Sociedade Brasileira de Educação Matemática

RIPEM

Determinants of school performance of 3rd year high school students in Mathematics in SARESP: an approach with hierarchical generalized linear models

Camila Fernanda Bassetto
Universidade Estadual Paulista
São Carlos, SP — Brasil
✉ camila.bassetto@unesp.br
0000-0002-2513-7455

Aquiles Elie Guimarães Kalatzis
Universidade de São Paulo
São Carlos, SP — Brasil
✉ aquiles@sc.usp.br
0000-0001-8534-3165

Marco Aurélio Kistemann Junior
Universidade Federal de Juiz de Fora
Juiz de Fora, MG — Brasil
✉ kistemann1972@gmail.br
0000-0002-8970-3954

2238-0345 ISSN

10.37001/ripem.v15i1.3846 doi

Received • 15/03/2024
Approved • 15/04/2024
Published • 03/03/2025

Editor • Gilberto Januario

Abstract: The present study aimed to identify the factors influencing students' academic performance considering data from the School Performance Evaluation System of the State of São Paulo (SARESP) from 2019. In addition to the micro data associated with performance in Mathematics in the third grade of high school, responses from parents to the socioeconomic questionnaire administered by SARESP were also considered. The Socioeconomic Level Index (INSE) of the schools that participated in the evaluation in 2019 was included in the sample. Variable selection was performed using the Lasso method, and the probability of a student achieving satisfactory classification in Mathematics proficiency was estimated through simulations in the RStudio software using generalized linear models. The results showed that maternal education and family income were significant factors affecting student proficiency. Concurrently, doing homework and liking the subject were factors contributing to better performance.

Keywords: Mathematics Proficiency. Hierarchical Generalized Linear Models. High School. SARESP.

Determinantes del rendimiento escolar de los alumnos de 3º de bachillerato en Matemáticas en la SARESP: una aproximación con modelos lineales generalizados jerárquicos

Resumen: El presente estudio tuvo como objetivo identificar los factores que influyen en el rendimiento académico de los estudiantes considerando datos del Sistema de Evaluación del Rendimiento Escolar del Estado de São Paulo (SARESP) de 2019. Además de los microdatos asociados con el rendimiento en Matemáticas en el tercer año de secundaria, también se consideraron las respuestas de los padres al cuestionario socioeconómico administrado por SARESP. El Índice de Nivel Socioeconómico (INSE) de las escuelas que participaron en la evaluación en 2019 se incluyó en la muestra. La selección de variables se realizó utilizando el método Lasso, y la probabilidad de que un estudiante obtuviera una clasificación satisfactoria en la competencia en Matemáticas se estimó mediante simulaciones en el software RStudio utilizando modelos lineales generalizados. Los resultados mostraron que la educación materna

y el ingreso familiar fueron factores significativos que afectaron la competencia del estudiante. Al mismo tiempo, hacer la tarea y disfrutar de la asignatura fueron factores que contribuyeron a un mejor rendimiento.

Palabras clave: Competencia en Matemáticas. Modelos Lineales Generalizados Jerárquicos. Escuela Secundaria. SARESP.

Determinantes do desempenho escolar de estudantes do 3º ano do Ensino Médio em Matemática no SARESP: uma abordagem com modelos lineares generalizados hierárquicos

Resumo: O presente estudo buscou identificar os fatores que atuam sobre o desempenho escolar do aluno considerando dados do Sistema de Avaliação de Rendimento Escolar do Estado de São Paulo (SARESP) de 2019. Além dos microdados associados ao desempenho em Matemática na terceira série do Ensino Médio, foram consideradas as respostas dos pais dos alunos dadas ao questionário socioeconômico aplicado pelo SARESP. O Índice de Nível Socioeconômico (INSE) das escolas que participaram da avaliação em 2019, foram inseridos na amostra. A seleção de variáveis deu-se a partir do método Lasso e a probabilidade de o aluno obter classificação satisfatória na proficiência em Matemática foi estimada por meio de simulações no software RStudio com modelos lineares generalizados. Os resultados evidenciaram que a escolaridade materna e a renda familiar foram significativas sobre a proficiência do aluno. Concomitantemente, fazer a tarefa e gostar da disciplina são fatores que contribuem para um melhor desempenho.

Palavras-chave: Proficiência em Matemática. Modelos Lineares Generalizados Hierárquicos. Ensino Médio. SARESP.

1 Introduction

Since the 1960s and the first quarter of the 21st century, interest in investigating the factors that interfere with school performance has gradually grown both nationally and internationally. On the international scene, these factors have been studied for decades and had as their starting point the so-called Coleman Report, published in 1966, which analyzed the causes for the differences in performance between American schools and had a strong impact on public policies in the United States and the Sociology of Education in general (Coleman, 1966).

In response to a request from the US government and Congress, the research carried out by Coleman and collaborators aimed to identify which of the school and family systems best explained the inequalities in students' school performance. To this end, the authors investigated the lack of equal educational opportunities for students on the basis of gender, race, religion or region of origin in public educational institutions at all levels in the country. The results of the Coleman Report led to the conclusion that most of the inequalities between students originate in the families and social context of the schools and that differences in infrastructure and equipment between schools, the quality of their teaching staff or their curricula, their location and socio-economic level did not justify the variation in the academic performance of students enrolled in different schools (Alves & Soares, 2007, 2008; Palermo, Silva & Novellino, 2014).

In view of the debates generated since its release, the Coleman Report spurred discussions and studies on the possible school effects of academic performance. Subsequent research has been aided by changes in education policy, as schools have become more autonomous and education systems more decentralized. Associated with the research, the development of models and software for the statistical analysis of data with variables measured

at multiple levels generated significant contributions.

In Brazil, research into the effect and quality of schools is quite recent, with the first results published in the 1990s, when the Anísio Teixeira National Institute for Educational Research (INEP, Portuguese initials), part of the Ministry of Education (MEC), began to make educational data available. Studies intensified with the consolidation of the Basic Education Assessment System (SAEB, Portuguese initials), the results of which attracted attention for revealing a picture of educational inequalities in the country, prompting the first Brazilian studies to analyze school performance (Fletcher, 1998; Barbosa & Fernandes, 2001; Ferrão *et al.*, 2001; Soares, Cesar & Mambrini, 2001).

We see external evaluation as something that can assess student performance and, consequently, the social quality of education, using its data to guide public education policies aimed at improving teaching. It is something macro, since it involves education systems in the development of guidelines for national or state education. To this end, it uses proficiency scales in which their ranges show the acquisition of competences in the curricular component being assessed (Borges, Castro e Almeida & Lima, 2022). In this way, the competences refer to a reference matrix that cannot be confused with curriculum proposals, teaching strategies or pedagogical guidelines (INEP/MEC, 2019).

