Sociedade Brasileira de Educação Matemática

RIPEM

Mathematics Knowledge Specific to Teaching as a possibility to advance in Mathematics teachers' education

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Abstract: Supported by analytical and exploratory research, a group of educators from GT07 from *Sociedade Brasileira de Educação Matemática* (SBEM) conducted studies and discussions with authors about Mathematics teaching in undergraduate education in 2022 and 2023. After identifying the specificities to teacher training for basic education, we systematized an understanding of what we called *mathematical knowledge specific to teaching* for teacher education, which requires deep understanding of mathematical concepts with flexibility and capacity to plan, explain and establish relationships aiming at teaching practice. From this perspective, we offer teacher education, managers and SBEM a possibility to devise a formative curricular path in Mathematics teaching undergraduate education that is in line with aspects required in teaching practice with a focus on diverse and inclusive public basic education.

Keywords: Mathematical Knowledge Specific to Teaching. Curriculum. Mathematics Education. Mathematics Teaching Degree Program. Formative Path.

Conocimiento matemático específico de la enseñanza como una posibilidad para avanzar en la formación de profesores de Matemáticas

Resumen: Apoyados por una perspectiva de investigación exploratoria y analítica, un grupo de educadores de GT07 de la *Sociedade Brasileira de Educação Matemática* (SBEM) realizó estudios y discusiones con autores sobre la enseñanza de las matemáticas en la educación superior para la enseñanza en los años 2022 y 2023. Al identificar las especificidades de la formación de profesores para la educación básica, se sistematizó una comprensión de lo que se denominó *conocimiento matemático específico de la enseñanza* para la formación, lo que requiere una profunda comprensión de los conceptos matemáticos con flexibilidad y capacidad para planificar, relacionar y explicar, con enfoque en la práctica de la enseñanza. Bajo esa perspectiva, presentamos a la SBEM, a los formadores de profesores y a los administradores una posibilidad para la construcción de una trayectoria formativa curricular en la enseñanza en línea con las necesidades de la práctica de la enseñanza, con enfoque en una educación básica pública, diversa e inclusiva.

***Palabras clave*:** Conocimiento Matemático Específico de la Enseñanza. Currículo. Educación Matemática. Licenciaturas en Matemáticas. Trayectoria Formativa.

Conhecimento matemático próprio da docência como uma possibilidade para avançar na formação de professores de Matemática

Resumo: Pautados numa perspectiva da pesquisa exploratória e analítica, um grupo de educadores do GT07 da Sociedade Brasileira de Educação Matemática (SBEM) realizou estudos e discussões com autores sobre o ensino da Matemática na licenciatura durante os anos 2022 e 2023. A partir da percepção das especificidades da formação docente para a Educação Básica, foi sistematizada uma compreensão do que se denominou por *conhecimento matemático próprio da docência* para a formação, o que exige sólido entendimento de conceitos matemáticos com flexibilidade e capacidade de planejar, relacionar e explicar, visando a prática de ensino. Nessa visão, apresenta-se à SBEM, aos formadores de professores e gestores uma possibilidade para a construção de um percurso formativo curricular na licenciatura em consonância com demandas da prática docente, na perspectiva da Educação Básica pública, diversa e inclusiva.

Palavras-chave: Conhecimento Matemático Próprio da Docência. Currículo. Educação Matemática. Licenciatura. Percurso Formativo.

1 Introduction

There has been a lot of discussion about the difficulties in Mathematics teaching in basic education, especially when it comes to the publication of results of exams and tests such as *Programa Internacional de Avaliação de Estudantes (Pisa)* and *Prova Brasil*. Several factors can be taken into account to explain said phenomenon: students' living conditions — hardship faced by poor people, who are the majority of students –; difficulties related to teachers' work, such as long hours, low salaries and a large number of students per group; conditions of schools and professionals' education.

With a focus on professional education, we will discuss a theme that is considered essential to this debate: the teaching of Mathematics in Mathematics teaching degree programs. Several studies point out that it is one of the main factors behind the identified difficulties (Gatti & Barreto, 2009; Gatti, 2010; Moreira & David, 2005; Moreira, 2012; Zaidan *et al*., 2021). We start with the premise that it is necessary for the study of Mathematics in Mathematics teaching degree programs to effectively consider the demands from the teaching profession, thus incorporating the specificities of teachers' professional practice.

