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RIPEM

The measures of central tendency and dispersion in mathematics textbooks for the final years of elementary education

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Abstract: This article analyses the activities related to measures of central tendency (MCT), measures of dispersion (MD), and their relationships in Brazilian mathematics textbooks intended for the final years of elementary school (6th to 9th grades) approved by PNLD 2020, with the proposed statistical literacy by Gal (2002) as a basis. A page-by-page document analysis was carried out on all 11 collections. We found 1.237 activities. The arithmetic mean and amplitude were the most frequently measured. The nature of the quantitative variable stands out in relation to the qualitative one, and fictitious data is quantitatively greater than real data. The connection between MCTs and MDs is scarcely explored in textbooks. Understanding their link is fundamental for decision making, forming critical citizens, and promoting statistical literacy.

Keywords: Measures of Central Tendency. Measures of Dispersion. Statistical Literacy. Final Years.

Las medidas de tendencia central y dispersión en los libros de texto de matemáticas para los últimos años de la educación primaria

Resumen: Este artículo analiza las actividades relacionadas con medidas de tendencia central (MTC), medidas de dispersión (MD) y sus relaciones en los libros de texto brasileños de matemática destinados a los últimos años de la enseñanza primaria (6º a 9º año) aprobados por el PNLD 2020, con la propuesta de literacia estadística de Gal (2002) como base. Se llevó a cabo un análisis documental página por página de las 11 colecciones. Se encontraron 1.237 actividades. La media aritmética y la amplitud fueron las medidas más frecuentemente. La naturaleza de la variable cuantitativa destaca respecto a la cualitativa y los datos ficticios son cuantitativamente mayores que los datos reales. Está claro que la articulación entre MTC y MD es muy tímida en los libros de texto. Comprender la articulación entre ellos es fundamental para la toma de decisiones, formar ciudadanos críticos y promover la Alfabetización Estadística.

Palabras clave: Medidas de Tendencia Central. Medidas de Dispersión. Literacia Estadística. Años Finales.

As medidas de tendência central e de dispersão nos livros didáticos de Matemática dos Anos Finais do Ensino Fundamental

Resumo: Este artigo analisa as atividades relacionadas às medidas de tendência central (MTC), medidas de dispersão (MD) e suas relações nos livros didáticos de Matemática brasileiros destinados aos anos finais do Ensino Fundamental (6º ao 9º ano) aprovados pelo PNLD 2020,

tendo o Letramento Estatístico proposto por Gal (2002) como base. Foi realizada uma análise documental página a página em todas as 11 coleções. Foram encontradas 1.237 atividades. A média aritmética e amplitude foram as medidas com maior frequência. A natureza da variável quantitativa se sobressai em relação à qualitativa e os dados fictícios são quantitativamente maiores que os dados reais. Percebe-se que a articulação entre as MTC e MD é muito tímida nos livros didáticos. Compreender a articulação entre elas é fundamental para tomada de decisão, formação de cidadãos críticos e promoção do Letramento Estatístico.

Palavras-chave: Medidas de Tendência Central. Medidas de Dispersão. Letramento Estatístico. Anos Finais.

1 Introduction

In a world where data (numbers in context) plays an increasingly significant role, understanding and evaluating statistical information is increasingly important for making decisions, engaging in debate in various situations, and developing a critical world perspective.

Statistics education in schools has focused primarily on measures of central tendency while neglecting the fundamental role played by variability in statistics (Moore, 1997). Without variability, statistics would not even exist. Furthermore, we emphasize that understanding statistics and its function depends on understanding the relationship between concepts.

For Batanero and Godino (2002), MCTs are represented by the set of values around which data are grouped: the arithmetic mean, median, and mode. While dispersion measures describe how close the values are to the central measures, such as amplitude, mean absolute deviation, variance, and standard deviation. Learning about variability is a challenge for students at all stages of education. Therefore, according to Garfield, Delmas, and Chance (2007), it is important to focus on building connections between measures of central tendency and measures of dispersion.

Much research has been developed on measures of central tendency that highlight students' difficulties (Luna & Carvalho, 2019; Fernandes & Junior, 2021; Cazorla *et al*., 2023, among others). Other studies have specifically investigated those measures in textbooks at different levels of education (García-García *et al*., 2021; Kus, 2022; Landtblom, 2023). Likewise, there is research on variability (Oliveira Junior & Pereira, 2018; Pereira *et al*., 2020, among others) and on measures of dispersion (Garcia, 2008; Büscher, 2022). However, we found only one study analyzing MDs in textbooks in the final years (Santana, 2023).

For Kus (2022), although measures of central tendency have a central role in statistical education, there is not much research on textbook analysis on MCTs and MDs in the final years, especially at the international level, which is essential to keep up with changes in a globalized, data-driven world.

The relevance of research into textbooks lies in its decisive influence on the pedagogical choices made by the teacher. This is mainly because the teaching of any topic is shaped by the context, level of difficulty, and format presented by the book (Ortiz, 2002). It is worth noting that the textbook is a significant resource for both students and teachers. Therefore, its use in the daily classroom is very relevant, as the book contributes to students' learning, enabling the acquisition and reflection of school knowledge and, consequently, expanding their understanding of reality. For Perovano, Amaral, and Mazzi (2023), the textbook is one of the primary materials used in classrooms throughout Brazil.

Based on these considerations, our objective is to analyze the activities related to measures of central tendency (MCTs), measures of dispersion (MDs), and their relationships in

Brazilian mathematics textbooks intended for the final years of elementary school (6th to 9th grades) approved by the National Textbook Assessment Program (Programa Nacional de Avaliação do Livro Didático - PNLD).

2 Statistical Literacy

The teaching and learning of combinatorics, probability, and statistics have been fundamental to statistical education. Knowledge of these topics has played a significant role in advancing contemporary society, providing methodological tools to explore possibilities, analyze variability, improve forecasts, and make decisions in situations of uncertainty. Those contributions have led educational systems in Brazil and several other countries to incorporate statistical education into mathematics curriculum guidelines at all levels of education (Coutinho, 2013; Guimarães & Gitirana, 2013; Lopes, 2014; Mendonça & Lopes, 2019; Lopes *et al*., 2019).

Statistical education in Brazil involves a range of research approaches. Guimarães and Gitirana (2013) highlight the analysis of: didactic approaches to statistical and probabilistic concepts and procedures in the curriculum in textbooks; state-of-the-art research; constructions, experimentation, and analysis of approaches to teaching statistics using teaching materials and educational software; and notions and statistical procedures of students and teachers in early childhood education, elementary education, youth and adult education, and higher education; and network assessments.

The theoretical framework of this research is anchored in the statistical literacy perspective proposed by Gal (2002), which refers to the ability to understand, interpret, and use statistical information to understand, interpret results and critically make decisions based on this information, expressing opinions and arguing about data on issues involving uncertainty and risk.

In 2002, Iddo Gal proposed a theoretical model composed of two components that promote statistical literacy: elements of knowledge and elements of disposition. The elements of knowledge comprise statistical knowledge, mathematical knowledge, literacy skills, context knowledge, and critical questioning. The elements of disposition form the critical stance, beliefs, and attitudes.