In the 21st century, the main interest is to identify factors, whether of a human, social, cultural or ethical nature, that act on students' school performance. Among other things, studies addressing investigations into how school performance affects individuals' future earnings, evaluating the impact of educational programs and analyzing the determinants of school performance are becoming increasingly frequent in the literature (Ferrão *et al.*, 2001; Jesus & Laros, 2004; Moreira, Andrade & Begolin, 2017; Soares, 2005; Riani & Rios-Neto, 2008; Rodrigues & Monteiro 2024; Rodrigues, 2023). The aim of these studies is to propose educational evaluation tools that will enable managers and other stakeholders to check whether schools are adequately meeting the challenges of economic transformations and the aspirations of Brazilian society.

Intensified in the early 1990s, the evaluation of educational systems became a relevant point in public policy proposals related to education. It is related to the need for better management of available resources, properly directing investments in public schools, and the need to diagnose public education with a view to effective improvements.

In recent years, the progress made in the field of education is noticeable, considering the methodological resources available and the current assessment criteria, such as the National Basic Education Assessment System (SAEB), the São Paulo State School Performance Assessment System (SARESP, Portuguese initials), the National High School Exam (ENEM, Portuguese initials), among others. These assessment systems have revealed a critical picture for education, which strengthens the need to investigate the factors that influence student performance in order to elucidate the process that produces the educational inequalities observed in the assessment results.

Since educational evaluation provides information that allows educators to identify practices that promote adequate results, it is considered a useful tool for drawing up public policies that can improve the Brazilian educational system. As pointed by Borges, Castro e Almeida & Lima (2022), the relevance of assessment was emphasized in the National Education Guidelines and Bases Law (LDBEN No. 9,394/96) (Brazil, 1996). According to this perspective, there is a centralization of the evaluative process, in which control is exerted through the assessment itself, defining contents and disciplines, goals and performance criteria, and evaluation systems are responsible for monitoring the achievement of these objectives.

The assessment of cognitive abilities and the analysis of performance are present at various stages of an individual's life. In educational assessments, various factors associated with variables internal and external to the school can influence school performance (Jesus & Laros, 2004; Riani & Rios-Neto, 2008; Laros *et al.*, 2010). In this sense, seeking to contribute to the literature regarding the investigation of factors that impact individual school performance, the objective proposed in this research consists of using hierarchical generalized linear models with data resulting from SARESP, corresponding to the grades and classifications obtained in Mathematics and the answers given to the socioeconomic questionnaire answered by the parents of the students participating in the 2019 assessment. The socioeconomic level index (INSE, Portuguese initials) of state schools in the state of São Paulo was also considered. The proposed models were estimated by simulation using RStudio software[1].

The results show that the variables associated with failure had a negative effect on Mathematics proficiency, demonstrating that students with academic delay due to failure have negative coefficients for these variables. On the other hand, maternal education was identified as a positive factor that influences students' academic performance.

2 School performance and hierarchical models

Individuals tend to live within organizational structures, such as families, schools, churches, cities, states and countries. In the educational environment, students are organized into classes, which in turn are defined within schools. This distribution suggests the use of a two-stage procedure for analyzing this data, the first consisting of a sample of schools and the second of samples of students within each school (Hox, Moerbeek & Van de Shoot, 2002). The existence of hierarchically structured data is not accidental and, when investigating the factors that affect student performance at school, this structure cannot be ignored (Goldstein, 2001). A statistical methodology that can be used to simultaneously analyze relationships at the student level, while taking grouping into account, consists of hierarchical linear models, also known as multilevel models (Hox, Moerbeek & Van de Shoot, 2002; Raudenbush & Bryk, 2002; Goldstein, 2001).

Multilevel regression models have been implemented in Brazil since the late 1990s to analyze data from large-scale educational assessments, as shown in the studies by Fletcher (1998), Ferrão *et al.* (2001), Alves and Soares (2008), Soares (2005), Andrade and Laros (2007), among others. In these studies, the authors identified the variables associated with students and schools that influence school performance. Considered one of the pioneers in the use of hierarchical linear models in Brazil, the educational research carried out by Fletcher (1998) found that the proportion of white and male students in the school had a positive effect on math performance. In the studies by Soares and Mendonça (2003), Riani and Rios-Neto (2008) and Brooke, Fernandes, Miranda and Soares (2014), the results indicated that male and black students perform poorly.

According to Barbosa and Fernandes (2000), the structure of the educational system is organized hierarchically, in which a group of students makes up a class and a group of classes gives rise to a school. In this context, the authors considered the multilevel approach specified at two levels, with students at level 1 and schools at level 2, and used 1997 SAEB data from the Southeast region of Brazil, which includes the states of Minas Gerais, Espírito Santo, Rio de Janeiro and São Paulo, to investigate the influence of family and school factors on the math proficiency of 8th grade students. Based on the results achieved, the authors state that the school

[1] This article is an excerpt from a Full Professor thesis defended at the São Paulo State University "Júlio de Mesquita Filho" (UNESP) - School of Sciences and Letters, Araraquara campus (FCL/Ar), written by the first author.

makes a difference when the student's academic performance is analyzed. Soares *et al.* (2001), using data from the 1997 SAEB, also found that students with a regular school career (no failures and no school dropouts), together with those who are male and study in private schools, achieved better results and showed higher proficiency compared to the proficiency of other students.

In order to identify the factors that can affect student performance in the Portuguese language, Jesus and Laros (2004) developed a hierarchical model using data from the 2001 Basic Education Assessment System (SAEB). The authors considered a two-level model and concluded that doing homework contributes positively to performance, while falling behind in school and the fact that the student works has a negative influence. Similarly, in addition to the mother's schooling, doing homework was a significant variable in explaining student performance in the studies by Machado, Moro, Martins and Rios (2008), Riani and Rios-Neto (2008) and Moreira *et al.* (2017).

Laros *et al.* (2010) and Laros *et al.* (2012) also studied the factors that affect student performance. They proposed a multilevel model with data from the 2001 SAEB and considered the empirical model presented by Jesus and Laros (2004), with two levels, that is, student and school. According to the results, the variable that most affected students' school performance was being behind at school, followed by whether the student liked studying math. The number of absences and the year repetition rate negatively affected academic performance. Unlike the other studies cited, Laros *et al.* (2010) suggest interventions that the family and school can take to increase student performance. As far as the family is concerned, the authors highlight encouraging study from an early age through games and playful activities and preventing children from entering and staying at school with an age-grade distortion.

