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Reading, interpretation, and construction of statistical graphs by high school students based on a sequence of activities

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Abstract: This article analyzes the results of a sequence of activities carried out with a basic education class. The teaching sequence included four activities and moments of collective discussion carried out in three meetings and structured by the reading, interpretation, and construction of statistical graphs considering the components of statistical literacy. The research, qualitative in nature, uses the activities the students completed, the recordings of the discussions held, and the researchers' notes registered in a field diary as data sources. The results show gaps in students' understanding of the graphic elements covered, mainly regarding the scale and types of graphics used and inconsistencies in preparing and interpreting statistical graphics. The data show that processes that lead students to understand and critically interpret different types of graphs must be reinforced, given the importance of teaching-learning statistics for citizenship formation.

Keywords: Statistical Graphs. Basic Education. Statistics Teaching. Statistical Literacy.

Lectura, interpretación y construcción de gráficos estadísticos por alumnos de la enseñanza secundaria a partir de una secuencia de actividades

Resumen: Este artículo analiza los resultados de una secuencia de actividades realizadas con una clase de educación secundaria. La secuencia de enseñanza tuvo cuatro actividades y momentos de debate colectivo, hechos en tres encuentros y estructurados por la lectura, interpretación y construcción de gráficos estadísticos, a la luz de los componentes de la alfabetización estadística. La investigación, de naturaleza cualitativa, tiene como fuentes de datos las actividades contestadas por los alumnos, las grabaciones de los debates y los apuntes de las investigadoras registrados en un diario de campo. Los resultados muestran huecos en la comprehensión de los elementos gráficos trabajados, sobre todo en lo que se refiere a la escala y a los tipos de gráficos utilizados, e incoherencias en la elaboración e interpretación de gráficos estadísticos. Los datos muestran que es necesario reforzar procesos que llevan los estudiantes a comprehender e interpretar críticamente los distintos tipos de gráficos, dada la importancia de la enseñanza y del aprendizaje de la Estadística para la educación de los ciudadanos.

Palabras clave: Gráficos Estadísticos. Enseñanza Secundaria. Enseñanza de Estadística. Alfabetización Estadística.

Leitura, interpretação e construção de gráficos estatísticos por alunos do ensino médio a partir de uma sequência de atividades

Resumo: Este artigo analisa os resultados de uma sequência de atividades realizada com uma turma da educação básica. A sequência de ensino contou com quatro atividades e momentos de discussão coletiva, realizados em três encontros e estruturados pela leitura, interpretação e construção de gráficos estatísticos à luz dos componentes do letramento estatístico. A pesquisa, de cunho qualitativo, possui como fontes de dados as atividades respondidas pelos discentes, as gravações das discussões realizadas e as anotações das pesquisadoras registradas em diário de campo. Os resultados mostram que há lacunas na compreensão dos elementos gráficos abordados, principalmente no que diz respeito à escala e aos tipos de gráficos utilizados, além de incongruências na elaboração e interpretação de gráficos estatísticos. Os dados evidenciam que há de se reforçar processos que levem os discentes a compreenderem e interpretarem criticamente diferentes tipos de gráficos, dada a importância do ensino-aprendizagem da Estatística para uma formação cidadã.

Palavras-chave: Gráficos Estatísticos. Educação Básica. Ensino de Estatística. Letramento Estatístico.

1 Introduction

In the current context, statistics plays a highly relevant role in society and is present in the most varied areas of knowledge. The various media use tables and graphs to present information, which requires citizens to seek statistical knowledge to understand the world around them and make conscious decisions.

Statistics uses graphical representation since it allows for organizing, tabulating, describing, comparing, and revealing various data in a summarized way (Guimarães *et al*., 2007). Therefore, it is necessary to better understand how these tools work to allow a critical analysis of the data presented in the most diverse situations.

Statistical graphs are configured as instruments that "help to reason about quantitative information" (Cazorla, 2002, p. 47). Without a doubt, even with a large amount of data, graphs allow for description and summary, enabling exploratory analysis. The author, agreeing with the ideas presented by Pinker (1990), highlights that:

> Graphs are an effective method of communication, as they take effective advantage of cognitive mechanisms, particularly perception. The preference for graphs in communicating information to the detriment of other non-pictorial forms (tables of numbers, lists of propositions, etc.) can be explained by the fact that pictorial presentation is more visually pleasurable. Furthermore, this author cites evidence suggesting that graphic formats present information more pleasantly so people can perceive and reason about it more easily. (Cazorla, 2002, p. 3)

The importance of graphs can also be assessed by their constant use in the most diverse everyday situations.

> In contemporary society, mass media such as magazines, newspapers, and television have frequently used graphs to report on the most varied subjects and mainly the print media has been using them to illustrate their journalistic arguments. However, the author emphasizes that it is necessary to understand that these graphs are directly linked to the intention of those who structure the article and can emphasize, mask, or

> omit particular aspects of the news. (Monteiro, 2006 *apud* Cavalcanti, Natrielli, & Guimarães, 2010, p. 735)

In this sense, Huyssen (2000, pp. 22-23) says, "We know that the media does not transport public memory naively; it conditions it in its own structure and form." Thus, statistics teaching represents a guiding instrument for a better critical understanding of different real-world situations.

We believe, aligned with Carraher, Schliemann, and Nemirovsky (1995), that when interpreting data represented in graphs, one must consider the influence of people's knowledge of the world, which will interfere with the interpretation of the data presented in the graphs. However, several elements related to statistical knowledge can and should be worked on as soon as possible so that we have civic education and are prepared to better interpret the data present at all times in the current situation.

