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First Mathematics Education laboratory in Higher Education in Portugal: stories built with words

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Abstract: This article aims to narrate the trajectories of the first Maths Education Labs (MEL) in higher education in Portugal. It seeks, in particular, to understand the perspectives inherent in its creation and evolution, to scrutinize the formative practices that took place there, and to identify the challenges facing its sustainability. It is a narrative investigation that, for data collection, opted for conducting interviews and documents focused on this MEL. Data analysis revealed that the MEL was established in 1989 to support students in initial training as well as teachers, in order to bring innovations to the teaching and learning of Mathematics. Over time, there was a need to revisit its principles, as the old function of supporting the construction of materials was no longer relevant. Among the main challenges are institutional difficulties and resource acquisition.

Keywords: Mathematics Education Laboratory. Teacher Training. Teaching and Learning of Mathematics.

Primer laboratorio de Educación Matemática en la Educación Superior en Portugal: historias construidas con palabras

Resumen: Este artículo tiene como objetivo narrar las trayectorias del primer laboratorio de educación matemática (LEM) en la educación superior en Portugal. Se busca, en particular, comprender las perspectivas inherentes a su creación y evolución, escrutar las prácticas formativas que tuvieron lugar allí e identificar los desafíos que enfrenta su sostenibilidad. Se trata de una investigación narrativa que, para la recopilación de datos, optó por realizar entrevistas y documentos centrados en este LEM. El análisis de datos reveló que el LEM se estableció en 1989 para apoyar a los estudiantes en la formación inicial y a los profesores, con el fin de aportar innovaciones a la enseñanza y el aprendizaje de las matemáticas. Con el tiempo, hubo necesidad de revisitar sus principios, ya que la antigua función de apoyar la construcción de materiales ya no era tan crucial. Entre los principales desafíos se encuentran las dificultades institucionales y la adquisición de recursos.

Palabras clave: Laboratorio de Educación Matemática. Formación de Profesores. Enseñanza y Aprendizaje de las Matemáticas.

Primeiro laboratório de Educação Matemática no Ensino Superior em Portugal: histórias construídas com palavras

Resumo: Este artigo objetiva narrar as trajetórias do primeiro laboratório de educação matemática (LEM) no ensino superior em Portugal. Busca-se, em particular, compreender as perspetivas inerentes à sua criação e evolução, perscrutar as práticas formativas que aí tiveram lugar e identificar os desafios que se colocam à sua sustentabilidade. Trata-se de uma investigação narrativa que para a recolha dos dados, optou-se pela realização de entrevista e por documentos focados nesse LEM. A análise dos dados revelou que o LEM foi estabelecido em 1989 para apoiar alunos na formação inicial e professores, de modo a trazer inovações ao ensino e à aprendizagem da matemática. Com o tempo, houve a necessidade de revisitar seus princípios, uma vez que uma das suas antigas funções, a de apoiar a construção de materiais, já não era mais crucial. No que se refere aos principais desafios, destacam-se as dificuldades ao nível institucional e a captação de recursos.

Palavras-chave: Laboratório de Educação Matemática. Formação de Professores. Ensino e Aprendizagem da Matemática.

1 When opening the doors

Maths education labs[1] can be privileged spaces for the development of transversal and specific mathematical skills, essential for the sustainability of humanity (Pereira *et al*., 2022; Kaushik Das, 2020). These structures can play an important role in training and professional development when present in higher education institutions dedicated to teacher training, with clear repercussions for Mathematics[2] learning by target students (Kaushik Das, 2020).

However, in Portugal, these labs have not been the target of systematic research that allows inference of their true resonance for training and professional development of teachers.

Contextually a Post-Doctoral research is underway at the University of Aveiro, in Portugal, titled *Maths Education Lab (lem@tic) of the University of Aveiro: repercussions for the training and practice of teachers who teach Mathematics*. Its main objectives are as follows: to historicize lem@tic and understand (in the web of constructed narratives) how the proposed training experiences that articulate training, research and extension can promote professional development; to identify concepts presented by teachers working with Mathematics in the 1st Cycle of Basic Education, who participated during initial training in activities proposed by lem@tic, in regard to the work developed by the Lab and its repercussions for their professional development; to analyze how the different training itineraries proposed by lem@tic were given new meanings by teachers and impacted upon the developed pedagogical practices.

The history of lem@tic is inevitably linked to other laboratory spaces that preceded and, consequently influenced it. This text focuses on the first Maths education lab (MEL), in Portugal, in the context of higher education, prior to that will be the main object of study of the ongoing research.

We opted for a narrative approach (Clandinin & Connelly, 2015) of the trajectories of that MEL, giving a voice to those who managed its creation and coordination for decades. This approach allows the conceptions of the people focused on that laboratory, while simultaneously, instigates a retrospective look at our own life stories, including academic and professional experiences, of which this MEL (now) is also a part, promoting itself, alongside intertwining

[1] We distinguish maths education lab from Maths Education Lab, opting for capital letters when referring to a particular lab;
[2] We also distinguish mathematics from Mathematics. With lower case, it refers to the scientific area. With a capital letter, it refers to the curricular area or discipline.

shared narratives.

In the section that concerns the presentation of the results, it was decided to use first-person narration. In some sections episodes report links to the experiences of a particular researcher, while, at others, the reflections are carried out jointly with the co-authors of the text and with authors who theoretically support the study.