Research in the field of school education shows that various factors can influence individuals' school trajectories and that education revolves around two axes, which must be monitored in order to capture the effects of each on student proficiency and the quality of teaching. The first axis encompasses the student and their individual and family characteristics, such as color/race, age, socioeconomic status, parents' level of education, etc. The second axis consists of the characteristics of the school where the individual is enrolled, such as the school's infrastructure, the level of education and commitment of the teachers, etc.

Moreira *et al.* (2017) used data from the Rio Grande do Sul School Performance Assessment System (SAERS, Portuguese initials) from 2007 to investigate the determinants of school performance in the state of Rio Grande do Sul. The authors concluded that the socio-economic characteristics of the students, represented by the items that the students have and do not have at home, are important determinants of school performance, while the characteristics of the schools, represented by the characteristics of the teachers, principals, and school infrastructure, do little to explain the school performance of students in Rio Grande do Sul.

The study by Palermo *et al.* (2014) considered proficiency in mathematics as the dependent variable and used data from the 2007 Prova Brasil (Brazil Test) for students in the 5th grade of elementary school in Rio de Janeiro. Following the methodology proposed by Barbosa and Fernandes (2001), Palermo *et al.* (2014) considered hierarchical modeling with two and three levels. For the two-level model, students and classes as well as students and schools were tested, and for the three-level model, students, classes and schools were considered, in that order. Regarding the level of the students, the authors identified a directly proportional relationship between socio-economic level, represented by the weighted sum of consumer goods available in the household and the parents' maximum schooling, on performance in Mathematics, suggesting that students with a socio-economic level above the

class average would have their proficiency increased by one tenth.

Studies carried out in Brazil on educational determinants emphasize the role of the family in children's education, showing that family factors, especially parents' level of education, have a major influence on educational performance. Research that includes, in addition to family factors, those related to school and society in general shows that, despite the important impact of characteristics, the other factors mentioned also have a significant effect on student proficiency. At the same time, based on the studies mentioned above, the impact of socio-economic status on students' school performance is evident and, for this reason, the study of any educational reality must necessarily consider the student's socio-economic status. Socio-demographic characteristics such as gender, color/race and the gap between the student's age and the age of the grade they are in, summarize lived experiences that have an impact on their learning.

The analyses developed are relevant because, as well as helping to formulate more effective public education policies, they make it possible to investigate the extent to which school factors minimize the influence of the family on the individual's educational outcome.

3 Data and model

External evaluations of students' educational performance are a frequent topic in most schools. The state of São Paulo, as well as taking part in national assessments, promotes the São Paulo State School Performance Assessment System - SARESP, whose purpose is to provide consistent information on the state of schooling in the state's public network, capable of guiding managers in monitoring policies aimed at improving the quality of teaching.

School Boards are defined as units of direct administration subordinate to the São Paulo State Department of Education and were created with the justification of eliminating the waste of human resources and promoting the effective application of financial resources. In the state of São Paulo there are a total of 91 School Boards, allocated to regions of the state, as shown in Chart 1.

Chart 1: School Boards allocated to the regions of the state of São Paulo

Region	School Board
Capital	Caieiras, Carapicuíba, Centro (SP), Centro Oeste (SP), Centro Sul (SP), Diadema, Guarulhos Norte, Guarulhos Sul, Itapecerica da Serra, Itapevi, Itaquaquecetuba, East 1 (SP), East 2 (SP), East 3 (SP), East 4 (SP), East 5 (SP), Mauá, Mogi das Cruzes, North 1 (SP), North2 (SP), Osasco, Santo André, São Bernardo do Campo, South 1 (SP), South 2 (SP), South 3 (SP), Suzano, Taboão da Serra.
Center	Americana, Bragança Paulista, Campinas Leste, Campinas Oeste, Capivari, Itu, Jundiaí, Limeira, Mogi Mirim, Piracicaba, Pirassununga, São João da Boa Vista, Sumaré.
Coast	Andradina, Castilho, Guaraçaí, Ilha Solteira, Itapura, Lavínia, Mirandópolis, Murutinga do Sul, Nova Independência, Pereira Barreto, Sud Mennucci.
Northeast	Caraguatatuba, Guaratinguetá, Jacareí, Pindamonhangaba, São José dos Campos, Taubaté.
North	Araraquara, Barretos, Franca, Jaboticabal, Ribeirão Preto, São Carlos, São Joaquim da Barra, Sertãozinho, Taquaritinga.
Northwest	Andradina, Araçatuba, Birigui, Catanduva, Fernandópolis, Jales, José Bonifácio, Penápolis, São José do Rio Preto, Votuporanga.

Southeast	Apiaí, Avaré, Botucatu, Itapetininga, Itapeva, Itararé, Miracatu, Piraju, Registro, São Roque, Sorocaba, Votorantim.
Southwest	Adamantina, Assis, Bauru, Jaú, Lins, Marília, Mirante do Paranapanema, Ourinhos, Presidente Prudente, Santo Anastácio, Tupã.

Fonte: http://www.educacao.sp.gov.br

In the schools belonging to the School Boards presented here, the assessment of student performance in Mathematics, and in other subjects, is based on the São Paulo State Curriculum, which refers to the National Curriculum Parameters (PCNs, Portuguese initials) and the theoretical assumptions behind the creation of the National High School Exam (ENEM); on the Reference Matrix for Assessment and on Statistical Methodologies for planning, collecting and analyzing the results.

Based on the learning expectations in terms of content, competencies and skills established for each year/grade of the subject in the São Paulo State Curriculum, the SARESP scale points are grouped into four levels: Below Basic (AB), Basic (BA), Adequate (AD) and Advanced (AV). Chart 2 contains information on the proficiency levels, score ranges and descriptions used by SARESP.

Chart 2: Scoring and description of proficiency levels in Mathematics

Proficiency level	Score and description
Below Basic (AB)	< 275: Insufficient mastery of the content, skills and abilities desirable for the year/grade they are in.
Basic (BA)	275 to < 350: Minimum mastery of the content, skills and abilities desirable for the year/grade they are in.
Adequate (AD)	350 to < 400: Full mastery of the content, skills and abilities desirable for the year/grade they are in.
Advanced (AV)	≥ 400: Knowledge and mastery above what is required of the content, skills and abilities desirable for the year/grade they are in.

Source: Sumário Executivo SARESP 2019 (São Paulo, 2019)

The first column shows the proficiency levels, which, combined with their score ranges, are classified as Insufficient (AB), Sufficient (BA and AD) and Advanced (AV). The second column contains a description of each proficiency level.