Chart 1: Statistical Literacy Model

ELEMENTS OF KNOWLEDGE	ELEMENTS OF DISPOSITION
Statistical knowledge Mathematical knowledge Context knowledge Literacy skill Critical questions	Critical stance Beliefs and attitudes
STATISTICAL	**LITERACY**

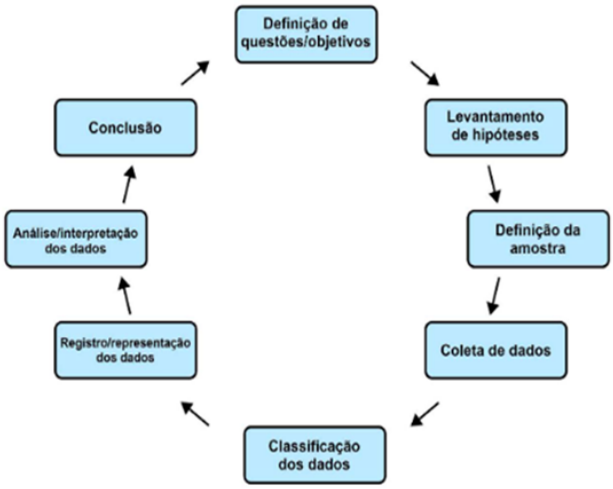
Source: Gal (2002)

Gal (2002) warns that developing only the cognitive dimension is not enough. Still, it is also necessary to develop the attitudinal dimension in understanding the world, in which school knowledge is at the service of forming our thinking, feeling, and understanding of our context.

To develop statistical literacy in students, teachers can experience the investigative cycle. Guimarães and Gitirana (2013) presented an investigative cycle scheme covering several research stages (Figure 1). They propose a cycle that begins with formulating the research question or objective, followed by hypotheses elaboration and reflection on the variables and

their relationships. They highlight the importance of carefully considering the population or sample to be investigated during data collection. After obtaining the data, a crucial phase involves classifying them and choosing the best way to represent them to analyze and compare the conclusions with the initial hypotheses. This suggests the possibility of new questions; thus, the investigative cycle begins again.

Figure 1: Investigative Research Cycle



Source: Guimarães and Gitirana (2013)

The researchers highlight the importance of students experiencing the entire investigative cycle to promote understanding and appropriation of the concepts involved in research and their interrelations. This allows a statistical investigation to be carried out based on the problematization of a situation that can be explored and answered through research, constantly evaluating each phase and its relationship with the whole. On the other hand, they also highlight the importance of moments of delving deeper into one or more phases of the cycle to learn the concepts involved.

Scientific research is evidenced in the Brazilian National Common Curriculum Base (Base Nacional Comum Curricular - BNCC). This normative document provides for the adaptation or systematization of mandatory content common to the country. It establishes the learning objectives that are intended to be achieved by defining competencies and skills that will be used to prepare State and Municipal curricula of public and private Brazilian schools (Brasil, 2018).

Both the BNCC (Brazil, 2018) and the Pernambuco Curriculum for Elementary Education [Currículo de Pernambuco do Ensino Fundamental] (Pernambuco, 2019) present mathematics in five thematic units, one of which is Probability and Statistics. That unit is proposed to be worked on in all years of basic education from the perspective that statistics teaching must develop students' ability to infer statistical information that circulates in society. To this end, it proposes working with the students to make them capable of carrying out a set of actions, including everything from data collection to analysis. This procedure aims for the individual to use statistical concepts to predict and explain phenomena that occur in different everyday situations. Chart 2 presents statistics skills in the final years of elementary school.

Chart 2: Statistics Skills Presented in the Mathematics Curriculum

6th grade of elementary school	Final Years
(EF06MA31) Identify the variables, their frequencies, and their constituent elements	**(EF06MA32)** Interpret and resolve situations involving research data on environmental

(title, axes, legends, sources, and dates) in different graphs.	contexts, sustainability, traffic, and responsible consumption, among other topics, presented by the media in tables and different graphs, and draft written texts to synthesize conclusions.
(EF06MA33) Plan and collect research data regarding social practices chosen by students. Use electronic spreadsheets to record, represent, and interpret information in tables, graphs, and text.	**(EF06MA34)** Interpret and develop simple flowcharts, identifying the relationships between the represented objects (for example, the position of cities considering the roads that connect them, the hierarchy of employees in a company, etc.).
7th grade of elementary school	**Final Years**
(EF07MA35) Understand the meaning of **average** statistics as an indicator of the research trend in meaningful contexts, calculate its value, and relate it intuitively with the **amplitude** of the dataset.	**(EF07MA36)** Plan and conduct research on social reality, identify the need to conduct a census or use a sample, and interpret the data to communicate them through written reports, tables, and graphs, with the support of electronic spreadsheets.
(EF07MA37) Interpret and analyze data presented in pie charts published by the media and understand when its use is possible or convenient.	
8th grade of elementary school	**Final Years**
(EF08MA23) Evaluate the suitability of different graphs to represent a set of research data.	**(EF08MA24)** Classify the frequencies of a continuous variable from a survey into classes so that they can summarize the data appropriately for decision making.
(EF08MA25) Obtain the values of central tendency measures from a statistical survey **(mean, mode, and median)** by understanding their meanings and relating them to the dispersion of data indicated by the **amplitude.**	**(EF08MA26)** Select reasons of different natures (physical, ethical, or economic) that justify sample and non-census research and recognize that sample selection can be done in different ways (simple, systematic, and stratified casual sample).
(EF08MA27) Plan and execute sample research, select an appropriate sampling technique, and write a report that contains appropriate graphs to represent the data sets, highlighting aspects such as **measures of central tendency, amplitude**, and conclusions.	
9th grade of elementary school	**Final Years**
(EF09MA21) Analyze and identify elements in graphs published by the media that can induce, sometimes purposefully, reading errors, such as inappropriate scales, captions incorrectly explained, and omission of important information (sources and dates).	**(EF09MA22)** Choose and build the most appropriate graph (columns, sectors, lines), with or without spreadsheets, to present a given data set, highlighting aspects such as **measures of central tendency.**
(EF09MA23) Plan and execute sample research on social reality and communicate the results through a report that evaluates **measures of central tendency and amplitude,** appropriate tables, and graphs constructed using electronic spreadsheets.	

Source: Brasil (2018)

Measures of central tendency and dispersion appear only from the 7th grade onwards. They relate to understanding and obtaining MCT values relating to the amplitude to draw conclusions about the statistical data, highlighting the research. Introducing them into mathematics classes requires the creation of an environment conducive to investigation, reflection, and action, as these elements are essential in constructing knowledge.

3 Previous studies on MCT and/or MD analysis in books

Cobo and Batanero (2004) investigated 22 textbooks aimed at high school students in Spain. They examined the meaning of the arithmetic mean, analyzing situations, definitions, properties, representations, and arguments associated with this concept. The authors' results indicate that the books focus on the mean mainly in terms of calculation without exploring its important properties. The research reveals the absence of elements in the manuals that could make the teaching of arithmetic averages more meaningful, such as activities that explore the meaning of the average as the best estimate of a value in the presence of measurement errors.

Anjos and Gitirana (2008) examined the concept of arithmetic mean in elementary school textbooks in Brazil. The results indicate that the treatment of this statistical concept is, according to the properties outlined by Strauss and Bichler (1988), satisfactory in certain aspects and inadequate in others. It is important to note that in approximately 75% of the activities analyzed, the arithmetic mean does not correspond to any value within the set. Properties not much explored include: a) the arithmetic mean is located between the extreme values of the distribution and b) the sum of the data deviations in relation to the mean is zero.

Carvalho and Gitirana (2014) investigated the meanings, properties, and representations of the arithmetic mean in Brazilian textbooks anchored in conceptual field theory. Their results indicate that most activities do not encourage reflection on the properties of the arithmetic mean in the studies by Straus and Bichler (1988). The most common meaning attributed to it is as a representative of a set of data, with its most frequent representation being carried out through natural language, commonly found in written text and, sometimes, complemented by formulas and symbols specific to the mathematical dimension of the arithmetic mean. Furthermore, the conceptual field of the mean in the books is fragmented and does not provide an understanding of the mean as a statistical measure.

Ocoró and Ocoró (2016) investigate how measures of central tendency (MCT) are approached in two seventh-grade Colombian textbooks. The results indicate that some properties of MCTs are not treated, the weighted average is not taught, and there is a lack of examples and illustrations linked to the activities. In addition, students are not offered more complex exercises that allow them to delve deeper into these measures.

