Sociedade Brasileira de Educação Matemática

RIPEM

Non-Equiprobable Sample Space: is it possible in childhood?

Sezilia Olmo de Toledo
Secretaria Municipal de Educação de Campinas
Campinas, SP — Brasil
professorasezilia@gmail.com
0000-0002-1340-7382

Márcio Eugen Klingenschmid Lopes dos Santos
Universidade Cruzeiro do Sul
São Paulo, SP — Brasil
marcioeugen@gmail.com
0000-0002-9812-5981

2238-0345 ISSN

10.37001/ripem.v15i1.3842 doi

Recebido • 15/03/2024
Aprovado • 29/05/2024
Publicado • 04/04/2025

Editor • Gilberto Januario

Abstract: This article presents an excerpt from a doctoral research project that aims to verify children's ideas about the probability of a certain event occurring during two draws, the first with replacement and the second without replacement, bringing to light probabilistic thinking expressed verbally and through written records. Data collection was carried out with 15 children from a 3rd grade class in the early years of primary school at a municipal public school in the city of Campinas, using video recording and a written record. During the activity, the children were able to recognize which event is most likely to occur and were able to work with change and the construction of new sample spaces. The results show the importance of exposing children to probabilistic ideas from an early age, because when they deal with these ideas, they reflect and build knowledge.

Keywords: Probabilistic Literacy. Childhood. Statistical Education. Early Elementary School. Probability.

Espacio muestral no equiprobable: ¿es posible en la infancia?

Resumen: Este artículo presenta un extracto de una investigación doctoral que tiene como objetivo verificar las ideas de los niños sobre la probabilidad de que un determinado evento ocurra con ocasión de dos sorteos, el primero con reposición y el segundo sin reposición, sacando a la luz el pensamiento probabilístico expresado de forma verbal y escrita. registros. La recolección de datos se realizó con 15 niños de una clase de 3er año de los primeros años de la Enseñanza Primaria de una escuela pública municipal de la ciudad de Campinas, mediante grabación de video y registros escritos. Durante la actividad, los niños demostraron que reconocían qué evento es más probable que ocurra y pudieron operar con el cambio y construcción de nuevos espacios muestrales. Los resultados indican la importancia de acercar a los niños a las ideas probabilísticas desde edades tempranas, ya que al abordar estas ideas reflexionan y construyen conocimientos.

Palabras clave: Alfabetización Probabilística. Infancia. Educación Estadística. Primeros Años de la Escuela Primaria. Probabilidad.

Espaço amostral não equiprovável: é possível na infância?

Resumo: Este artigo apresenta um recorte de uma pesquisa de doutorado que objetiva verificar as ideias das crianças a respeito da chance de ocorrência de um determinado evento durante a ocasião de dois sorteios, o primeiro com reposição e o segundo sem reposição, trazendo à tona o pensamento probabilístico expresso verbalmente e através do registro escrito. A coleta de dados foi realizada com 15 crianças de uma turma de 3.º ano dos anos iniciais do Ensino Fundamental em uma escola pública municipal da cidade de Campinas com o uso de vide

gravação e registro escrito. Durante a atividade, as crianças demonstraram reconhecer qual evento é mais provável de ocorrer e se mostraram capazes de operar com a mudança e a construção de novos espaços amostrais. Os resultados indicam a importância de aproximá-las das ideias probabilísticas desde cedo, pois, ao lidarem com estas ideias, refletem e constroem conhecimentos.

Palavras-chave: Letramento Probabilístico. Infância. Educação Estatística. Anos Iniciais do Ensino Fundamental. Probabilidade.

1 Introduction[1]

Probability is a part of people's lives and is present in their daily lives, for example, estimating the likelihood of certain events occurring during the day, inferring the likelihood of rain, finding a parking space near the entrance to the market, finding all the traffic lights open when you're running late on your commute, or even winning the lottery. These and other questions involve probability and are commonplace in people's daily lives, even if they are not aware of it.

Providing children and young people with a statistics education that discusses ideas of probability means broadening their knowledge to exercise critical citizenship. For a citizen to understand clearly the world in which he or she lives, he or she must master the reasoning involved in this content.

Although probabilistic ideas are present in people's daily lives, Frei, Rosa and Biazi (2023) found that the thematic units of statistics and probability are the ones that teachers find the most difficult to teach, a fact justified by the insufficient time allocated to them in their academic training, which does not provide adequate preparation for their teaching.

According to Santos, Kalhil and Ghedin (2015), in pedagogy courses, the time devoted to training and the mathematical content dealt with in the early years emphasizes methodological proposals and teaching methods, leaving aside the foundations of mathematics as a science and a school subject. However, statistics content needs to be learned during initial training, as well as the cognitive and didactic processes involved in the teaching-learning process.

The study by Souza, Mendonça and Lopes (2013) points to the absence of pedagogy courses and reports that many teachers are uncertain when asked about their preparation to teach statistics. In addition, the researchers point out that the initial training of teachers influences the way they work, as they are unlikely to build knowledge with their students differently than what they received, which makes us think about the relevance of continuing education for this profession.

Oliveira and Lopes (2013) validated the need to provide moments of reflection on mathematics teaching in training spaces, as teachers only teach what they know. When they feel confident about the relationship between the theory and practice of the content they want to teach, they discuss, analyze, and plan situations for the classroom.

The National Common Core Curriculum [BNCC] (Brazil, 2018), the current guiding document, provides a thematic axis with objects of knowledge and skills for working with the topics from the first year, emphasizing the importance of studying statistics and probability from the beginning of primary school.

[1] This article is part of a doctoral thesis currently being completed by the Postgraduate Program in Science and Mathematics Teaching at Cruzeiro do Sul University, written by the first author and supervised by the second author.

