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Dialogues evidenced in Modelling practice in the Early Years of Elementary School

Karen Ramos
Universidade Federal do ABC
Santo André, SP — Brasil
✉ karen.ramos@ufabc.edu.br
0000-0001-7233-6265

Regina Helena de Oliveira Lino Franchi
Universidade Federal do ABC
Santo André, SP — Brasil
✉ regina.franchi@ufabc.edu.br
0000-0002-9549-1134

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Abstract: In this paper, we aim to identify how the dialogue can be evidenced in a Mathematical Modelling practice about air pollution, developed with fifth-year elementary school students. The data were gathered from researcher's field notes and audio and video recordings. The analysis was made by identification and characterization of dialogical acts according to the Investigative Cooperation Model and analyzing the quality of communication according to references about dialogue in education. As results, we found that it was possible to highlight in every step of the modelling process, elements that qualify the communication among the participants, stablishing an investigative and dialogical environment, opening possibilities to cooperation and critical reflection about the theme studied.

Keywords: Mathematics Education. Mathematical Modelling. Dialogue. Critical Mathematics Education.

Diálogos evidenciados en la práctica de modelación en los primeros años de la escuela primaria

Resumen: En este artículo buscamos identificar cómo se puede evidenciar el diálogo en una práctica de Modelación Matemática, desarrollada con la temática de la contaminación del aire, en el quinto año de la Educación Primaria. Los datos se obtuvieron de notas en el cuaderno de campo del investigador y de grabaciones de audio y video. El análisis se realizó identificando y caracterizando actos dialógicos según el Modelo de Cooperación Investigativa y analizando las cualidades de la comunicación según referentes sobre el diálogo en educación. Como resultado, fue posible resaltar elementos en todas las etapas del proceso de modelación que califican la comunicación entre los participantes, estableciendo un ambiente dialógico de investigación, abriendo posibilidades para la cooperación y la reflexión crítica sobre el tema estudiado.

Palabras clave: Educación Matemática. Modelación Matemático. Diálogo. Educación en Matemática Crítica.

Diálogos evidenciados em prática de Modelagem nos anos iniciais do Ensino Fundamental

Resumo: Neste artigo busca-se identificar de que modo o diálogo pode ser evidenciado em uma prática de Modelagem Matemática, desenvolvida com o tema "poluição do ar", com alunos do quinto ano do Ensino Fundamental. Os dados foram obtidos a partir de anotações do caderno de campo da pesquisadora e gravações em áudio e vídeo. A análise foi feita mediante identificação e caracterização dos atos dialógicos de acordo com o Modelo de Cooperação

Investigativa e análise das qualidades de comunicação segundo referenciais sobre diálogo em educação. Como resultados, foi possível evidenciar em todas as etapas do processo de modelagem elementos que qualificam a comunicação entre os participantes, estabelecem um ambiente de investigação dialógico e abrem possibilidades para cooperação e reflexão crítica sobre o tema estudado.

Palavras-chave: Educação Matemática. Modelagem Matemática. Diálogo. Educação Matemática Crítica.

1 Introduction

Mathematical modelling[1] has been practiced in school contexts and constitutes an area of research in mathematics education. It is characterized by approaching real-life situations or problems using mathematics to represent and analyze information, seeking to understand the context and find solutions to the problems studied.

In modelling practices, communication between participants is fundamental. As activities occur in groups, opportunities for interaction occur at various times, from choosing the topic to be investigated to searching for and analyzing the solutions for the problems raised, which can be observed in modelling practices developed in different contexts and education levels, as Ferruzzi and Almeida (2015) and Silva and Silva (2021) reported. This communication must open space for participants to present ideas and consider the contributions presented to make their choices and propose solutions, building knowledge collectively. Thus, modelling can favor the creation of dialogic environments in the classroom.

This article covers the modelling practice of[2] the Air Pollution project, developed in the fifth-grade elementary school class in 2022. The students had access to the report from the Companhia Ambiental do Estado de São Paulo (Cetesb) [São Paulo Environmental Company], through which they realized that, during social isolation during the COVID-19 pandemic, the air quality was good at the measuring station near the school. The amount of particulate matter (PM), a pollutant released into the air by cars, was low and increased after this period. Based on this observation, they decided to research alternatives to replace cars with other means of transport that pollute less.