Landtblom (2018) investigated the definitions and tasks related to the concept of fashion in Swedish textbooks. The results indicate a predominance of procedural tasks and some conceptual ones, but both with quantitative data. This suggests a risk that students' understanding of mode is mainly focused on procedural knowledge and quantitative data due to the significant influence of textbooks on understanding the concepts. There were also examples of inappropriate use of data levels and ambiguity in definitions, which can lead to misconceptions. These findings highlight the importance of clear and precise language in educational materials.

Kus's (2022) investigation looked at how a collection of mathematics textbooks from Australia and Turkey approach measures of central tendency. The analysis considered the organization of concepts over the years, the mathematical and statistical approach to measurements, and the use of context, representations, and technology. The researcher observed

that the order of presentation of concepts differs between the two countries, with both books emphasizing the mathematical aspect more than the statistical one. Australian books often use contexts of everyday life and technology, while Turkish books do not. Those differences point to the need to develop more comprehensive curriculum materials.

Furthermore, Kus (2022) also revealed that the textbooks selected from Australia and Turkey introduced the concepts of mean, median, mode, and amplitude in different school years. In Australia, it started with the concept of mode, which is included relatively early in books compared to Turkey. Then, they moved on to the concepts of mean and median, which is close to the natural progression of students in relation to MCTs. In the Turkish curriculum, the concepts of mean and amplitude are introduced in the sixth grade, and the concept of mode in the seventh grade. Australian textbooks emphasize the relationship between mean, median, mode, and amplitude with various visual representations (e.g. column charts, dots (dot-plot), sectors, or tables).

In a more recent study, Landtblom (2023) analyzed tasks related to mean, median, and mode in 17 collections of Swedish textbooks for students between 10 and 13 years old. Tasks were examined considering context, mathematical properties, input and output objects, and transformations. The analysis reveals that most tasks focus on averaging and procedural transformations with quantitative values. In general, Swedish textbooks focus heavily on procedural knowledge, limiting opportunities for students to develop a thorough conceptual understanding regarding measures of central tendency.

Santana (2023) analyzed activities related to dispersion measures, especially amplitude, in all Brazilian mathematics textbook collections for the final years of elementary school approved by the PNLD 2020. The author noted that although the amplitude is not recommended in the BNCC (2018) for the 6th grade, activities were found in eight book collections. The books for the 7th, 8th, and 9th grades have more questions about amplitude and other measures of dispersion, such as mean absolute deviation, variance, and standard deviation. It was also evident that the books present a small number of activities that relate to measures of central tendency and amplitude, mainly in real contexts or in research.

Thus, this article seeks to analyze the activities related to measures of central tendency (MCTs), measures of dispersion (MDs), and their relationships in Brazilian mathematics textbooks approved by the PNLD 2020 for the final years of elementary school (6th to 9th grades).

4 Methodology

The article presents itself as documentary research carried out through the analysis of previously published materials, such as textbooks. We decided to carry out a census statistical analysis to obtain an explicit or implicit comprehensive view of the activities on MCTs and MDs in the books. In this approach, all four volumes (6th to 9th grades) of each of the 11 textbook collections approved by the PNLD 2020, a total of 44 volumes, were examined page by page. The textbooks were named from A through K, as our objective is to conduct a qualitative analysis of the activities related to the object under study.

The categories that guided the analysis of the textbooks were established based on two sets of content: Measures of Central Tendency and Measures of Dispersion. The MCTs were divided into Arithmetic Mean, Mode, and Median. The MDs were divided into Amplitude, Deviation from the Mean, Absolute Mean Deviation, Standard Deviation, Variance, Variation, and Coefficient of Variation. To analyze these measures, we consider the thematic unit (Numbers, Algebra, Geometry, Quantities and Measures, and Probability and Statistics), the

nature of the data (Real or Fictitious), the nature of the variable (Nominal, Ordinal, Discrete or Continuous), Skills (Compare, Calculate, Interpret), Attitudinal Posture (Decision Making, Critical Positioning, Mathematics, Statistics) and whether it was inserted in specific research.

We list other subcategories in relation to the arithmetic mean since research (Marques *et al*., 2011; Carvalho, 2011) highlights different meanings, invariants, and situations for this concept. This research was based on the meanings defined by Batanero (2000), the invariants proposed by Strauss and Bichler (1988), and graphic, tabular, and written representations.

Regarding the median, the subcategory was used to quantify the sample data, i.e., whether it was an even or odd quantity. This approach is due to studies by Cazorla, Magina, and Santana (2021) and Fernandes and Junior (2021), who noticed that students find it more challenging to determine the median when the sample data is even.

5 Results and Discussion

We identified 1.237 activities that address measures of central tendency and dispersion inside and outside the chapters or sections dedicated to the thematic unit of Probability and Statistics. Table 1 shows that 91.3% of the activities are in the Probability and Statistics thematic unit, and no activities related to the Geometry thematic unit were found. On the other hand, in relation to the school year, activities are concentrated in the 8th grade (49.5%), followed by the 9th grade (22.2%) and 8th grade (21.3%).

Table 1: Percentage of activities related to the thematic unit and school grade

Thematic Unit	6th grade	7th grade	8th grade	9th grade	Total
Numbers	3.0	2.3	0.3	-	69
Algebra	-	0.4	0.2	-	8
Quantities and Measures	2.3	0.2	-	-	32
Probability and Statistics	1.7	18.4	49.0	22.2	1.128
Total	87	264	613	273	1.237

Source: Research data

Under the activities heading, we include exercises for explaining, solving, and constructing data and solving the problem. In Table 2, we observe that exercises for explanation appear throughout the introduction of the concepts and correspond to 69.8%; exercises for solving are the activities proposed after the examples of the concepts, corresponding to 19.6%; and exercises for data construction and resolution refer to questions that students must prepare or research to resolve individually or in groups, 10.6%.

Table 2: Percentage of type of activities related to the school grade

Activity	6th grade	7th grade	8th grade	9th grade	Total
Exercise for Explanation	0.6	3.8	10.7	4.5	242
Exercise for Resolution	6.1	16.3	33.6	13.8	863
Exercises for Data Construction and Resolution	0.3	1.2	5.4	3.7	132
Total	87	264	613	273	1.237

Source: Research data

The textbooks still present an approach strictly focused on solving activities. In turn, it is necessary to clarify that the number of activities that encourage students to create and develop their own questions about MCTs and MDs is limited. The results in Table 2 are consistent with those found in studies by García *et al*. (2021), which also present many questions to be solved.

Regarding the number of activities related to measures of central tendency, Table 3 presents the quantities related to the mean, median, and mode, with a slight predominance of the mean, in line with the object of study prioritized in similar research (Cobo & Batanero, 2004; Carvalho & Gitirana, 2014; García *et al*., 2021; Kus, 2022; Landtblom, 2023).

Table 3: Percentage of activities related to MCT and school grade

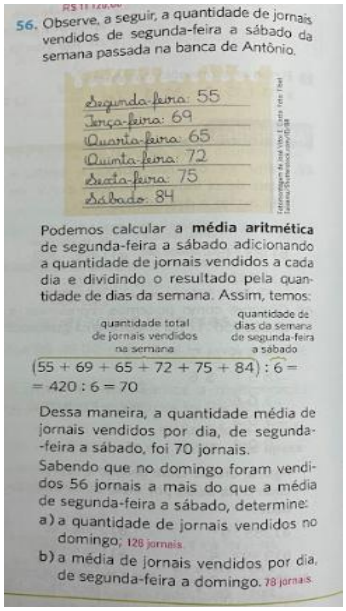
MCT	6th grade	7th grade	8th grade	9th grade	Total
Arithmetic Mean	4.0	17.2	21.7	9.1	493
Mode	-	-	13.6	5.9	187
Median	-	-	13.6	5.6	184
Weighted Arithmetic Mean	-	3.4	3.9	1.0	81
Average Value	0.3	0.7	-	-	10
Total	42	200	504	209	955

Source: Research data

Although the BNCC does not recommend that 6th-grade mathematics textbooks address arithmetic mean, we found activities proposed in the thematic axis Numbers. The activities proposed after addition, multiplication, and division involve arithmetic mean, focusing on calculating the algorithm instead of understanding it as a statistical measure.