In 2019, the São Paulo State Department of Education (SEDUC/SP) held the 22nd edition of SARESP, characterized as a large-scale assessment of basic education that has been applied since 1996. Municipal, state and private schools take part in this assessment, as well as the State Technical Schools (ETE), administered by the Paula Souza Technological Educational Center and linked to the São Paulo State Development Department. SARESP involves students, parents, schools, principals, coordinators, teachers, applicators and inspectors, and uses two assessment instruments, the first characterized by the application of tests, and the second corresponding to the application of questionnaires directed at parents and students, principals, coordinating teachers and other teachers (Brazil, 2019).

The questionnaires answered by parents and students make it possible to collect information on the students' social, economic, cultural and family background, on their school career, study habits and perceptions and expectations of teachers and school management, and on the profile and aspects of school management and teaching practice.

Given the large number of variables provided by the questionnaires administered to students and their parents, the Group Lasso method (Izbicky, 2018) was implemented in the RStudio software in order to reduce the size of the data.

In Lasso, one seeks:

$$\hat{\beta}_{L_1,\lambda} = argmin_{\beta} \sum_{i=1}^{n} \left(y_k - \beta_0 - \sum_{j=1}^{d} \beta_j x_{i,j} \right)^2 + \lambda \sum_{j=1}^{d} |\beta_j| \quad (1)$$

in which d is the predictors number, β_0 is the intercept, L_1 indicates the vector sparsity using L_1 norm represented by $\|\beta_j\|_{L_1}^2 = \sum_{j=1}^{d} |\beta_j|$, $\sum_{i=1}^{n} (y_k - \beta_0 - \sum_{j=1}^{d} \beta_j x_{i,j})^2$ is the model mean squared error and $\lambda \sum_{j=1}^{d} |\beta_j|$ is the penalization factor by Lasso.

The method selected the variables associated with the questions shown in Chart 3.

Chart 3: Questions selected via Group Lasso method

Variables	Alternatives
Sex	(A) Feminine. (B) Masculine.
Shift	(A) Morning. (B) Afternoon. (C) Night.
Q10A. Have you ever been failed?	(A) No. (B) Yes, once. (C) Yes, twice or more.
Your parents or guardians: **Q13A.** Do they help with the lesson or check that you have done it?	(A) Yes, a lot. (B) Yes, a little. (C) No.
Q22A. Do you like studying mathematics?	(A) Yes. (B) No.
Q23A. Do you do your math homework?	(A) Always or almost always. (B) Sometimes. (C) Never. (D) I do not have any homework.
Q24A. Do you like studying Portuguese?	(A) Yes. (B) No.
What's your favorite way to study? **Q27A.** Paying attention in class? **Q28A.** Reading my notes at home? **Q31A.** Doing research on the Internet.	(A) It's not what I like/do. (B) I like it just a little. (C) I like it a lot.
Q19P. Up to what grade/level did the mother study?	(A) Never studied or did not complete the 4th grade/5th year (former primary school). (B) Completed 4th grade/5th year, but not 8th grade/9th year (former secondary school).

	(C) Completed 8th grade/9th grade, but not high school. (D) Completed high school, but not higher education. (E) Completed higher education. (F) Completed postgraduate studies. (G) Do not know.
Q23P. What is your household income?	(A) Up to 1 MW (R$ 998.00). (B) From 1 to 2 MW (R$ 998.01 to R$ 1,996.00). (C) From 2 to 3 MW (R$ 1,996.01 to R$ 2,994.00). (D) From 3 to 5 MW (R$ 2,994.01 to R$ 4,990.00). (E) From 5 to 8 MW (R$ 4,990.01 to R$ 7,984.00). (F) From 8 to 15 MW (R$ 7,984.01 to R$ 14,970.00). (G) More than 15 MW (more than R$ 14,970.01). (G) (H) I don't know/don't want to answer.
You have it at home: **Q27P.** Books (novels, poetry, etc.).	(A) Yes. (B) No.

Source: Socio-economic questionnaire applied by SARESP

Since this research considers a generalized linear model approach (McCullagh, 2019), it is necessary to categorize the variables. For the questions presented in Chart 3, the notation given in Chart 4 was used.

The combination of these variables made it possible to develop different logistic models with the aim of estimating the probability of a student in the 3rd grade of primary school being classified as Sufficient in Mathematics proficiency.

Using the encoding of selected variables by Lasso method, the resulting logistic model is given as in equation (2).

$$P\left(Y_{ij}=1 \mid X=x_{ij}\right)=\frac{e^{\left(\beta_0+\beta_k x_{kij}\right)}}{1+e^{\left(\beta_0+\beta_k x_{kij}\right)}}, k=1,\ldots,31 \quad (2)$$

Chart 4: Coding of variables

Question	Notation	Coding	Question	Notation	Coding
Classification	Y_{ij}	0, if Insufficient. 1, if Sufficient.	Q24A	x_{7ij}	0, if B 1, if A
Sex	x_{1ij}	0, if girl. 1, if boy.	Q27A	x_{8ij}	0, if A 1, if B or C
Shift	x_{2ij}	1, if morning 0, if afternoon or night.	Q28A	x_{9ij}	0, if A 1, if B or C
Q10A	x_{3ij}	1, if A. 0, if B or C	Q31A	x_{10ij}	0, if A 1, if B or C

Q13A	x_{4ij}	0, if C 1, if A or B	Q20P	x_{11ij}	0, if A or B 1, if C or D2, if E or F
Q22A	x_{5ij}	0, if B 1, if A	Q23P	x_{12ij}	0, if A or B. 1, if C or D 2, if E or F 3, if G
Q23A	x_{6ij}	0, if C or D 1, if A or B	Q27P	x_{13ij}	0, if B 1, if A

Source: Devised by the author

Equation (2) appears somewhat complex and difficult to interpret. Therefore, the logit transformation, given in expression (3), is used to understanding the results.

$$logit\left(x_{ij}\right) = ln\left[\frac{e^{\left(\beta_0+\beta_k x_{kij}\right)}}{1+e^{\left(\beta_0+\beta_k x_{kij}\right)}}\right] = \beta_0 + \beta_k x_{kij},\ k = 1,\ldots,31 \tag{3}$$

For the models used, the deviance value was considered as a selection criterion, in order to identify the one with the best fit to the data and, at the same time, the greatest predictive power. The model selected by this criterion will be used to estimate the parameters associated with each of its predictor variables.