2 Building paths

This study was assumed to be qualitative in nature, considering the subjectivity inherent to the research problem (Bogdan & Biklen, 2007) and adopting the perspective explained in Borba (2004) that the truth is not rigid. We agree with the author when he states that the characteristics of a qualitative investigation should not be seen as fixed rules, because the focus of the research itself is constantly evolving, allowing it to be seen from different perspectives.

We opted for a narrative approach (Clandinin & Connelly, 2015); this is valuable for understanding and attributing meaning to lived experiences. But narrative was assumed, simultaneously, as a methodological strategy and as a phenomenon to be investigated (Clandinin & Connelly, 2015).

It should be clarified that the concept of 'experience' proposed by Larrosa (2002, p. 21) was adopted, understanding it as "what happens to us, what happens to us, what touches us". Having *experience* of something is, according to Contreras and Lara (2010, p. 27), being immersed in events or actions that bring their own lessons, their own learning, their own knowledge "and it is a condition of 'experience' to be involved in a doing, in a practice, an immersion in the world that comes to us, that involves us, that commits us or, sometimes, demands of us or imposes on us". By adopting a Larrosian approach, *experience* was assumed to be something that *shudders* us (Larrosa, 2014). Based on this principle, this investigation should not be understood "as 'content' with the intention of exhausting a subject or as a 'thesis' with the intention of establishing a position, but rather as a trace and effect of what we did and what happened to us" (Larrosa, 2019, p. 19). Working from a perspective that places the category *experience* as central, according to the author, means understanding that *experience* does not begin, but rather tries to access it, entering the middle of it, nor does it end, but rather, it continues.

Narrative, in turn, is understood as "a way of using language in the processes that organize life and lived experiences" (Maffioletti, 2016, p. 52). Narrating, as highlighted by Weller and Zardo (2013, p. 132), implies "When considering that individuals interpret the world from a given perspective, from certain interests, motivations, desires, among others, reality cannot be conceived under the principle of universal validity".

In this context, the importance of providing teachers with opportunities to access and reflect on their *experiences* is recognized, mainly through narrative processes that involve life story episodes because, in this way, "reflection on their professional development" (Oliveira, 2011, p. 232).

To collect data as part of the ongoing research, in Portugal a survey was executed of public higher education institutions (HEIs) that provide teacher training courses. Based on this information, the websites of the mapped Universities and Higher Education Schools (ESE) were accessed and teachers linked to Mathematics education were contacted by email to inquire on the existence of a MEL at the institution. Subsequently, contact was established with the coordinators of these identified structures to present the ongoing investigation, and invite them to participate in the study through a narrative interview. As Weller and Zardo (2013, p. 133)

highlight, this type of instrument provides essential elements to understand the "procedural structures of the life courses or trajectories of the researched subjects". Also according to the authors, "the act of remembering and narrating the lived experience in a sequential way allow access to the particular perspectives of subjects in a natural way" (Weller & Zardo, 2013, p. 133).

It was considered important to conduct the interview in person, especially to establish bonds with collaborators, as established by Clandinin and Connelly (2015), and to gain comprehensive experience of the Maths education labs. It was essential to explore the documentary field, because these sources also tell a story. Therefore, during the scheduled visit, the coordinators of the structures were asked for official documents, projects or other files they had created regarding the respective MEL.

The data collected was subject to analysis and interpretation, which also following a narrative perspective. As Creci (2016) highlights, this narrative path takes into account the three-dimensional process, involving temporality (diachrony), personal and social interactions and the place (scenario) where the phenomenon to be investigated and narrated is located. In addition, it attributes meanings to the experiences narrated by research collaborators.

3 Narratives about MEL: telling experiences

In this section, we present, analyze and interpret the results obtained.

It is crucial to highlight the existence of a relationship between the problematic of the ongoing investigation and the life stories of the three authors of this article, which, according to Clandinin and Connelly (2015), gives the investigation the status of a *puzzle*.

From this perspective, we consider it relevant to begin by addressing the relationship, which I myself, as the originator, establish with Maths education labs. This space was chosen as a place of professional activity and as a field of investigation, since I was a teacher trainer, and I thus recognized its potential in the training and professional development of teachers who teach Mathematics. Below is the narrative of the identification path of MELs in public teacher training institutions in Portugal. We end this point around the first of these structures in that country.

3.1 MEL and my life story: building relationships

When carrying out this writing, I refer to my master's thesis, later published in a book (Silva, 2014), and my doctoral thesis (Silva, 2018). In these texts, I reflected on how the topic in question relates to my life story. Revisiting these works now is important, as it allows me to give new meaning and expand upon understandings previously recorded.

I observed since my decision to become a teacher and start the degree course in Mathematics in 2004, I identified with the area of Mathematics Education, recognizing its importance in the process of training teachers in the most diverse school levels. I realized that the reflections provided in the curricular components, throughout the course, and the integration between theory and practice, as highlighted by Ponte (2005) and Delmondes, Lima and Neto (2024), were fundamental to the journey of establishing professional identity and the reframing of teaching and learning Mathematics.

