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"Let's Travel" - A Proposal for a Game about Probability, for the Sixth Grade of Elementary School

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Abstract: This qualitative study has designed to examine how the game "Let's travel" could help with the development of the following cognitive demands: understanding the principles of randomness, sample space and comparing probabilities. Twenty-eight students from the Sixth Grade of Elementary School participated in this research. The theoretical principles that this activity was based on were the curriculum guidelines, the use of games as a teaching method, the guiding principles of teaching and learning about probability and cognitive demand. We found that the majority of students were able to recognize the sample space and we could also see that they intuitively associated the idea of randomness with luck. We also found that most students could compare probabilities by analyzing the sample space, without making any calculations. Our analysis of the results showed that this game, with intervention and support from the teacher, was a valid way of helping the students develop the cognitive demands mentioned above.

Keywords: Probability. Cognitive Demand. Games. Elementary School.

¿Vamos a viajar? Una propuesta de juego sobre probabilidad en 6º de Educación Primaria

Resumen: Este estudio cualitativo tuvo como objetivo identificar las contribuciones de la aplicación del juego *¿Vamos a viajar*? en el desarrollo de demandas cognitivas: comprensión de la aleatoriedad, espacio muestral y comparación de probabilidades. En esta investigación participaron 28 estudiantes del 6to año de Educación Primaria. Los supuestos teóricos incluyeron lineamientos curriculares, el uso de los juegos como metodología de enseñanza, los principios rectores de la enseñanza y el aprendizaje, la probabilidad y las exigencias cognitivas. En cuanto a la formación del espacio muestral, se encontró que la mayoría de los estudiantes supieron reconocerlo. En cuanto a la comprensión de la aleatoriedad, se comprobó, de forma intuitiva, la asociación con la idea de suerte. Y, en cuanto a la comparación de probabilidades, se observó que la mayoría de los estudiantes realizaron comparaciones, mediante análisis del espacio muestral, sin realizar cálculos. El análisis de los resultados mostró la validez de utilizar el juego mencionado, combinado con un proceso de intervención pedagógica, con el fin de favorecer el desarrollo de las demandas cognitivas antes mencionadas.

Palabras clave: Probabilidad. Demandas Cognitivas. Juegos. Enseñanza Primaria.

Vamos viajar? Uma proposta de jogo sobre probabilidade no 6º ano do Ensino Fundamental

Resumo: Este estudo qualitativo objetivou identificar as contribuições da aplicação do jogo *Vamos viajar*? no desenvolvimento das demandas cognitivas: compreensão de aleatoriedade, espaço amostral e comparação de probabilidades. Participaram, desta pesquisa, 28 estudantes do 6º ano do Ensino Fundamental. Os pressupostos teóricos contemplaram as orientações curriculares, o emprego de jogos como metodologia de ensino, os princípios direcionadores de ensino e aprendizagem de probabilidade e as demandas cognitivas. Em relação à formação do espaço amostral, verificou-se que a maioria dos discentes conseguiu reconhecê-lo. Quanto à compreensão da aleatoriedade, constatou-se a associação com a ideia de sorte, de maneira intuitiva. E, sobre a comparação de probabilidades, observou-se que a maior parte dos estudantes estabeleceu comparações, mediante análise do espaço amostral, sem efetuar cálculos. A análise dos resultados evidenciou a validade do uso do referido jogo, aliado a um processo de intervenção pedagógica, de modo a favorecer o desenvolvimento das demandas cognitivas anteriormente citadas.

Palavras-chave: Probabilidade. Demandas Cognitivas. Jogos. Ensino Fundamental.

1 Introduction

We often come across everyday applications of probability theory, such as demographic studies, incidence of diseases, vaccination effectiveness, calculations relating to insurance or the chance[1] of winning lotteries. It is useful for citizens to develop an appreciation of probability, in order to help them critically analyze situations such as those mentioned. This skill requires that they are able to complete real-world tasks and functions involving risk and uncertainty, efficiently. It involves the ability to access, use, interpret and communicate information and ideas related to probability (Alsina & Vásquez, 2016).

Probability means "quantifying the possibility of an event occurring; and it should be treated as a measurement" (Alsina & Vásquez, 2016, p. 46). It can be used as a tool in statistics, where you make predictions by identifying the characteristics of experiments and random phenomena. Normally, one creates a model that allows you to measure uncertainties or to understand what they mean from a scientific, professional or social perspective (Lima & Borba, 2019).

Given that there are countless random situations in many different contexts, it is important that the teaching of Mathematics is not limited to purely deterministic circumstances; otherwise, this will create problems for students when dealing with many kinds of everyday activities (Fischbein, 1975). Children, however, use elements of probability from a very early age; for example, whenever they play games that involve dice, roulette wheels or drawing options randomly. It is important, therefore, that they study probability from very early years (Fischbein, 1975; Bryant & Nunes, 2012).

Fischbein (1975), in fact, concluded in his research that children have very intuitive and firm ideas about probability. In further support of this view, Alsina and Vásquez (2015) stated that children under the age of seven have an understanding of chance and, as they grow, they create a "distinct and organized conceptual structure that provides the foundation for them to fully develop probabilistic logic" (Alsina & Vásquez, 2015, p. 13).

[1]The word 'chance' is being used as a synonym for probability.

In view of this, it is important to design various activities for use in schools that allow students to experience situations involving risk and uncertainty. One option for teaching probability, therefore, is the use of games. Given the above, the guiding question for this study was: How can the game "Let's Travel" help with the development of cognitive demand (understanding the principles of randomness, sample space and comparing probabilities) for students in the Sixth Grade of Elementary School.

This was a way to examine the concepts of probability at school using games. To this end, a lesson was created, based on the guidance suggested by Grando (2000, 2004, 2015), that focused on the playing of a game, for Sixth Grade students in Elementary School, in 2023, at a private school in Porto Alegre, Rio Grande do Sul. This proposal was designed to explore three of the four cognitive demands recommended by Bryant and Nunes (2012), specifically: understanding randomness, understanding the sample space and comparing probabilities.