The selected model is given as in the formula (4).

$$P\left(Y_{ij}=1 \mid X = x_{ij}\right) = \frac{e^{\left(\beta_0+\beta_1 x_{1ij}+\beta_2 x_{2ij}+\beta_3 x_{3ij}+\beta_4 x_{4ij}+\beta_5 x_{5ij}+\beta_7 x_{7ij}+\beta_{11} x_{11ij}+\beta_{12} x_{12ij}\right)}}{1+e^{\left(\beta_0+\beta_1 x_{1ij}+\beta_2 x_{2ij}+\beta_3 x_{3ij}+\beta_4 x_{4ij}+\beta_5 x_{5ij}+\beta_7 x_{7ij}+\beta_{11} x_{11ij}+\beta_{12} x_{12ij}\right)}} \tag{4}$$

Considering the hierarchical generalized linear model, with level 1 represented by the students and level 2 by the schools, estimates were obtained for the parameters in order to know the probability of the student achieving the Sufficient classification in Mathematics proficiency. The results are shown in the following section.

4 Results

For the regions of the state of São Paulo, defined as shown in Chart 1, a descriptive analysis was made of the levels of proficiency in Mathematics achieved by the students enrolled in the schools that took part in SARESP in 2019.

4.1 Descriptive analysis of regional proficiency in mathematics

Table 1 shows the descriptive values associated with the proficiency levels in the state's regions, as well as the percentage of students in each of them.

In the Capital region, of the total of 39,046 students, the highest percentages were classified at the lowest levels of proficiency in Mathematics, that is, AB and BA, showing insufficient mastery of the content, skills and competencies associated with the 3rd grade of secondary school. For students belonging to level AB, approximately 45%, the minimum and maximum scores observed were 159.2 and 274.9, respectively, with an average of 240.3.

The standard deviation of 24 points means that most students scored between 216.3 and 264.3. Similarly, with a percentage close to 48%, of students at Basic level, whose average

score was 306.3. Also in this region, only 6.5% of the students were classified at the AD level of proficiency in Mathematics, demonstrating sufficient mastery of the content, skills and competencies programmed for the 3rd grade of secondary school. In this group, the average score was 366.6 and the variability was 13 points above or below the average. In the AV level, which encompasses students who demonstrate above-average mastery of the content, skills and competences of the 3rd grade of secondary school, only 170 students were classified, which represents less than 0.5% of the total number of participants in this region.

For students enrolled in public schools in the Center region, of the total of 13,259 students who took part in SARESP in 2019, the highest percentage was observed at BA level, given by 52%. Among these students, the average score was 308.5. Almost 38% of students were at level AB. Compared to the Capital region, students in the Center region achieved better results, given the higher percentages at AD and AV levels. The minimum score in this region was higher than in the capital.

The total number of students who took part in the 2019 edition of the assessment was 3,034. Table 1 shows that, of the total number of students, the highest percentage reached level BA of proficiency in Mathematics, equal to 51%, followed by level AB, with 41.3% of students. At level AB, while the average score was 241.8 with a variability of approximately 23 points, suggesting that most students scored between 218.5 and 246.6, the lowest score observed was 163.6, lower than that achieved by most students at this level of proficiency. Less than 7% of students in the Coast region scored at AD level, with a maximum score of 399.5, and 0.7% at AV level. Compared to the Capital and Center regions, students at AV level in the Coast region achieved the highest average score, close to 413 points.

Considering the students belonging to the School Boards that make up the Northeast region, the total number of students taking part in the 2019 SARESP was 5,641. As in the Center and Coast regions, in the Northeast the highest percentage of students reached level BA of proficiency in Mathematics. While at this level the percentage of students was close to 54%, at levels AB, AD and AV these figures were 33.9%, 10.6% and 1.2% respectively. With similar average scores and variability, in the Northeast region, the minimum score among students at level AB was higher than those achieved by students in the regions previously analyzed.

Table 1: Descriptive analysis of regional performance in mathematics

Region	Proficiency level	Min. score	Average	Max. score	Standard deviation	Percentage of students
Capital	AB	159.2	240.8	274.9	24.0	45.34
	BA	275.0	306.3	349.9	20.1	47.77
	AD	350.0	366.9	399.9	12.9	6.47
	AV	400.0	412.2	434.6	9.2	0.49
Center	AB	163.1	242.8	274.9	23.6	37.59
	BA	275.0	308.5	349.9	20.5	52.04
	AD	350.0	367.4	399.9	13.2	9.61
	AV	400.2	411.3	432.5	8.6	0.76
Coast	AB	163.6	241.8	274.9	23.3	41.35
	BA	275.0	307.6	349.9	20.2	51.09
	AD	350.2	366.5	399.5	12.9	6.85

	AV	400.6	413.1	429.4	10.2	0.70
Northeast	AB	171.6	244.4	274.9	23.2	33.89
	BA	275.0	309.5	349.9	20.5	54.23
	AD	350.0	369.0	399.5	13.58	10.65
	AV	400.2	413.4	432.9	9.6	1.22
Northwest	AB	160.2	243.9	274.9	23.7	29.75
	BA	275.0	309.6	349.9	20.6	53.75
	AD	350.0	369.4	399.9	13.4	14.4
	AV	400.6	413.7	432.9	9.0	2.09
North	AB	165.7	243.7	274.9	23.5	36.37
	BA	275.0	308.6	349.9	20.1	52.37
	AD	350.0	368.3	399.8	13.3	10.04
	AV	400.1	414.7	434.6	10.1	1.22
Southeast	AB	164.7	244.4	274.9	23.1	35.96
	BA	275.0	308.5	349.9	20.4	53.11
	AD	350.0	367.6	399.0	13.0	10.08
	AV	400.1	413.2	434.6	10.3	0.84
Southwest	AB	159.6	243.1	274.9	23.8	35.85
	BA	275.0	308.0	349.9	20.0	52.07
	AD	350.0	368.7	399.8	13.5	10.67
	AV	400.1	413.4	430.3	8.8	1.41

Source: Devised by the authors

In the Northwest region, which had 4,199 students participating in the 2019 SARESP, approximately 54% were classified at BA level, demonstrating minimal mastery of the content, skills and abilities of the 3rd grade of secondary school, but with the ability to interact with the next grade. Almost 30% of the students were at AB level. The Northwest region recorded the highest percentages of students at AD and AV levels, with 14.4% and 2.0% respectively, suggesting a better performance from students in this region compared to those in the Capital, Center, Coast and Northeast regions.