Although I didn't initially intend to be a teacher, I found myself in the process of studying for a degree, and simultaneously taught Basic Education. One of the main reasons for awakening my interest in teaching was the ability to perceive Mathematics as a living dynamic (Silva, 2018). Playfulness is one of the elements that embodies this vision, and I sought this

approach throughout the course and my teaching work, to give new meaning to the usual concept that many students had of Mathematics (Silva, 2014).

Teaching a Basic Education class from the outset of my degree, although challenging, proved to be essential for relating theory and practice throughout the course, which was a great difficulty for several colleagues. To help overcome these difficulties, I invoked situations that occurred in my Mathematics classes that made it possible to highlight this articulation. Revisiting these episodes and reflecting on them, in light of discussions and scientific elements, contributed to the development of essential skills for the formation of my professional identity. I realized that, agreeing with Larrosa (2019, p. 12), "the way-of-being-a-teacher is generated by being a teacher, in a certain way of being a teacher, of playing-a-teacher or of living-a-life - of-teacher in the daily exercise of the 'teacher's job' […]". Thus, in line with Cabrita (2018), we recognize the relevance of establishing an early relationship between the future teacher and the in-service teachers, since the school represents their future space for professional activity.

Thus, during my undergraduate degree, I had my first contact with the Maths education lab at the institution where I was studying. At that time, despite understanding the importance of the interconnection between mathematical, pedagogical elements and professional practice, as highlighted by Ribeiro and Ponte (2020), I realized that the design of this MEL, especially in the way it was presented at that time, was more aligned with a lab/archive deposit approach, as categorized by Rodrigues and Gazire (2015).

I recall an instance when my colleagues and I visited the institution's Maths education lab. The intention of this visit, one of the few carried out during the entire degree, was to learn about the structure of the space and the materials available. We questioned when would we have the opportunity to analyze and use them? Can we take them to our internship activities, considering that many schools do not have access to materials of this type? In our perception, the laboratory was, at least at that moment, more similar to a museum, whose structure and exposed materials we admired, from afar. A 'look but don't touch' scenario?

I was bothered by the lack of opportunities available in this Maths education lab for us to transform every day or intentionally organized situations into meaningful moments of mathematical learning, as Cabrita (2004) argues. To achieve this objective, the author highlights the importance of emphasizing no overuse of complex language and calculation skills, but problem-solving activities, preferably within playful contexts and related to everyday life. The teaching materials offered by this MEL could help to embody these intentions.

Revisiting this frustration led me to reflect on what Larrosa (2019, p. 27) pointed out - "the school is to the teacher what the bakery is to the baker, the kitchen is to the cook or the shoe is to the shoemaker: your workshop, your laboratory". School culture must always be understood as essential for teacher training, training itineraries and training structures. Therefore, we understand that there should not be a Maths education lab disconnected from the school environment. must be 'school'.

Driven by the conviction that a Maths education lab should constitute a training environment, enhancing training and generating impacts on professional development (Silva, 2020), I completed the course and continued teaching in Basic Education. In 2010, she took on the position of professor and trainer of teachers who teach Mathematics at the Universidade do Estado da Bahia, also becoming the institution's MEL coordinator.

Motivated by the challenges inherent to this role and by understanding the importance of the Maths education lab for teacher training, I have since sought to transform it for undergraduate students, with training experiences different from those I experienced during my

Initial formation. It is the impetus behind my interest in undertaking the ongoing post-doctoral internship.

3.2 In search of MELs in higher education institutions in Portugal

Motivated by the following question: *Which public higher education institutions in Portugal offer teacher training courses that have Maths education labs?*, as an initial response, we mapped these institutions. In relation to the 24 HEIs identified, we explored their respective websites and sent emails to the respective Mathematics teachers who taught on teacher training courses. We asked about the presence of a MEL at the institution and similar structures in other Portuguese HEIs. We found that many teachers were unaware of the country's Maths education labs. This finding highlights the importance of increasing the visibility of the work developed in existing structures and discussing the impacts of their practices on the training and professional development of teachers who teach Mathematics in Portugal. In this context, we recognize that our developed practices prove to be particularly relevant.

Cross-referencing the responses obtained with the information collected on the websites and made available by the study supervisor made it possible to identify the following Mathematics education laboratories: at the Escola Superior de Educação do Instituto Politécnico de Viana do Castelo (ESE-IPVC), at the Universidade do Minho (UMinho) and at the Universidade de Aveiro (UA).

We did not interpret the geographic proximity of the three facilities, all located in the North of the country, as a coincidence. It is possible that partnerships, at various levels, between those responsible for these structures (at least in their genesis) and their life stories have influenced their creation and the practices that take place there. We will resume this discussion by exploring the narratives.

Subsequently, we established contact with the professors responsible for creation and/or coordination, with the aim of presenting the research project and inviting them to participate, granting us an interview together with documents focused on MEL. All teachers responded positively. We proceeded to set dates and times for visits to the Higher Education Schools and Universities where these labs are located.

This text, due to the established page limit, focuses on the Maths education lab at ESE i de Viana do Castelo (MEL-VC), as it was the first Mathematics education laboratory in Portugal and given by one of the collaborators of this investigation — Isabel Vale in interview. Throughout the narrative, documents (photos and texts) provided by the Professor are mentioned.