This article, in order to answer the question proposed, will begin by presenting the theoretical basis of the study. Next, we will describe the steps used to apply the game mentioned previously in the lesson. Finally, we will discuss the data gathered and form some conclusions about the experience.

2 Theoretical References

The theoretical basis of this study can be divided into four areas. The first covers the main curriculum guidance on teaching probability. Then, there is a discussion on the use of games as a teaching method in Mathematics, specifically to study probability. The third area consists of guiding principles on teaching and learning probability, from the perspective of different researchers. Finally, we describe the four cognitive demands identified by Bryant and Nunes (2012).

2.1 Curriculum Guidance on Teaching Probability

Various countries have updated their curriculums in recent years. This has generally involved incorporating probability and statistics into the curricula for Basic Education, from early years onwards (Ballejo, Braga & Gea; 2021). Since then, researchers in this area of Mathematics have sought ways to improve the teaching and learning of these subjects.

According to Alsina (2021), the principle milestone for this internationally, was when the National Council of Teachers of Mathematics (NCTM) included a section on "Data and Chance" in the Curricular Standards for Mathematics at the end of the eighties. This document explained how they believe that probabilistic and statistical literacy need to be addressed more progressively. It also recommended that Probability and Statistics should be included at Pre-K level (Alsina, 2021), which corresponds, in Brazil, to students of three and older.

Alsina and Vásquez (2017) highlighted that, in relation to probability, the NCTM recommended that schools create informal exercises to explore the ideas on this subject, in order to encourage children to think about it in relation to their own experiences. This explains how essential it is, therefore, to include probability at various different stages of education, and teachers have to pay attention when planning lessons to the student's profile and the degree of complexity required at each grade (Batanero, 2019).

It is important to clarify that probability is taught as part of Basic Education in various countries, including Spain (Ministerio de Educación, Cultura y Deporte MECD - (Spain,

2014)), Australia (Australian Curriculum Assessment and Reporting Authority - ACARA (Australia, 2010)), New Zealand (New Zealand Curriculum (New Zealand, 2007)), Brazil (National Core Curriculum - BNCC - (Brazil, 2018)) and others.

The BNCC (Brazil, 2018) stipulates the knowledge, skills and abilities that students must develop as part of their Basic Education. Just like the rest of the international community, the above document states that Probability and Statistics should be a single thematic unit that runs throughout Elementary Education.

Furthermore, the BNCC explains that the study of probability during the final years of Elementary School should be about continuing the work begun in early years, in more depth. For this purpose, the document recommends that teaching involves the carrying out of experiments and simulations to encourage students to compare theoretical (Laplacian) and frequentist probability. The aim, therefore, is to "improve their ability to enumerate the elements of the sample space, which is also associated with counting problems" (Brasil, 2018, p. 272). It is important, therefore, to stress that there is a need for educational exercises that can teach students the different meanings of probability because of its essential complexity, both in the cognitive and epistemological sense (Cavalcante, Lima & Andrade, 2021).

However, making the teaching of probability in Mathematics a mandatory part of the curriculum does not ensure that a student's education will allow them to understand the role of chance and uncertainty in their own lives, as opposed to having a deterministic conception of the world. Batanero et al. (2016) explained this by highlighting the fact that, due to the irreversible nature of random experiments and their different outcomes, the process of teaching and learning about probability can be difficult for both teachers and students. This is not a problem that occurs in other branches of Mathematics, such as arithmetic, geometry or quantities and measurements, where the operations on objects can be reversed.

According to Batanero and Godino (2002), the first issue can arise at the very beginning, because probability can be wrongly associated with ideas of luck, providence or ignorance. Because of its relationship with chance and uncertainty, probabilistic reasoning can be complex (Batanero & Godino, 2002). Corrêa and Lopes (2020, p. 100) recommended that, when discussing the principle of uncertainty, it was important to prioritize the teaching of probability, in order to produce citizens who can deal with uncertainty naturally and can solve issues and make decisions, as "few things in our lives are known with certainty".

In this case, it is important that lesson plans and techniques are quite diverse when teaching probability, in order to allow students to experience successive experiments and investigate issues, simulations and other ideas. In line with this, the next section discusses the use of games to teach mathematics, probability in particular.

2.2 Games for Teaching Mathematics / Probability

Fun activities are ones that students enjoy because of the activity itself, e.g. listening to music, singing, dancing, playing games etc. (Raupp & Grando, 2016). Grando (2000), when examining these, stated that, based on the views of various theorists in the field of psychology, such activities can help with the cognitive, affective, social and moral development of children.

Alsina (2011), specifically referencing the ideas of Piers and Erikson (1982)[2], Bettelheim (1987)[3], Winnicott (1971)[4] and Vygotsky (1995)[5], defined a game as something

[2] Piers, M.W. y Erikson, E.H. (1982). Juego y desarrollo. Barcelona: Critical
[3] Bettelheim, B. (1987). No hay padres perfectos. Barcelona: Crítica, 1994.

that, whether free-form or structured, is an activity that establishes a connection between fantasy and reality, and encourages social and intellectual development. It is important, therefore, to incorporate such practices into school activities and use them as a teaching strategy to encourage students to develop hypotheses, make conjectures, reflect, analyze and summarize the issues created by the game itself (Grando, 2000). It is important, in order to do so, that teachers appreciate that games should be used as a means to achieve a certain purpose and not as an end in themselves. Similarly, Kishimoto (2017) emphasized that an educational game must be equally fun and educational. If it is only the former and not the latter, it is purely entertainment and loses any educational purpose. However, if its educational character overshadows the fun aspect, students will only see it as a normal teaching activity.

When the teacher includes some fun aspects in their teaching, it encourages the students to engage more with the lesson and they are likely to learn more. Christensen, Horn, Johnson (2012) explains that using games in the classroom can contribute to students' learning, when they involve resolving problems, making decisions, forming hypotheses and creating strategies, which they describe as 21st century skills. According to Rezende, Carrasco and Silvas-Salse (2022), games in education are usually based around a system of rules. This provides an opportunity to activate those brain functions that stimulate the affective, cognitive and motor skills of those taking part. In particular, games in the school environment help to develop social skills (Raupp & Grando, 2016), and stimulate different types of intelligence in the students (Ortiz & Denardin, 2021).