When we propose activities involving probability, we expect children to express some ideas about it, using their everyday experiences to make more confident choices and judgments, which brings us back to the concept of literacy. Gal (2002) explains that the term literacy refers to the mastery of basic reading and writing skills linked to people's experiences in society. Literacy has in its essence elements of criticality, argumentation, and scientific speculation, and subjects use mathematical, statistical, and scientific elements to make informed decisions in relation to the phenomena they are asked to analyze.

Gal (2005) advocates the introduction of probability from the beginning of schooling since its ideas are part of people's daily lives, such as the chance or predictability of an event occurring, the uncertainty present in its results, the calculation or communication of probabilities, and randomness, especially when they require decision-making, a skill expected of competent and knowledgeable citizens.

The research began in 2022 and ended in 2023, with the children in the 3rd grade. The children participating in the study went through a period of the COVID-19 pandemic when they were in school in 2021 in rotation, affecting part of the 1st grade. This scenario may have influenced the data collected, since the children were used to doing all the activities of the day alone or with distancing measures.

Considering this, the present study aims to verify the children's ideas about the probability of a certain event occurring during two draws, the first with substitution and the second without substitution, bringing out the probabilistic thinking expressed verbally and through the written record. Data were collected through video recording, written activity, and a final record through drawing and writing. The activity was carried out with a class of 15 children in the 3rd year of primary school at a municipal public school in the city of Campinas, all of whom had signed a free and informed consent form for the use of their productions, dialogues, videos and images for research purposes.

This was the last activity developed with the class. They had already participated in four activities in grade 2 and three in grade 3, and this was the last one. Therefore, with this activity, we were looking for an answer to the following question: in the continuous work with probability, do the children acquire their language and use it appropriately in different situations?

Intending to contextualize the activities carried out at the time of the research, we chose a text narrated in the 1st person singular, because the activities were carried out by the 1st author of this article, who was not the class teacher and did not work at the school.

2 Theoretical support

Gal (2005) argues that the acquisition of a linguistic repertoire related to probability contributes to the formation of probabilistically literate citizens. In education, two basic reasons for learning probability are emphasized: first, because it is part of mathematics and statistics, and second, because it is seen as fundamental to preparing students to understand the random events that pervade people's daily lives.

Probabilistic literacy should be introduced from the very beginning of schooling because concepts of probability that involve randomness, phenomena with the idea of measuring chance, predictability of an event's occurrence, uncertainty, and calculating or communicating probabilities are commonplace in people's lives, especially when they require decision-making (Gal, 2005).

The author considers *probabilistic literacy* in terms of the elements of knowledge and dispositions. The elements of knowledge relate to big ideas, probability calculations, the language used for probabilistic communication, the different contexts in which probability is used, and critical questions about the concept. The disposition elements relate to critical attitudes, beliefs, attitudes, and personal feelings about uncertainty and risk. They are linked to context and individual reality, as decisions are influenced by the environments and contexts to which people are exposed.

When we think about risk and uncertainty, we are confronted with the difficulty that people have when faced with decision-making situations, as they are sometimes unable to assess the consequences of each choice.

Borovcnik (2016) argues that probabilistic literacy is related to risk, defining it as a situation of inherent uncertainty about outcomes associated with an impact, and that people need to be able to use relevant concepts and methods in the context of everyday problems to make more confident decisions in their lives.

Since the consequences of decisions directly affect people's lives, having a knowledge of probability allows them to recognize the different effects of each decision, to make better decisions in their lives, to expose themselves to less risk, and to make better choices considering the context and everyday problems.

Jones, Langrall, Thornton, and Mogill (1997) also state that probability is part of everyday life because children participate in situations in which probabilistic thinking is evoked and, without having systematized knowledge of calculation or proportion, they make choices and justify them based on their everyday experiences. In this way, prior knowledge must be considered, as it is the basis for future constructions, and school is the space where they are challenged to interact with their peers, argue their conceptions, listen and (re)construct knowledge.

In this context, Jones *et al*. (1997) propose four constructs for dealing with probability. First, sample space is considered, defined as the ability to identify the full set of outcomes in a one-step experiment (e.g., tossing a coin) or a two-step experiment (e.g., tossing two coins).

Probability is defined as the ability to identify and justify which of two or three events is more or less likely to occur. Typically, probability tasks studied with children involve equally probable outcomes, but this was not the case in Jones *et al*.'s (1997) study, which considered non-equiprobable sample spaces.

When comparing probabilities, children's understanding is measured by their ability to determine and justify: (a) which probability situation is more likely to produce the target event in a random draw; or (b) whether two probability situations offer the same chance of producing the target event.

Finally, the conditional probability structure is measured by the ability to recognize when the probability of an event is or is not changed by the occurrence of another event, i.e., it is the most sophisticated structure within probability knowledge.

The four proposed constructions are similar to what we see in the classroom, and the use of these justifications in the face of children's arguments is appropriate. When we want to observe probabilistic reasoning, the sample space of an event must be considered to determine the probability of occurrence and its comparison in order to make assertive choices. Sometimes the probability of an event is changed in light of a previous event, so children need to be exposed

to, for example, drawing lots without replacement so that they can perceive the change in probability.

In this learning scenario, the role of the teacher is fundamental. Your lesson needs to be planned and objectives outlined; you need to know and feel confident about what you are going to teach; you need to give children opportunities to express themselves; and you need to mediate learning by asking questions to check for possible misunderstandings. Your questions should be formulated in a way that helps the students to think, to think and to build knowledge.

According to Lopes, Grando and D'Ambrosio (2017), when teachers discuss numerous situations and create an investigative scenario characterized by time, space and manipulable materials, they consequently encourage their students to construct their own knowledge.

In this sense, D'Ambrosio (2013) advocates hermeneutic listening, which is understood as the teacher listening to the student to believe that they themselves can learn something new and add new perspectives to their knowledge. This kind of listening encourages and values students' efforts in their constructions.

The more opportunities our students have to be exposed to situations involving probability, the more they will use its language to measure chance and the likelihood that an event will occur. A probabilistically literate person can make decisions based on probability concepts and is able to make better choices in life.