In a research project conducted from 2018 to 2021 by a group of 54 Higher Education teachers affiliated with GT07 from Sociedade Brasileira de Educação Matemática (SBEM) from all Brazilian regions[1], the researchers analyzed degree program pedagogical projects of 172 Mathematics teaching degree programs that implemented Resolution CNE/CP 02/2015. The Resolution was massively supported by associations of education professionals, who could give opinions and put forward a considerable number of proposals advocated by the teaching movement. In the resolution, initial education should encompass pedagogical and mathematical education in its 3,200 proposed hours, as well as a wide range of knowledge such as the inclusion of disabled people, diversities and practice as a curricular component.

At the final stage of the research in 2021, it was possible to observe that Resolution CNE/CP 02/2015 generated advances in the curricula of 172 Mathematics teaching degree

[1] The research referred to here is titled *A Licenciatura em Matemática no Brasil em 2019: análises dos projetos dos cursos que se adequaram à Resolução CNE/CP 02/2015*. Conducted by a team of 54 researchers, it was organized by Ana Cristina Ferreira, Enio Freire de Paula, Flávia Cristina de Macêdo Santana, Flávia Cristina Figueiredo Coura, Patrícia Sandalo Pereira and Vandoir Stormowski. It is available at www.sbem.org.br.

programs in institutions in all of the analyzed Brazilian regions. Thus, we highlight the following aspects:

- courses offer an average of 3,306 hours (with the law requiring a minimum of 3,200 hours);
- curricula are basically structured around subjects and contemplate Practice as a Curricular Component (PCC), Supervised Training Practice, and contents that can be linked to the fields of Mathematics and Education, among others;
- out of the total number of curricular hours of Mathematics teaching degree programs, the study of Mathematics averages 1,452 hours, with 87% of that time dedicated to subjects encompassing pure Mathematics, while 13% is allocated to subjects classified as review courses;
- it is possible to consider that degree programs no longer adhere to a rigid curriculum structure under the 3+1 model, but they are still organized around knowledge blocks and keep the same logic, with the study of Mathematics done separately, alongside the study of other themes, as articulated by Moreira (2012), Gomes (2016) and Gatti, Barretto, André & Almeida (2019);
- in most of the analyzed degree programs, supervised teaching practice is structured across four courses; in other programs, it is divided into two or three courses;
- in around 40% of the degree programs, PCC appears structured as part of different courses; in around 25% of the degree programs, it appears as part of specific courses; and in around 28% of the analyzed degree programs, it appears in both ways; there are still some forms that are different from the aforementioned ones and even some proposals in which it is not mentioned;
- most degree programs refer to extension projects;
- interdisciplinarity appears with different intensity levels and is addressed in different ways in most Mathematics teaching degree programs, although some programs do not refer to it, and others just mention it with no specific actions being described;
- about the university-school relationship, a small number of programs explicitly acknowledge the importance of training in these spaces; most only make a brief reference to this relationship, without specifying any proposals. There are mentions of initiatives in this sense, but there is no articulated view of initial and continuing education, as proposed by the Resolution;
- a set of themes related to diversity and inclusion proposed by the Resolution are referred to in several degree program pedagogical projects, which shows that there is some progress in terms of preparing future teachers for the diversity existing in schools.

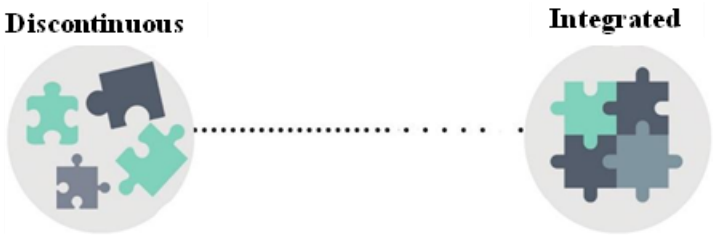
However, the aforementioned Resolution, like those who preceded it, does not prescribe any changes to the study of Mathematics in Mathematics teaching degree programs. This content takes a considerable part of the curriculum and is centered around a scientific-academic perspective (Moreira & David, 2005), usually taught through expository lessons. Obviously, this is the study that leaves a mark and characterizes teachers' education.