Abreu and Silva (2023) also wrote on this topic. They said that the use of games in a classroom helps students to develop empathy, encourages discussion and cooperation, as well as respect for the different points of view that classmates may hold. The teacher, therefore, must prepare a detailed plan in order to ensure the planned objectives are achieved. This should allow the students to be creative, be involved and learn and understand the ideas on the various topics. It should allow the students to interact with each other and with the teacher(s), in order that those involved can develop their cognitive processes, critical thinking and argumentation skills, as well as explore their social and emotional skills. Ortiz and Denardin (2021, p.18) recognized that the use of games in the classroom can lead to competitiveness. "Society is already competitive and we do not believe that there needs to be any extra competitiveness in the classroom, so it is important for the teacher to put it into context and minimize it, by emphasizing that students should work collaboratively and cooperatively".

The purpose of the game, therefore, is to encourage students to develop a specific area of knowledge and learn about a specific subject. According to Grando (2000), games in Mathematics can make it easier to learn or revise concepts. They can also provide opportunities for students to develop their problem solving, both working with others and independently.

The teacher, therefore, must design the game as a teaching method with a specific intent, either to introduce a concept or to put a fundamental principle into context. The method, in this case, is a set of guidance on conducting an educational practice, by employing "concrete, specific and different strategies, approaches and techniques" (Moran, 2018, p. 4).

Alsina's (2011) vision is one where games are properly and rigorously incorporated into the Mathematics curriculum. That said, it is important to determine the objectives that need to be achieved and describe how to evaluate these. Only when this is done, can a game

[4] Winnicott, D.W. (1971). Realidad y juego. Barcelona: Gedisa, 1993.
[5] Vigotsky, L.S. (1995). El desarrollo de los procesos psicológicos superiores. Barcelona: Crítica.

be more than just a resource and become a method that can be applied to education (Alsina, 2021).

Grando (2000, 2004) recommended that games should be included in teaching methods, in order to develop problem solving. He believes that a game can help students to learn mathematical concepts by instigating a similar process to other problem-solving methods but more effectively. In order to achieve this, it is important for the teacher and the students to discuss their reasoning and what they have understood while playing the game, both teacher to student and between classmates (Grando, 2015).

In fact, it is important that the classroom environment is suitable for investigation, and allows the students to come up with theories, predict what will happen and make and change strategies. There are seven stages listed by Grando (2000, 2004, 2015) that need to be covered when using gaming methods in the classroom, these are:

- *Familiarization with the material:* the first step is to ensure that they have examined the material for the game;
- *Understanding the rules:* the rules may be explained or read out by the teacher at this point or even demonstrated as a dry run*;*
- *Playing to embed the rules:* encourage them to try to play by themselves, so that they can learn the rules. This will help them to understand the mathematical principles;
- *Recording the game:* next, the students should be told to write down points, procedures or calculations using the right language. This process is designed to encourage them to analyze the play and come up with strategies*;*
- *Verbal Intervention by the Teacher:* while they are playing, the teacher should ask questions or make comments, to encourage them to think about what they are doing*;*
- *Written intervention:* at this stage, the students should provide written answers to the problems, to encourage them to analyze the game. The teacher should have a prepared list of questions for this that will direct the students to the mathematical concepts being taught;
- *Playing "competently*": playing the game once again to try out the strategies that they have come up with previously. This allows students to play the game as well as they can and have the opportunity to think about moves that they had not originally considered.

It is important to mention that, historically, probability has often been involved in games of chance. The game "Tali" (the bone game) is a type of dice game that was played with astragali dice (the ancient version of a die was an irregular tetrahedron), which were the first example of a probabilistic device (Viali, 2008). It is, therefore, quite natural to investigate the concept of probability by using gaming as a teaching method. The next section, therefore, examines this by providing some guiding principles on teaching and learning about probability.

2.3 Guiding Principles for Teaching and Learning about Probability

Alsina and Vásquez (2017) suggested that probability should begin to be taught in early years schooling by using situations that the children will recognize, so that they can form a connection with concepts that deal with uncertainty. Batanero (2019) reinforces this by saying that it is fundamental to include this, because the concepts related to probability can be taught at various stages of education. You just need to ensure you target the type of students you are dealing with appropriately and provide the right level of complexity.

However, this subject is often taught according to the textbook, where the process focuses on presenting information and repetition, rather than introducing it in context to allow students to be able to learn inductively and associate it with their own experience (Alsina & Vásquez, 2017). Cavalcante, Lima and Andrade (2021) explained that undergraduate teacher training tends to treat probability poorly. It does not go beyond elementary techniques and tasks. Cavalcante (2018) also found that, in relation to this, the Introduction to Probability course for the Mathematics Teaching degree program only addressed the classical meaning, which is out of step with the standards set by the BNCC (Brazil, 2018).

Alsina (2021) makes five recommendations on teaching statistics and probability to improve practice in Early Childhood Education and the Early Years of Elementary School. These are: plan and organize the teaching of these subjects based on mathematical processes; take into account the experience of both students and teachers when designing activities; consider different contexts when suggesting teaching methods, from the most formal to informal, taking into account the students' stage of education; ensure that their learning is progressive - covering concrete aspects to the more abstract ideas; and, finally, have clear criteria when choosing your particular teaching method.

According to Batanero (2013), activities based on games of chance can help students to understand the fundamental principles of this subject gradually, such as randomness, chance, the sample space etc. He also stressed that it is essential to understand what the students believe about what they experience, so that they can be directed to consider non-deterministic explanations (Batanero, 2013).