3 Development and analysis

I was at the school for the 3rd time in 2023. I introduced the lesson by making it clear that there was no right or wrong in the activities, but rather the path they had taken to arrive at the answers. I began by reading the problem The Box of Chocolates (Diniz & Carvalho, 2021), a problem that deals with a draw with replacement and then without replacement, to introduce and provoke discussion before the dynamics that would take place.

I read the problem aloud and gave the children only the picture, printed in black and white. I asked them to look at it and tell me how many candies of each were in the picture (brigadeiro and beijinho), pointing out that one of the shapes in the box was empty. I asked them if there was a difference between taking the candy out, looking at it, and giving it back, or taking the candy out and eating it immediately. They realized that there was, and said that they could take the same candy out more than once if it was returned.

Then, starting another activity, I gave each child a blank sheet, folded in half, and asked them to write their name on it and use only one part to make the record. I presented an opaque bag with colored stars inside. I agreed with the class that I would take a star out, show it to them, they would write it down, and I would put it back in the bag and draw the next one. This went on until the 10th drawing.

As soon as I started, one student asked what color stars were in the bag. I told her that she would find out during the drawing, and she also told me that I could pick the same star more than once, which was noticed by other children.

When the drawing was over, I handed out an activity sheet and asked them to put the sheet aside and begin Activity 1, which asked them to paint which stars they thought were in the bag. Only three children painted with colors that hadn't been drawn. Below, in Chart 1, is the discussion of Question 1, which was videotaped.

Chart 1: Transcript of the oral discussion - Drawing of stars - Question 1

Q*: Emanuel, you think there are 4 roses and 6 yellows, why?* Emanuel*: Because there were twice as many, no, not twice... I think it was twice as many as the pink, so I put 4 roses and 6 yellows.* Miguel*: I put 7 yellows because there are many yellows, and 3 pinks because I don't think there's more than 3.* Q*: Guys, I saw that you used numerous colors. Tell me about that.* Victor*: I just did.* Q*: Do you think there are all these colors in the bag? (nods). What about you, Luís?* *Luís: You just gave me the idea that there are these colors.* Q*: What about you, Angelo?* Angelo*: Also.* Heloá*: Pink was the one that I liked the least.* Q*: So you think there are 3 pinks and 7 yellows? [I was visualizing the painting of the stars in the child's activity].* Heloá*: Yes.* Pietro*: Because the pink appeared a few times and the yellow more often.* Q*: So you think there are 4 pink and 6 yellow? (nods) [I asked the question based on the visualization of the star picture in the children's activity].* Emilly*: I think there are 5 yellows and 5 pinks because 5 plus 5 is 10.* Maria*: Same possibility, but it's a matter of luck.* Isabela*: Because if Sezilia took them out and put them in, there would only be two stars in each bag (referring to the pink and the yellow). Because if you took them out and put them in...* Q*: Is that possible?* Isabella*: I think it's possible. But the bag wasn't very full.* Maria*: The bag was too full to have two stars.*

Source: Prepared by the authors

The children acknowledged that there were ten stars in the bag for the drawing, but Isabela said that there were only two stars, one pink and one yellow, since they were the only colors drawn, which shows us how important the manipulatives and the visualization of the stars were for her.

We can also see the importance of the interaction between the children because at the end of the conversation, Maria showed that she was paying attention to Isabela's conclusions and then debated, referring to the appearance of the bag.

Some students said that there would be different colors of stars from the ten drawn, believing that there would be other colors hidden in the bag. Some children justified the choice of more yellow stars by saying that since they had been drawn more often, there would probably be more of them in the bag.

Emanuel, Miguel, and Pietro justified choosing more yellow stars because they had been drawn more often, demonstrating perceptual and quantitative judgment at a transitional level (Jones *et al*., 1997). Chart 2 shows the tabulation of the written record of Activity 1.

Chart 2: Written records, Star draw - Question 1

Question 1: THERE ARE 10 STARS IN THE BAG. 10 STARS WERE DRAWN. AND THEN RETURNED. WHAT COLORS OF STARS DO YOU THINK ARE IN THE BAG? PAINT.

Which stars	Number of children
3 pink stars and 7 yellow stars	1
4 pink stars and 6 yellow stars	3
5 pink stars and 5 yellow stars	8
4 yellow stars, 4 blue stars and 2 pink stars	1
2 yellow, 2 pink, 2 green, 2 black, and 2 purple	1
2 yellow, 2 pink, 2 black, 2 orange, 1 red and 1 blue	1

Source: Prepared by the authors

About half of the children believed that there were an equal number of yellow and pink stars. All of the children indicated in their records that there were yellow and pink stars, and three children indicated that there were other colors of stars.

According to Jones *et al*. (1997), there is a greater emphasis in the classroom on activities related to equiprobable events. Perhaps for this reason, most of the children ended up saying that both colors had an equal chance and attributed this fact to luck, as Maria did in the oral discussion.

After answering the first question, I asked them to take back the sheet of paper on which they had written down the colors they had drawn in the first moment and instructed them to use the blank part to write down the second stage of the activity. I drew lots again, but this time every star drawn would not be returned. I took them out one by one and asked them to record the drawing. After removing six stars and leaving them exposed, I asked them to return to the activity sheet and draw in Question 2 which stars they thought were in the bag before the draw. Chart 3 shows the transcript of the drawing of the stars without replacement.