As a summary of the results, the research shows the existence of an effort by the teams behind Mathematics teaching degree programs to implement the proposals brought forward by Resolution CNE/CP 02/2015. After understanding that there are limitations in terms of analysis and study because programs are based on PPC, the research identified that Mathematics

teaching degree program formative paths vary, which is summarized as follows:

- *Discontinuous formative path:* structured around well-defined knowledge blocks with little articulation among them, and it does not establish a clear link between mathematical knowledge and the professional perspective. It is present in most degree programs;
- *Integrated formative path:* it takes the opposite direction because it suggests activities aiming to articulate courses, encourages the use of participatory methodologies and the study of themes related to teaching practice. This type of path is present in a considerably small number of programs;
- *Intermediate formative path:* it is structured around knowledge blocks and introduces some activities to articulate them, but it could not be classified as discontinuous or integrated. It can be found in a significant number of programs, here represented by the dotted line between those classified as discontinuous and integrated, as shown in Figure 1.

Figure 1: Formative paths



Source: Caldatto *et al.* (2021, p. 406)

Moreover, it was possible to notice that most Mathematics teaching degree programs are located in Mathematics Departments, in which Mathematics educators' presence and actions have been just a mere complement – and they are even voiceless when it comes to decision-making. Therefore, mathematicians compose most of the teams which teach degree programs and make decisions about them.

After the research was finished and the book was published — it is available on the website of SBEM-Brazil —, a new group was formed in 2022 do devise a new research project[2]. The same team that started the first research invited researchers to get involved in the sequel and crafted a new project to study a theme which, from the group's viewpoint, defines a degree program and can be considered its *heart*: Mathematics teaching in teachers' initial education. Thus, a new research project was formulated, and its main question was: "What type of Mathematics do we refer to when we think about Mathematics teachers' education?".

When it comes to methodology, the new research group decided to conduct exploratory research, in which "when facing an issue or theme that is not very well-known or defined, the researcher performs a study in order to obtain clarifying or consistent information or data about it" (Fiorentini & Lorenzato, 2009, p. 69). A literature review was conducted based on existing platforms in order to select studies that refer to teachers' mathematical education. Furthermore, we read and discussed some selected works and had conversations with some authors who were

[2] Title: "Um currículo para a Licenciatura em Matemática do ponto de vista da Educação Matemática – a necessária renovação da formação de professore(a)s de Matemática, tendo em vista a profissão docente na Educação Básica". Team: Alana Nunes Pereira – UFES - Alegre; Ana Cristina Ferreira – UFOP; Cirléia Pereira Barbosa – IFMG - Formiga; Elenilton Godoy – UFPR; Eliane Matesco Cristovão – UNIFEI; Enio Freire de Paula – IFSP - Presidente Epitácio; Flávia Cristina Figueiredo Coura – UFSJ; Jussara de Loiola Araújo – UFMG; Maria Auxiliadora Vilela Paiva – IFES; Marlova Estela Caldatto – UTFPR - Pato Branco (em memória); Patrícia Sandalo Pereira - UFMS; Samira Zaidan – UFMG (Coordenadora); Vania Cristina da Silva Rodrigues – UFTM.

invited to the group as special guests.

Finally, as a summary, we collectively produced an "essay on future Mathematics teachers' mathematical education based on mathematical knowledge that is specific to teaching" (Cristovão *et al*., 2023, p. 86). The essay contains understandings of the mathematical knowledge required by professional practice, so it was possible to understand clearly the differences identified in the prevailing academic knowledge that is present in Mathematics teaching program curricula. Finally, as a result of the study, we formulated the views and propositions that will be presented here.

2 Resuming existing studies to advance in the development of Mathematics knowledge specific to teaching

Knowledge of teacher education has been considered in research projects for some decades. Studies like the ones performed by Ponte (2005), Ball, Thames and Phelps (2008), Diniz-Pereira (2011) and Fiorentini and Oliveira (2013) acknowledge that, from a perspective aiming to break with technicism, knowledge related to teacher education needs to be more in line with professional practice and with social demands for Mathematics teaching.