Statistics is a relevant school content and constitutes a cultural element, contributing to more assertive decisions. Moretti and Arruda (2002) state that citizenship is a condition linked to education. "Citizens will not be educated at school to assume rights and fulfill duties, but above all, to be equipped to question social order and exclusion" (Moretti & Arruda, 2002, p. 429).

The publication of the National Mathematics Curriculum Parameters made the importance of statistics teaching more evident (Brasil, 1997). Later, in 2017, with the National Common Curriculum Base (BNCC), a normative document that currently guides teaching in the country, the teaching of probability and statistics became mandatory from the primary to the secondary school (Brasil, 2017). Statistics-related content is distributed throughout the school years in the thematic unit Probability and Statistics.

Despite the advances already achieved, a distinctive look at the teaching of statistics is necessary so that the school can actually provide civic, critical, and reflective education. From this perspective, and looking at graphic representations, it is worth asking: How do basic education students read, interpret, and construct statistical graphs? Therefore, in this study, we analyze the results of a sequence of activities conducted with a basic education class, focusing on reading, interpretation, and construction of statistical graphs in light of the components of statistical literacy.

2 About Statistical Literacy

With the implementation of the BNCC, the need to work on statistics content since the early years of elementary school has become part of the integral education of Brazilian citizens so that they can receive information, understand it, and use it. According to Rumsey (2002), for students to be good statistical citizens and develop researcher skills, they need to understand statistics enough to consume the information they encounter on a daily basis. Furthermore, it is important to know how to think critically about this information and make the best decisions based on it.

According to Gal (2002), the ability to interpret and critically evaluate information and/or statistical data is a fundamental component of statistical education. These ideas, therefore, lead us to the discussion regarding statistical literacy (SL), which the author conceives as:

> a) the person's ability to interpret and critically analyze statistical information, arguments relating to data or stochastic phenomena, which may be found in diverse contexts, and, where relevant, b) the person's ability to discuss or communicate their reactions to this statistical information, such as their understanding its meaning, their opinions about the implications of the information or their thoughts about accepting the conclusions given. (Gal, 2002, p. 2-3, our translation)

In this way, statistical literacy is strongly anchored in developing "the ability to understand and critically analyze statistical results that permeate our lives daily" (Wallman, 1993, p. 1, our translation). We, therefore, recognize the urgent and healthy need to expose students to situations that are not merely applying formulas and rules.

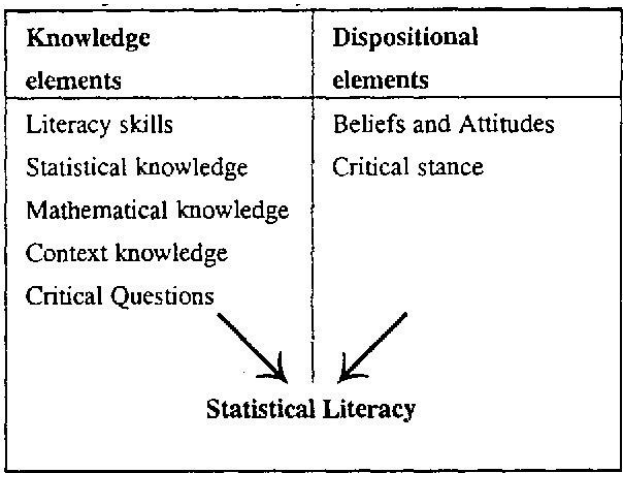
Gal (2002) also proposes an organization through five abilities of basic statistical knowledge that are essential for a citizen to be considered statistically literate:

> 1. Knowing why data is needed and how they can be produced 2. Familiarity with basic terms and ideas related to descriptive statistics 3. Familiarity with basic terms and ideas related to graphical and tabular displays 4. Understanding the basics of probability 5. Knowing how statistical conclusions or inferences are reached. (Gal, 2002, p. 10, our translation)

Given the competencies described above, it is imperative that individuals not only carry out complex calculations and construct graphs and tables but also know that data is necessary, among other things, in decision-making. Furthermore, they must be able to interpret the impacts of information on their lives.

Considering the statistical literacy concept, we have the model proposed by Gal (2002), according to Figure 1, which lists the knowledge-oriented and dispositional elements:

Figure 1: Statistical Literacy Model



Source: Gal (2002, p. 4).

As for the elements of knowledge, *literacy skills* are literacy in its most general sense since statistical messages can be presented through oral or written texts. In this way, literacy and SL are interconnected. We define *statistical knowledge* as knowledge of how data can be produced, why they are necessary, and the conclusions reached, in addition to familiarization with basic ideas and terms of descriptive statistics, including representations in graphs and tables, their interpretation, and basic notions of probabilities. *Mathematical knowledge*, in turn, is the mathematical ideas and procedures involved throughout the processes worked on;

however, it is not the central point, as the teaching of statistics, from a SL perspective, cannot be seen as "mere application of formulas to a set of data that has no real meaning" (Santana & Cazorla, 2020, p. 3). *Context Knowledge* constitutes the meaning and basis for the interpretation of the results obtained. *Critical issues* are related to the critical analysis of statistical information, including data presentation.

Regarding dispositional elements, Gal (2002) highlights that the concepts of *critical stance, beliefs, and attitudes* are interconnected. The *critical stance* is the attitude of questioning the information that reaches us. *Beliefs and attitudes* are the basis for people's critical positioning and involve the interest in "thinking statistically" –and not influenced by particular cases, for example– and the motivation to investigate more and more in search of answers to their questions.