Chart 3: Transcript of the oral discussion - Drawing stars

Q*: Now I'm going to do another drawing; every little star I take off, I'm not going to put back on. It will be left out. I'm going to draw 6 this time. How many are there?*
Students*: 10.*
Q*: So how many will be hidden?*
Student*: 4.*
Q*: You'll know what 6 are. Then I'll take them out and leave them out. Is it different from the first time?*
Student*: Yes.*
Q*: Why?*
Heloá*: Because before you showed it and put it back, so it was a little different.*
Miguel*: Each time it passes, the chance is less.*
Q*: And what's that like? What happens?*
Miguel: If I go there and I take 3 yellows and 4 pinks, there's less chance of a pink and more chance of a yellow.
Q*: They are out, right? (nods)*
Maria*: Because now you're not giving them back, there's a little less chance of each color falling out.*

Q*: Why?*
Maria*: Because now you're not giving them back, so they're out.*
Helloisa*: I think it's different because if you take it and put it in, it's one thing. Now if you take it and put it out, it's something else.*
Q: *And what's different about taking it out?*
Helloisa*: Because if you put it inside, no one will see it, and if you put it outside, they will see it.*
Q*: Can I take it out again?*
Student*: No.*
Helloisa*: Because it's like a game, if you lose, you're out.*

Source: Prepared by the authors

After the interview, the children showed that they understood that the first draw was different. In this second draw, the removed star could not be drawn again because it would be exposed outside the bag. In addition, the children recognized that not putting the star back in the bag would change the probability that each color of star would be drawn.

According to Jones *et al*. (1997), conditional probability is the most sophisticated structure because it tests the level at which children recognize the change in probability without substituting items. Based on the dialog, the children were able to express the change in the measure of probability in situations where there was no substitution.

The language used by the children brings us back to Gal's (2005) studies, as they are able to communicate their ideas about chance using everyday expressions that refer to the chance of a certain event occurring. Figure 4 shows the transcript of the dialog on question 2:

Chart 4: Transcript of the oral discussion - Drawing of stars - Question 2

Helloisa*: 3.*
Isabella*: And pink too 3.*
Q*: How many will be left to paint?*
Heloisa*: 4.*
Q*: And for the 4 you need to tell me what color you think they are. They're hidden in the bag... You're going to paint them, OK?*
Maria*: I have to paint the ones in the bag, okay?*
P*: Guys, you have to paint everything. There are 10 little stars drawn; you're going to paint what's outside plus what's inside.*
[...] Pause - children painting the stars.
P*: Come on, Maria! Why did you paint them like that?*
Maria*: Because if before I thought there were 5 yellow stars and 5 pink stars, then there were 3 yellow stars and 3 pink stars, so there are 4 more, which are 2 pink stars and 2 yellow stars.*
Q*: What about you, Heloisa? I noticed that you separated outside and inside. Explain how you painted.*
Heloisa*: I did practically the same thing as her. There are already three here. And if there are 5 yellows and 5 pinks, that's 10. There have to be 3 here and two here. That was my tactic.*
Q: *Pietro, explain to me why you think there are these colors inside.*
Pietro*: I don't know.*
Q*: And you, Emanuel, why did you paint it that way?*
Emanuel*: Because I added one more to the pink that came from here and three, which was the amount of yellow. Because this one and this one are practically the same amount (compare questions 1 and 2).*
Q*: I saw that you had painted other colors before, now you haven't. Why did you do it this way? Why did you do it this way?*
Eduardo*: I don't know either.*
Q*: Luís, do you think there will be a blue star?*

Luiz*: I think so.* Q: *So you think that there are 3 yellow stars and 3 pink stars that came out and that 4 blue stars will appear?Do you think everything inside will be blue?* Luiz*: I think so.*

Source: Prepared by the authors

At first the children didn't understand what they had to do in the activity, but with a more detailed explanation they understood the task.

The justifications for painting the stars in the bag were based on the ones that had been drawn, as they believed that the one that had been drawn the most was also the most numerous before the drawing. Only one student believed that the remaining four stars were a different color than the ones drawn.

Heloisa, who initially asked several questions to understand the activity, made an interesting recording as she separated the stars that had been drawn from those that were still in the bag. The teacher-researcher's dialogue with the students was important, as she asked questions that stimulated the children to understand the proposal and to think about their answers, thus building knowledge.

In this question, the children were asked to estimate which colors of stars could be in the bag based on the initial drawing of six stars. Gal (2005), when dealing with the elements of knowledge in probability, lists as one of them the calculation of probabilities as a way of calculating or estimating the probability of events occurring to understand the phenomenon analyzed. It points out that probability is estimated or judged and that different sources are used, including non-probabilistic information. The children demonstrate mental operations and find justifications for their estimates. Figure 5 shows the tabulation of the children's written records for question 2.

Chart 5: Written records, Star draw - Question 2

Question 2: USING THE SAME BAG, 6 STARS WERE DRAWED OUT OF THE BAG. WHICH COLORS DO YOU THINK MAKE UP THE SET OF STARS IN THE BAG? PAINT.

What colors of stars	Number of children
4 pink stars and 6 yellow stars	5
5 pink stars and 5 yellow stars	9
3 pink stars, 3 yellow stars and 4 blue stars	1

Source: Prepared by the authors.

More than half of the children believed that there were the same number of stars of both colors. Only five got the number of yellow and pink stars right.

According to the BNCC (Brazil, 2018), the estimation of equally likely events should be worked on in 5th grade, starting with the enumeration of the sample space. This activity revealed that the children had already developed hypotheses in situations where the chance of

a certain event happening could be equiprobable, and when they were challenged with questions that made sense to them, they were able to build knowledge beyond what was expected for their stage of schooling. A remarkable situation. Chart 6 illustrates the transcript of the dialog after the draw.

Chart 6: Transcript of the oral discussion - Drawing of stars - Question 3

Q*: You did it. Heloisa?* Heloisa*: No, because I put 5 yellow ones and 5 pink ones. (made a disappointed expression)* Q*: And you, Pietro, did you get it?* Pietro*: No! Because I did everything wrong.* Q*: And you, Milena, did you do it?* *She nodded (with a happy expression)* Q*: Why did you choose that?* Student*: Because she wanted me to.* Q*: Say it, Milena!* Milena*: Because, yes!* Emanuel*: I got 4 roses and 6 yellows right in the 2.* Q*: And why do you think you got it right?* Emanuel*: Well, it fell there because it's right.*

Source: Prepared by the authors

I think it's important to refer to Pietro's answer: "Because I did everything wrong". In this activity, it was necessary to estimate and justify, without being bound by right and wrong. In this statement, it's clear how present the need to get the question right, presenting expected answers, is in the classroom. This was confirmed by Heloísa's expression of disappointment and Milena's happiness, even though she couldn't justify the reason for her choice of colors.