While constituting the field of Mathematics Education, Pontes (2005) discusses the concept of *professional development* and links it to knowledge obtained from education:

> It is no longer just about identifying what teachers have to know for their professional practice, but also taking into consideration the nature of this knowledge and how it is constituted. Based on experience and also through reflective processes, we should aim to understand how the knowledge is communicated through beliefs, conceptions and images related to education, Mathematics, curriculum, students, learning and institutional processes (Ponte, 2005, p. 270, our translation).

Fiorentini and Oliveira (2013) point out that social teaching practice takes place within a set of relationships and contexts, and it requires formative training articulated with reality. In addition, the authors discuss what *place* Mathematics holds in the curriculum that is currently used to train teaching professionals:

> When we say that teachers need to know thoroughly multiple types of Mathematics, especially school Mathematics, we mean that it is not enough for teachers to master mathematical procedures and know how to use them in demonstrations or to solve problems and do exercises. For Mathematics teaching, it is important for teachers to know how to justify the procedures, to know other procedures that have been historically and culturally produced, to know current concepts and ideas and their historical evolution [...] (Fiorentini & Oliveira, 2013, p. 924-925, our translation).

Ball, Thames and Phelps (2008) performed studies and research, mainly through the observation of teaching practice and systematization of demands, so that they could constitute knowledge for teacher education. It is important to highlight their concern about the search that every Mathematics teacher needs to perform in order to find a balance between Mathematics rigor and the thinking produced by students, that is, to bring mathematical concepts to students' comprehension level.

They emphasized the wide set of themes and subthemes contained in pedagogical practice, the large number of aspects teachers need to deal with and, at the same time, there are aspects related to mathematical knowledge that need to be explained to those learning it for the

first time. Thus, they aimed to understand what kind of training teachers need to receive. Their studies resulted in analyses and typologies of knowledge that form teacher education, and among them, they saved a specific place for the mathematical knowledge aimed at professional practice: "Knowledge of content for teaching — what makes it special?"

Our studies corroborated Plínio Moreira's formulations through the perception and recognition of a set specificities of the knowledge that is particular to teaching professionals, which was called School Mathematics:

> [...] part of the conclusions expressed by Ball, Thames and Phelps (2008)[2], in order to consolidate the idea that there is a set of studies already systematized that point out the existence of a type of Mathematics that is particular to teachers' work in basic school, and this Mathematics (which some have called teachers' Mathematics, school Mathematics or Mathematics for teaching) is not in line with the one that, under the reference of content training, has been taught in Mathematics teaching degree programs following the 3+1 model and its variants (Moreira, 2012, p. 1143, our translation).

The author added that:

> [...] overcoming the 3+1 model requires structuring a new Mathematics formative training which defines its knowledge according to their relevance to professional practice – the goal of a Mathematics teaching degree program –, not just based on criteria of relevance related to academic Mathematics (Moreira, 2012, p. 1144, our translation).

Giraldo (2018) emphasizes that the construction of curricular projects does not consider research results properly:

> In this sense, it is necessary to pay special attention to the idea that we have possibly been (inadvertently or not) training teachers based on an anachronic type of school, still based on a paradigm of acquisition of ready knowledge – a school that completely ignores social and cultural transformations and ways of communicating and producing knowledge (p. 38, our translation).

Regarding the consideration that knowledge is produced in teaching practice, Davis and Renert (2014) referred to *Mathematics for teaching* as emerging Mathematics:

> [...] a **willingness** to engage with the Mathematics knowledge that *allows teachers to structure learning situations, interpret students' actions consciously and respond in a flexible way* in order to enable students to understand and expand their possibilities of interpretation through their access to powerful connections and appropriate practices […] Teachers should have a thorough understanding of **emerging** Mathematics (Davis & Renert, 2014, p. 117, our translation).

Within the context of the main question that guides this research, "What type of Mathematics do we refer to when we think about Mathematics teachers' education?", it is possible to emphasize the perception that there are specificities to the Mathematics knowledge studied by those who will become teaching professionals when we take pedagogical practice into consideration. Thus, they point to the inadequacy of today's dominant training in Mathematics teaching degree programs, in which the study of Mathematics is performed in an

isolated way through a formal and academic approach, and there is little articulation with the professional perspective.