For the North and Southeast regions, it is possible to observe similar behavior between the two regions at levels AB, BA and AD, with percentages close to 36%, 52% and 10%, respectively. In the North, 5,419 students took part in the test. Of these, 36.4% obtained an average score of 243.4 and were classified at the worst level, that is, AB. Another 52.4% belonged to level BA, 10% to level AD and 1.2% to level AV.

In the Southeast, the number of students in the 2019 SARESP was 5,734, who were classified in proficiency levels in the following proportions: 36% in AB, 53.1% in BA, 10% in AD and 0.84% in AV. For the two regions, the characteristics shown in Tables 1 are very similar, with the biggest difference being between the medians of the AV level. While in the North this value was 413.1, in the Southeast it was slightly lower at 409.4. In the Southwest region, 4,898 students took part in the 2019 SARESP, 35.85% of whom were classified at AB

level, 52% at BA level, 10.7% at AD level and 1.4% at AV level.

For the schools, grouped according to the Education Board to which they belong, we collected the values for the Socioeconomic Level Index (INSE, Portuguese initials), made available by the São Paulo State Secretariat, through Open Education Data[2]. The purpose of this indicator is to contextualize the results obtained in evaluations and exams in the field of basic education, thus providing insight into the social reality of schools and educational systems. At the same time, the INSE is calculated with the aim of serving as an auxiliary instrument in the execution, supervision and evaluation of public policies, with the aim of promoting the improvement of quality and equity in the education system.

According to Soares and Collares (2006) and Alves *et al.* (2014), socioeconomic status is a theoretical concept used to categorize individuals into different social strata or classes.

Although there is no consensus in the literature on the dimensions that should be incorporated into its operationalization, those related to occupation, income and educational level are often included. According to these authors, evaluating the results of large-scale educational assessments without considering the Socioeconomic Level Index (INSE) of school units can result in distorted associations, given the presence of associations between the elements that compose it (Soares & Collares, 2006; Alves *et al.*, 2014). The authors also add that the proposition of this index represents a response to the demand that has emerged in recent years, coinciding with the dissemination of educational results by educational institutions. In this sense, the INSE values were obtained by School Board and then calculated for the regions of the state of São Paulo.

Table 2 shows the descriptive statistics for the INSE by region.

According to the values shown in Table 2, it can be seen that, in 2019, the Coast and Center regions had, in that order, the lowest and highest average values for the INSE, equal to 5.075 and 5.317, respectively. Also, according to Table 2, the Southeast region recorded the lowest and highest values for the INSE in that year, 3.98 and 5.92, which also resulted in the greatest dispersion of the values observed for the School Boards in relation to the average value calculated.

Table 2: Descriptive analysis of regional INSE

Region	Minimum	Average	Maximum	Standard deviation
Capital	4.59	5.188	5.87	0.214
Center	4.67	5.317	5.88	0.228
Coast	4.58	5.075	5.56	0.212
Northeast	4.36	5.262	5.92	0.264
Northwest	4.58	5.299	5.74	0.212
North	4.76	5.259	5.80	0.221
Southeast	3.98	5.230	5.92	0.302
Southwest	4.56	5.278	5.84	0.231

Source: Devised by the authors with data from SARESP 2019

[2] https://dados.educacao.sp.gov.br/.

4.2 Regional school performance

The hierarchical model given in expression (1) was considered for all the regions of the state of São Paulo, that is, Capital, Center, Coast, Northeast, Northwest, North, Southeast and Southwest, established according to the School Board, allowing for a regional analysis. The estimates shown in Table 3 were obtained via simulations in the RStudio *software*.

According to the values shown in Table 3, it can be seen that, in all regions, male students are more likely to score sufficiently in Mathematics when compared to girls, as the estimate for the SEX1 variable was greater than 1. This is true for students enrolled during the morning, when compared to those who study in the afternoon or evening, which can be seen from the estimates greater than 1 for the SHIFT1 variable. The probability in question is also higher for students who have never failed and who enjoy studying math, compared to those who have failed one or more times and who do not enjoy studying math. Table 3 shows that, in all regions, for students who have never failed, the probability of being classified as Sufficient in Mathematics proficiency is twice as high as for students who have failed the subject at least once. And for those who enjoy studying mathematics, the chance of obtaining such a classification is almost three times greater when compared to the probability of the other students. This is due to the fact that the estimates for variables Q10A1 and Q22A1 were close to two and three, respectively. On the other hand, when compared to students who do not receive homework help from their parents, those who do are less likely to get a Sufficient rating, given the estimate value of less than one for the Q10A1 variable.

One explanation for this result could be that students who need help from their parents have more difficulty than those who are able to cope with school lessons on their own. The same was true for students who enjoyed studying Portuguese. When they prefer this subject, they consequently dislike studying mathematics.

Considering maternal schooling and family income, it can be said that, for students enrolled in schools located in the regions of the state of São Paulo, the likelihood of scoring sufficiently in math proficiency, according to 2019 SARESP data, is higher for those whose mothers have completed primary and secondary education, compared to students whose mothers have not even completed primary education. This probability also increases as family income rises, but up to the 15 MW threshold.

Table 3: Regional estimates associated with the selected model

Variable	Estimate	Variable	Estimate	Variable	Estimate
Capital					
Intercepto	0,264	Q22A1	2,591	Q23P2	1,583
SEXO1	1,348	Q24A1	0,660	Q23P3	1,019
PERIODO1	1,574	Q20P1	1,245	INSE1	1,188
Q10A1	2,255	Q20P2	1,171	INSE2	1,356
Q13A1	0,676	Q23P1	1,352	INSE3	1,869
Center					
Intercepto	0,277	Q22A1	3,025	Q23P2	2,240
SEXO1	1,219	Q24A1	0,809	Q23P3	0,622
PERIODO1	1,547	Q20P1	1,330	INSE1	1,051