Batanero and Godino (2002) list four recommendations when teaching probability. These are: provide a variety of experience to encourage students to distinguish between random and deterministic phenomena; encourage them to make predictions about how the random phenomena will behave, what the result will be, and what the probability is; ensure that the results of an experiment are recorded, so that there is an opportunity to compare the predictions with the results; explain the unpredictability of each result, as well as the possible variation in small samples, so that students will try to compare the results and provide them with opportunities to analyze the overall results obtained by the class, so that they can compare, in terms of reliability, small and large samples (Batanero & Godino, 2002).

Vasques et al. (2019) created a model for teaching probability, specifically for early years, based on five dimensions. Firstly, Probability Activities are tasks that encourage students to explore and think about probability and which help them to learn new information. Probability Reasoning is designed to help them identify scenarios that involve probability, so that they can recognize it and consider which ideas and beliefs are incorrect. The aim of this, then, is to formulate, interpret, accept or verify pronouncements or statements that have an element of uncertainty. Probability Connections is where they need to identify associations between probability and the ideas, concepts, definitions, properties and procedures that relate to other mathematical concepts, at all stages of schooling. Probability Communication, meanwhile, is intended to help them learn by encouraging interaction, negotiation and discussion in the classroom. Finally, the last topic, Probability Language (verbal, numerical, symbolic, tabular and graphic) is designed to show that they understand the concepts correctly.

Regardless of the principles used by the teacher, it is important that students have a variety of learning activities that can help them to develop the key concepts relating to probability. These constructs could include random events, randomness, sample space, chance, equiprobable events, probability calculations or comparison and correlation. These

ideas may be related to the cognitive demands identified by Bryant and Nunes (2012), which are described in the next section.

2.4 Cognitive Demand

The researchers, Peter Bryant and Terezinha Nunes, suggested that probability learning is based on four cognitive demands, specifically: understanding randomness, understanding the sample space, comparing probabilities and understanding the relationships between events (Bryant & Nunes, 2012).

The first aspect is the ability to understand the interdependence of successive events. This is essential to understanding the nature of random phenomena and experiments. Although it is possible to list all potential outcomes, it is not possible to predict which ones will actually occur, nor the order in which they will occur. In the opinion of Bryant and Nunes (2012), any teaching approach must consider the concepts of random events, chance and equiprobability. In particular, the researchers noted that children aged 10 and over are particularly interested in the concept of fairness and equiprobability. Teachers, therefore, should try games that encourage the children to discuss this idea (Bryant & Nunes, 2012).

In relation to the second cognitive demand, understanding the sample space, it is essential to identify all of the possible outcomes of a given event. No kind of probability problem can be solved without understanding the sample space (Bryant & Nunes, 2012). The sets or subsets in any scenario have to be quantified using combinatorial reasoning. Navarro-Pelayo, Batanero and Godino (1996) also stated that combinatorial reasoning is essential for calculating probability, and this is not limited to just one tool. These researchers identified that there was a relationship between the sample space of a compound experiment and combinatorial operations when using a tree diagram in combinatorics or probability. Furthermore, if you want to take inventory of all possible events in a given sample space, you need to use a combinatorial construction process (Navarro-Pelayo et al., 1996).

The third demand is about comparing and quantifying probability. According to Bryant and Nunes (2012), there are some problem scenarios that require you to make probability comparisons, based on "more" or "greater" relationships, for example. In other cases, however, one might need to calculate the probability of an outcome. This calculation would be expressed as the ratio between a specific result and the set of possible results, i.e. the classic definition of probability (Batanero, 2005). Both activities require an understanding of randomness and sample space.

The last demand, referred to as ‘correlation’, is about recognizing whether an event is dependent or independent, according to whether there is a random or de facto relationship between them. Correlational thinking does not necessarily imply a cause and effect relationship, but it does depend on understanding the link between events and whether they are random or not (Bryant & Nunes, 2012).

The authors believe that these demands are all related and they must be included when planning the teaching of probability.

Next, we will introduce the methods used to carry out the activities based on the game “Let's travel”.

3 Methods and Procedures

This is a qualitative form of study. This approach involves investigating phenomena in their context and in all their diversity, taking into account the knowledge and actions of the

research participants (Bogdan & Biklen, 1994).

The school in which we conducted this study has six Sixth Grade classes with around 30 students in each class. It is located in the city of Porto Alegre, Rio Grande do Sul, Brazil. The plan was applied to all these classes and one of the authors is a teacher of one of them. However, we only analyzed the results of the students in one of these groups for this study. There were 28 students present for this activity, 10 girls and 18 boys, aged between 11 and 12 years old. We also gathered data from a field diary completed by the teacher/researcher.

The activity described below was carried out in the second semester of 2023, during two consecutive class periods, lasting fifty minutes each. The students worked in pairs, threes or fours for this activity; however, each one made their own written record.

It should be noted that, because the study was carried out on students in the Sixth Grade of Elementary School, they had not yet been taught anything involving the correlation demand. This is generally considered a more complex topic and is more appropriately taught at a later stage.

It is important to mention that the students in this school learn probability from the First Year of Elementary School and this activity took place at the end of the school year. They had, therefore, already studied some probability, including, by this stage, equiprobable sample spaces, in accordance with the BNCC standards (Brasil, 2018).

It is important to mention that the game “Let's travel” involves conditional probability. However, as the research participants were students in the Sixth Grade of Elementary School, this study was designed to focus on the demands of sample space, randomness and comparing probabilities, as recommended by Bryant and Nunes (2012). The aim was to introduce them to the idea of a non-equiprobable sample space, using the game in question. The next section describes the questions posed by the written material to explore these aims, as well as an analysis of the answers given by the students to these questions.

4 Description and Analysis of the Activities

4.1 Understanding the Game “Let’s Travel”

The lesson plan utilizing the game “Let's travel” was designed according to the guidelines issued by Grando (2000, 2004, 2015). First, the students had the chance to examine the material that was given out to each group, in order to familiarize themselves with the activity. This consisted of a normal die (six sides, numbered from 1 to 6), one playing piece (plastic bottle top) for each of the players and a board made by the authors, as shown in Figure 1.