After a year and a half of studies and debates with guest researchers (2022-2023), the research team worked to analyze an aspect that emerged and showed relevance: the large distance between the Mathematics education provided in teaching degree programs and demands from teaching practice in basic school. It was possible to perceive the existence of specificities to the mathematical knowledge used to train those who will perform teaching work, that is, teachers. Therefore, we formulated an understanding of the *Mathematics knowledge that is specific to teaching,* comprehended as the knowledge that opposes the Mathematics knowledge studied in Mathematics teaching degree programs today, which is commonly known as academic. In this sense, we agree with Moreira and David (2005) as follows.

The *Mathematics knowledge that is specific to teaching* is understood as the knowledge that prepares and provides teachers with tools in their initial education for professional practice. Therefore, it is knowledge from Mathematics teaching degree programs. The following aspects constitute the *Mathematics knowledge that is specific to teaching*: understanding Mathematics as a historical construction; valuing the relationships between Mathematics and other subjects, as well as its employment in themes and problems related to social life; seeing that mathematicians' professional practice has different characteristics from pedagogical teaching practice, and they stem from different degree programs as well – bachelor's and teaching degree programs; understanding that teacher education means preparing for teaching and preparing students of different ages from different realities, in a society that is more and more technological and in which merely transmissive teaching is not possible anymore; preparing teachers to learn how to act within a set of demands that are specific to the teaching profession, as well as comprehending learning assessment as a continuous and inclusive process.

Based on the considerations produced by the research, we understand that the study of Mathematics in Mathematics teaching degree programs should have specificities related to professional training and, in this sense, it is essential to the have Basic Education curriculum as a starting point. Thus, it is possible to consider a Mathematics teaching curriculum structured in a helical way, that is, from simple to complex, so that it starts with knowledge from Basic Education and then reaches Higher Education Mathematics.

Furthermore, in a democratic and technological society, it is vital for teaching to take place through participatory methodologies, in which students understand and take part in their own formation, break with the expository logic of classes and with the view of underlying banking education (Freire, 1993). Formative activities (in the form of courses/subjects and others) should be articulated with mathematical knowledge connected with dynamic and democratic principles of pedagogical-didactical knowledge, as expected from teaching practice in any education level. In this sense, it is important to aim at the construction of thought-provoking and participatory knowledge with a wealth of didactical resources according to existing teaching conditions.

Teacher education that encompasses an understanding of *Mathematics knowledge specific to teaching* stems from criticism of how Mathematics is studied in current teaching degree programs. The aforementioned criticism is expressed in order to acknowledge that, in current curricula, there is prevalence of isolation of the study of Mathematics in relation to other fields of knowledge (Zaidan *et al.*, 2021), a dominant model organized in a fragmented way, distant from the demands from teaching practice. Moreover, we emphasize the current prevalence of methodologies focused on expository lessons, a practice that disadvantages students' participation in the construction of knowledge. In addition, it is important to

conceptually acknowledge Ethnomathematics as a field of study, research and diversified social practices (D'Ambrosio, 2005).

Thus, we start to understand that the Mathematics studied in teaching degree programs has more academic characteristics and perspectives that are particular to another professional practice: mathematicians' practice. A type of Mathematics constituted by "a set of practices and knowledge related to a scientific body of knowledge produced by professional mathematicians and socially acknowledged as such" (David, Moreira & Tomaz, 2013, p. 57). Thus, it is clear that:

> [...] if teachers' mathematical knowledge matters in their teaching practice at school, it is necessary to start with the Mathematics that is demanded in classroom practice to reach the mathematical knowledge necessary for training, not to start with a type of Mathematics preestablished by the training process, expecting that, after teachers graduate, they "bring" it to school classrooms. In other words, it is necessary to know what teachers do and what difficulties they face in their practice in order to, based on that, structure training knowledge. Thus, we would be closer to training teachers who learn to exercise their teaching practice and also learn during the exercise of their practice (p. 57, our translation).

Based on a professional view of teacher education, the *Mathematics knowledge that is specific to teaching* entails knowing Mathematics for teaching, explaining it to and training students under different conditions and at different ages, in diversified contexts, in institutions that are different, too; such professional practice demands specific training, the type of knowledge that requires deep understanding of the essential concepts of Mathematics with flexibility and ability to employ them and establish relationships with them (Cristovão *et al*., 2023). We believe that all these elements are fundamental to Mathematics teacher training processes, so that it is possible to break with today's formative model.