Q10A1	2,368	Q20P2	1,197	INSE2	1,173
Q13A1	0,710	Q23P1	1,455	INSE3	1,339
Coast					
Intercepto	0,305	Q22A1	2,830	Q23P2	1,820
SEXO1	1,432	Q24A1	0,672	Q23P3	0,674
PERIODO1	1,419	Q20P1	1,293	INSE1	1,364
Q10A1	2,096	Q20P2	1,044	INSE2	1,953
Q13A1	0,714	Q23P1	1,303	INSE3	3,838
Northeast					
Intercepto	0,408	Q22A1	2,721	Q23P2	1,539
SEXO1	1,192	Q24A1	0,679	Q23P3	7,255
PERIODO1	1,403	Q20P1	1,311	INSE1	0,902
Q10A1	2,644	Q20P2	1,290	INSE2	1,335
Q13A1	0,626	Q23P1	1,267	INSE3	1,719
Northwest					
Intercepto	0,329	Q22A1	3,229	Q23P2	2,332
SEXO1	1,452	Q24A1	0,952	Q23P3	1,067
PERIODO1	1,383	Q20P1	1,243	INSE1	1,102
Q10A1	2,611	Q20P2	1,210	INSE2	1,098
Q13A1	0,606	Q23P1	1,547	INSE3	1,245
North					
Intercepto	0,429	Q22A1	2,749	Q23P2	1,382
SEXO1	1,280	Q24A1	0,743	Q23P3	0,394
PERIODO1	1,263	Q20P1	1,324	INSE1	1,094
Q10A1	2,058	Q20P2	1,273	INSE2	1,149
Q13A1	0,608	Q23P1	1,521	INSE3	1,500
Southeast					
Intercepto	0,335	Q22A1	2,905	Q23P2	1,389
SEXO1	1,224	Q24A1	0,785	Q23P3	0,879
PERIODO1	1,612	Q20P1	1,395	INSE1	0,838
Q10A1	2,565	Q20P2	1,216	INSE2	1,040
Q13A1	0,690	Q23P1	1,244	INSE3	1,120
Southwest					
Intercepto	0,277	Q22A1	3,025	Q23P2	2,240
SEXO1	1,219	Q24A1	0,809	Q23P3	0,622
PERIODO1	1,547	Q20P1	1,330	INSE1	1,051

Q10A1	2,368	Q20P2	1,197	INSE2	1,173
Q13A1	0,710	Q23P1	1,455	INSE3	1,339

Source: Devised by the authors based on estimated values

According to Table 3, the estimates obtained for variables Q23A1, Q23A2 and Q23A3 suggest that the probability of students with a family income of between 2 MW and 5 MW achieving a Sufficient rating in Mathematics is approximately 1.5 times the probability of those with a family income of less than 2 MW. For students with a family income between 5 MW and 15 MW, this probability is 1.6 times the probability of those with the lowest income. Family income above 15 MW was not significant in explaining this probability.

In relation to the school variable, INSE, it can be seen that the probability increases as the socio-economic level of the school increases, showing that schools with better conditions and financial resources contribute positively to the performance of their students.

5 Towards a conclusion

Although it is in the process of being developed in Brazil, large-scale educational assessment has proved important in recent decades, as a tool to help identify factors that affect student performance at school. Over the last 25 years or so, the priority objective of assessment systems has been to find mechanisms that can effectively and efficiently improve the quality of education offered in society.

In turn, educational evaluation aims to provide elements that make it possible to diagnose the situation of the educational system in a given location and, at the same time, to subsidize appropriate policies and guidelines in the municipal, state and national context, with a view to continuously improving the quality of education.

According to Souza and Magalhães (2024):

> (...) it becomes clear that the challenges associated with assessment methods have their roots in the teacher training process. However, these challenges are not related to the lack of consistent and proven theories that could enhance assessment practices. Instead, they are problems of practical application, as pre-service teachers do not receive adequate guidance in the courses intended to provide these foundations. This reinforces the need for additional investments in teacher education programs, which play a fundamental role in improving the quality of education. (Souza & Magalhães, 2024, p. 6).

It is also widely recognized in the specialized field that students' academic performance, measured by proficiency in standardized tests, is the result of a complex interaction of factors that act concomitantly at the various levels of their social insertion, that is. socio-economic aspects in the family environment, relationships and teaching practices in the school environment, among others. As Soares (2005) argued when proposing a conceptual model linking five structures, the student's cognitive performance is associated with the student themselves, their family, the school, the system they are part of and the society in which they live. In this sense, the aim of this research was to use hierarchical generalized linear models to identify the factors that affect a student's propensity to achieve satisfactory levels of proficiency in mathematics.

In this research, SARESP data for the 2019 edition was considered for the development of the proposed methodology, specifically the grades obtained in Mathematics by 3rd grade

high school students from the public school system. The answers given by the students and parents of the participating students to the questionnaires applied by SARESP constituted significant data for the proposed analysis. In the generalized hierarchical models, the response variable was characterized as categorical and represented the student's chance of being classified as Sufficient in Mathematics proficiency, according to the score obtained in the assessment.

The database used in this research was organized and composed according to the regions of the state of São Paulo, that is, Capital, Center, Coast, Northeast, Northwest, North, Southeast and Southwest. The schools were grouped according to the School Board to which they belonged. Due to the number of variables available, the Group Lasso method was used to reduce the dimensionality of the data. Different models were developed by combining the variables that remained in the database structured for this research. Deviance was then calculated as a selection criterion and parameter estimates were obtained via the RStudio software for the selected model.

With the model selected, estimates for the parameters were obtained, considering students within schools. In order to develop the proposed analysis, it was necessary, in addition to information about the students, who represent level 1 of the multilevel model, to also have variables associated with the schools, which constitute level 2 in the hierarchy. In this context, we collected the Socioeconomic Level Index (INSE) of the state schools in São Paulo whose students took part in the assessment in 2019 and, at the same time, the parents answered the socioeconomic questionnaire applied by SARESP.

The results obtained in this study corroborate the conclusions of studies in the literature, such as Soares and Mendonça (2003), Andrade and Laros (2007), Laros *et al.* (2010) and Palermo *et al.* (2014). These authors found that the variables associated with failure had a negative effect on the student's math proficiency, while the mother's schooling and socioeconomic status contributed positively to a satisfactory result. Brooke *et al.* (2014) and Machado *et al.* (2008) showed that students with an educational age/grade gap caused by failure had negative regression coefficients associated with these variables, and that the mother's schooling had a positive effect on school performance.

The effective contributions of studies such as the one presented throughout this article seek to obtain knowledge to support solutions to problems that are a constant concern for all those involved in the education system. Understanding the extra- and intra-school factors that affect the likelihood of a student achieving an adequate result on the SARESP, directly influencing the level of classification in which they find themselves, is important for subsidizing the agents of the Education Department in defining educational policies aimed at improving basic education in Brazil.

Thus, based on the results achieved, we hope to have contributed to the literature that deals with the identification of factors that impact on student educational performance and to highlight the importance of formulating and implementing public policies aimed at education that simultaneously improve the quality of the education offered to society and reduce the impact of socio-economic characteristics on student performance.

References

Alves, M. T. G. & Soares, J. F. (2007). Efeito-escola e estratificação escolar: O impacto da composição de turmas por nível de habilidade dos alunos. *Educação em Revista, 45*, 25-59.

