]He was a French-born mathematician, recognized by De Moivre's formula, a pioneer in the development of Analytical Geometry and Probability Theory
[25]https://books.google.com.br/books?id=3EPac6QpbuMC&redir_esc=y.
[26]https://vdocuments.site/a-rare-pamphlet-of-moivre-and-some-of-his-discoveries.html.
[27]https://vdocuments.site/a-rare-pamphlet-of-moivre-and-some-of-his-discoveries.html.

In 1740, the English mathematician Thomas Simpson[28] (1710-1761) published *The Nature And Laws of Chance*[29], based on the ideas of Abraham de Moivre. His innovative ideas, such as the use of continuous probability distributions and the formalization of the theory of random measurement errors, generated intense debates among mathematicians at the time. Simpson's contributions were fundamental to the development of mathematical analysis and probability, influencing later mathematicians such as Laplace and Gauss. These works, even though questioned and debated, contributed to the advancement of Mathematics. In 1759, Joseph-Louis Lagrange[30] (1736-1813) applied differential calculus to Probability Theory (Silva, 2020).

In 1763, the most important work of Thomas Bayes[31] (1702–1761), an English Presbyterian pastor and mathematician, was published in the journal *Philosophical Transactions of the Royal Society of London*. Entitled *Essay Towards Solving a Problem in the Doctrine of Chances*[32], the paper presented a problem about calculating the probability of unknown events, based on their past frequency. Bayes' theorem, described in the aforementioned work, is a crucial device for improving probabilistic forecasts when new events arise, which makes it possible to calculate the conditional probability based on available information. Bayes conceived probability as a prior subjective assessment of risk in relation to expected gain. His solving procedure reflected an axiomatic view and emphasized the subjective nature of probability. This epistemological approach makes Bayes' contribution so relevant to this day.

Pierre-Simon Laplace[33] (1749-1827) was a French mathematician who contributed to the development of Probability Theory in the early 19th century. In his work *Théorie Analytique des Probabilités*[34], published in 1812 and reprinted in 1814 and 1820, the author addressed the generative functions and expressions used in the theory. He also discussed the calculation of probability and its applications, including the method of multiplying independent events, the probability of compound events and the application of probability to mistakes of observations of celestial bodies. Laplace also adapted Buffon's needle problem to estimate the value of π (Morales, 1985). In his work *Essai Philosophique sur les Probabilités*[35], published in 1814, he pondered the epistemology of probability and emphasized its application in everyday life and in several areas, such as Error Theory, Actuarial Mathematics and Statistical Mechanics (Viali, 2008).

In the 19th century, in 1809, the German mathematician Johann Carl Friedrich Gauss[36] developed the least squares method to overcome mistakes in astronomical observations, with the aim of measuring the curvature of the Earth and improving the accuracy of geodetic measurements. Although there is controversy as to who actually formulated the model, some authors call it the Gauss-Moivre-Laplace model[37]. Gauss realized that as the number of measurements increased, the estimates clustered around a central average and formed a bell-shaped curve known as a normal distribution. It not only illustrates the distribution of errors in

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[28]He was a British mathematician and inventor, creator of Simpson's formula, used to approximate definite integrals. He is well known for his work on interpolation and numerical integration methods.

[29]https://wellcomecollection.org/works/ftbgrgqu/items.

[30]He was a mathematician and astronomer born in Turin, Italy. He had great prominence in analysis, Number Theory and Celestial Mechanics

[31]The first to use inductive mathematics and establish a mathematical basis for probability inference.

[32]https://royalsocietypublishing.org/doi/pdf/10.1098/rstl.1763.0053.

[33]He was a French mathematician, astronomer and physicist. Studied Probability Theory and differential equations.

[34]https://archive.org/details/theorieanaldepro00laplrich.

[35]https://archive.org/details/bub_gb_wAdmU2e2unAC.

[36]He has contributions in Differential Geometry, Geodesy, Number Theory, Analysis, Probability Theory, etc.

[37]Expression used to indicate that other mathematicians, such as Moivre and Laplace, also came to the same conclusion.

experimental measurements, but is also a powerful tool in statistical analysis, widely used in several areas of knowledge, such as Physics, Economics and Biology (Zindel, 2018).

In 1837, the renowned French mathematician and engineer Siméon Denis Poisson[38] (1781-1840) published a work entitled *Recherches sur la Probabilité des Jugements in Matières Criminelles et in Matière Civile*[39], in which he presented the Poisson Distribution, related to rare events with a low probability of occurrence and a large number of attempts. This distribution is widely used in areas such as Engineering, Finance and Life Sciences. It is applied to calculate the probability of events in a interval of time or space, such as counting cars at an intersection or industrial equipment failures. It is also useful when analyzing biological data and counting sells or individuals in a study field, especially when rare events are crucial to decision-making and the conditions of occurrence are certainly known.