SBEM, government departments, universities and society in general were introduced to this view of essential transformation of Mathematics teaching degree programs regarding the Mathematics knowledge that is studied in them by understanding the importance of articulating it with demands from professional life.

Then, as a summary, and in order to understand the Mathematics in teacher training, the team that conducted the research presented in this article comprehended and formulated their view of the *Mathematics knowledge that is specific to teaching.* The need for expanding studies was taken into consideration and, for the next three years, we aim to collect and systematize Mathematics Education research works conducted since 2006. Our goal is to collect the theoretical knowledge already produced aiming at the formulation of teaching materials for a Mathematics teaching degree program based on it.

The ideas presented in this text are related to (an essential) part of Mathematics teachers' initial education, but we do not ignore the need for rethinking other *knowledge specific to teaching* such as Education theories and supervised teaching practice. However, we emphasize that our studies were focused on the Mathematics studied in Mathematics teaching degree programs because we understand it is crucial to professional practice.

Thus, the view presented here generated outcomes to be harnessed in the formulation of formative paths in Mathematics teaching degree program curricula, which will be discussed next.

3 Some proposals in the perspective to advance in Mathematics teaching degree programs

Besides understanding the specificities of teacher education, we know that the fields of Education and Mathematics Education have conducted crucial research and produced formulations that could compose a teacher training project that takes *Mathematics knowledge specific to teaching* as its central part, aiming at an integrated formative path. As bases for reflection and for proposing changes in this sense, we will point out aspects that could be considered essential to the formulation of a curriculum for Mathematics teaching degree programs.

Firstly, we should comprehend training from the perspective of teaching, that is, professionals who will become a Mathematics teacher after graduating, who see themselves as Basic Education educators, who have deep knowledge of Mathematics to teach and see school as a space for human development.

The basis for the view that is intended for a teaching project is the permanent articulation that Mathematics teaching degree programs should have with competent professional practice. In order for that to occur, there are several ways, but the essential aspect, what differentiates this proposal from the current projects, is the understanding of *Mathematics knowledge specific to teaching*, which should be central to initial education.

Mathematics knowledge aimed at professional practice as *Mathematics knowledge specific to teaching,* which should be studied in Mathematics teaching degree programs, is inseparable from other forms of knowledge that compose training; Mathematics teachers who learn in order to explain, to teach under different conditions and within diversified contexts by using proper methodologies, dialogical relationships, with a specific look at learning difficulties, and assessing students constructively and continuously. It is also an interdisciplinary perspective because it encompasses the articulation of Mathematics with other fields of knowledge such as physics, geography and arts, and it is transdisciplinary because it also involves studying social themes that are part of daily life.

A proposal for the development of the aforementioned perspective should take into consideration: (i) studying the essential concepts of Mathematics in didactical activities/courses that present mathematical definitions with justifications, demonstrations, proofs and properties that are inherent to the field; (ii) perceiving the historical construction of knowledge and the relationships it has established with social practices; (iii) specifying students' learning difficulties that have already been identified by a set of research studies in Mathematics Education, discussing and developing teaching possibilities with the use of diverse didactical resources; (iv) analyzing books and teaching materials aiming to comprehend the proposed approaches and their possible uses; and (v) understanding learning assessment in a continuous way that aids students in their difficulties.

An aspect to be considered is that, in the transition from Middle School to High School, we observed that mathematics content approaches should be presented in progressive organization and complexity, and notation should become gradually more formal, in a way that allows progressive growth from childhood to youth. For Mathematics teaching, this understanding is important because it points out the need for a continuous construction of concepts and of the notation that is particular to Mathematics, going from simple to more organized and formal.

For example, in the study of numbers, there is the construction of the concept of number in kindergarten and its gradual transition to the set of Real numbers in Middle School, then to Complex numbers in High School. The same holds true for Algebra and Geometry, with the introduction of algebraic and geometric thinking, so that their gradual construction over the years allows an understanding of equations, functions, space and shapes.