3 Methodological Path

In this work, we will present the results of a teaching sequence presented in a second-grade high school class in a Sergipe state school in the first semester of 2023. According to Santana (2010, p. 113), teaching sequences are "a set of situations created and arranged in such a way that concepts previously selected to be worked on are addressed." To investigate how high school second graders at a state school in Sergipe read, interpret, and construct statistical graphs, we proposed a sequence of activities designed per Gal's (2002) ideas. To this end, we addressed aspects of graphic representations, such as the types of graphs and tables, understanding the information contained in graphs, graph construction, scale, and errors.

The actions performed are part of the deployments of the project *Development of statistical literacy to promote the scientific and technological education of basic education students.* This project resulted from a partnership between the Federal University of Sergipe (UFS), the Anhanguera University of São Paulo (Unian/SP), and the Federal University of Rio Grande (Furg) and was financed by an agreement between the Foundation for Support for Research and Technological Innovation of the State of Sergipe (Fapitec/SE) and the National Research Council (CNPq). One of the objectives of the project is to encourage students to pursue scientific careers, qualify teachers for teaching through scientific research, and strengthen interaction between higher education institutions and primary and secondary schools.

The sequence included four activities and moments of collective discussion when we asked questions about the propositions and aspects of the statistical graphs outlined based on the students' statements. In short, the activities carried out dealt with the interpretation and construction of graphics, as well as aspects of adequacy and intentionality. The activities completed by the students throughout the three meetings, the recordings of the discussions held, and the notes made in the field diary are thus constituted as data sources. In this way, we consider this research qualitative in nature since "the central interest of this research is in an interpretation of the meanings attributed by the subjects to their actions in a socially constructed reality, through participatory observation, that is, the researcher is immersed in the phenomenon of interest" (Moreira, 2002, p. 2).

Twelve pairs were formed to1 participate in the activities and discussions. We emphasize that although the students were teamed in pairs to start the peer discussion, they had individual answer sheets for individual registers that could differ from each other. In our study,

1 The pairs were formed according to the students' wishes and maintained throughout all activities.

for participants' anonymity, the students were designated A1, A2 through L1, L2, each pair with the same initial letter; i.e., students A1 and A2 formed a pair, as did students F1 and F2.

Finally, the collected data, organized by activity, formed our database for analysis, as did the recordings of the discussions and the notes from the researchers' field diary. During this process, we observed the basic education students' resolutions to categorize their solutions in light of the components of Gal's statistical literacy (SL) (2002).

4 The actions carried out: some reflections

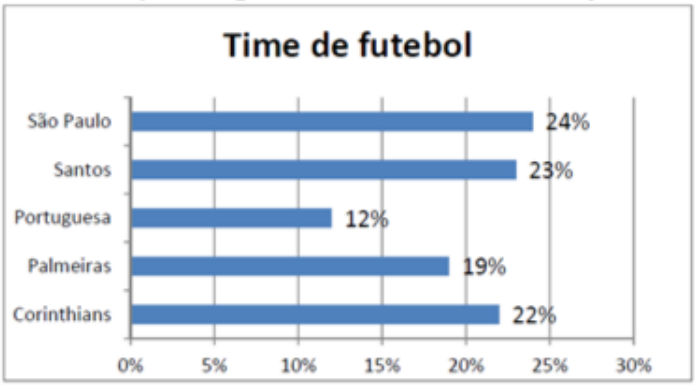
At the first meeting, two activities were performed. Activity 1 verified the students' ability to observe and transcribe a bar graph into a table. They identified whether they knew how to calculate2 percentages and relate absolute and relative values from a presented graph, thus mobilizing *literacy skills* and *mathematical knowledge*.

Figure 2: Activity 1

ACTIVITY 1

A survey was carried out with 300 people from São Paulo about the soccer team. Look at the graph below and build a table with the information collected.

Graph 1: Largest soccer fans in a community



Source: Observatório de Educação

a) Which team was the most mentioned among the interviewees? How many interviewees support this team?
b) Suppose that this survey was expanded, and 100,000 people from all over Brazil were interviewed, asking which São Paulo team supports or has the greatest affinity. If the percentages were maintained, how many people would have responded São Paulo? And Santos?
c) Would you suggest another type of chart? Justify.

Source: Adapted from Sera (2016)

Immediately after completing the activity, students were asked about their first impressions; they considered it easy. Although the statement requested that students represent the data in the listed bar graph in table form, several did not pay attention to the indication, making it necessary for the researchers to reinforce, throughout the activity, that the table construction was also part of the activity. Seventeen students filled in tables, five indicating the

2 Students were allowed to use calculators.

percentages already shown in the graph. Two students indicated the absolute values in the table, and ten students recorded the absolute and relative values as below.

Figure 3: C2's answer [3]

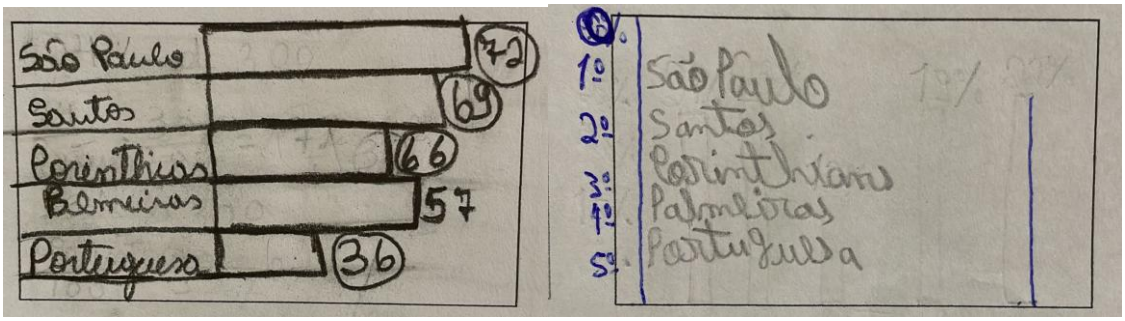
Teams	Percentage %	People
São Paulo	24%	72
Santos	23%	69
Portuguesa	12%	36
Palmeiras	19%	57
Corintians	22%	66
Total	100%	300

Source: Research Collection

Students showed signs of *statistical knowledge* when given the graphic representation in the activity. The ten students who still indicated absolute values based on the statement in Activity 1 realized that, as it was a survey carried out with 300 people, it would be possible to find the respective absolute values. In this case, we can highlight that the students showed *mathematical knowledge* and *literacy skills*, taking the available data as references.