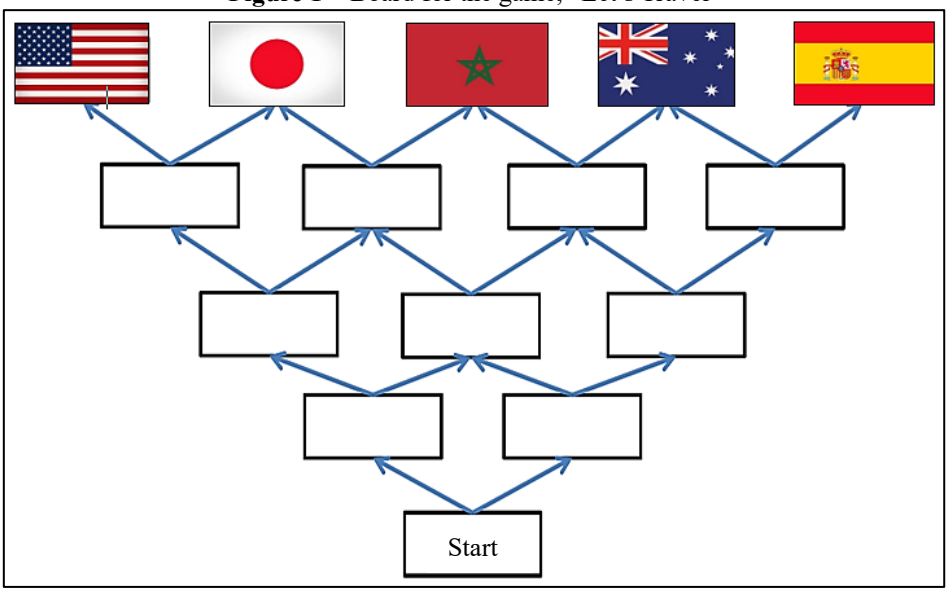
Once they had been given the materials, the students were asked whether they recognized the flags on the board, and were then asked to say what they knew about these countries. The main information they gave mainly revolved around the capital, where it was on a map of the world, its currency and population. The students also noticed that all of these countries participated in the 2022 World Cup in Qatar.

Next, in line with Grando's recommendations (2000, 2004, 2015), the teacher/researcher explained the rules of the game to the class as follows:

- First, each player had to choose a country from those on the board (Figure 1). The aim of the game was to reach this country with their piece.

- At the beginning of the game, all the pieces were placed in the "Saída (Start)" rectangle. They may move from one rectangle to another, in the direction of the arrows.
- Each player rolls a die to determine the initial order of play, according to the ascending order of the number thrown on the die. If there is a tie, the players repeat the process in the same way to decide who goes first.
- Once the order is decided, the first player throws the die. If the number rolled is even, the player must move his piece to the right and, if it is odd, to the left. The winner is the player who lands on his chosen country. It is possible, therefore, to have more than one winner of a game.

Figure 1 – Board for the game, "Let's Travel"



Source: authors.

The students were asked whether it would be possible to add another rule about using the die, which was appropriate for the game, and was based on the principles of uncertainty, randomness and the sample (Vásquez et al., 2019), in order to encourage them to think about probability. According to Bryant and Nunes (2012), children over 10 years old have ideas about randomness and are able to make connections between what is fair and what is unfair. These authors stress, therefore, that it is important to explore this idea in the classroom, using games (Bryant; Nunes, 2012).

Some of the most interesting comments made by the students were: "make 1 to 3 go right and 4 to 6 go left (or vice versa)", "decide on three numbers for each side", "prime numbers go to the right and non-prime numbers to the left (or vice versa)", "divisors of 4 to the right and non-divisors of 4 to the left (or vice versa)" and, also, "multiples of 2 to the right and non-multiples of 2 to the left (or vice versa)".

It should be mentioned that the teacher/researcher tried out two of these moves to help them understand the rules better. Each group also had a dry run, so that they could discuss any problems they had as a class and ensure that everyone was confident about what they were doing. While this was going on, the students were encouraged to try to identify how the game worked, as recommended by Grando (2000).

Each student was given a sheet with a picture of the board to write down the number rolled and the route taken by their piece, so that they could analyze its progress on each

round. Grando (2000) emphasized that this stage is when students need to be looking at the moves they are making to encourage them to understand the mathematical concept. To make sure they did, the teacher intervened by helping them with any problems and asking them questions to get them to analyze the game. The previously mentioned sheet also had questions for the students to write down answers to. This was to capture some of their thoughts on the cognitive demand involved in understanding randomness and sample space, as well as comparing probability. This material was gathered up at the end of the class for later analysis. These questions are discussed below.

4.2 Putting the Game into Practice

The students were asked to answer eight questions, by writing their answers on the sheet provided, based on their experience of playing the game "Let's travel", We were, therefore, able to analyze the game using the records made by the students, together with the notes from the teacher/researcher's field diary.

The first question asked which country each player had chosen to travel to. Of the 28 students, six chose the United States, eight Japan, five Morocco, four Australia and five Spain. As we can see, there was an even balance of choices between the available options.

Next, they were asked why they chose this country. In most cases, the students expressed an interest or preference in a particular country, rather than considering the rules of the game. Table 1 shows the reasons they gave, together with the absolute frequency.

Table 1 – Distribution of the Reasons for Choosing a Particular Country

Reason for Choosing the Country	Number of Students
It is interesting / I like it	7
It is the country with the best chance	6
I would like to know more / visit / live there	3
Because of the country's culture	1
It is a developed country	1
I already know this country and I would like to go again	1
Because the dollar is cheaper	1
Because it is a beautiful country	1
I have family in that country	1
Other	6
Total	**28**

Source: Research Data

It is important to mention that the six players who said they chose Morocco, said that the reason was because there was a better chance of reaching that country. Two of these records are presented in Figure 2 and they show, as Alsina and Vásquez (2015) have stated, that students in this age group already understand some of the elements related to probability.

Figure 2: Reasons for choosing Morocco

2) Por que você escolheu este país? Porque, nós achamos que tinha mais probabilidade de chegar nesse.