We sought to understand what the narratives reveal about the history of the coordinated Lab, particularly in terms of the logic of the conception and evolution of MEL-VC, the activities carried out over time, as well as the future perspectives of its maintenance, expanding upon the understanding of this pioneering structure in Portugal. The interview explored these dimensions.

Throughout the text, we will refer to E | F | T-IV-7-12-2023, relating to the interview (E) given by Isabel Vale on the indicated date, to photos (F) and texts (T) provided by her on the aforementioned date.

3.3 The Viana do Castelo MEL: the first Maths education lab in the context of higher education in Portugal

Isabel Vale agreed a date for my visit to ESE de Viana do Castelo. This meeting would

allow the development of a close relationship with the participant, something necessary in the context of a narrative interview (Clandinin & Connelly, 2015).

Professor Isabel Vale is a collaborator in this research. She has a PhD in Mathematics Didactics and is a retired professor at the Higher School of Education of the Instituto Politécnico de Viana do Castelo (ESE-IPVC). Throughout her career, in the pedagogical and investigative context, she taught classes in initial and postgraduate training courses, promoted numerous ongoing training actions, actively participated in research and extension projects, resulting in a vast publication of reports, articles and books. We note that her research interests are related, to a certain extent, to the work she develops at MEL-VC and that they include, in addition to the training of teachers who teach Mathematics, Mathematics didactics, a special focus on solving problems, patterns, creativity, visualization and connections. She is presently dedicated to developing tasks and strategies, inside and outside the classroom, to promote a more active learning of Mathematics.

I was excited to meet Isabel Vale and the MEL coordinated by her, I leave for ESE-IPVC. The journey, from Aveiro, where I currently reside due to my post-doctorate, to Viana do Castelo, turned out to be a beautiful experience. Contemplating the landscape that presented itself during the journey inspired me to revisit some notes that would guide the interview.

Upon arriving in Viana do Castelo, I was warmly welcomed by the Professor, who insisted on picking me up at the train station and invited me to share a coffee. The kindness and conversation that took place along the way served to bring us closer and, at that moment, I realized that a script would not be necessary to guide the interview.

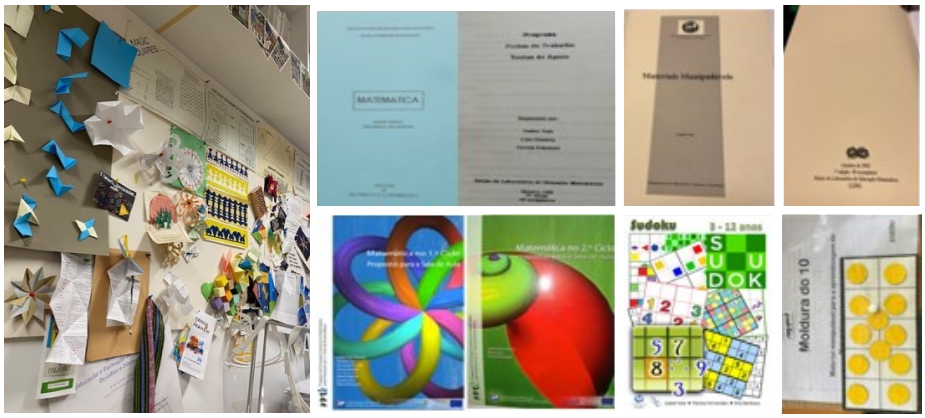
My interview with the Professor was narrated, and (auto) biographically oriented, where a set of previously thought out ideas were present. The Maths education lab is and has always been part of her life path and professional career. When talking about this space, she shows great affection, revealing how her experiences over the last 35 years at the helm of this laboratory have significantly marked her professional development, influencing not only her role as a trainer of teachers who teach Mathematics, but also her teaching practices. research and community outreach.

After having breakfast, we headed to ESE-IPVC. It was a rainy day and, therefore, I was unable to admire, at that moment, the city, which, in my previous research, appeared to be very beautiful. From what I read, it is the capital of the district of Viana do Castelo, which is located in the coastal region of Northern Portugal. It is a city with a very rich history and architecture, beautiful coastal landscapes and cultural traditions.

We arrived at ESE-IPVC, after a brief journey fueled by our conversation. During the journey to the Laboratory, I met some co-workers who also participate in activities in the space. I couldn't access MEL-VC at that time because a class was taking place there. Then, we went to the Professor's office. Upon arrival, I was immediately captured by countless photographs displayed on the room's cabinets. These images told part of his story, including participation in conferences, meetings with colleagues, including other collaborators in this research, and several significant moments, many of them experienced in the Lab itself.

After being introduced to some of these people, even if only through photographs, and some materials produced at MEL-VC, as shown in Figure 1, we sat down and continued our conversation. The productions carried out at the beginning of MEL-VC's operation were mainly publications from various Sebentas in support of the different Curricular Units of the different courses taught at the time at ESE. Another more recent aspect has been the development of some materials such as the '10 Frame' or educational games, the last being the 'Sudoku' game.

Figure 1: Some productions carried out between 1991-2022 and posted in LEM



Source: Own authorship

Interestingly, sitting down seemed to be the act that signaled the formal start of the interview. In fact, the dialogue had already occurred, in a light and affectionate way, during the coffee, the journey to ESE and the Professor's office and the entrance to that space. Many stories had already been shared up to that point. But, from then on, I was introduced to the history of MEL-VC in another way.