Four students did not understand the task and constructed another inconsistent graphical representation. Three students only wrote down the information in the graph in question, as shown in Figure 4. In those cases, we could not verify the *literacy skill* of understanding the table construction indication. In the construction of other graphs, we noticed gaps in the *statistical knowledge*. Such gaps remain evident in the subsequent activities when, in fact, the construction and/or interpretation of statistical graphs was requested and will be analyzed in greater depth below.

Figure 4: D1's and A1's answers



Source: Research Collection

Drawing a parallel with teachers' answers in Sera (2016), in both cases, we observed representations that indicated only relative frequencies and representations that signaled absolute and relative frequencies, in addition to cases that showed a lack of understanding of what was requested, resulting in the construction of a column chart.

[3] The students' protocols were rewritten because of the translation, in addition, we chose to reproduce the spellings of the names of the soccer teams exactly as in the protocols of the participants, such as "Corintians", which corresponds to the Corinthians soccer team.

Regarding statistical knowledge, the absence of a title in the graphic representations constructed by students suggests a gap in this component. According to Gal (2002), the component involves familiarity with statistical terms and the representation of graphs and tables.

On the other hand, we found that all the students' answers related to the calculation of percentages were correct, which may suggest focusing on calculation processes and, consequently, mastering aspects linked to *mathematical knowledge*. Although mathematical knowledge is necessary from a statistical literacy perspective, this type of element should not be the only one provided to the detriment of others, as it is insufficient for correct evaluation, analysis, and critical positioning before data and stochastic information.

Regarding the suggestion for other types of graphs, only seven students did so, as, in general, they said the graph presented was adequate.

> *No, because this type of graph makes it much easier to find out which team had the highest percentage. By looking at it, you can already indicate which was the highest or which was the lowest.* (C1, audio transcription, 2023)
>
> *No, because this one is easier to understand*. (L1, audio transcription, 2023)

Furthermore, among the seven students who made suggestions, two suggested the pyramid chart, simply claiming to be another form of representation; two students suggested being in descending order; two others did not understand what was requested and suggested a topic for possible research; and one of the students, H2, suggested a table, justifying that it would have more information. We understand that H2's argument may be related to the content studied in statistics, including frequency tables, which, the students report, contain much information and favor data visualization. One student highlighted that he liked the graph in Activity 1, specifically, as it presented the relative frequency on the bar, avoiding confusion when looking at the scale. However, in general, he preferred the tables: "*If I didn't have the number, we would have to see more or less the size to reach 19* [%]" (H2 when referring to the percentage relative to Palmeiras, audio transcription, 2023).

About the choice between table and graph, student J2 highlighted:

> *I think the graph is simpler. The table has a lot of information put together; and sometimes people get lost in something there, they look at it in the wrong way. The graphic, just by looking at it, you already know more or less what it is.* [...] The *table can confuse the [absolute] frequency with the percentage.* (J2, audio transcript, 2023)

Regarding the answers given in Sera (2016), only 1 of the 15 participants' teachers spoke negatively about indicating another type of graph. For the students in this study, 17 out of 24 did not suggest another type of graph. This situation may be related to students' lack of knowledge of other graphic representations, highlighting gaps in the *statistical knowledge*.

Activity 2 proposed the construction of a graph based on the data presented in a table. According to Gal (2002), it can contribute to the development of both *statistical knowledge* and *mathematical knowledge* in terms of understanding representations and using mathematical procedures and ideas.

Figure 5: Activity 2

ACTIVITY 2

Consider the table below containing data from a survey conducted by the França de Pesquisas Institute regarding local and national fan groups present in our state.

Table: Supporters of Sergipe clubs

Football club	Number of fans	%
Confiança	93,500	54.49
Sergipe	60,940	35.51
Itabaiana	14,300	8.34
Frei Paulistano	2,200	1.28
Estanciano	660	0.38
Total	171,600	100.00

Source: Adapted from Instituto França de Pesquisa, 2020

Now build a graph containing this information.

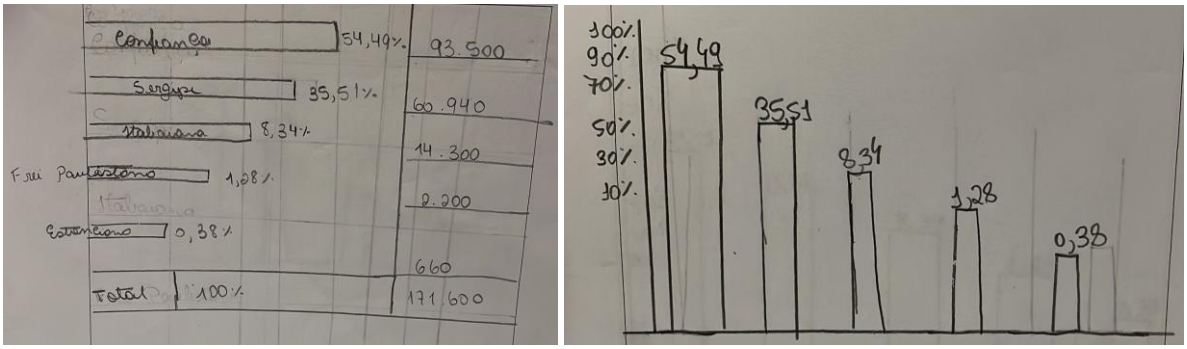
Source: Own elaboration

While analyzing the students' answers, we realized that only bar and column graphs were chosen for representation. Eleven students chose bar graphs, while 13 chose column graphs, and, once again, none presented a title.