Why did you choose this country? Because we thought we had a better chance to arrive as this [country].

2) Por que você escolheu este país? Porque ele estava no meio e parecia ser fácil de chegar.

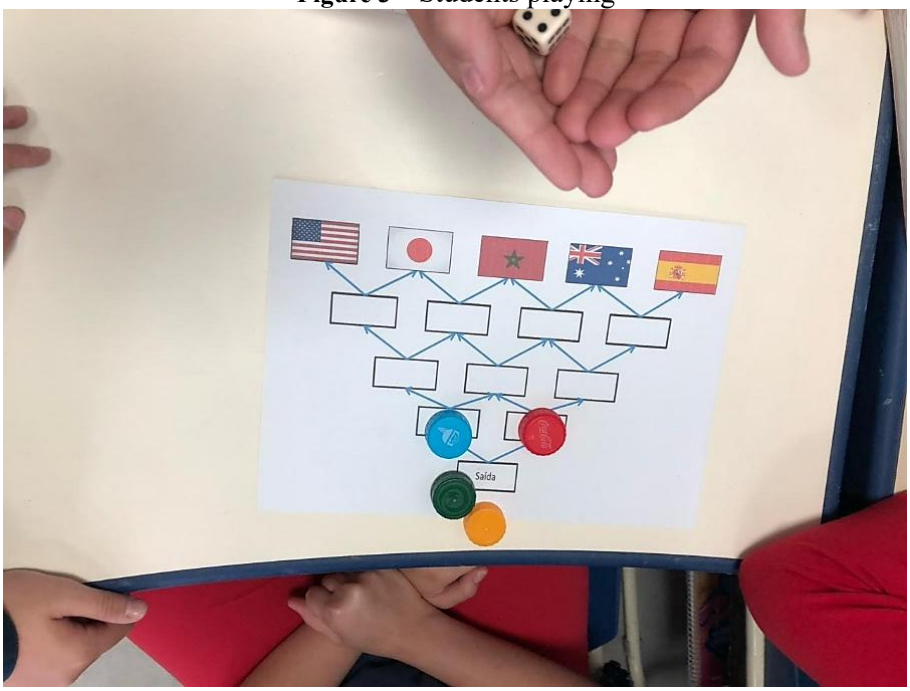
Why did you choose this country? Because it was in the middles and seemed easy to go to.

Source: Research Data

Furthermore, it is reasonable to treat their choices and reasons very positively, because they had not been given any other information, nor any prior explanation about the use of probability in the game. This shows that it is important to use scenarios that do not involve equal probabilities, because everyday situations do not, in most cases, involve equiprobable examples. Similarly, Bryant and Nunes (2012) emphasized that it was important for children to learn about probability in the early years of schooling, so that the students can properly interpret and resolve the problem situations that they encounter in their lives.

These questions helped the children to understand the game and think about the choices they were making. It encouraged them to think about the game mathematically, as was recommended by Grando (2000, 2004, 2015). We, therefore, tried to encourage the students to compare probabilities (Bryant & Nunes, 2012) and to analyze whether every option had the same likelihood of winning, but without making any calculations. Figure 3 shows a group of students playing.

Figure 3 – Students playing



Source: Research Data

When the game was over, the class was asked who had reached their chosen country. In response, 12 said that they had arrived, while 16 stated that they had not achieved the objective.

Japan and Morocco were the countries that the class had been most successful at reaching. Although Spain and Australia were the countries that players had the least chance of reaching, Spain and the United States were the ones that the students were most disappointed about. Table 2 shows these results.

Table 2 – Distribution, showing how many did or did not reach their destination.

Country Chosen	Arrived	Did Not Arrive
United States	2	4
Japan	5	3
Morocco	4	1
Australia	1	3
Spain	0	5
Total	**12**	**16**

Source: Research Data

When they were asked if anyone else in their group had managed to reach their chosen country, 19 said that one had reached their destination, and Morocco was the most common. We then tried to address the fourth area recommended by Grando (2000, 2004, 2015) by asking a further question. This area is about recording the game, so that students have an opportunity to think about their moves and come up with strategies to get better results.

To achieve this, the class was next asked whether the game "Let's Travel" was a fair or unfair game and they were also asked to give reasons. The result was that 12 students believed it was fair and five of these used the word "luck" to justify their opinion. These students appeared to equate randomness with luck, which is similar to the results in the research carried out by Batista and Borba (2016) and Ballejo, Braga and Viali (2021). The comparison between randomness and luck appears to be based on the students' intuitive conception of probability (Batanero, 2005).

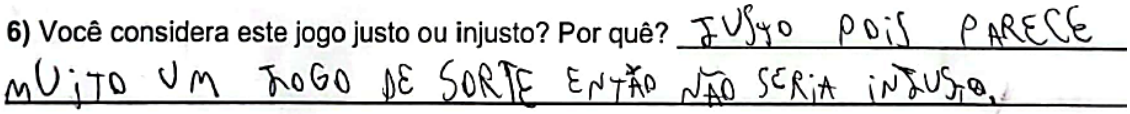
Figure 4 presents two examples of responses by students as to whether the game was fair or not.

It is important to remember that the objective of this activity was not to calculate the probability of arriving in any country, as represented by the flags, but to realize that the outcomes are not equally probable. It is important, therefore to stress that six students decided that the game was fair because each side of the die has the same probability. Figure 5 shows one of their justifications, which uses the argument that the chance of rolling an even or odd number on a six-sided die is the same. One could say that this student believes that the sample space is the sides of the die, rather than the routes to each country. Similarly, Batista, Henriques and Borba (2021) stated that students do not always appreciate the difference between equiprobable and non-equiprobable events and have the mistaken idea that all events have generally the same chance of occurring.