3.3.1 Genesis of MEL-VC

Prof Isabel Vale began by revealing her motivations behind creation of the MEL at ESE de Viana do Castelo and the logic underlying this structure, together with principles, pursued objectives, proposed training practices and challenges that faced its implementation.

It commenced in 1982, when the Portuguese State sent a group of higher education professors, (with diverse academic knowledge, to Boston University, the main city in the state of Massachusetts, in the United States of America (USA). They were to obtain master's degrees in different areas of education and would become teachers at the recent Higher Education Schools. At that time, attention was given to the USA, due to its innovative ideas in educational terms. These were inspired by NCTM's *An Agenda for Action* (1980), which defended new perspectives on the teaching and learning of Mathematics, with emphasis on problem solving and the exploration of manipulative teaching materials. The objective of this initiative was an intensive training of these teachers, so when they gained their doctorates, they would lead training courses and influence teaching and learning paths based on their learning. (E-IV-7-12-2023).

In the context of Mathematics, Prof Isabel Vale highlights the participation of Professor Domingos Fernandes. He was one of the professors who completed his master's degree at Boston University, and who, at the time, worked as a teacher at ESE in Viana do Castelo. It was under his leadership and that of other colleagues at the institution that the idea of creating a Maths education lab emerged in 1989, in order to bring innovations to the teaching and learning of Mathematics.

The Viana do Castelo MEL was created with the authorization of the ESE Installation Committee, which provided a room for the installation of the MEL. In 1989, Prof Isabel Vale highlighted that support from the Calouste Gulbenkian Foundation made it possible to equip the MEL with the purchase of teaching materials such as manipulatives and books, as well as some furniture (see Figure 2).

Figure 2: LEM-VC at the time of its creation

Source: F-IV--7-12-2023

Subsequently, between 1990 and 1991, funding granted by the Institute for Educational Innovation allowed the acquisition of more materials, including audiovisual materials. According to your narrative.

> *(...) This entire trajectory began with the use of purchased and other constructed materials, in a very small room. Before the materials became popular, everything was very expensive and had to come from abroad. The Laboratory was also designed to build materials, such as the Geoplan, which, at that time, was made with a wooden board and nails; It was a carpentry activity in the classroom* (E-IV-7-12-2023).

Only in 2004, "with the expansion of the ESE building, the [MEL-VC] moved to a larger space [see Figure 3], with a room consisting of two interconnected spaces, from an open space perspective, having been furnished and equipped with different materials and equipment, including, in particular, computers, [acquired] within the scope of the Continuous Training Program in Mathematics (PFCM) for the 1st and 2nd cycles [of basic education], allowing to become, to date, the largest Maths education lab in the country" (T-IV--7-12-2023).

Figure 3: Main space of the LEM-VC currently and with the two rooms open

Source: Research collection

In relation to the objectives that guided the creation of the LEM, Prof Isabel Vale highlights, when accessing the Lab project, the:

> *Support initial training students and their respective teachers and cooperating teachers of pedagogical practice, from the different courses taught by ESE, within the scope of Mathematics education. On the other hand, establish a connection with the district's schools, responding to the needs of teachers working in Basic Education and possibly Secondary Education and providing teaching material* (T-IV--7-12-2023).

Based on Prof Isabel Vale's understandings and perspectives, we realized that, in the genesis of MEL-VC, there was an understanding of this structure as a physical space that would promote the teaching and learning of Mathematics through its use and, in particular, the use of (re)constructed materials. As indicated by the Professor, "the idea was more of an experiment, construction of materials, because there was nothing" (E-IV-7-12-2023).

"A very important function was to support the schools, as they also did not have materials" (E-IV-7-12-2023). He continues: "I remember the first Mathematics degree students taking the Geoplan to the schools, during the internship, and the teachers commenting 'Here they come and play in the classroom'" (E-IV-7-12-2023).

At that time, MEL-VC sought to give new meaning to these concepts. We tried to find ways to make the use of teaching materials influence teachers' practices and, consequently, the teaching and learning process of Mathematics.

This search also finds support in marks from the Professor's life story:

> *I went to university when I was 16, I got in very early. And I didn't have good teachers there. I had teachers who came in with chalk and turned to the board writing and speaking and only turned around to leave. When I saw that teacher using only the Greek alphabet, the first thing I did was try to know the alphabet, because I didn't even know it, nor knew how to copy it. In other words, there was no dialogue, there was nothing* (E-IV-7-12-2023).

This experience during her initial training seems to have led Prof Isabel Vale to question the nature of Mathematics teaching. She realized that there was potential for something different, for other approaches, incorporating materials that could support teaching practices. In this direction, she continues her reflection:

> *And what problem did teaching have in my time? Expository classes, purely prescriptive. The teacher said - "today we are going to give the area of the rectangle". However, he didn't even explain what the area was - "the rectangle is like this, its sides have such and such lengths and I want to know the area. For this we use this formula and that's it. Imagine that this is a field and I want to know what the area of this field is, just use the formula". And then we would decorate, train... and forget. [...]. I couldn't teach that very theoretical Mathematics without understanding, while I was a secondary school teacher and later at ESE [...]. Because future teachers would take the subjects [during the course] but when they went to teach, they wouldn't know how to explain Mathematics to children [...]. And I started to realize that learning wasn't happening, so teaching had to be different. I tried, whenever possible, to use physical support or drawing. will reinforce my ideas* (E-IV-7-12-2023).