Regarding the bar and column graph, it is noteworthy that the students did not differentiate between them in a discussion held after carrying out the activity. In addition to these types of representation, they claimed to remember pyramid charts, "pizza," and "thin line," referring to pie charts and line charts, respectively. Thus, despite the mention of other types of graphic representations, only two were presented in the solutions.

Regarding graph construction, the representations contained errors in the scale, as shown in Figure 6 below. There, the scale is not constructed or does not correspond to equal intervals for equal variations, and the proportion for the respective percentage is not observed. The number of fans was also confused with the percentage in the second graph, as seen below.

Figure 6: Graphics with errors

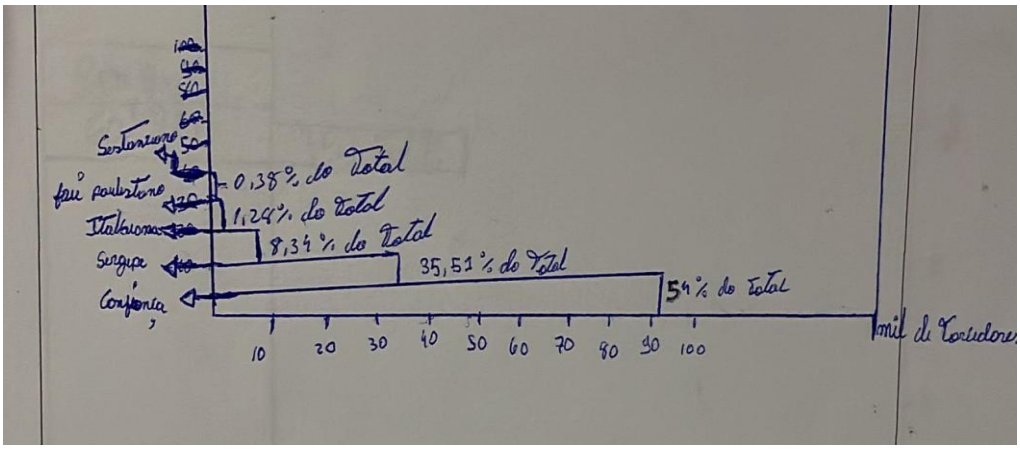


Source: Research Collection

As seen below (Figure 7), C2 made an error when relating the values in percentages and the thousands of fans on the horizontal axis. We noted that C2 indicated the thousands of fans on the horizontal axis, adding the corresponding percentage next to the bar. In the case of Confiança (line 1 of the table), more than 90 thousand fans were correctly indicated, corresponding to 54% of the interviewees. However, only the percentage values were taken as a reference when recording the data relating to the Sergipe and Itabaiana teams (lines 2 and 3

of the Activity 2 table), and the scale was not used properly. Thus, the graph does not show the thousands of fans, 60,940 and 14,300, respectively.

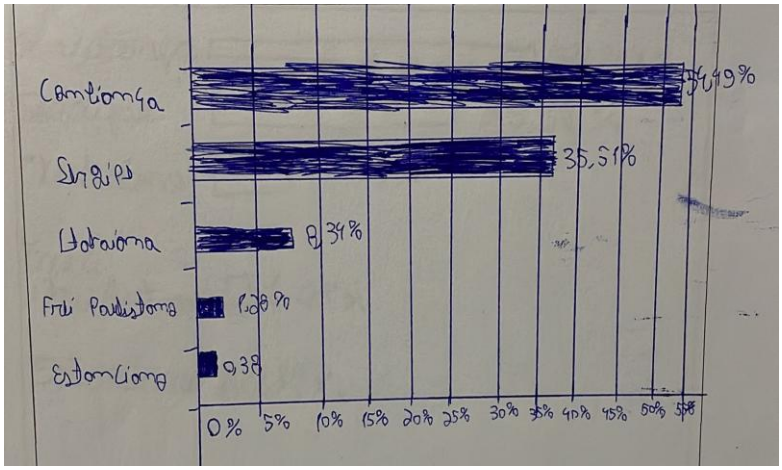
Figure 7: C2's response to Activity 2



Source: Research Collection

Five students presented the data correctly arranged, observing the proportions and scales satisfactorily. Below, in Figure 8, a bar graph exemplifies such cases.

Figure 8: Data arranged correctly in graphical representations

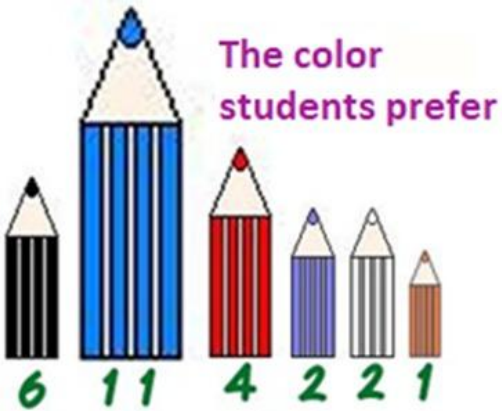


Source: Research Collection

This activity revealed several gaps in the identification and preparation of the characteristics of the statistical graphs, which denotes, once again, difficulties regarding familiarity with the basic terms and ideas related to graphic and tabular displays, which demands a need to develop the SL element called *statistical knowledge*.