Chart 7: Written records, Star draw - Question 3

Question 3: DID YOU FIND WHICH COLORS OF STARS WERE IN THE BAG? EXPLAIN.		
	Number of children	Justification
No	1	Because I put it in a different sequence
	5	Because I put 5 roses and 5 yellows
	1	Because I didn't think before I did it
	1	Because I put 4 blue, 3 pink and 3 yellow
	2	No justification
Yes	1	Because there are 3 yellows and 1 rose
	2	Because I put 6 yellow and 4 pink
	1	Because there are 4 pinks
	1	I got both questions right

Source: Prepared by the authors

Some of the children got the number of colors of the stars right, but the goal was to check their reasoning. We expected them to write down what they thought, but they surprised

us by writing down their estimates. One of the children wrote that she had put them in a different order, showing us that she knew how to use mathematical language better. Again, most of the children focused on the probable event, believing that there were half of each color of star. One of them thought that she didn't get it right because she didn't think before she did it. In the classroom, when a child doesn't come up with the correct answer, the teacher sometimes tells them to try again and think more, and we see that this behavior extends to questions that don't require a correct answer but an argued one.

For question 4 there was no dialog, only reading. The children made a written record, as shown in Figure 8.

Chart 8: Written records, Star draw - Question 4

Question 4: DRAW 10 STARS INTO THE BAG, THE COLORS OF WHICH ARE RED AND BLUE, SO THAT THE BLUE STAR HAS THE GREATER CHANCE OF BEING DRAWN.	
Stars drawn	Number of children
6 blue stars and 4 red stars	8
8 blue stars and 2 red stars	3
9 blue stars and 1 red star	1
10 blue stars and 10 red stars	2
More blue stars and 1 red star	1

Source: Prepared by the authors

All but two of the children were able to make a draw bag as requested. According to the BNCC (Brasil, 2018), by the third grade the children should have acquired the ability to estimate an event that has a higher probability of occurring. In the proposed question, the children were asked to make a new probability bag, different from the one they had worked with. Only two children were unable to make a bag in which one of the colors had a greater chance of being drawn, drawing an equiprobable event, as we saw happen several times during the activity.

After they finished answering the four questions, I handed out the second activity sheet. It was handed out only at this point because there was an illustration of the composition of the draw bag.

Chart 9: Transcript of the oral discussion - Drawing of stars - Question 5.A

Q*: Question 5, letter A: Which color star has the best chance of being drawn?* Students*: Yellow.* Q*: Why?* Students*: Because yellow has more.*

Source: Prepared by the authors.

The children were able to identify an event that was more likely to occur after checking the stars in the bag, a skill suggested by the BNCC (2018) for third grade.

The justification for the probability of an event is constructed in a quantitative way, according to Jones *et al*. (1997), as the children compared the number of yellow and pink stars and confirmed that there were more yellow stars. Figure 10 shows the written record of the question.

Chart 10: Written records Star draw - Question 5.A

5 - KNOWING THAT THE BAG HAS THE FOLLOWING COMPOSITION, ANSWER THE FOLLOWING QUESTIONS:

5.A - WHICH COLOR STAR HAS THE BEST CHANCE OF BEING DRAWN? JUSTIFY.

Color	Number of children	Justification
Yellow	7	Because you have more
	1	More likely to go out more often
	2	It has more chance, because it has more
	1	Because it's a lot
	1	Because there are 6
	1	There are more of them, but that doesn't mean the color pink has no chance of being drawn
	1	Because there are more of them
Pink	1	Because there are fewer of them

Source: Prepared by the authors

In the written records, we found that almost the entire class chose the yellow star as the one with the greatest chance of being drawn, justifying it by comparing quantities or even explaining the amount shown by the color yellow.

From the justifications, we can see that the children used expressions related to probability, making use of terms related to everyday life and those used in school. Gal (2005) argues that students need to understand how to communicate probability orally and express their feelings about it.

One of the students, even though she had chosen the yellow star, explained that the rose also had a chance of being drawn, showing that she understood the uncertainty present in a draw. In short, we can see the importance of working with probability and the vocabulary used with children of this age group.

The ability to predict events that are more likely to occur is provided for in the BNCC (Brazil, 2018) for 3rd grade, so we see that the children are able to estimate an event that is more likely to occur, comparing the probability of each one and presenting a justification based on quantitative judgment, according to Jones *et al*. (1997).

Only one student chose the color pink, justifying the fact that there was less of it.

Chart 11: Transcript of the oral discussion - Drawing of stars - Question 5.B

Q*: After taking 2 yellow stars out of the bag and not returning them,. If I take out 2 yellow stars, how many are left?* Pietro*: 4.* Q*: Four yellow and 4 pink. Which color has the greatest chance of being drawn?* Mary*: None!* Louis*: Because there's the same amount.* Michael*: Because there's a 50% chance of drawing yellow and a 50% chance of drawing pink.* Q*: Why 50%?* Q*: Because there's the same amount.*

Source: Prepared by the authors

In this dialogue, we see that the children showed that probability changes when the sample space changes, that they recognized an equally probable event, and that they assigned numerical probabilities according to Jones *et al*. (1997).

In the moments of dialogue, it is possible to perceive hermeneutic listening, since the children were asked to explain how they thought, to give justifications, to listen to their classmates and to have the opportunity to put themselves in front of others, not being judged, but being listened to in a respectful way. This type of attitude is defended by D'Ambrosio (2013) when he talks about the attitude of the teacher who listens to his students and reframes his own solutions based on what they say.