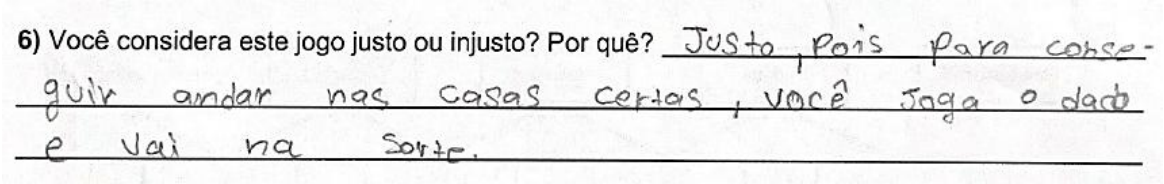
because they need to be able to understand how proportionality relates to this and that can be a problem (Bryant & Nunes, 2012). Therefore, it is important to include activities that have outcomes that are not equally probable from the early years of Elementary School onwards. Campos and Carvalho (2016) believe that teachers at this level of education should

not limit themselves to covering only a sample space that is equiprobable.

Figure 4: Describing the game as fair, based on the idea of luck



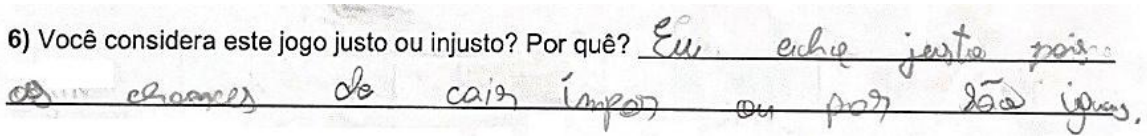
Do you consider this game fair or unfair? Why? Fair, because it looks like a game of luck, so it wouldn´t be unfair.



Do you consider this game fair or unfair? Why? Fair, because to move places you roll a dice and "goes on your luck".

Source: Research Data

Figure 5: Describing the game as fair, based on the probability of the dice being odd or even



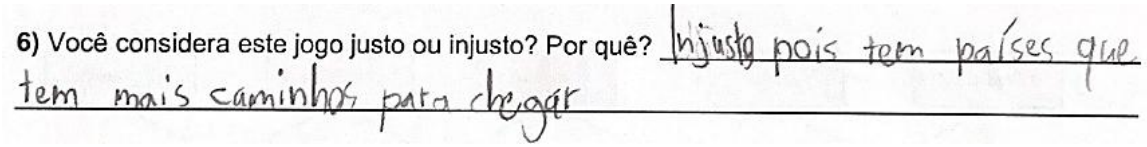
Do you consider this game fair or unfair? Why? I think it´s fair, because the chances of rolling an odd or even number are equal.

Source: Research Data

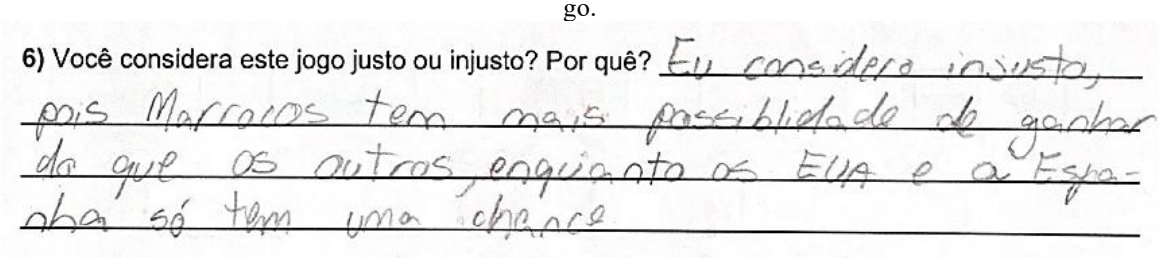
Children and adults can have difficulty quantifying and comparing probability,

Out of all the players, sixteen thought that the game was unfair. The reasons were mostly to do with the fact that there were fewer routes to the corner countries (United States and Spain) than to the center country (Morocco). Figure 6 describes the responses of two of the research participants who stated the game was unfair.

Figure 6: Describing the game as unfair, from looking at the pathways.



Do you consider this game fair or unfair? Why? Unfair because there are countries with more than one way to go.



Do you consider this game fair or unfair? Why? I think it is unfair, because Morocco has a greater chance of winning than the others, while the USA and Spain have only one chance.

Source: Research Data

It should be mentioned that there was a written intervention at this point of the activity, as recommended by Grando (2000, 2004, 2015). It should also be noted that the

teacher/researcher had to provide some guidance, where it was agreed with the class that the game would be considered fair if there was the same chance of reaching each country. This is in line with Borovcnik (2016) who stated that a fair game is one in which all players have the same probability of winning. Bryant and Nunes (2012) emphasized that it is vital to understand the concept of randomness, in relation to individual events, to ensure that a game is fair.

Next, the class was asked if there was a better chance of reaching any particular country, together with reasons. The aim at this point was to encourage the students to reflect, in line with the seventh item suggested by Grando (2000, 2004, 2015), which is about playing "competently". Here, the aim was to get the students to think about their previous strategy, analyze it and come up with a new one.

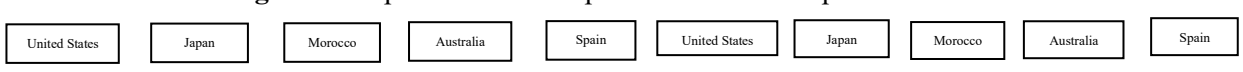
In this case, the main aim was to see whether the students realized that they were more likely to reach the most central country. In relation to this, Batista, Henriques and Borba (2021, p. 4) stated that "it is not enough to simply list all the possibilities for the sample space, because events are not always equally probable. You need to associate the number of elements of an event to the complete set, which is the sample space". It should be noted that, although the students were not asked to list all of the elements of the sample space, they were expected to appreciate that there were more routes to the central rectangle (Morocco). The class was, therefore, asked to reconsider whether it was equally likely to reach each country, without, however, making any calculations, as recommended by Bryant and Nunes (2012).