In this article, we seek to answer the following question: *How can dialogue be highlighted in mathematical modelling practices in the early years of elementary school?* To this end, we proposed the following objectives: identifying and characterizing dialogues in the practice in question. For the analysis, we used references from mathematical modelling, critical mathematics education (CME), and dialogues in education. Below, we present a summary of these references.

2 Mathematical modelling in the early years of elementary school

There are several ways to understand and implement modelling in teaching (Menezes, Braga, Santo, & Bairral, 2022). The researchers' conceptions of modelling are similar in that they involve bringing real-life situations into the classroom to be investigated, but they present peculiarities according to each author's vision. Considering aspects pointed out by other authors in specific situations and which do not conflict with the conception used, we considered Franchi's conception (2020, p. 201) for this study:

[1] We use modelling and mathematical modelling with the same meaning.
[2] This article is part of the master's dissertation –in multipaper format, written by the first author and supervised by the second– defended in the Postgraduate Program in Teaching and History of Sciences and Mathematics at the Federal University of ABC.

> Mathematical modelling is a pedagogical proposal characterized by the creation of an environment for investigation into a theme or problem situation through mathematics, which enables the learning of mathematics, the development of the student's potential, as well as their capacity for critical reflection in the context of the theme or problem situation.

When we think about modelling, specifically in the initial elementary school years, several questions arise: What are math classes like in this cycle? What are the goals? What do the curriculum documents say? Would modelling be an alternative for this level of education?

There is concern in classroom practices in the early years with introducing mathematical content linked to fundamental operations, with the idea of equipping students with knowledge of the respective algorithms and the appropriate use of these operations in mathematical problems. We can draw a parallel between teaching reading and writing and teaching mathematical operations or concepts involving numbers. More than teaching reading and writing, it is necessary to teach literacy (in a broad sense). It is also not enough to teach calculations. It is necessary to seek mathematical literacy, which involves not only knowing the operations with understanding but also applying them in different contexts beyond school.

With a similar focus, the National Common Curriculum Base – BNCC – (Brasil, 2018, p. 266, mentions the term "mathematical literacy":

> Elementary education must be committed to the development of **mathematical literacy**, defined as the skills and abilities to reason, represent, communicate, and argue mathematically to favor the establishment of conjectures, the formulation, and resolution of problems in a variety of contexts, using concepts, procedures, facts, and mathematical tools. Mathematical literacy also ensures that students identify that mathematical knowledge is fundamental for understanding and acting in the world and perceive the intellectual game nature of mathematics as an aspect that favors the development of logical and critical reasoning, stimulates investigation, and can be pleasurable (fruition).

As for the initial years of elementary education,

> Regarding calculation, it is necessary to add to the execution of the algorithms of the operations the ability to perform calculations mentally, make estimates, use a calculator, and decide when it is appropriate to use one or another calculation procedure.
>
> Therefore, the BNCC assumes that learning in mathematics is intrinsically related to understanding, i.e., to the apprehension of the meanings of mathematical objects, without leaving aside their applications. The meanings of these objects result from the connections that students establish between them and the other components, between them and their daily lives, and between the different mathematical themes. (Brasil, 2018, p. 276)

Tortola (2016, p. 44) raises questions about the mathematics education of students in the early years and points out the need for changes: "Have we provided conditions so that they not only study the content but can use mathematics beyond the school context and are capable of producing new mathematical and scientific knowledge?" For him, "Looking at mathematical modelling in this school period, however, requires (re)thinking about issues that encompass its practice and understanding" (p. 43).

In the early years of elementary school, many mathematical concepts are approached, but the methodologies do not always meet the needs of lifelong learning. Tortola (2016, p. 238) argues: “How do we expect our students to identify problems, whether in or outside of school, if they have always received ready-made and well-defined problems, with all the data and information available in their statements?”

Possibilities for changes in pedagogical practices are also highlighted by Silva (2018, p. 9) when he presents modelling specifically to pedagogy students, i.e., prospective teachers for the initial years:

> From the mathematics education perspective and conceived as a teaching methodology, mathematical modelling has been highlighted, within the scope of basic education, as a change from current educational practices, corresponding to more dynamic, attractive actions and the establishment of relationships between mathematics and students' daily lives.

Students must have opportunities beyond those posed with already formulated problems and data presented and prepared to be solved with the information provided. This can be achieved by addressing issues in the student's daily life, such as going to the supermarket. To know whether he or she has enough money to buy two chocolates, he/she will have to investigate the amount, check whether the money is enough and whether there will be a change. The cashier will not ask whether he/she has enough money. This is one of the differences in modelling. He/She is the one who problematizes, collects data, establishes the relationships with information processing, and gives the solution. The initiative to solve routine students' issues is encouraged, incentivized, and supported by the group and the teacher.