Moreover, the study of contents should occur in a way that students are able to understand them through contextualization, according to the environment and the social moment that is being experienced and in relation to other fields and themes present in society. Thus, we recommend that school Mathematics should be the starting point of every course in a Mathematics teaching degree program, with a focus on demands from teaching practice.

From this view of teaching and training, contents and teaching methods are inseparable from technologies, that is, the content to be taught should not be separated from how it is taught. In this sense, it is recommended that every course/subject and activity should harness a set of technologies that foster observation, manipulation, visualization, perception and sensitization based on dialogical relations. This is how we aim to carry out future teachers' learning so that it is put into practice in schools, too. Also, it is important to highlight digital technologies such as software and online searching.

Therefore, according to Benedito and Ferreira (2022, p. 18-19), the use of technologies in teacher training and practice, especially in Mathematics, should "not be based only on mastering and handling technological devices, but also on the awareness of the importance of using technology as a teaching methodology". Silva and Santos (2023, our translation) highlight mobile learning as an option to be used with students in the classroom.

Secondly, we understand that a curricular project should be formalized within a text discussed and agreed upon by the teams that developed it, but it needs to have flexibility in a way that it does not contain only preestablished indispensable and fixed contents, but there should also be contents and methodologies that could be modified according to each student group each year, that is, according to the educational moment. In other words, curriculum is understood as a formative path that can be continuously adapted and improved based on the subjects that are involved and on existing conditions, from a cultural and social perspective that sets students' knowledge in the action.

Consequently, the development of a degree program pedagogical project should consider that all professionals involved in it constitute a collaborative team that establishes a continuous relationship with the planning and development of courses/subjects and didactical activities — under the supervision of the degree program coordinating board, of Núcleo Docente Estruturante (NDE) or any other appointed commission — and who periodically carries out the necessary planning and replanning.

Thirdly, it is necessary to consider that even though students starting a Mathematics teaching degree program did not usually obtain the desirable mathematical knowledge in Basic Education, they have interest in teaching and in the field. There are several reasons behind the difficulties that originate in Basic Education, but it is important to emphasize that they are strengthened by the current Mathematics teaching degree program model. Thus, a curricular project for Mathematics teaching degree programs needs to address these demands and point out ways to overcome them.

Therefore, it is essential to prepare students' introduction to a Mathematics teaching degree program. Starting college is a very special moment usually filled with joy and apprehension. Several student reports show they feel scared, especially due to the autonomy they start to have in college in comparison with the experience they previously had in Basic Education. It is very important for them to understand the considerable autonomy they have in Higher Education so that they become responsible for their formative actions. And it is extremely necessary to present a proposal that enables them to obtain information and that presents the possibilities that both the degree program and the university/institution have to offer. Besides, it is necessary to understand that their transition demands dialogue and guidance.

According to this proposal for Mathematics teaching degree programs, the coordinating team should consider students' difficulties with Mathematics and their introduction to Higher Education Mathematics. The team needs to work from a perspective to expand and give new meaning to the concepts they constructed in High School, aiming to comprehend the meaning students understand when they arrive, to carry out the study and the debate in order to expand their comprehension.

It is vital for a degree program team to understand the significant gaps and difficulties related to the mathematical knowledge that students bring to Mathematics teaching degree programs and to develop strategies to boost the construction of knowledge focused on learning for teaching. It means breaking with revision course models based on exercise lists and tests. Instead, these spaces should be used as opportunities to learn and to learn how to teach by experiencing classes, activities and tasks that foster the comprehension of the topics being studied, in a pleasant and encouraging environment in which students are protagonists, not merely passive spectators.

The fourth aspect to be presented here is that the development of the proposal should also consider a close relationship with Basic Education school teams linked to universities or federal institutions. In other words, we understand that training teachers for Basic Education is a process that requires articulation between professors from the Higher Education institutions responsible for the Mathematics teaching degree programs and teachers who work in Basic Education schools.

Thus, it is essential to propose and establish effective, professional and well-financed ways of articulating Mathematics teaching degree programs with schools' experienced teachers who are willing to participate, which could foster initial teaching practice. Their participation also means in-service training for all of those involved and possibilities to address issues related to school practice since university is a study and research center. This articulation refers to the understanding of (school-university) coeducation, and this bond should be institutionally strengthened and professionalized as an integral part of the project.