Figure 2 presents an example of an activity explaining the arithmetic mean algorithm and then asking students to calculate the average value. Of the 11 collections analyzed, only collection H, in the 6th-grade volume, does not include activities related to MCTs or MDs.

Figure 2: Activity on the concept of arithmetic mean in the 6th grade


56. Observe, a seguir, a quantidade de jornais vendidos de segunda-feira a sábado da semana passada na banca de Antônio.

Segunda-feira: 55
Terça-feira: 69
Quarta-feira: 65
Quinta-feira: 72
Sexta-feira: 75
Sábado: 84

Podemos calcular a **média aritmética** de segunda-feira a sábado adicionando a quantidade de jornais vendidos a cada dia e dividindo o resultado pela quantidade de dias da semana. Assim, temos:

quantidade total de jornais vendidos na semana / quantidade de dias da semana de segunda-feira a sábado

$(55 + 69 + 65 + 72 + 75 + 84) : 6 =$
$= 420 : 6 = 70$

Dessa maneira, a quantidade média de jornais vendidos por dia, de segunda-feira a sábado, foi 70 jornais.

Sabendo que no domingo foram vendidos 56 jornais a mais do que a média de segunda-feira a sábado, determine:

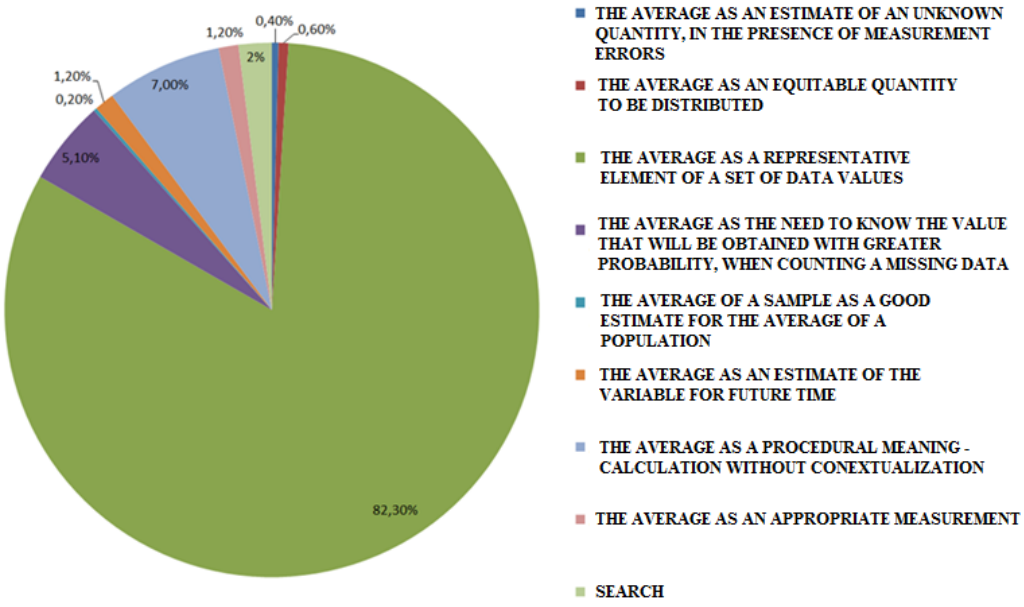
a) a quantidade de jornais vendidos no domingo; 126 jornais.

b) a média de jornais vendidos por dia, de segunda-feira a domingo. 78 jornais.

Source: Coleção F (2018, 6º ano, p. 50)

For the subcategories of the arithmetic mean, we analyzed the meanings presented in Batanero (2000) and Carvalho (2011). We also added the "Research" category for activities involving research development.

Graphic 1: Arithmetic mean and its meanings



Source: Research Data

Graph 1 presents the percentages of meanings found in questions relating to the arithmetic mean. Most activities (82.3%) use the average to represent a set of data values. The average as a procedural meaning, i.e., calculation without contextualization, appears in only 7%. Thus, the collections did not change their focus over the years since Carvalho (2011) obtained very similar results years before, with 86.8% the average as a representative element of a set of data values and 8.6% the average as procedural meaning.

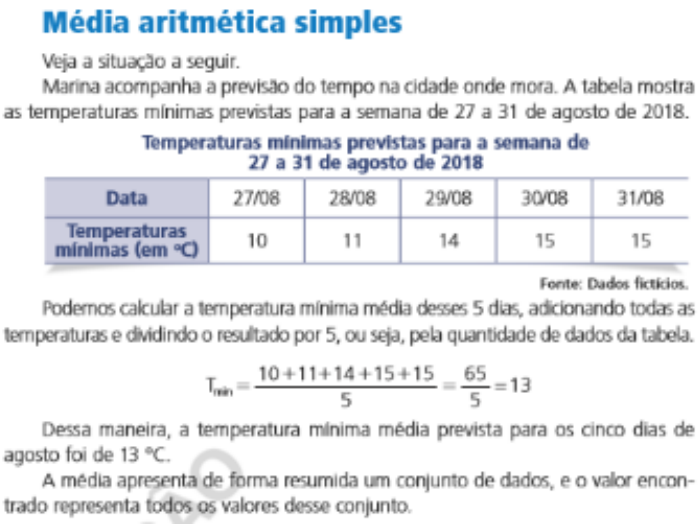
Figure 3 presents an example of an activity in which the average value has the representative element of a set of data on the temperature of a city as meaning.

Regarding the properties by Strauss and Bichler (1998) in relation to the arithmetic mean. Graph 2 shows that 62.5% of the activities related to it encompass a property in that the average does not necessarily need to coincide with one of the values from which it was calculated. For this property, Figure 3 serves as an example, as the average was calculated in relation to temperatures 10º C, 11º C, 14º C, 15º C, and 15º C, with an average of 13º C, which is not necessarily equal to any of the values from which it was calculated.

The average as a representative value of the data, i.e., the value closest to all, presented 19.4% of the activities. Then, the average is influenced by one and all values, which consists of 8.9% of the activities.

Regarding the type of representation, the activities on arithmetic mean were classified as tabular or written graphs. Thus, according to Table 4, the representation that appeared most frequently in 46% of the activities was tabular, followed by written natural language (38.2%) and graphic (15.8%).

Figure 3: Example of average as a representative element of a set of data values



Média aritmética simples

Veja a situação a seguir.

Marina acompanha a previsão do tempo na cidade onde mora. A tabela mostra as temperaturas mínimas previstas para a semana de 27 a 31 de agosto de 2018.

Temperaturas mínimas previstas para a semana de 27 a 31 de agosto de 2018

Data	27/08	28/08	29/08	30/08	31/08
Temperaturas mínimas (em °C)	10	11	14	15	15

Fonte: Dados fictícios.

Podemos calcular a temperatura mínima média desses 5 dias, adicionando todas as temperaturas e dividindo o resultado por 5, ou seja, pela quantidade de dados da tabela.