In modelling, after collecting information about the problem, students talk, give opinions, analyze, develop strategies, and collectively decide on possible solutions to the problem at stake. In this way, they use critical thinking, since decision-making is based on the criteria that students evaluate and indicate. Tortola (2016, p. 42) presents this issue in the use of modelling in the early years, relating it to performance in society:

> Mathematical modelling is proposed as an alternative to pedagogical practices associated with the classroom. It aims to promote the teaching and learning of mathematics and an education in which students are prepared to act critically and autonomously in society.

According to Ramos and Franchi (2024), during the modelling process, students use mathematical concepts that they already have or look for concepts that are not yet known, but necessary for the resolution. Thus, modelling favors the learning of mathematical concepts that acquire meaning when used in practice. This may be valuable when concepts are being constructed. Jocoski (2020, p. 21), when characterizing modelling at this stage of teaching, highlights that importance: “At this age group, mathematical modelling in mathematics education is concerned with helping children form concepts.”

Another interesting aspect is the interdisciplinary nature of these practices, highlighted by authors such as Borgo and Burak (2011), Franchi (2020), and, regarding the initial years, Teodoro and Kato (2021). In modelling, an interdisciplinary approach is fundamental since knowledge from different areas may be necessary to solve the problems at issue. At that level, the teacher who teaches mathematics also teaches other subjects. In this way, the versatile teacher, acting as a mediator and challenging students to seek answers, can encourage them to

look at different aspects of problems and consider different knowledge. Having the freedom to manage the planning and development of classes, the teacher can work on subject content related to the problems being studied.

There may be different objectives when developing a modelling practice. We can mention those based on the CME, whose main name is Ole Skovsmose. CME is concerned with the sociopolitical role of mathematics education and its influence on power relations in society. One of its main pillars is education for democracy (Skovsmose, 2013). When modelling is developed according to the CME, we say that it is in accordance with the socio-critical perspective of modelling (Kaiser & Sriraman, 2006). Its objective is to critically understand the world to promote the development of critical skills for acting in society and citizenship. Developing modelling practices from a socio-critical perspective is in line with the BNCC (Brazil, 2018) guidelines for elementary education, especially regarding mathematical literacy, which allows students to identify the importance of mathematical knowledge for understanding and acting in society.

In modelling practices, especially those developed from a socio-critical perspective, communication between participants stands out. As it is a practice carried out in groups, dialogue is present as a form of respectful and democratic communication, enabling the collective construction of knowledge. Below, we present references regarding dialogue in education.

3 Dialogue in pedagogical practices

In relationships between teachers and students, there may be different communication patterns. One of them, which is not always established in so-called traditional classes but can happen in investigative practices such as modelling, is dialogue. Dialogical communications have specific characteristics that we intend to emphasize and that concern the democratic and respectful way communication is conducted between everyone.

Dialogic communication is important to understand the student's thoughts, worldview, and questions, enabling conversation *with the other* in a movement of exchange and acceptance. Ambrósio (2013, p. 1078) states that

> If the teacher considers the student's thoughts, listens to them, and lets them conjecture in their own way instead of imposing an already established way of thinking, they give the learners the opportunity to construct their own knowledge, which will be much more significant and lasting. Hence the importance of respect, modesty, and trust.

Freire (1993, p. 42) affirms that dialogue is a requirement of human existence, as through it, we present our worldview to others:

> Dialogue is the meeting between men, mediated by the world, to designate it. If by saying their words, by calling out to the world, men transform it, dialogue imposes itself as the path by which men find their meaning as men; dialogue is, therefore, an existential necessity.

Freire and Shor (1986) argue that through dialogue, we identify ourselves as social, cultural, and historical beings belonging to a specific society, full of historically constructed culture. Therefore: “Dialogue must be understood as something that is part of the very historical

nature of human beings. It is part of our historical progress on the path to becoming human beings" (p. 64).

Democracy is also cited by Freire (1993) when discussing dialogue. He says it is necessary to know how to speak, listen, give voice to all involved, understanding the other person's worldview, exchanging knowledge, and acting to transform society into a more democratic environment. Education, something specifically human, becomes an intervention in the world.