The ability to recognize equally probable outcomes is foreseen in the BNCC (2018) for the 5th grade, but during our study, we were able to verify that when children are faced with situations in which they can dialogue, reveal their way of thinking and manipulate materials, they are able to answer this type of question, even though they are still in the 3rd grade.

Chart 12: Written records Star draw - Question 5.B

5.B - AFTER TAKING TWO YELLOW STARS OUT OF THE BAG AND NOT RETURNING THEM, WHICH COLOR HAS THE HIGHEST CHANCE OF BEING DRAWN AT THIS MOMENT? JUSTIFY.		
Color	Number of children	Justification
Yellow	1	Because there are 4, the same number as the rose
	1	Because there are more
	1	Because there are two more yellow ones
None	5	Because there's the same amount
	2	Because 50% are yellow and pink
	1	50%, because there are 4 yellow and 4 pink
	1	50% and 50%, which means the same quantity

	1	There's 50% of one and 50% of the other, and 50+50=100
Both	1	Both can be removed
	1	Because the yellow one came out twice, and there may be more to come

Source: Prepared by the authors

In this question, the children were asked to understand the change in the probability of an event occurring as we moved from a 60% - 40% sample space to an equiprobable sample space. Again, a large part of the class recognized that the two colors of stars would have the same chance of being drawn, but with different justifications.

Although one of the children answered that the yellow one had a greater chance, he made it clear in his explanation that the two had the same amount.

The measure of half, 50%, was present among the justifications, remembering that this content is not worked on in a 3rd grade class, as they have not yet seen the content of fractions. It is possible that this vocabulary comes from everyday experience, which shows us how important it is to take children's prior knowledge into account. The use and attention given to everyday language to express probability is defended by Gal (2005), who believes that children are capable of communicating measures of chance even before they know the vocabulary related to probability.

Most children assign a numerical measure of probability by comparing the number of pink and yellow stars to identify an equally likely event. According to Jones *et al*. (1997), these children's justifications would be at a numerical level of reasoning related to comparing probabilities.

Chart 13: Transcript of the oral discussion - Drawing of stars - Question 5.C

Q*: What would you suggest doing so that the two colors of stars have the same chance of being drawn? How many yellow ones?*
Students*: 6*
Q*: And how many pink ones?*
Students*: 4.*
Q*: What can I do to make them both have the same chance?*
Students*: Draw 2 yellow ones.*
Q: *Take 2 yellow ones? What else can I do?*
Luis*: Take 1 off the yellow one and put one on the pink one.*
Q*: Take 1 off the yellow one and put one on the pink one. What about you, Heloisa?*
Heloisa*: Can you add them up?*
Q*: Yes!*
Heloisa*: Add two roses.*
Q*: Look how many possibilities you're talking about.*
Emanuel*: Take away two roses and take away 4 yellow ones.*
P*: That's another possibility. Write down how you do it so that you have the same chance of taking away the yellow and the pink.*

Source: Prepared by the authors

The children came up with different alternatives to make the drawing of the stars equiprobable, using different strategies, including addition and subtraction. One student talked about replacing a yellow star with a pink one so that both would have the same number.

According to Lopes, Grando, and D'Ambrosio (2017), when teachers discuss numerous situations and create an investigative scenario characterized by time, space, and manipulable materials, they consequently encourage their students to construct their own knowledge.

Jones *et al*. (1997) state that a child is able to determine changes in probability measures within conditional probability. This is what we saw in this dialogue, as they went further by changing the probability of drawing each color of the star with different strategies as they felt prompted by the questions.

BNCC (2018) talks about recognizing equally probable events, but not about constructing or changing equally probable sample spaces. Once again, we affirm that when children are offered meaningful situations in which they are challenged to propose alternative solutions, they can go beyond what is suggested in the curriculum.

This type of knowledge can be built when the teacher asks critical questions that challenge students to think about their statements and the information they provide to their peers. According to Gal (2005), critical questions help students reflect on their judgments when estimating probabilities.

Chart 14: Written records Star draw - Question 5.C

5.C - WHAT WOULD YOU SUGGEST DOING SO THAT THE TWO COLORS OF STARS HAVE THE SAME CHANCE OF BEING DRAWN? EXPLAIN.	
What to do	Number of children
Remove two yellow ones	6
Put a rose in place of the yellow	4
Remove 4 yellow and two pink	1
Remove yellow	1
Add two pinks	1
Count 1 by 1	1
Blank	1

Source: Prepared by the authors

In the written record, their ability to change the sample space was confirmed, as they were able to operate with the stars, changing the chance of them being drawn, making the sample space equiprobable. Only two children did not provide answers for this change, revealing that they need to experience more situations to acquire this type of knowledge.

Question 5.D was read out, and the children immediately responded by saying that there was no chance of drawing a red star, as this color didn't exist in the draw bag. Below is a tabulation of the written answers shown in Chart 15.

Chart 15: Written records Star draw - Question 5.D

5.D - WHAT ARE THE CHANCES OF ME DRAWING A RED STAR? WHY?		
What are the chances	Number of children	Justification

None	7	Because there's no red inside the bag
	3	There's only yellow and pink; there's no red
	1	There's no way to get red out
	1	There's no chance, because there's no red
	1	Red has no chance, because there's no red
1	1	There may be 1, but you didn't get it, but maybe
	1	Pink 40% and yellow 60%

Source: Prepared by the authors

The children were able to recognize an impossible event, justifying the non-existence of the red star or even listing those present in the draw bag.

The notion of chance, classifying an event as impossible, is a skill proposed by the BNCC (Brazil, 2018) for the first grade. According to Jones *et al*. (1997), even a child who presents subjective justifications (the most elementary argument) about the probability of an event occurring is already capable of classifying an event as impossible.