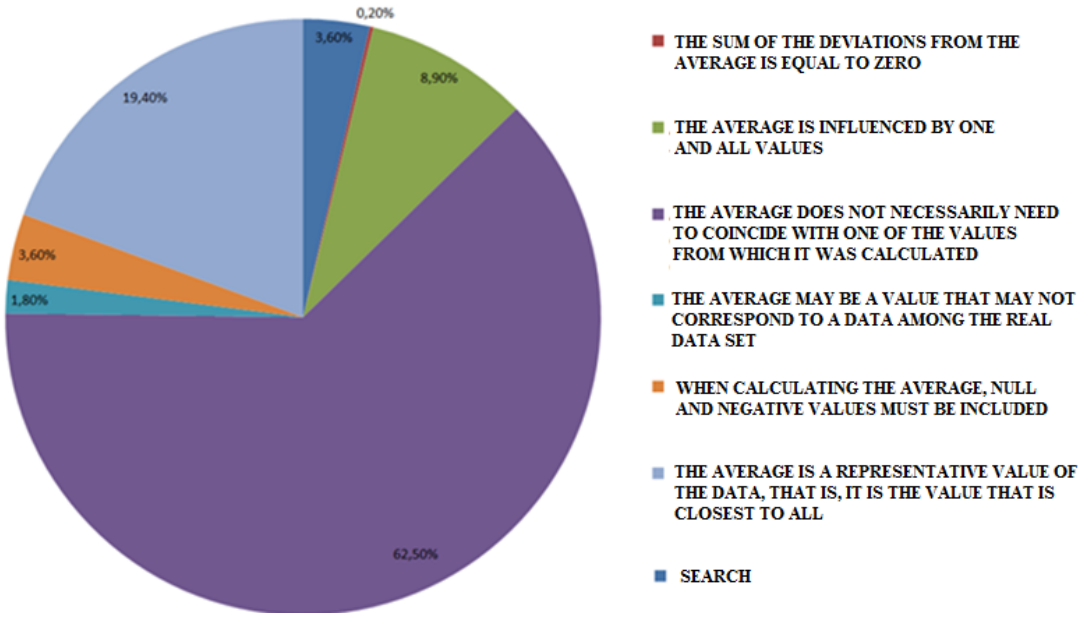
$$T_{min} = \frac{10+11+14+15+15}{5} = \frac{65}{5} = 13$$

Dessa maneira, a temperatura mínima média prevista para os cinco dias de agosto foi de 13 °C.

A média apresenta de forma resumida um conjunto de dados, e o valor encontrado representa todos os valores desse conjunto.

Source: Coleção H (2018, 8º ano, p. 218)

Graph 2: The arithmetic mean and its properties



Source: Research Data

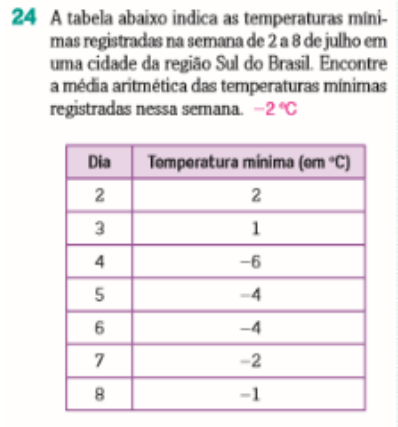
Table 4: Arithmetic mean activities and the type of representation

Representation	Percentage	Total
Graphic	15.8	78
Tabular	46.0	227
Written	38.2	188
Total	100.0	493

Source: Research data

Figure 4 shows the request to determine the arithmetic mean as an example of an activity with tabular representation. Furthermore, with this example, we can verify the property that says that when calculating the average, null and negative values must be considered.

Figure 3: Example of average as a representative element of a set of data values



24 A tabela abaixo indica as temperaturas mínimas registradas na semana de 2 a 8 de julho em uma cidade da região Sul do Brasil. Encontre a média aritmética das temperaturas mínimas registradas nessa semana. −2 °C

Dia	Temperatura mínima (em °C)
2	2
3	1
4	−6
5	−4
6	−4
7	−2
8	−1

Source: Coleção C (2018, 8º ano, p. 81)

The measure of central tendency mode is the only one that can be applied to qualitative and quantitative variables (Groth & Bergner, 2006). Table 5 shows that in 56.2% of the activities on mode, the nature of the variable is continuous, and 33.7% is discrete. Those data corroborate the study developed by Landtblom (2018) in which activities with quantitative data predominate.

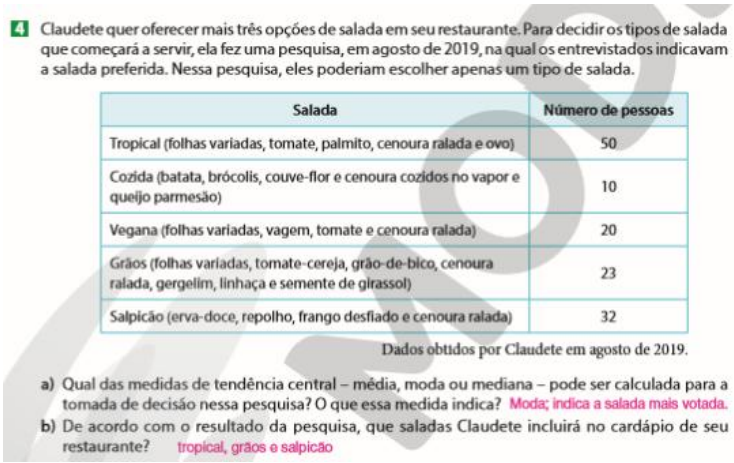
Table 5: Percentage of activities related to mode and the nature of the variable

	Nominal	Ordinal	Discrete	Continuous	Unspecified	Total
Mode	4.8	0.5	33.7	56.2	4.8	
Total	9	1	63	105	9	187

Source: Research data

To exemplify, we chose the activity in Figure 4, where the mode is determined for nominal qualitative data. Questions of this type are essential to ensure that students' learning is not limited when working on the concept of mode with only quantitative data. Furthermore, this activity expands the discussion for decision-making, as the restaurant owner will need to discover which type of salad she will put on her menu based on her customers' feedback.

Figure 4: Example of activity on nominal qualitative variable mode



4 Claudete quer oferecer mais três opções de salada em seu restaurante. Para decidir os tipos de salada que começará a servir, ela fez uma pesquisa, em agosto de 2019, na qual os entrevistados indicavam a salada preferida. Nessa pesquisa, eles poderiam escolher apenas um tipo de salada.

Salada	Número de pessoas
Tropical (folhas variadas, tomate, palmito, cenoura ralada e ovo)	50
Cozida (batata, brócolis, couve-flor e cenoura cozidos no vapor e queijo parmesão)	10
Vegana (folhas variadas, vagem, tomate e cenoura ralada)	20
Grãos (folhas variadas, tomate-cereja, grão-de-bico, cenoura ralada, gergelim, linhaça e semente de girassol)	23
Salpicão (erva-doce, repolho, frango desfiado e cenoura ralada)	32

Dados obtidos por Claudete em agosto de 2019.

a) Qual das medidas de tendência central – média, moda ou mediana – pode ser calculada para a tomada de decisão nessa pesquisa? O que essa medida indica? Moda; indica a salada mais votada.

b) De acordo com o resultado da pesquisa, que saladas Claudete incluirá no cardápio de seu restaurante? tropical, grãos e salpicão

Source: Coleção B (2018, 9º ano, p. 95)

For the concept of median, when analyzing the activities, we considered the amount of data observed. This is because, to determine the median and organize the data into a list, the

amount of data influences how it is calculated. Thus, we checked whether the quantity was even or odd in all activities. However, activities that involved research, construction, and elaboration were classified as unspecified. Table 6 shows a greater concentration of activities with even data to the detriment of odd amounts.

Table 6: Percentage of activities related to the median and amount of data

	Odd	Even	Unspecified	Total
Median	34.3	56.5	9.2	100.0
Total	63	104	17	184

Source: Research data

Figure 5 presents an activity in which students were asked to determine the median of an odd amount of data. The concept of median is simpler for those cases, but one must remember to organize the data into a list.

Figure 5: Example of activity on median for odd amount of data

9. Em uma partida do campeonato escolar de basquete, estavam em quadra as jogadoras de um dos times, cujas alturas estão indicadas ao lado.

a) Qual dessas jogadoras é a mais alta? Eliana.

b) Determine a média, a moda e a mediana das alturas dessas jogadoras. Média: 155,8 cm; moda: 157 cm; mediana: 157 cm.

c) Em certo momento da partida, Eliana foi substituída por Isabele, que tem 165 cm de altura. Estime o que ocorreu com a média, a moda e a mediana das alturas das jogadoras do time em quadra após a substituição: aumentou, diminuiu ou permaneceu a mesma?