Recognizing the path for students to approach Mathematics could be much more active, effective and affective, than just through memorization, Prof Isabel Vale directed her attention to problem solving and the use of materials to support the educational process. When narrating her stories, we realize that her statements reflect the genuine desire to provide students with more solid and more enjoyable learning and that she believes in the power of these resources in these processes to instill knowledge.

But, she also mentions, for the Mathematics class to be guided by the principle that the

teaching material serves to support learning and for it to be truly consistent, it is crucial that the teacher has adequate training, which allows him to identify the potential, constraints and weaknesses of these materials, as well as the why, how and when of their use in the teaching and learning process (Vale, 2018; Vale, 2002; Vale & Barbosa, 2014).

In fact, selecting or (re)creating and exploring the most appropriate teaching materials for each context demands a series of skills that involve aspects of the disciplinary area, pedagogical and professional practice, as highlighted by Silva (2014). Adopting an approach focused on the use of the aforementioned materials professionalize teaching ability, and represent a significant impact on training and professional development, and reflection experiences teaching Mathematics, as revealed by Cabrita (2004) and Silva (2020). However, such an approach includes a reflection, supported in practice, on training itineraries as well as on the respective support structures, with the Mathematics education laboratory being a privileged environment.

Several investigations related to MEL-VC and developed by Prof Isabel Vale and collaborators, prove the positive impacts of the use of carefully selected teaching materials and included in innovative methodological approaches in learning Mathematics, in the context of teacher training (Barbosa & Vale, 2023; Vale & Barbosa, 2021, 2023; Vale *et al*., 2022;).

As pointed out by Silva (2018), in these courses, the materials perform a dual function. One, they inspire active methodological approaches, when future teachers are in practice; and two, they allow us to overcome conceptual difficulties that many of these students feel concerning Mathematics, and giving new meaning to concepts.

In this regard, Prof Isabel Vale said — “When the student has some difficulty, that's when the teacher comes in — 'is there no other way to do it?'” (E-IV-7-12-2023). From what was shared, we realized their understanding of the material's support in solving tasks and developing mathematical reasoning. On the other hand, “it is not limited to just kids” (E-IV-7-12-2023), guiding, as we have already seen, their practices in teacher training courses. She also mentions the use of teaching material does not diminish Mathematics — “Sometimes, they think that, when using material, we are dealing with lesser Mathematics. But it's not the case! It’s the same Mathematics, just seen in a different way” (E-IV-7-12-2023). Indeed, based on our experiences as trainers, we found that, at times, prejudices persist regarding the use of teaching materials.

After a morning of talking to reach an understanding that the class at MEL-VC would last a little longer, we decided to take a lunch break and then return so I could visit the Laboratory. We chose to have lunch at a restaurant located right in front of ESE-IPVC, as the rain persisted. The restaurant, which was very pleasant, provided an environment conducive to the continuation of our conversation, which covered MEL-VC, professional trajectories and aspects of our personal lives. During lunch, we discussed Portugal, including recommendations of places I could visit during my stay in the country, and I tried to imagine all those scenarios through Isabel Vale's affectionate eyes.

We lost track of time and MEL-VC was busy again for afternoon activities. The Lab would only be released for my visit at the end of the afternoon, but that did not represent a problem. As the main reason for my trip was to meet Professor Isabel Vale and ‘her’ Mathematics education laboratory, I was willing to wait as long as necessary for this to happen. As a result, our conversation had to continue in his office.

We resumed the dialogue about the materials and prof Isabel Vale highlighted recognition that the materials are not the panacea for all the challenges that arise in the teaching

and learning processes of Mathematics, but that there are times when they are necessary. She stated: “there are students who don’t use them because they don’t need them. But now the materials are there. It's needed? Go get it. Because the purpose is to [support] the move from concrete to abstraction. I will not always use materials, only when necessary, because it is more laborious" (E-IV-7-12-2023). This statement highlights the relevance both for the professional practices of future teachers and for the work of the teacher herself.

Developing the ability to discern when it is appropriate to use a certain material and which is most appropriate, considering students' learning levels and spectra, are examples of fundamental skills to be developed – “perhaps manipulating a physical material, at a given moment, for the student, [be] the most interesting. It is the teacher who needs to understand and address this” (E-IV-7-12-2023).

Therefore, as previously discussed, it is essential that adequate training be given which interweaves the disciplinary area, pedagogical and professional practice. This highlights the need for interconnection between the MEL and the school, where the school understanding is fundamental to its action.

3.3.2 Evolution of MEL-VC

The principles and goals of the Viana do Castelo Maths education lab have continued to evolve since conception, adapting to changes in the school and society.

The initial idea of a space for the construction and provision of teaching materials, shared by Prof Isabel Vale when addressing the origin of the Lab, needed to be reevaluated, considering that, over time, school institutions began to build or purchase their individual teaching materials — for example the Ministry of Education started investing in projects and training programs. With the availability of teaching materials, it became necessary to rethink MEL-VC, revisiting its initial principles and objectives, since the old function of supporting the construction of materials was no longer as crucial for many institutions.