In the justifications, we also see that children use the language of probability, with the terms possibility and chance. One child did not refer to the red star as impossible to remove but gave a numerical justification, assigning a percentage to each of the colors of stars in the draw bag. According to Jones *et al*. (1997), this child was at a numerical level of reasoning, i.e., he was basing his justification on measures of quantity comparison.

We can see here that the more children are asked about probability, the more they offer answers using appropriate language, showing us the importance of providing tasks in which probabilistic language should be used, but also of questioning children and listening to and challenging their formulations, creating a space for inquiry in the classroom.

Chart 16: Transcript of the oral discussion - Drawing of stars - Question 5.E

Q*: What happens if I put five more pink stars in the bag? Does the chance of drawing the yellow one or the pink one remain the same?* Students*: Nooooo! Because there will be pinker.* Luís*: The pink one will get more.*

Source: Prepared by the authors

The children were able to see that by adding new stars to the bag, the chance of drawing a certain color would change, demonstrating that they were able to determine changes in probability measures, according to Jones *et al*. (1997).

The children became increasingly comfortable participating in the activities, putting themselves forward, making comments, listening and arguing about their classmates'

constructions. In this way, the development of an investigative environment in the classroom fostered the formation of critical citizens.

Chart 17: Written records Star draw - Question 5.E

5.E - WHAT HAPPENS IF I PUT 5 MORE PINK STARS IN THE BAG? DOES THE CHANCE OF DRAWING THE YELLOW ONE OR THE PINK ONE REMAIN THE SAME? EXPLAIN.		
The chance remains the same	Number of children	Justification
No	4	Because pink will have more
	1	Because it gets pinker, the rose will have more
	2	Because the rose has a better chance
	4	Because there will be 9 pinks and 6 yellows
	1	It will stay the same, because it has more pink and no more yellow
	1	There will be 15 stars
	2	No answer

Source: Prepared by the authors

In their written records, the children said that the probability would change with the addition of five pink stars, making it more likely to be drawn. Only one child responded by giving the number of stars in the bag and two left the activity blank.

We noticed that the children offered increasingly elaborate justifications based on comparing the number of yellow and pink stars, the final number of each in the drawing bag, or even saying which one would have the most, demonstrating a move toward numerical justification according to Jones *et al*. (1997).

It is noteworthy how much participation in the socialization of each question contributed to increasing the complexity of the children's arguments, as they became more elaborate with each discussion and/or recording.

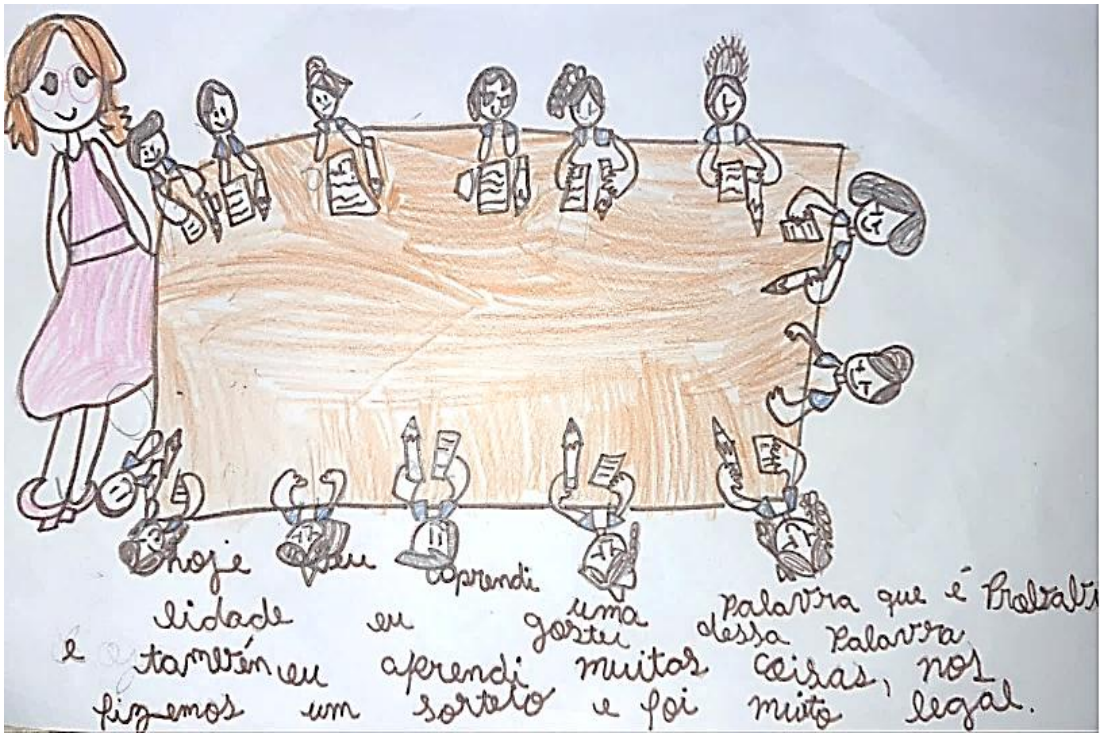
After this activity, the children were asked to make a representation of the activity on a sheet of paper. They were told that this could be in the form of a drawing and/or writing. Below, we will look at some of these recordings.

Figure 1 shows the moment when the activity was carried out, the organization of the room, and the student mentioning that he had learned the word probability. We can also see that the children are happy to take part in the activity.

Figure 2 shows that experiences outside the classroom are significant and blend in with the learning acquired. In his production, the student referred to the multiverse and defined the term probable, learned in class, as something that could happen.

Figure 3 shows the illustration of the bag with the stars and the sweets draw card from the story read to introduce the subject of randomness. The student's statement in his entry reminds us of the fact that, during a draw, you can't be sure what will come out.

Figure 1: The timing of the activity



Transcript: Today I learned a word called probability. I liked this word, and I also learned many things. We did a drawing, and it was very nice.