8. a) Resposta esperada: Da produção de ovos no Brasil nos meses de cada trimestre de 2017.

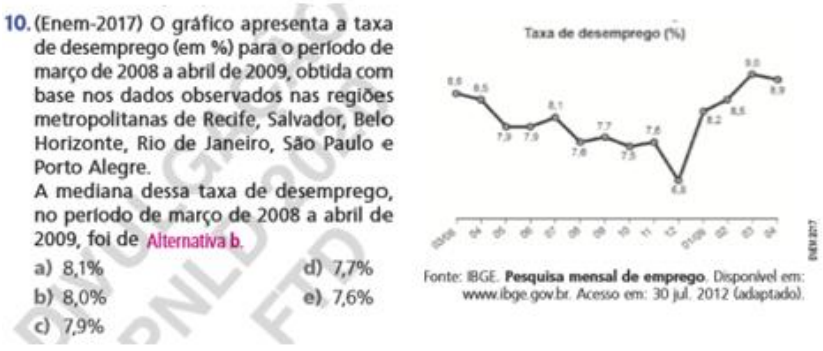
- Agora, realize os cálculos da média, da moda e da mediana das alturas das atletas desse time após a substituição e verifique se suas estimativas estavam corretas.

Média: 156,4 cm; moda: 157 cm; mediana: 157 cm.

Source: Coleção I (2018, 9º ano, p. 202)

Figure 6 presents an activity in which students are requested to determine the median of an odd amount of data. First, students must organize the data and then calculate the average of the two central elements.

Figure 6: Example of activity on the median for an even amount of data



10. (Enem-2017) O gráfico apresenta a taxa de desemprego (em %) para o período de março de 2008 a abril de 2009, obtida com base nos dados observados nas regiões metropolitanas de Recife, Salvador, Belo Horizonte, Rio de Janeiro, São Paulo e Porto Alegre.

A mediana dessa taxa de desemprego, no período de março de 2008 a abril de 2009, foi de Alternativa b.

a) 8,1%
b) 8,0%
c) 7,9%
d) 7,7%
e) 7,6%

Fonte: IBGE. **Pesquisa mensal de emprego**. Disponível em: www.ibge.gov.br. Acesso em: 30 jul. 2012 (adaptado).

Source: Coleção I (2018, 9º ano, p. 202)

It is worth noting that the data in this question are real, taken from the website of the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística - IBGE), which is fundamental, as we must work with real and reliable data so that students understand the meaning of statistics. On the other hand, as indicated in Table 7, we identified

that 83.2% of activities involve fictitious data, and only 14.9% involve real data. Activities that were research or elaboration were classified as unspecified.

Table 7: Percentage of activities by data type

	Real	Fictitious	Unspecified	Total
Data	14.9	83.2	1.9	100.0
Total	185	1.029	23	1.237

Source: Research data

We believe real and meaningful data must be used to understand reality better. This can help students take concrete social actions and convert reflections into effective initiatives. In this sense, the activities will begin to mobilize and promote statistical literacy with quality.

Now, we will present the dispersion measures found in final-year textbooks. Table 8 depicts that the dispersion measure with the highest percentage is amplitude (54.9%), which is emphasized in the 7th, 8th, and 9th grades in the BNCC. However, other measures of dispersion were also found in the activities proposed in the textbooks of the final years, such as deviation in relation to the mean, absolute mean deviation, standard deviation, variance, variation, and coefficient of variation, which are concentrated in the 8th and 9th grades, highlighting the importance of students knowing other measures of dispersion.

Table 8: Percentage of activities related to the MD and school grade

Measurements of dispersion	6th grade	7th grade	8th grade	9th grade	Total
Amplitude	11.0	15.6	16.3	12.0	155
Deviation from the mean	-	0.7	1.8	0.7	9
Absolute mean deviation	-	0.3	3.2	3.2	19
Standard Deviation	-	-	7.1	3.5	30
Variance	-	-	5.0	5.7	30
Variation	5.0	6.4	1.8	-	37
Variation coefficient	-	-	0.7	-	2
Total	45	65	101	71	282

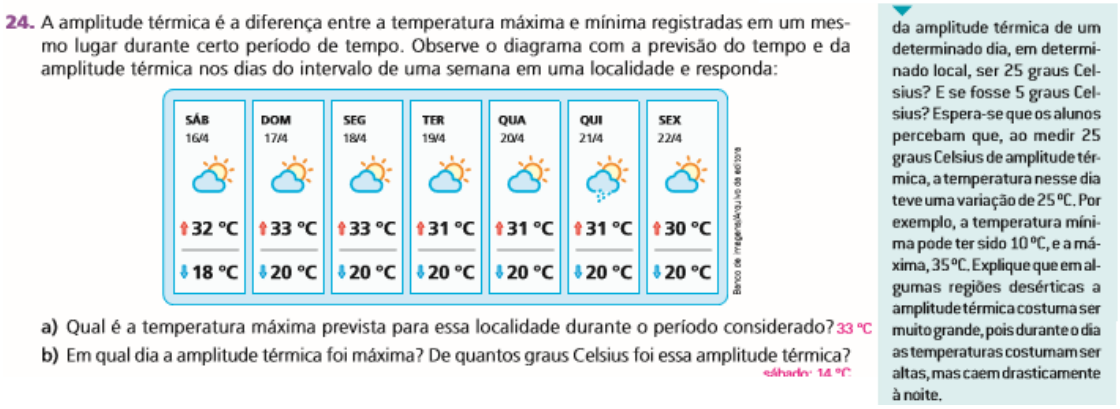
Source: Research data

The term variation is concentrated in the 6th and 7th grades, and the coefficient of variation appeared only in the 8th grade and in a single collection. We noticed that this grade has the greatest concentration of activities on measures of dispersion.

Furthermore, since the 6th grade, we found activities (11%) with ideas associated with thermal amplitude (Figure 7) in eight of the eleven collections. These activities appear in the thematic unit of Quantities and Measures or Numbers, emphasizing only calculation. Furthermore, the U-shaped guidelines suggest that teachers review the concept of amplitude by giving examples if necessary.

Table 9 shows the distribution of questions on amplitude. They appear to be related to MCTs in different situations. The connection between the MCTs and amplitude is very discreet.

Figure 7: Example of activity on amplitude in the 6th grade



24. A amplitude térmica é a diferença entre a temperatura máxima e mínima registradas em um mesmo lugar durante certo período de tempo. Observe o diagrama com a previsão do tempo e da amplitude térmica nos dias do intervalo de uma semana em uma localidade e responda:

SÁB 16/4	DOM 17/4	SEG 18/4	TER 19/4	QUA 20/4	QUI 21/4	SEX 22/4
↑32 °C	↑33 °C	↑33 °C	↑31 °C	↑31 °C	↑31 °C	↑30 °C
↓18 °C	↓20 °C	↓20 °C	↓20 °C	↓20 °C	↓20 °C	↓20 °C

Banco de imagens/Arquivo da editora

a) Qual é a temperatura máxima prevista para essa localidade durante o período considerado? 33 °C

b) Em qual dia a amplitude térmica foi máxima? De quantos graus Celsius foi essa amplitude térmica? sábado; 14 °C

da amplitude térmica de um determinado dia, em determinado local, ser 25 graus Celsius? E se fosse 5 graus Celsius? Espera-se que os alunos percebam que, ao medir 25 graus Celsius de amplitude térmica, a temperatura nesse dia teve uma variação de 25 °C. Por exemplo, a temperatura mínima pode ter sido 10 °C, e a máxima, 35 °C. Explique que em algumas regiões desérticas a amplitude térmica costuma ser muito grande, pois durante o dia as temperaturas costumam ser altas, mas caem drasticamente à noite.