Finally, we explain that the view we present here does not ignore a whole set of complex situations that reach all teaching degree programs in Brazil: (i) there is a severe lack of encouragement towards the teaching profession due to the current working conditions in schools (especially low salaries); (ii) there are enormous challenges related to social inequality in Basic Education that became widespread in the past few decades; (iii) there is a history of low prestige in teaching degree programs. Such discredit is caused by the constant comparison with the bachelor's degree, which is usually prioritized by professors themselves. It is known that there are complicating factors in institutions where teaching degree programs share courses/subjects with other degree programs. Even so, we insist that training should be focused on teacher education.

4 Political-pedagogical references for teacher education

Some political-pedagogical references need to be clarified in the construction of this view of Mathematics teaching degree programs based on professional practice and following the principles of Education and Mathematics Education. Here, they were just systematized to corroborate the theoretical perspective we adopted.

1. A project that aims to break with a theoretical perspective of technical rationality to build a perspective of critical rationality (Diniz-Pereira, 2014).
2. The perception of basic school as a social project so that everyone could exercise their

right to Education and development in contact with the socially accumulated knowledge, in the socialization with their peers, in terms of educational age (child, teen, young person, adult), through the access to cultural goods and to creativity and inventiveness.

3. School as a sociocultural space for diversity and inclusion, for diversities to coexist and for the formulation of knowledge.
4. Initial teacher education that takes the teaching profession, professionalization and professionalism as references rather than being focused on the acquisition of academic Mathematics isolated from a teaching professional perspective.
5. Mathematics as socially and historically constructed knowledge, so that different types of Mathematics could be considered; Ethnomathematics as a research program that acknowledges different ways of mathematizing and problematizing the specific nature of mathematical knowledge.
6. Learning as a complex process, considering students' conditions in articulation with their development as critical subjects in dialogical, respectful and ethical relationships.
7. Teacher training as a continuous process that should be continued throughout the profession, thus fostering permanent reflection on practice and pertinent studies.
8. Teachers as sociocultural subjects that have rights and duties, who fight for dignified conditions for their work, personal and professional development.
9. Teaching practice as a daily, complex and transformative socio-political-cultural practice, and Basic School teachers as coeducators of future teachers.
10. Training from an investigative, interdisciplinary and transdisciplinary perspective.
11. Centrality of participatory and dialogical teaching and assessment methodologies.
12. Education and Mathematics Education as fields of teacher education and research on the multiple aspects related to teaching practice, on the analysis of practice and on the preparation of Mathematics educators.

5 The importance of carrying out experimental projects in Mathematics teaching degree programs

At the end of the study, when we analyzed understandings of specificities of teaching practice and the mathematics teaching knowledge that is specific to practice, we asked: "are there prepared teams to develop a proposal of such nature?"

We know that innovations require planning, practice and analyses of practice. Transformations in social practices, including pedagogical practices in Basic and Higher Education, require knowledge, time and experimentation because in the preview of what we aim to achieve, there is nothing ready, which requires that both practice and studying take place simultaneously. Therefore, we propose that the understandings presented here – results from several mathematics educators' research and studies – should be used as a basis for new initial and continuing education projects, which could be monitored, analyzed and even more researched.

We believe that SBEM – an institution that gathers mathematics educators who have been conducting research and studies that, among their objectives, highlight teaching education – is coherently a space to propose our view. There has been considerable criticism to Mathematics teaching degree programs in studies on Education and Mathematics Education,

there are several propositions and studies that are the results of practice and experimentation that have been carried out, theories that have been studied and/or are being formulated, all of which show paths that are still unknown and ignored by universities, institutions and authorities.

Based on all of the aforementioned aspects, we assume it is necessary to dare, to propose new initial education practices that address the existing propositions and the criticism in order to break with the fragmented teaching education that does not clearly focus on professional practice. The current Mathematics teaching degree program project is confusing. According to a student who is going to graduate soon: "The Mathematics teaching degree program doesn't make you a bachelor or a teacher." We want to construct a type of Mathematics teaching degree program in which teachers can be prepared for teaching and, to do so, it is necessary to carry out studies, research and practical proposals in line with the demands and challenges posed by the profession, which are specific to pedagogical teaching practice.

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