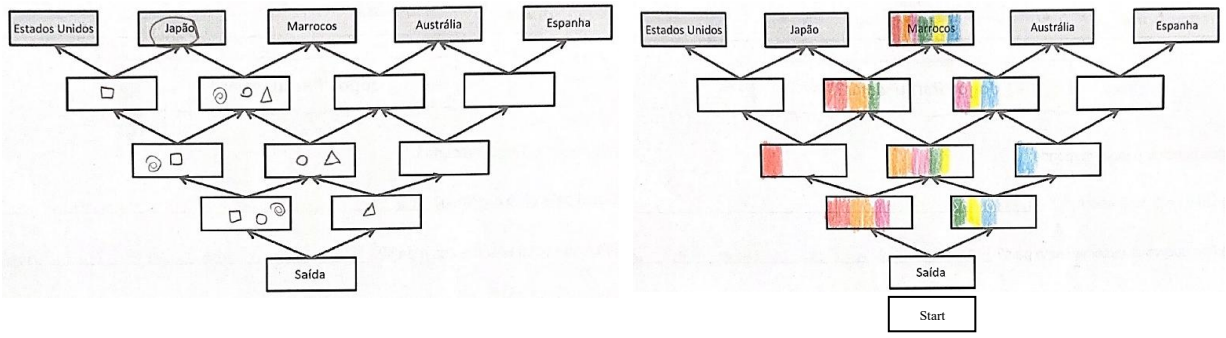
Out of all the respondents, 21 realized that there was a better chance of reaching one particular country, while seven left this question blank. Among those who answered correctly, 11 used the argument that Morocco was the most central; five stated that there were more ways to reach that country; one realized that, you can take paths on both the right and the left to get to Morocco; one realized that you had to roll only odd or even numbers respectively to get to the United States and Spain; one stated that to get to Morocco you could go either left or right; while two only said that there was more chance of getting to that country. Out of those who did not realize that the outcomes were not equally probable, three said that the game was random, because it involves dice; two mistakenly said that there was a 20% chance of reaching each country; one justified themselves by saying that the game's "Start" rectangle was in the middle and one gave no reason.

The class, therefore, made progress and realized that the outcomes of the game were not equally probable, because the majority of students decided that the best chance of reaching a country was with Morocco. You can see this progress by comparing this result with the answers given in the first question, because there the choice of countries was quite balanced. The use of gaming methods with a pedagogical approach did actually help the students to "learn mathematical structures that are often difficult to assimilate" (Grando, 2000, p. 28). It also encouraged them to "think, reflect, analyze and understand mathematical concepts" (Grando, 2000, p. 28).

For the final part of their written work, each student was asked to make a version of the Board that showed all the possible routes from the "Start" rectangle to the rectangle for their original "chosen country". They also needed to show the number of routes. This task was designed to identify the country with the best chance of being reached by understanding the sample space and comparing the probabilities. Figure 7 illustrates all the possible paths to reach Japan and Morocco, completed by two students respectively.

Figure 7: Depiction of all the possible routes to Japan and Morocco





Source: Research Data

On the left side of Figure 7, we can see that the student used different symbols to represent the four possible routes to reach Japan. While, on the right side of the figure, they used the colors red, orange, pink, green, yellow and blue to show the six possible paths to Morocco. The other students used similar ways to represent all the possible routes to the United States, Australia and Spain. We can say, therefore, that each student was able to identify the branches of the possibility tree for a given country, and each branch of the tree was an element of the sample space. If we join all the depictions together, we would get the complete possibility tree.

It should be noted that the sample space contains 16 paths. By the end of the written activity, the students had identified all the possible paths to reach each of the countries between them, with the guidance of the teacher/researcher; while the tree they produced explained the numbers involved. The objective of this activity was to identify the sample space for this game, and the relationship between the number of elements for an outcome and the complete set (the sample space), in order to demonstrate that these outcomes are not equally probable.

Finally, the students played the game again and were allowed to change their choice of country. It is worth mentioning that some groups played more than one round, in order to check which was the best option. This was another positive aspect of the game: the students had an opportunity to experiment and make and test hypotheses. Grando (2000), for example, explains that when games are used to teach, it encourages students to think about what they have done if they make a mistake, not just consider the best strategy to win the game.

Grando (2000, p. 45) also notes that students become more "competent" at playing the game each time they "play and reflect on their moves and possible moves, in other words they begin to think about the game from different aspects and angles that they had not originally thought of."

5 Final Thoughts

Our objective with the game "Let's travel" was to introduce the idea that a sample space is not always equally probable. To begin with, we found that the students believed that the sample space was made up of the five countries featured in the game. We could see this from their reasons for choosing their country. Only six students realized that there was a better chance of reaching Morocco, but none of them talked about the possible paths.

As the game progressed, however, we found that, in relation to the formation of the sample space, the majority of students began to realize that the sample space corresponded to the routes that could take them to each country. It is interesting to note that the students needed to apply combinatorial reasoning to appreciate this and when they misunderstood the

sample space it was difficult for them to understand probability. It is also worth mentioning that the questions raised by the written activities and the contribution of the teacher/researcher were important in helping them to recognize the correct sample space.

As for understanding randomness, we could see that they intuitively associated this with the idea of luck. This association can influence what they learn and, therefore, teachers have to take this into account when planning the lesson, in order to provide them with learning opportunities to experience the subjective, classical, frequentist and axiomatic meanings of probability, as the students' progress.

In relation to the third demand, the activity centered on the game "Let's Travel" also involved comparing probability. This occurred when the students analyzed the possible routes to each of the five countries. Most of the students were able to compare them correctly after analyzing the sample space, without needing to calculate the probabilities. This game proved to be a valid way to develop the mentioned cognitive demand of the students. The written activity on the game showed how they improved their understanding of the concepts related to probability.

Developing the first three cognitive demands is considered to be a process and it takes more than just one gaming activity to achieve this. It is important to stress, therefore, that the students need other types of learning situations that will allow them to understand randomness, sample space and comparing probabilities, together with the support and intervention of the teacher, in order to encourage them to think about what they believe and what they already know.

Our analysis of the results showed that this game, "Let's Travel", together with the contribution of the teacher, was a valid way of helping the students develop the cognitive demands previously mentioned. We also suggest that further studies examine the use of games that can help students to consider correlation demand.

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