Dialogue is one of the key points for Freire's (1993) liberating and critical pedagogy. Recognizing our human condition in constant transformation enables us to act to transform the world. For some students, this dialogue is an opening to others' world, allowing them to exchange information, educate themselves, and build new knowledge. Freire points out some elements necessary for the constitution of dialogue: love for the world and fellow human beings, modesty, hope, faith, trust, and true thinking. For Streck, Redin, and Zitkoski (2010, p. 237), "the Freirean challenge is to build new knowledge from the dialogical situation that provokes the interaction and sharing of different worlds, but which share the dream and hope of building together our being more."

In the learning environments presented by Alrø and Skovsmose (2010), collective work and communication are highlighted. In some –the so-called landscapes for investigation–, activities based on repetition exercises, which the authors call the exercise paradigm, are avoided. Investigation activities, generally organized based on projects, are open and are constituted according to needs during their development. The student is the protagonist.

The authors present the dialogical acts –communications with certain qualities that enable learning and can appear in landscapes for investigation– and raise some of these acts in the investigative cooperation model (IC model) but indicate that others can also occur. They are: getting in contact, perceiving[3], identifying, advocating, thinking aloud, reformulating, challenging, and evaluating. Below, we present situations in which these acts may arise.

In the IC model, at the beginning of the investigation, students and teachers *get in contact* and start cooperating. In this movement, the teacher *perceives* students' understanding and the way they transform issues of reality to be resolved using mathematical tools. The teacher *identifies* students' stances in relation to the problems investigated.

The students *advocate* by exposing ideas, considering opinions, and generating new paths in the investigation. When they *think aloud*, they spontaneously present their ideas, which can be validated or investigated to be possibly derived to other paths and led to a *reformulation* of the understanding of the group and the teacher and the change in the course of the investigation in search of alternatives. With this information, the teacher can *challenge*, question, and propose possibilities. They can also *assess* the routes taken and, with the students, define new paths to find solutions. "The acts of communication included in the investigative cooperation model bring students and their perspectives to center stage in the educational process. New learning tools become available, and new qualities of learning become possible" (Alrø & Skovsmose, 2010, p. 72).

According to Dalvi, Rezende, and Lorenzoni (2020, p. 21), "Mathematical modelling, from a socio-critical perspective, favors dialogical communication." In the investigation

[3] When using the CI Model, we use terms identical to those described by Alro & Skovsmose (2010) to name dialogical acts. In the English version, one of them was changed: the term *perceive*, used by the authors as *location* in the English language, was replaced by *perceiving*, as it better suits the meaning usually attributed in Portuguese language publications and the analyzes made in the article.

process, participants must communicate, show their points of view on the topic, listen to classmates' opinions, analyze the information available, and seek strategies to achieve their objectives –they act in cooperation, build knowledge, and critically reflect on solutions and implications in the context studied. Besides the dialogic characteristics, the environment encourages reflection and criticism, contributing to the development of citizenship.

This article analyzes the dialogues in a modelling practice during the Air Pollution project. For this purpose, we used the IC model proposed by Alrø and Skovsmose (2010) and the references on mathematical modelling, CME, and dialogues in education. In the next section, we will present the methodological procedures.

4 Contextualization and methodological procedures

The Air Pollution project, in which the practice of mathematical modelling is included, was carried out in the fifth-grade classroom of the Escola Municipal de Educação Infantil e Ensino Fundamental Darcy Ribeiro [Municipal School of Early Childhood Education and Elementary Education], located in Santo André, SP. The activities of the entire project were carried out in the regular classes of the class teacher, who is also the researcher, totaling 15 meetings of 90 minutes each between May and September 2022. Thirty 10-year-old students participated, working in groups.

The data were obtained from the transcriptions of the audio recordings of the work of one of the groups and the teacher's interventions when she was in that group for monitoring and guidance. The group comprises four students: Ma, Mi, Mu, and Ro. We sought to identify dialogues between students and between students and the teacher; and characterize these dialogues according to the IC model presented by Alrø and Skovsmose (2010). We used italics to put in evidence the terms used in this model – *getting in contact*, *perceiving identifying*, *advocating*, *thinking aloud*, *reformulating*, *challenging*, and *evaluating*.

In a previous work (Ramos & Franchi, 2023), we analyzed dialogues evidenced in the data relating to this group in just one of the modelling stages. Here, we seek to highlight the dialogues throughout the developed project, numbering the stages as they occurred:

Stage 1 - Initial investigation and topic selection.

Stage 2 - Problematization.