Source: Personal collection

Figure 2: The multiverse and probability

Transcript: I was dreaming about the multiverse and today I learned that something will probably happen.

Source: Personal collection.

Figure 3: What will happen?

Transcript: I've learned that I don't know what's coming without seeing it.

Source: Personal collection.

Figure 4 shows that, while I was explaining the image of the story being read, the student was apparently distant, thinking about what she had done when she left the house. The use of the word will refer to probability or doubt, as she wasn't sure if the door had been closed, causing her concern. At the same time, I noticed that she was paying attention to what was happening as she drew the explanation that was being given.

Figure 4: Is the door closed?

Transcript: - Raffle: 2 chocolate sweets and 3 coconut sweets, an empty mold
-Did I lock the door?

Source: Personal collection

Figure 5 shows the moment of the draw. The student has drawn the researcher with the draw bag in her hand, and the sentence shows us that she has understood that the word probability refers to the chance of something happening. The vast majority of children illustrated the moment of the draw in their records.

Figure 5: What is probability?

Transcript: - Let's draw stars.
Probability is the chance of something happening.

Source: Personal collection

4 Considerations

The article responded to the initial objectives proposed, verifying the children's ideas about the probability of a certain event occurring during two draws, the first with substitution and the second without substitution, bringing out the probabilistic thinking expressed verbally and through the written record, and also confirming that participation in such activities promotes the use and learning of probabilistic language.

During the activity, the children were able to identify an impossible event and a certain event, as well as the most likely and least likely events.

During the discussions, in which the researcher encouraged dialogue, the children expressed themselves and told us what they thought about each question. These collective discussions allowed the children to listen to each other, which promoted the construction of increasingly sophisticated individual and collective knowledge.

While the activities were being written down, the children could talk to each other, but with each explanation by the researcher, they remained silent in order to pay attention to every detail of what was being said and thus answer any questions they might have in order to resolve the problem.

While the explanations and questions were going on, the children developed their knowledge, because sometimes they didn't understand, but when they listened to their classmates, they finally understood, a fact that can be seen in the written record. Some children needed more concrete experiences to understand some concepts, operate and discuss probabilistic tasks, which should be offered at different times and with diversified activities, since a certain type of task may not reach everyone in the class.

Luvison and Santos (2013) point out that it is important for the teacher to be attentive to each child's learning time, listening to them and encouraging them to participate and reflect in this mathematical investigation environment.

The BNCC indicates the objects of knowledge and skills that should be built in the first, second and third grades, starting with the notion of chance, moving to the analysis of the idea of randomness in everyday situations and the analysis of the idea of chance in everyday situations: sample space. In carrying out the activities, we found that when children are challenged, they can build on the knowledge listed for their grade level, proving once again that the earlier probability is part of the classroom, the more knowledge children will build.

References

Borovcnik, M. (2016). Pensamento probabilístico e alfabetização em probabilidade no contexto de risco. *Educação Matemática Pesquisa*, *18*(3), 1491-1516.

Brasil. Ministério da Educação. (2018). *Base Nacional Comum Curricular: educação é a base*. http://basenacionalcomum.mec.gov.br/wp-content/uploads/2018/02/bncc-20dez-site.pdf

D'Ambrosio, B. S. (2013). O professor-pesquisador diante da produção escrita dos alunos. *Revista de Educação PUC-Campinas*, *18*(3), 249-258.

Diniz, E. & Carvalho, I. (2021). *O Clubinho*. Editora dos Autores. https://educacaoemteia.digital/2021/08/29/e-book-o-clubinho/

Frei, F., Rosa, J. S. & Biazi, A. H. (2023). Professores de Matemática estão preparados para o ensino de Estatística e Probabilidade? *Revista Internacional de Pesquisa em Educação Matemática*, *13*(2), 1-17. https://www.sbembrasil.org.br/periodicos/index.php/ripem/article/view/3378/2371

Gal, I. (2002). Adults' statistical literacy: Meanings, components, responsibilities. *International Statistical Review*, *70*(1), 1-51. https://doi.org/10.1111/j.1751-5823.2002.tb00336.x

Gal, I. (2005). Towards "probability literacy" for all citizens. In G. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 43-71). Kluwer Academic Publishers.

Jones, G. A., Langrall, C. W., Thornton, C. A. & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics*, *32*(2), 101-125.

Lopes, C. E., Grando, R. C. & D'Ambrosio, B. S. (2017). Experiences situating mathematical problem solving at the core of early childhood classrooms. *Early Childhood Education Journal*, *45*(2), 251-259. https://doi.org/10.1007/s10643-016-0775-0

Luvison, C. da C. & Santos, C. A. (2013). "Nunca vai cair o número 1, porque não tem 0 no dado": jogo, linguagem e resolução de problemas possibilitando a aprendizagem matemática. In A. M. Nacarato & R. C. Grando (Orgs.), *Estatística e Probabilidade na Educação Básica: professores narrando suas experiências* (1. ed., pp. 91-110). Mercado de Letras.

Oliveira, D. de & Lopes, C. E. (2013). A prática docente em estocástica, revelada por professoras que ensinam matemática nos anos iniciais do ensino fundamental. *Educação Matemática Pesquisa*, *15*(Número Especial), 909-925.

Santos, E. O. dos, Kalhil, J. B. & Ghedin, E. (2015). A formação matemática no curso de Pedagogia: o que revelam as matrizes curriculares. *Revista de Educação Matemática e Científica, 3*(1), 25-41.

Souza, L. de O., Mendonça, L. de O. & Lopes, C. E. (2013). A ação pedagógica e o desenvolvimento profissional de professores em Educação Estocástica. In C. de Q. S. Coutinho (Org.), *Discussões sobre o Ensino e a Aprendizagem da Probabilidade e da Estatística na Escola Básica* (1. ed., pp. 121-142). Mercado de Letras.