Alves, M. T. G. & Soares, J. F. (2008). O efeito das escolas no aprendizado dos alunos: Um estudo com dados longitudinais no ensino fundamental. *Educação e Pesquisa, 34*(3), 527-544.

Andrade, J. M. e & Laros, J. A. (2007). Fatores associados ao desempenho escolar: Estudo multinível com dados do SAEB/2001. *Psicologia: Teoria e Pesquisa, 23*, 33–41.

Barbosa, M. E. F. & Fernandes, C. (2000). Modelo multinível: Uma aplicação a dados de avaliação educacional. *Estudos em Avaliação Educacional, 22*, 135-154.

Barbosa, M. E. F. & Fernandes, C. (2001). A escola brasileira faz diferença? Uma investigação dos efeitos da escola na proficiência em matemática dos alunos da 4ª série. In C. Franco (Org.), *Promoção, ciclos e avaliação educacional* (pp. 155–172). ArtMed.

Borges, A. P. N. R., Castro e Almeida, S. P. N. & Lima, K. (2022). Avaliação Educacional: O Saeb, seus pressupostos, finalidades e repercussões. *Revista Internacional de Pesquisa em Educação Matemática, 12*(4), 1-19.

Brasil. (1996). Lei n. 9.394, de 20 de dezembro de 1996. Estabelece as diretrizes e bases da educação nacional. *Brasília, DF*.

São Paulo (Estado). Secretaria da Educação. (2019). *Sumário Executivo SARESP 2019*. Secretaria da Educação do Estado de São Paulo.

Brooke, N., Fernandes, N. S., Miranda, I. P. H. de & Soares, T. M. (2014). Modelagem do crescimento da aprendizagem nos anos iniciais com dados longitudinais da pesquisa GERES. *Educação e Pesquisa, 40*(1), 77-94.

Coleman, J. S., Campbell, E. Q., Hobson, C. J., McPartland, J., Mood, A. M., Weinfeld, F. D. & York, R. L. (1966). *Equality of Educational Opportunity* (Vol. 1). U.S. Department of Health, Education, and Welfare, Office of Education.

Ferrão, M. E., Beltrão, K. I., Fernandes, C., Santos, D., Suarez, M. & Andrade, A. do C. (2001). O SAEB – Sistema Nacional de Avaliação da Educação Básica: Objetivos, características e contribuições na investigação da escola eficaz. *Revista Brasileira de Estudos de População, 18*(102), 111-130.

Fletcher, P. (1998). *À procura do ensino eficaz*. Ministério da Educação e Cultura, Departamento da Avaliação da Educação Básica.

Goldstein, H. (2001). Modelos de realidade: Novas abordagens para a compreensão de processos educacionais. In C. Franco (Org.), *Promoção, ciclos e avaliação educacional* (pp. 84-99). ArtMed.

Hox, J., Moerbeek, M. & Van de Schoot, R. (2017). *Multilevel analysis: Techniques and Applications* (3ª ed.). Routledge.

Izbicki, R. & dos Santos, T. M. (2018). *Machine Learning sob a ótica estatística*. UFSCar/Insper.

Jesus, G. R. de & Laros, J. A. (2004). Eficácia escolar: Regressão multinível com dados de avaliação em larga escala. *Avaliação Psicológica, 3*(2), 93-106.

Laros, J. A., Marciano, J. L. & Andrade, J. M. de. (2010). Fatores que afetam o desempenho na prova de matemática do SAEB: Um estudo multinível. *Avaliação Psicológica, 9*(2), 173-186.

Laros, J. A., Marciano, J. L. & Andrade, J. M. de. (2012). Fatores associados ao desempenho

escolar em Português: Um estudo multinível por regiões. *Ensaio: Avaliações Políticas Públicas em Educação*, *20*(77), 623-646.

Machado, A. F., Moro, S., Martins, L. & Rios, J. (2008). Qualidade do ensino em matemática: Determinantes do desempenho de alunos em escolas públicas estaduais mineiras. *Revista EconomiA, 9*(1), 23-45.

McCullagh, P. (2019). *Generalized linear models*. Routledge.

Moreira, K. da S. G., Andrade, P. de A. & Begolin, I. P. (2017). Determinantes da proficiência em matemática no Rio Grande do Sul: Uma análise a partir de modelos hierárquicos. *Ensaios FEE*, *38*(1), 7-34.

Palermo, G. A., Silva, D. B. do N. & Novellino, M. S. F. (2014). Fatores associados ao desempenho escolar: Uma análise da proficiência em matemática dos alunos do 5º ano do ensino fundamental da rede municipal do Rio de Janeiro. *Revista Brasileira de Estudos de População, 31*(2), 367-394.

Raudenbush, S. W. & Bryk, A. S. (2002). *Hierarchical linear models: Applications and Data Analysis Methods* (2ª ed.). Sage Publications.

Riani, J. de L. R. & Rios-Neto, E. L. G. (2008). Background familiar versus perfil escolar do município: Qual possui maior impacto no resultado educacional dos alunos brasileiros? *Revista Brasileira de Estudos de População, 25*(2), 251–269.

Rodrigues, F. C. & Monteiro, M. A. A. (2024). Um modelo para avaliação do argumento de prova em contextos de ensino baseado em argumentação coletiva. *Revista Internacional de Pesquisa em Educação Matemática, 14*(1), 1-20.

Rodrigues, F. C. (2023). *Da argumentação a prova: Produção de argumentos matemáticos produzidos por alunos ingressantes em um curso de formação de professores* (Tese de Doutorado). Universidade Estadual Paulista, Bauru, SP.

RStudio. (n.d.).

Soares, J. F., César, C. C. & Mambrini, J. (2001). Determinantes de desempenho dos alunos do ensino básico brasileiro: Evidências do SAEB de 1997. In C. Franco (Org.), *Promoção, ciclos e avaliação educacional* (pp. 121-153). ArtMed.

Soares, T. M. (2005). Modelo de três níveis hierárquicos para a proficiência dos alunos de 4ª série de língua portuguesa do SIMAVE/PROEB-2002. *Revista Brasileira de Educação, 29*, 73–87.

Soares, T. M. & Mendonça, M. C. M. (2003). Construção de um modelo de regressão hierárquico para os dados do SIMAVE-2000. *Pesquisa Operacional, 23*(3), 421–441.

Souza, F. dos S. & Magalhães, D. L. (2024). Análise das percepções sobre avaliação no Instituto do Noroeste Fluminense de Educação Superior da Universidade Federal Fluminense com o suporte do software CHIC. *Revista Internacional de Pesquisa em Educação Matemática*, *14*(1), 1-25.