In Activity 3, students received a pictogram that presented the students' favorite colors represented by colored pencils. Along with the graph, students received four questions: the type of graph, the information it presented, whether the object chosen for representation was appropriate, and whether it was constructed correctly. Students were then expected to recognize the pictogram and information about the color preference of the pencils. Furthermore, there was an expectation for recognition of the error in its construction due to the scale adopted. This activity can favor the development of the SL components listed by Gal (2002) and encourage reflection on *how the data was produced* and which *ideas related to graphical displays* are necessary for the correct data preparation and understanding.

Figure 9: Activity 3 Chart



Source: Secretaria de Educação do Estado do Paraná *apud* Pietropaolo, Garcia Silva & Amorim (2018, p. 10)

However, the students did not recognize that the graphic representation was a pictogram. Instead, 16 students called it a bar graph, four did not know how to classify it, and four called it a column graph. Furthermore, L2 said it was a "straight graph," and A1 said it was a "pyramid graph." Although A1 classified the graph as a pyramid, "he stated that that representation was not a graph" (Field Diary, 2023), which shows that he still found it difficult to recognize some graphic representations.

This question was also asked in Sera (2016), and although the respondents were mathematics teachers at the time, we can see some similarities in their answers. Regarding the type of graph, "seven participants out of ten considered it to be a column graph" (Sera, 2016, p. 148).

Despite differences regarding the graphic representation, although they used different ways of expressing themselves, all students identified that the information presented by the graphic was *students' color preference*. As this information appears in the image, we can see that the students correctly read the graph, highlighting the presence of the component *literacy skills*.

Most students considered the choice of the pencil to illustrate the graph appropriate, stating that it was "within the context" (A1) of the information and "presented it in an interactive and easy-to-interpret manner" (Field Diary, 2023). Some students, even though they agreed with the choice of pencils, could not justify it. It is interesting to highlight that, during the discussion, student C1 stated that "the pencil symbolizes well, but it could be [replaced by] a stain, a blue stain, [...] but the choice is also great [...], it could be a can of paint, blue, black…" (Field Diary, 2023).

Eighteen students answered that the graph construction was incorrect, while four considered it adequate. The justifications for the errors were diverse, such as considering that the error resulted from the data not being arranged in descending order. However, six students stated that the error was due to the scale, which was the expected answer. In other words, in this case, most students did not demonstrate *statistical knowledge*.

While one student, in his speech, correctly exemplified an error he found —according to him, "the red pencil is bigger than the black one, and it should be the other way around" (E2, Field Diary, 2023)— two others stated that "the image does not present characteristics of a graphic" (Field Diary, 2023), reiterating the lack of familiarity with pictograms. Once again,

we can observe some gaps in the *ideas related to graphical displays* (Gal, 2002) and in *statistical knowledge.*

Another aspect discussed concerns the adequate presentation of the activity. The students pointed out in their speeches that the colors used in the graph were similar and could confuse those who could not distinguish colors well. According to them, it would be interesting for the name of the color to appear above the pencil to facilitate interpretation by students who have disorders such as color blindness, which highlights students' *critical stance* before situations that can make reading and interpreting data difficult.

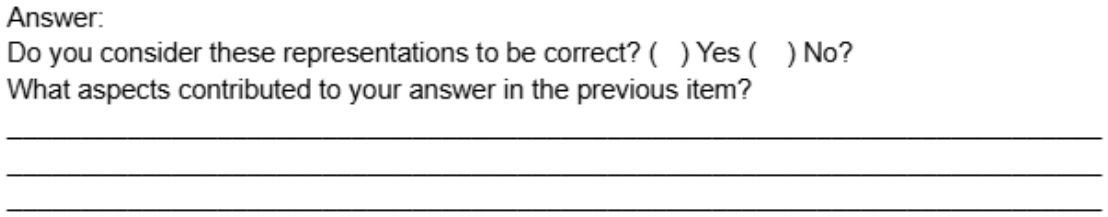
This activity made us reflect on the need to study other graphic representations besides the usual ones, such as bars, columns, and sectors. These become more familiar to students as they are presented more commonly through the media and teaching materials.

In Activity 4, students received different graphic representations: lines, columns, sectors, bars, and pictograms. Among them, there were correctly constructed graphs and others containing errors. Students should identify which graphs were constructed correctly and which contained errors. They also had to justify their choices.

At this point, we highlight the possibility of observing how students interpret and evaluate statistical information. This activity can develop *literacy skills*, *statistical knowledge*, *mathematical knowledge*, *context knowledge*, and *critical issues*.

After presenting each graphical representation, students should say whether it was correct and justify their answers based on elements noted in the statistical graph in question, as explained below in Figure 10.

Figure 10: Activity 4 Questions



Source: Own elaboration

Most students answered item *a* correctly, with more hits than errors. Despite correctly marking the justifications in item *b*, these did not match the error found or were incompatible with the choice in the previous item.

Graph 1 was constructed correctly, while Graph 2, despite presenting the correct scale and values corresponding to those presented in the table, has a label error on the axes, which was detected by 22 and six students, respectively. In the first case, the majority could correctly justify that the graph presented corresponds to the accompanying table. In the second, only six students noticed that, on the horizontal and vertical axes, the labels "Number of fines" and "Politician" appear, incompatible with the information presented in the table.