5 Modern Probability Period

The Modern Probability Period began at the end of the 19th century with the formalization of Probability Theory. The χ^2 or chi-square distribution, developed independently by Ernst Carl Abbe[40] (1840-1905) and Friedrich Robert Helmert[41] (1843-1917) in 1875, was named and popularized by Karl Pearson[42] (1857-1936), in 1900. This continuous distribution has k degrees of freedom and is widely used in statistics, as well as in many areas of science. It allows testing hypotheses about the distribution of data and is a particular case of the gamma distribution, which models various positive continuous quantities. The formalization of Probability Theory provided statisticians and mathematicians with tools and techniques that deepened the understanding of probability and its applications in several areas of knowledge, such as Physics, Economics and Engineering. The chi-square distribution illustrates how this theory has evolved and how it has become a fundamental tool in statistics as well as in other fields.

Pafnuty L'vovich Chebyshev[43] (1821-1884), a Russian mathematician and founder of the St. Petersburg Mathematical School, developed the Chebyshev's inequality in 1867. It is a comparison by inequality which states that, for a broad class of probability distributions, no more than a certain fraction of values can be more than a certain distance from the mean, proving the Law of Large Numbers. Chebyshev's inequality is essential since it allows us to assess the quality of estimates obtained from sample data and provides a measure of dispersion in relation to the mean of a random variable. It is a generalization of the Bienaymé-Chebyshev inequality and is a fundamental result of Probability Theory.

Later, in 1906, came the contribution of Andrei Andreiwich Markov[44] (1856-922), a Russian mathematician who developed the Theory of Stochastic Processes and the concept of the so-called Markov Chains. These chains are sequences of random events that depend only on the previous event in the sequence, which makes his analysis simpler. In addition, Markov also applied the method of continuous fractions in the field of probability, a useful mathematical

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[38]French mathematician known for his work on definite integrals, Electromagnetic Theory and probability.
[39]https://archive.org/details/recherchessurlap00pois
[40]He was a physical and German professor at the University of Jena, Germany. He is recognized for his work on the Theory of Optics and Diffraction.
[41]He was a German geodesist very important for Geodesy (science that studies the shape and dimensions of the earth) and for the Theory of Errors.
[42]He was a British statistician who pioneered the development of modern statistical methods.
[43]Russian mathematician founder of the St. Petersburg School, remembered mainly for his studies related to Number Theory and the approximation of functions.
[44]Born in St. Petersburg, he was a Soviet mathematician with important contributions in various fields of mathematics, including differential equations, topology, mathematical logic, and foundations of mathematics.

technique for approximating functions and evaluating integrals. His contributions are fundamental to understanding randomness and its application in many fields, such as Engineering, Economics and Social Sciences.

In 1908, the statistician William Sealey Gosset[45] (1837-1937) proposed the use of the T-distribution in his article published in the journal *Biometrics*, under the pseudonym *Student*. This distribution is widely used in statistical inference to test hypotheses about the mean of a population in cases of small samples or unknown variance. Gosset's discovery enabled significant advances in Statistical Theory for small samples and in areas such as confidence interval estimation and regression analysis (Viali & Bittencourt, 2007).

At the beginning of the 20th century, the mathematician Jules Henri[46] apparently random events have causes that originate from minimal disturbances. In his book "The Calculus of Probabilities", published in 1912, Poincaré developed a mathematically rigorous basis for understanding the cause-effect relationship and the importance of information in decision-making. His comprehensive and rigorous approach to these topics makes him possibly the first mathematician to treat them this way (Zindel, 2018).

Félix Édouard Justin Émile Borel[47] (1871-1956) also contributed to the Calculus of Probability. In his work entitled *Le Hasard*[48], published in 1914, he presented the axiomatization of calculus, an important step that made the theory more accurate and rigorous. Borel published more than fifty articles on the subject between 1905 and 1950 and stood out as the author of several textbooks. He was also a pioneer in the study of strategy games and wrote a collection of articles on game theory. One of his books on probability included a mental experiment that became known as the “Infinite Monkey Theorem.”

In 1919, the mathematician Richard Edler von Mises[49] (1883-1921) proposed a problematic perspective for probability called The Collective, which was based on infinitely long random sequences. He advocated an objective frequency approach to probability calculation. According to Mises, the Collective consists of uniform sequences of events that respect the axioms of randomness and convergence. The axiom of randomness affirms the independence of events, while the axiom of convergence states that the relative frequency of an event tends towards a limit value as the probability of occurrence increases. Mises concluded that probability is the limiting frequency of an event in a Collective that follows these axioms. But, this approach presents practical problems as the application of infinite random sequences is difficult and not all events can be treated as infinite sequences of results.