As she mentioned, “Today we have several materials [Figure 4], including technologies. We have everything [...], but, in the end, it is up to the teacher to make his or her choices in the classroom” (E-IV-7-12-2023).

Figure 4: Isabel Vale presenting LEM-VC materials

Source: Research collection

However, after overcoming initial challenges related to the lack of teaching materials, new challenges emerged.

Despite having a variety of teaching materials, many teachers chose not to use them. In the case of future teachers, it is observed that they often only use these teaching materials to satisfy the trainers' expectations or respect their suggestions. According to Prof Isabel Vale:

> *When we talk about internships, they often use them because we are the ones who guide them to use these materials and not just rely on paper, pencils and talking. Why don't they use it willingly, as more questions will arise, the class may get out of their control. Planning a class with materials involves thinking very carefully and the selection of materials and respective tasks and the organization of students' work. Although, in several undergraduate subjects, they have to construct materials, whether games, films, materials from everyday life, etc. that allow them to see the applicability of Mathematics and themselves to identify the Mathematics present in these different creations. Realize that what they do and what they will propose to their future students makes sense, and is present in various aspects of our lives beyond school Mathematics and textbooks* (E-IV-7-12-2023).

Throughout his narrative, there is also evidence of a concern with developing an enthusiasm for Mathematics and its teaching in future teachers. “We only truly learn when we are motivated. We are focused on promoting emotional involvement, enabling the construction of something that brings them pleasure and, at the same time, includes Mathematics. I believe that we only get involved when the work attracts us, do you agree?” (E-IV-7-12-2023).

Let us focus on the questions: Why do future teachers often fail to use the materials on their own? Do we get more involved when the work attracts us? During the interview, these questions were answered. According to Prof Isabel Vale, the choice not to use teaching materials at certain times may be related to the perception that *it's work* (E-IV-7-12-2023).

In several situations, the *work* mentioned above refers both to the exploration of the material itself and the logistics of using it in a large class and its management. Indeed, the use of these teaching materials triggers a collaborative dynamic that breaks the silence, considered essential for the learning process — and noted by Prof Isabel Vale,

> *Organizing the classroom with the use of these materials demands a different attitude, as it often involves things that break the rules. The concept of 'making noise' does not just refer to disrupting the class, but to collaborative work. The teacher needs to understand that collaborative work involves dialogue, right? The classroom should be a space for dialogue and questioning* (E-IV-7-12-2023).

In this context, she highlighted, “the Laboratory is a philosophy, but it is still a physical space where we study Mathematics every day: When we make students do a folding, when they start to carry out a Gallery Walk, which can be inside or out [...]” (E-IV-7-12-2023). We understand, therefore, that the focus of MEL-VC became the didactic approach that guides the use of materials. The Laboratory, therefore, guides the training and practice of Mathematics teachers, encouraging student engagement, whether in the university or school environment, and cultivating a bond with Mathematics.

As registered in the Project, MEL-VC is

> *A space where Mathematics is thought learned and taught which aims to produce, develop and disseminate mathematical, didactic and pedagogical knowledge. It contributes, in particular, to improving the training of (future) teachers and educators, by allowing them to experience active, innovative Mathematics teaching and learning strategies and to use and/or produce innovative and differentiating teaching resources in Mathematics education. It is also a teaching resource center to support the ESE educational community and provide services to the community in which it operates.* (T-IV-7-12-2023)

3.3.3 MEL-VC training practices

Based on these insights Prof Isabel Vale was asked about training practices developed in/by MEL-VC. Even before addressing this topic directly, her narrative offered clues concerning what was and is being done in space.

Over the years of MEL-VC's existence, there has been a constant search to facilitate various practices related to training, research and community outreach. In this regard, the intention to establish links between the laboratory and the community, in particular between in-service teachers and future teachers, becomes evident in her thinking.

In this Lab, in relation to training, currently, "all activities focused on teaching are carried out. [Whenever] I have to do work, it's there!" (E-IV-7-12-2023), since the space offers necessary resources and materials. In addition to the subjects in the field of Didactics, Number Theory, in the area of Mathematics, is taught in this space. Even in subjects of this nature, students begin to use teaching materials to explore concepts such as Change of Base — "We do all the work at MEL. It is a space that must always be open. That was always the idea and it lasted for a long time. It is not just a physical repository, but a space where everything happens, where Mathematics is done" (E-IV-7-12-2023). But it also provides opportunities for students to move freely in the space, getting involved in different activities and working collaboratively, accessing different resources, whether technological or bibliographic, with the supervision of the teacher who is in the space.

In addition to teaching activities, the people involved in the Laboratory develop various research projects — "I understand that it is the people who make up the Laboratory who are carrying out the investigations. The Maths Education Lab offers support for the investigations to occur" (E-IV- 7-12-2023). This reflects the vision that it is people who drive the initiatives necessary for MEL-VC to be a place of experience and not a museum, as it was for me during my undergraduate studies, for example. According to Prof Isobel Vale (n.d., p. 02), "designing and supporting research and intervention projects that contribute to increasing knowledge in the area of Mathematics education [...]" was a responsibility.