Of the 18 students who marked item *a* that the second graph was correct, nine justified their choice by the correspondence between the table and the graphical representation, and the others justified it incorrectly. We observed that students mobilized to compare the data listed in the tables and graphs, seeking to capture characteristics that would identify a consistent representation. We thus note elements relating to *familiarity with basic terms and ideas related to graphical and tabular displays.* We also stress the three pairs who noted terms incompatible with the table, showing knowledge about the graphic representation in question; in addition,

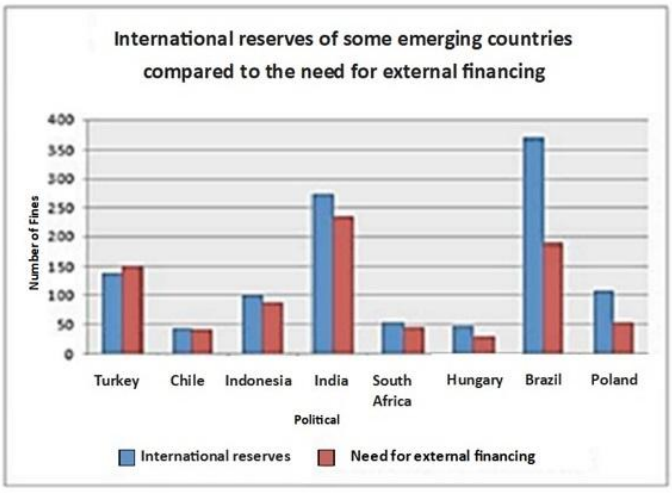
observing the table as a guide for the construction of the graphs, similarly to Activity 2, denotes an understanding of the graph production.

Figure 11: Graphs 1 and 2 of Activity 4

Smartphones sold in Brazil	
Period	Number of smartphones sold (in millions)
1st trimester 2014	10,6
2nd trimester 2014	13
3rd trimester 2014	14
4th trimester 2014	16,2
1st trimester 2015	14,1
2nd trimester 2015	11,3
3rd trimester 2015	10,8

International reserves and the need for external financing in some emerging countries		
Country	**International Reserves**	**Need for external financing**
Turkey	137	150
Chile	41	39
Indonesia	98	86
India	272	234
South Africa	53	44
Hungary	45	27
Brazil	369	189
Poland	107	51

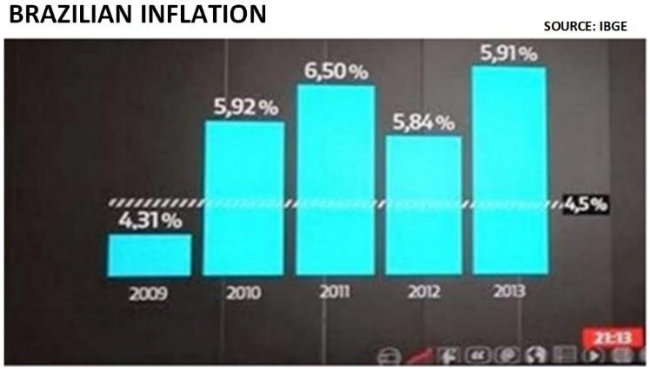
Values in billions of dollars



Source: Research Collection

Graph 3, presented in a television news program (Figure 12), presents a construction error in which the heights of the columns and their percentages are incompatible. Because this difference was quite visible, students were expected to detect the error more easily.

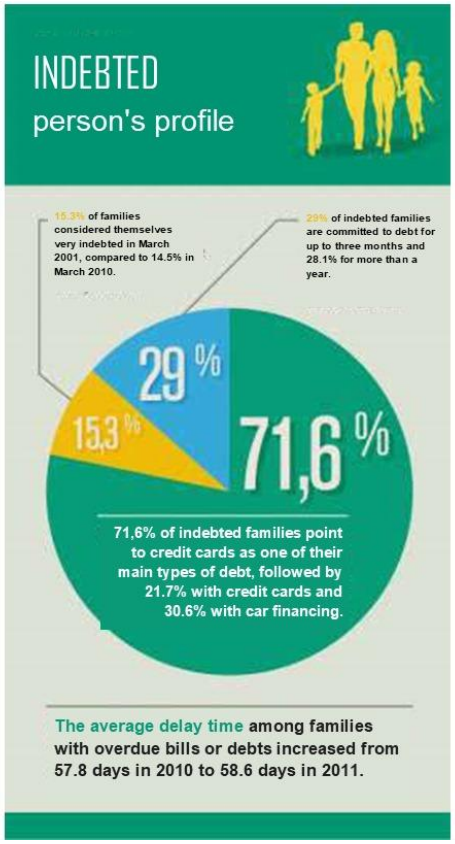
Figure 12: Graph 3 of Activity 4



Source: Research Collection

However, 12 students answered item *a* correctly, indicating that the graphical representation was incorrect, and only four correctly explained the error between the presented percentages and the column heights. Furthermore, four other students stated that the graph was incorrect because a table associated with it was not presented, which shows a frailty in the *statistical knowledge*.

Figure 13: Graph 4 of Activity 4



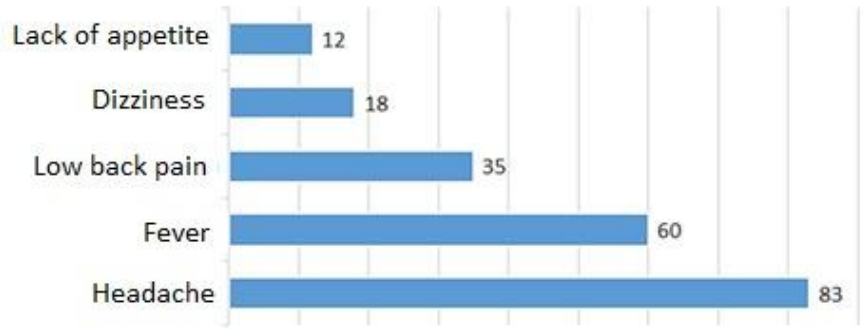
Source: Research Collection

The graphical representation presented, as shown in Figure 13, consists of a sector chart. This graph was constructed incorrectly since the sum of the sector percentages exceeds 100%. In this type of graph, the first expected action to check if the representation is correct would be to add the values.