Source: Coleção J (2018, 6º ano, p. 249)

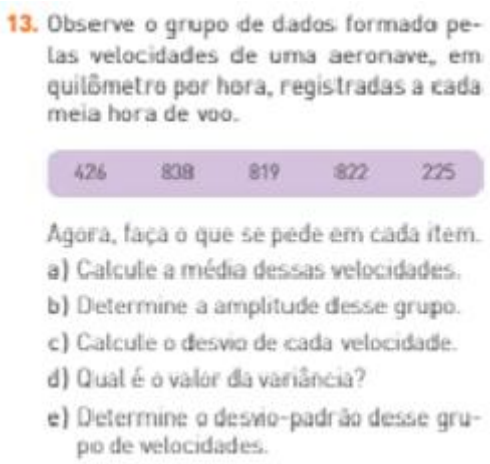
Table 9: Percentage of activities regarding amplitude

	Frequency	**Percentage**
Amplitude	81	52.2
MCT and Amplitude	51	33.0
MCT connected with Amplitude	7	4.5
Research with MCT and Amplitude	16	10.3
Total	155	65

Source: Research data

Figure 8 presents an activity that focuses only on calculating the mean, amplitude, deviation (regarding the mean), variance, and standard deviation without reflection on the reason for those calculations. Such an activity is insufficient, as it does not explore the relationship between data, i.e., it involves just calculating and applying the algorithm.

Figure 8: Example of activity on amplitude and other MDs



13. Observe o grupo de dados formado pelas velocidades de uma aeronave, em quilômetro por hora, registradas a cada meia hora de voo.

426 838 819 822 225

Agora, faça o que se pede em cada item.

a) Calcule a média dessas velocidades.

b) Determine a amplitude desse grupo.

c) Calcule o desvio de cada velocidade.

d) Qual é o valor da variância?

e) Determine o desvio-padrão desse grupo de velocidades.

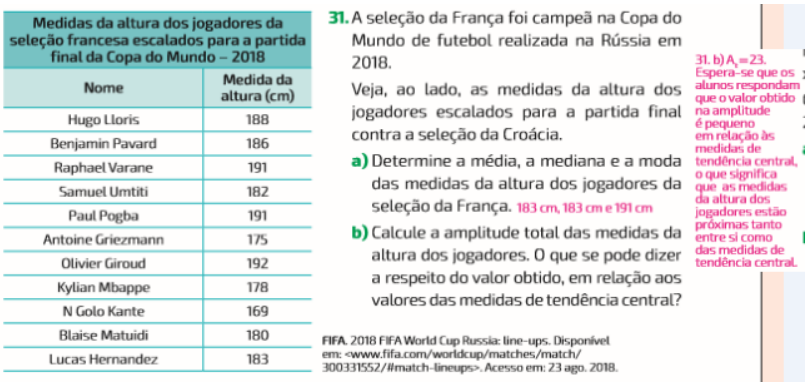
Source: Coleção E (2018, 8º ano, p. 202)

However, teaching guidelines suggest that the amplitude can be used to present the meaning of the MCTs. Therefore, the activities must be well designed so students can better understand the link between the MCTs and the MDs.

Figure 9 presents a situation with real data and establishes a link between the measures.

Gal (2002) emphasizes that students must understand how this measure quantifies data dispersion for decision making. Furthermore, in item b, students are asked to indicate this measure in relation to the MCTs, i.e., students will realize that the smaller the amplitude, the closer the data are to each other and the MCTs, which means that the data are homogeneous.

Figure 9: Example of activity on amplitude and other MDs



Medidas da altura dos jogadores da seleção francesa escalados para a partida final da Copa do Mundo – 2018

Nome	Medida da altura (cm)
Hugo Lloris	188
Benjamin Pavard	186
Raphael Varane	191
Samuel Umtiti	182
Paul Pogba	191
Antoine Griezmann	175
Olivier Giroud	192
Kylian Mbappe	178
N Golo Kante	169
Blaise Matuidi	180
Lucas Hernandez	183

31. A seleção da França foi campeã na Copa do Mundo de futebol realizada na Rússia em 2018.

Veja, ao lado, as medidas da altura dos jogadores escalados para a partida final contra a seleção da Croácia.

a) Determine a média, a mediana e a moda das medidas da altura dos jogadores da seleção da França. 183 cm, 183 cm e 191 cm

b) Calcule a amplitude total das medidas da altura dos jogadores. O que se pode dizer a respeito do valor obtido, em relação aos valores das medidas de tendência central?

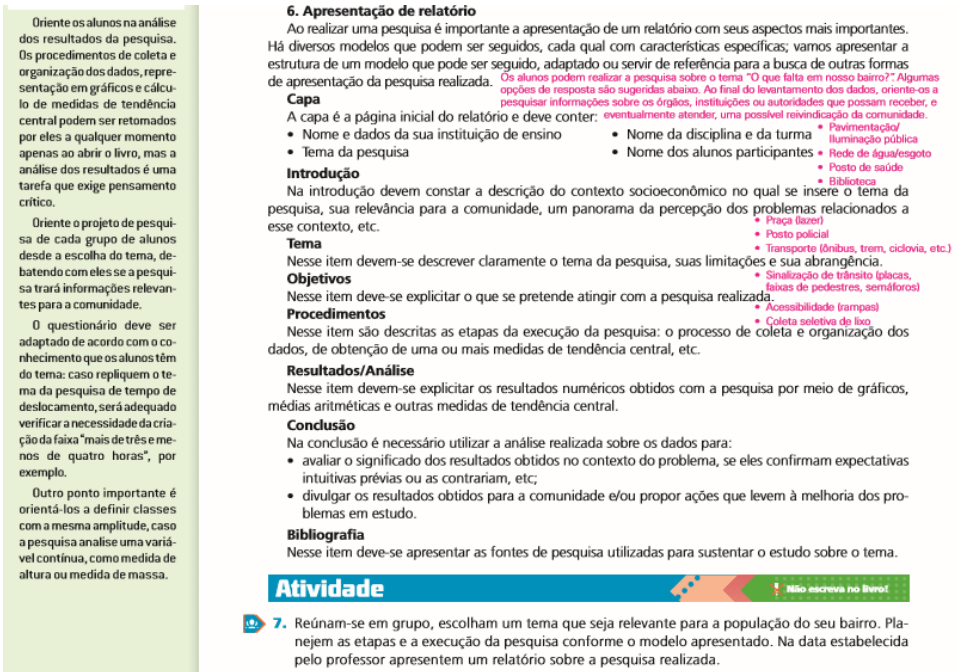
FIFA. 2018 FIFA World Cup Russia: line-ups. Disponível em: <www.fifa.com/worldcup/matches/match/300331552/#match-lineups>. Acesso em: 23 ago. 2018.

31. b) $A_r = 23$. Espera-se que os alunos respondam que o valor obtido na amplitude é pequeno em relação às medidas de tendência central, o que significa que as medidas da altura dos jogadores estão próximas tanto entre si como das medidas de tendência central.

Source: Coleção G (2018, 8º ano, p. 184)

According to the BNCC, experiencing an investigative process can enable the development of statistical thinking. When experiencing the investigative cycle, students must formulate questions involving data from reality, collect, organize, and present information, and observe and interpret situations from the social environment. When analyzing the activities, we realized that the number of activities that involve research as a guiding element to produce statistics is still limited. Figure 10 presents an activity with a model for developing statistical research in the classroom, the request for the analysis of the MCTs, and, of course, its possible link with the MDs.

Figure 10: Activity on research with MCT and MD



Oriente os alunos na análise dos resultados da pesquisa. Os procedimentos de coleta e organização dos dados, representação em gráficos e cálculo de medidas de tendência central podem ser retomados por eles a qualquer momento apenas ao abrir o livro, mas a análise dos resultados é uma tarefa que exige pensamento crítico.