It is important to note that Prof Isabel Vale highlighted the lack of specific publications about the Lab itself. However, there are numerous publications by her in (co)authorship, — books, articles in national and international magazines, and publications in the proceedings of scientific events — that provide a broad view of the work carried out in her context. These articles focus on training, research and community outreach activities. Highlight projects such as 'MATRIX – Math Trails with an Inclusive Perspective on Students' Experiences', 'MaSCE3-Math Trails in School, Curriculum and Educational Environments of Europe', 'Curriculum Innovation and Success in Mathematics', 'Standards- Mathematics and standards in basic education: perspectives and curricular experiences of students and teachers', and 'MatCid-Mathematics in everyday life: Mathematics and the city', among other examples (T-IV-7-12-2023).

Regarding community extension activities, Prof Isabel Vale revealed that, in an initial phase, the Lab maintained links with the community, but, over time, this interaction decreased. The opening to the community, as illustrated in Figure 5, was encouraged by the students, but this practice was interrupted. As stated, "There were occasions when Lab activities were taken outside the ESE space, such as in Praça da República, in the city center" (E-IV-7-12-2023). Currently, we propose other activities, 'World Children's Day', 'Science on Vacation', 'International Mathematics Day', 'A trail through the city' among others (T-IV-7-12-2023).

Figure 5: Activity promoted by LEM for Mathematics Day, in 1993

Source: Vale (2023)

Based on one of her statements, I questioned Prof Isabel Vale about the reasons why "the relationship with the community has diminished". She replied:

> *In the beginning, MEL was very important in supporting schools because access to training and materials, at the time, were scarce and teachers were eager to learn more. In addition, several training courses were held on new trends in teaching and learning Mathematics in addition to providing materials. Currently, they don't ask for them and I don't really know why - whether it's because they have the materials or because they don't use them. However, the reality today is very different; access to materials to be used in classrooms is difficult. Above all, the digital resources that publishers make available are immense, so there is no longer this need. However, today, LEM continues to support students who are carrying out their internship and during the design and implementation of their research to obtain a Master's degree* (E-IV-7-12-2023).

3.3.4 Challenges to the sustainability of MEL-VC

Regarding the main challenges that MEL-VC faces, Prof Isabel Vale highlighted difficulties at the institutional level related to: support for carrying out activities; approval of a regulation for the aforementioned Maths education lab in Viana do Castelo; creation of its logo; and provision of space on the institution's website.

These steps are fundamental for the institution of the Lab, enabling it to be included in funding applications and giving it the visibility it deserves. As she points out, "so far, this has not stopped us from working. In MEL, there has been a personal investment of many years. If this space exists, it was thanks to everyone's efforts. It's a shame not to have this visibility, because a lot of people, as academics, students, teachers, associations, organizations, to whom we could be useful, do not even know that we exist" (E-IV-7-12-2023). Despite everything, the sustainability of the MEL is achieved through the application for projects, with a different purpose, which are channeled into the MEL. Last July, the proposal by Ana Barbosa (a colleague) for a Blended Intensive Program Erasmus+, allowed the acquisition of two new resources: a 3D printer, an interactive touch board, and respective mobile support. The 3D printer extended a project on the Engineering Design cycle to solve authentic problems.

Nevertheless, she emphasizes, "the big challenge is related to financial investments" (E-IV-7-12-2023). Obtaining finance is a very difficult, and presently the biggest financial challenge is acquiring furniture that is more versatile.

Furthermore, she states, “in the context we are living in, it is difficult to mobilize teachers” (E-IV-7-12-2023). Another point is the visibility of what is done in the MEL space. “Disclosure occurs, in general, when students take materials and practices to the school where they carry out their internship and through academic master's degrees that take place in that space” (E-IV-7-12-2023).

After an extensive and productive day, we ended our conversation in the office. I turned off the recorder, which recorded almost five hours of audio recording. Noticing that the MEL-VC was beginning to be vacated, after the class that took place there, indicated the end of our conversation.

We headed to the Laboratory, which was duly introduced to me. Part of what I witnessed is shared in this article, through some of the many photographs I captured. The intention is to present to the reader, through the fragments of this narrative, a perspective, albeit limited, of what I was able to witness.

I confess that I was truly impressed, not only with the structure itself, but with all the work of the ESE-IPVC professors linked to MEL-VC and Isabel Vale, in particular, over the last 35 years heading the first Maths education lab in Portugal.

4 Some final considerations

If the MEL acts dynamically at the level of training, research and community outreach, it plays a crucial role in the training of Mathematics teachers. It promotes the development of specific and transversal mathematical, curricular and didactic skills. In particular, it promotes the autonomy and improvement of the) teacher's pedagogical practice. It offers, depending on the way it is structured, support for planning and implementation of classes, providing different approaches and maintaining a focus on society, school, student, together with the process of learning Mathematics. This occurs through the articulation between academic Mathematics, school Mathematics and the Mathematics of *life*, recognizing their approaches and importance without the constitution of hierarchies (Cabrita, 2004; Cabrita & Correia, 2001; Silva, 2020; Pereira *et al*., 2022; Silva, 2023a; Silva, 2023b).

The presence of a space like this in teacher training courses becomes fundamental for a solid education in Mathematics. The dialogues between training, research and community outreach reinforce our initial thesis that the Maths Education Lab transcends its physical dimension, becoming a philosophy.

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