Data analysis showed that six students performed this addition, found the error, and justified it correctly, showing *statistical knowledge* and *mathematical knowledge*. We also noticed a *critical stance* at this point; once the student thought about checking whether the values were correct, he initially needed to ask himself whether that sum would, in fact, be 100%. The other students classified the graph as correct and presented justifications that did not have a theoretical basis.

Graph 5 (Figure 14) shows the symptoms presented by patients from a sample of 150 individuals in the last six months. Despite being a simple bar graph, a critical detail had to be considered: it was not specified whether each patient would have reported a single symptom, which apparently did not happen since the sum of the values presented in the bars is 208, instead of 150, the total number of individuals interviewed.

Figure 14: Graph 5 - Symptoms presented by patients from a sample of 150 individuals in the last six months



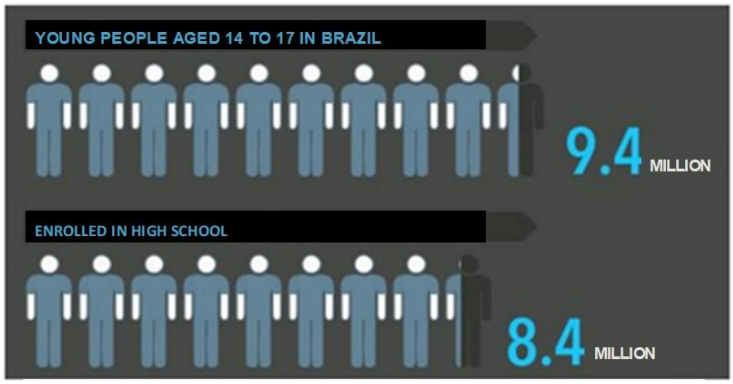
Source: Research Collection

According to the students' answers, we noticed the existence of different understandings. One part added up the values and concluded that it exceeded 150, thus considering the graph incorrect. Thus, they justified that the sum was not 150, which would be enough to consider the graph incorrect. No student questioned whether patients could have mentioned more than one symptom; that is, they did not notice the lack of information or point out conjectures about aspects inherent to the research methodology. However, during the discussion, this question favored an expansion of students' statistical knowledge regarding the *critical issues* and *context knowledge*, as they came to the conclusion that when symptoms are at stake, an individual can present one or more simultaneously and that it would be necessary to know how the research had been carried out.

Finally, Graph 6 (Figure 15) is a pictogram that represents the number of young people and the number enrolled in Brazilian high schools according to the 2010 census. As a pictogram had already been discussed in a previous activity (Activity 3), we expected students to pay attention to the error in the last figure, which presents different painted parts for the same value, 400 thousand. However, 16 students stated that the representation was correct and did not provide a justification or even justified it incorrectly. Eight students said the graphic representation was incorrect but did not provide an adequate reason.

Figure 15: Graph 6 of Activity 4

Figure 11: Number of young people and number enrolled in Brazilian secondary education, according to the 2010 census.



Source: IG News Portal (http://www.ultimosegundo.ig.com.br)

Source: Research Collection

Later, during the discussion, L2 realized that he had marked it wrong and was able to identify the error in the proportions of the representations, as we can see in the following excerpt:

> **P:** *Is this graph constructed correctly?*
>
> **Students together:** *Yes.*
>
> **P:** *But why so? We must give a justification for either affirming or denying it.*
>
> **L2:** *Because of the little people there —look, there are nine little people on top and half of one there— the correct result corresponds. Below, there is eight, and the other… Ah, but the middle one isn't working there… It's not possible below. [...] The one at the bottom wouldn't be 400 thousand, it would be a little less.* (Audio transcript, 2023)

This demonstrates the student's *critical stance* and *mathematical knowledge*, as his consideration involves proportions. It also encompasses *literacy skills* since he could correctly read the data presented in the representation. With this graph, we realized students have difficulty interpreting data presented as pictograms.

5 Final considerations

We found that students present gaps in understanding the graphic elements covered through the proposed activities, especially concerning the scale and types of graphics used. Furthermore, interpreting graphs to analyze possible inconsistencies in their preparation highlights many doubts and uncertainties regarding how they should be. According to Gal (2002), such gaps are related mainly to the elements of knowledge of statistical literacy: *literacy skills*, *statistical knowledge*, and *mathematical knowledge*. As for a *critical stance*, which would allow us to question the representations presented in a well-informed way, we noticed a limitation linked to the fact that they did not have a more extensive understanding of graphics and greater familiarity with ideas related to graphic and tabular displays. In this way, the *critical stance* was affected due to the other elements of the SL.

Our data show that processes that lead students to understand and critically interpret different graphs must be reinforced, given the importance of teaching-learning statistics for citizenship formation. We also highlight that an education attentive to the identification of the basic elements of a graphical representation, the appropriate types of representation and interpretation of the data presented, including those that present incompatibilities in the elaboration of the graphics, and a critical analysis of the forms of presentation, taking into account their intentionalities, are essential for the development of skills related to the SL.

Finally, we could note similarities between the answers given by teachers in Sera's (2016) work and students' resolutions and explanations in the discussions. Therefore, we consider it essential that teachers are included in statistical education formative courses to develop their own and their students' SLs.

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