Oriente o projeto de pesquisa de cada grupo de alunos desde a escolha do tema, debatendo com eles se a pesquisa trará informações relevantes para a comunidade.

O questionário deve ser adaptado de acordo com o conhecimento que os alunos têm do tema: caso repliquem o tema da pesquisa de tempo de deslocamento, será adequado verificar a necessidade da criação da faixa "mais de três e menos de quatro horas", por exemplo.

Outro ponto importante é orientá-los a definir classes com a mesma amplitude, caso a pesquisa analise uma variável contínua, como medida de altura ou medida de massa.

6. Apresentação de relatório

Ao realizar uma pesquisa é importante a apresentação de um relatório com seus aspectos mais importantes. Há diversos modelos que podem ser seguidos, cada qual com características específicas; vamos apresentar a estrutura de um modelo que pode ser seguido, adaptado ou servir de referência para a busca de outras formas de apresentação da pesquisa realizada.

Os alunos podem realizar a pesquisa sobre o tema "O que falta em nosso bairro?". Algumas opções de resposta são sugeridas abaixo. Ao final do levantamento dos dados, oriente-os a pesquisar informações sobre os órgãos, instituições ou autoridades que possam receber, e eventualmente atender, uma possível reivindicação da comunidade.

Capa

A capa é a página inicial do relatório e deve conter:

- Nome e dados da sua instituição de ensino
- Tema da pesquisa
- Nome da disciplina e da turma
- Nome dos alunos participantes

- Pavimentação/ iluminação pública
- Rede de água/esgoto
- Posto de saúde
- Biblioteca
- Praça (lazer)
- Posto policial
- Transporte (ônibus, trem, ciclovia, etc.)
- Sinalização de trânsito (placas, faixas de pedestres, semáforos)
- Acessibilidade (rampas)
- Coleta seletiva de lixo

Introdução

Na introdução devem constar a descrição do contexto socioeconômico no qual se insere o tema da pesquisa, sua relevância para a comunidade, um panorama da percepção dos problemas relacionados a esse contexto, etc.

Tema

Nesse item devem-se descrever claramente o tema da pesquisa, suas limitações e sua abrangência.

Objetivos

Nesse item deve-se explicitar o que se pretende atingir com a pesquisa realizada.

Procedimentos

Nesse item são descritas as etapas da execução da pesquisa: o processo de coleta e organização dos dados, de obtenção de uma ou mais medidas de tendência central, etc.

Resultados/Análise

Nesse item devem-se explicitar os resultados numéricos obtidos com a pesquisa por meio de gráficos, médias aritméticas e outras medidas de tendência central.

Conclusão

Na conclusão é necessário utilizar a análise realizada sobre os dados para:

- avaliar o significado dos resultados obtidos no contexto do problema, se eles confirmam expectativas intuitivas prévias ou as contrariam, etc;
- divulgar os resultados obtidos para a comunidade e/ou propor ações que levem à melhoria dos problemas em estudo.

Bibliografia

Nesse item deve-se apresentar as fontes de pesquisa utilizadas para sustentar o estudo sobre o tema.

Atividade — Não escreva no livro!

7. Reúnam-se em grupo, escolham um tema que seja relevante para a população do seu bairro. Planejem as etapas e a execução da pesquisa conforme o modelo apresentado. Na data estabelecida pelo professor apresentem um relatório sobre a pesquisa realizada.

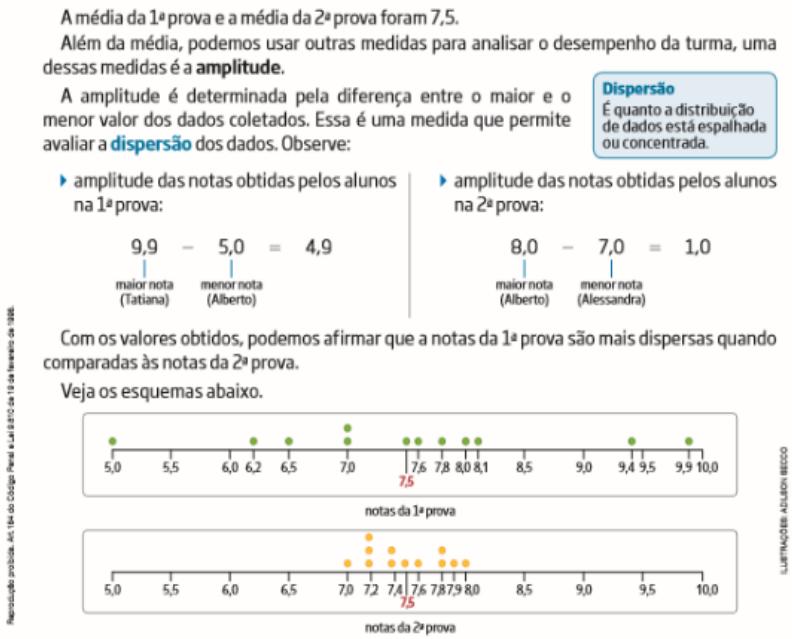
Source: Coleção J (2018, 9º ano, p. 248)

The textbook collections presented research activities focused on the 8th and 9th grades. Only one collection had this kind of activity in the 6th grade. It is important to highlight that research from the investigative cycle can be experienced from the initial years onwards. We

believe such activities promote the investigator's sense of solving a problem or making decisions.

Only Book D offers a remarkable graphical approach to understanding the variability relationship by exploring the dot-plot graphic. This example is relevant for students to understand the correlation between the mean and amplitude of the data, as shown in Figure 11.

Figure 11: Activity connecting mean and amplitude in the dot-plot



Source: Coleção D (2018, 7º ano, p. 283)

Although it is noticeable that in sample 2, the calculated value of the amplitude presents a smaller dispersion, the representation in the dot-plot graph helps us visualize that the yellow dots are more concentrated around the average than the green dots. This type of representation was listed in the study by Kus (2022) when analyzing mathematics books in Australia. The author provided evidence of more types of graphic representations in all classes in the final years. Dot plots provide opportunities for intuitive reasoning about measures of central tendency and variability (Büscher, 2022).

In general, textbooks also focus on activities that only require calculation (76.7%), followed by interpreting activities (22.4%) and activities to compare data (0.9%). Furthermore, few questions promote reflection and understanding of concepts; only one addresses decision making, and ten approach students' critical positioning.

6 Final Considerations

In this study, we analyzed the activities related to the MCTs and MDs and their relationships proposed in all 11 collections of mathematics textbooks for the 6th to 9th grade of elementary school approved by the PNLD 2020. The results highlight how those measures are proposed -and probably taught- in the final years of elementary school in Brazil.

Arithmetic mean activities lead, followed by mode and median, as found in studies by Ocoró and Ocoró (2016), Kus (2022), and Landtblom (2023). The arithmetic mean is presented in most activities as a representative element of a set of values, and most indicate that it does not necessarily need to coincide with one of the values from which it was calculated. Besides, the tabular representation stood out in relation to writing and graphic.

The mode is proposed in most activities with quantitative variables. The context for qualitative variables is still little explored, corroborating Landtblom (2018). The median is proposed with even quantities instead of odd quantities.

Amplitude activities are the majority in relation to other measures of dispersion, despite being proposed in the BNCC. We believe that it is important for students to know other MDs to support their learning about variability in decision making.

Unfortunately, activities that combine MCTs with MDs are still very scarce in textbooks. Therefore, it will be up to teachers to overcome this gap. In addition, working with research that involves the phases of the investigative cycle, using real data in significant problem situations, can be a valuable tool for understanding and solving social problems in which students are involved, providing coordination between MCTs and MDs. The dot plot can be a valuable resource to highlight this relationship.

Thus, considering the statistical literacy proposed by Gal (2002), we believe that understanding situations that lead students to understand MDs and link them with MCTs is key to statistical education teaching and learning.

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