In 1921, the economist John Maynard Keynes[50] (1883-1946) published *A Treatise On Probability*[51], making significant contributions to Probability Theory and the study of decision-making. Keynes questioned the classical theory of probability and proposed a “logical-relational” approach. Despite complex mathematical formulations, the book is considered a classic in the logical interpretation of probability and presents an essentially philosophical approach. Its importance lies in Keynes' emphasis on rationality and uncertainty, and in his

[45]English chemist and statistician known for his study of *Student's t-distribution*.
[46]Mathematician, physicist and philosopher of French science who brought contributions to Geometry, Differential Equations, Electromagnetism, Topology and Philosophy of Mathematics.
[47]He was a French mathematician and politician who made contributions to Probability and Mathematical Analysis.
[48]https://ia800703.us.archive.org/30/items/lehasard00boreuoft/lehasard00boreuoft.pdf.
[49]Austrian Jewish scientist and mathematician born in Lemberg. He has contributions in the area of Fluid Mechanics, Statistics, Aerodynamics and Probability Theory.
[50]He was an economist whose ideas shaped modern economic theory and economic policy.
[51]https://www.gutenberg.org/files/32625/32625-pdf.pdf.

conception of probability as a union between evidence and conclusions derived from a sound logical argument (Zindel, 2018).

In 1922, Ronald Aylmer Fisher[52] (1980-1962) developed the *F-Distribution* as part of his Theory of Analysis of Variance (ANOVA). This distribution is used to test the equality of variances between groups in a data set and is especially useful in scientific experiments. In 1934, George Waddell Snedecor[53] (1881-1974) tabulated the distribution and symbolized it with the letter F in his book *Statistical Methods*, dedicating it to Fisher. Snedecor, known for his contributions to Analysis of Variance, also founded the first statistics department at an American university. Since then, the *F-Distribution* has been widely used in statistical inference, Analysis of Variance and Regression in several areas such as Engineering, Natural and Social Sciences (Viali & Bittencourt, 2007).

The Russian mathematician Andrey Nikolaevich Kolmogorov[54] (1903-1987) laid the modern foundations of Probability Theory on a rigorous axiomatic basis. In his book *Grundbegriffe der Wahrscheinlichkeitsrechnung*[55] (Fundamentals of Probability Theory), published in 1933, the author presented three fundamental axioms: non-negativity, additivity and normalization. These axioms enabled the proof of important theorems such as the Law of Large Numbers and the Central Limit Theorem, and unified different perspectives on probability. Kolmogorov also contributed to the study of stochastic processes, such as Markov processes, in works published in 1938.

6 Conclusions and implications

As we explore this historical evolution in detail, we realize that probability is a field in constant development, dependent on the advancement of mathematical knowledge and the incorporation of new ideas and theories. This historical understanding plays a fundamental role in the teaching this field in Elementary School, as it allows students to perceive the evolution of concepts and theories over time and to understand the importance of Mathematics for probability.

By studying the history of probability students can develop a critical and reflective view of the theory. They understand that probability is not just a set of mathematical rules, but that it is a powerful tool for dealing with uncertainty, assessing risks and making informed decisions in various situations. Historical understanding broadens students' perspective and shows them that this is a constantly evolving discipline, subject to revisions and improvements over time.

The history of probability familiarizes students with terms and concepts specific to this area. They learn to use precise language to describe uncertain events and estimate probabilities by applying appropriate words and numerical values. The ability to express with appropriate vocabulary and coherent argumentation contributes to the development of probabilistic literacy. This perspective emphasizes that understanding probability goes beyond the mere manipulation of formulas and calculations. It involves the ability to interpret and communicate probabilistic information in a meaningful way in real contexts. Students need to understand how probability is applied in practical situations, such as decision-making, risk assessment and interpretation of statistical data (Gal, 2005).

By relating the acquisition of probabilistic language to the historical discussion of

[52]Evolutionary biologist statistician and English geneticist, pioneer in the application of statistical procedures for the design of scientific experiments.
[53]American statistician with contributions in correlation, analysis of variance, experimental design, etc.
[54]Significantly contributed to Probability Theory, Topology, Logic, Classical Mechanics and more.
[55]https://archive.org/details/kolmogoroff-1933-grundbegriffe-der-wahrscheinlichkeitsrechnung.

probability and its teaching, teachers broaden students' understanding of the concept. Students understand the evolution of language over time, the incorporation of different perspectives and meanings, and the fundamental role of probabilistic language in understanding and applying probability in various areas of knowledge. This approach offers students a more comprehensive and in-depth perspective, and enables them to understand the importance of probabilistic language in developing critical thinking, as wall as in making informed decisions (Ortiz & Alsina, 2017; Batanero, Henry & Parzysz, 2005).

Therefore, the teaching of probability should be conducted in a progressive and longitudinal way, allowing students to develop a solid and effective understanding of the concept. Teachers need to be prepared to guide the teaching process, as well as to promote the construction of strategies and forms of reasoning that help students make appropriate decisions in everyday and professional situations that involve uncertainty (Ortiz & Alsina, 2017).

Lastly, the link between the history of probability and the teaching of probability promoted in the classroom is crucial for students to understand the conceptual evolution of this subject, to establish connections between the constructs learned and their practical application, to develop a critical and reflective perspective of the theory and to acquire the probabilistic language necessary to become probabilistically literate citizens. This progressive and contextualized teaching process contributes to a more comprehensive and appropriate understanding of probability, and prepares students to deal with uncertainty as well as to make informed decisions in their lives.

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