Stage 3 - Investigative research to choose possible solutions.

Stage 4 - Group choice of the proposed solution to be investigated.

Stage 5 - Research and data collection on the solution the group proposed.

Stage 6 - Record the data investigated and the justifications for the choice.

Stage 7 - Plenary session to present the groups' solutions and critically reflect on the topic.

The stages above are an adaptation of Burak's (2004) five steps: choosing the topic, investigative research, problem posing, problem solving, and critical analysis of solutions. Our adaptation aims to portray what happened in the classroom while developing the modelling project. They encompass our going back and forth in a non-linear modelling process (Burak, 2004).

Finally, we carry out a global analysis of the characterized dialogues, also considering, in addition to the IC model, other references regarding dialogue in pedagogical practices, such as Faustino (2018), Freire (1993), and Freire and Shor (1986).

Below, we present the analysis of the dialogues evidenced in the modelling practice.

5 Identification and characterization of dialogues evidenced in practice

We analyzed moments of communication between participants in the modelling practice. We consider these moments as dialogues because, in them, the relationships are horizontal: students and teachers are equally important, and everyone speaks with respect and listens to others' points of view: “Listening, in the sense discussed here, means the permanent availability on the part of the subject who listens to openness to the other's speech, to the other's gesture, to the other's differences” (Freire, 1993, p. 61). Furthermore, they represent moments in which one seeks to “construct new meanings in a collaborative investigation process” (Alrø & Skovsmose, 2010, p. 120). They also consider the three ideal aspects raised by the authors to relate dialogue to learning: “1- Carry out an investigation; 2- Take risks and 3- Promote equality” (p. 123). Taking risks refers to the unpredictability of the consequences that may occur because of the nature of the activity and the democratic form of communication.

The theme of the Air Pollution project arose from the interest in diseases of the respiratory system, based on the study of textbook content. The students decided to study air pollution and its effects on our health. At this stage, all groups first investigated (Stage 1) the types of pollution and recorded the data obtained. Later, based on information about the large amount of particulate matter in the air that comes from car smoke, they chose the theme (Stage 1) “Air pollution produced by cars used for transportation.” We understand that there was a delimitation of the general theme initially chosen. The groups were asked to make a written record of what they had researched. As this was the beginning of the project, the students were somewhat reluctant to work in groups and had difficulty making decisions that took everyone's opinions into account. Helping them and providing information about written records, the teacher *got in contact with* and *perceived* that students decided to divide the writing into parts (paragraphs) that would be the responsibility of each person in the group. The teacher then sought to guide the writing so that the text is not fragmented and encourages everyone's participation.

> Teacher: *What is air pollution? You are going to write about what air pollution is. What was the second one?* [mentioning the subject to be addressed]
> Mu: *The second was car pollution.*
> Teacher: *So how does car pollution happen? What does it affect? It's a text with the information you've already collected, which is all here. Right?!* [mentioning students' notes on initial investigative research on the topic]
> Mu: *Right! Air pollution...*
> Mi: *So, guys, we're going to start the text. Air pollution reduces life expectancy. Causes of pollution. Vehicle pollution.*
> Mi: *Yes. And it will start with our friend Mu. First paragraph and then it's me.*
> Ro: *So, Mu, where are we going to start?*
> Mu: *Air pollution reduces life expectancy, causes pollution and vehicle pollution.*
> Mu: *We will first write the text and then we will read it.*
> *Ma: We started; should each of us write a paragraph?*
> Teacher: *No. It is a group text, not an individual text. You will write together; you will decide together what will be written.*
> Mi: *Ah, I understand.*

This dialogue reveals that the participants also *got in contact* with the teacher; they thought about what she said and *advocated* the contents to be placed in the text. They also *advocated* the way they planned to construct the text. We interpret that by saying "*Ah, I understand,*" they are considering the possibility of *reformulating* the way to construct the text based on the interventions made.

In Stage 2 **(**Problematization), students examine Cetesb reports on air quality from June 2021 to June 2022 to compare changes in air quality during social isolation due to the COVID-19 pandemic and afterward. They were asked to develop questions about a problem for which they should seek solutions. This type of request differed from what they were used to doing in the classroom, and the difficulty in organizing information and asking questions was visible. Several times, students *thought out loud* about the structure of the question and then began to answer. However, despite the difficulty, the group managed to establish a dialogue regarding different opinions.

> Ma: *What do we have to do?*
> Mi: *We must do a super calculation of the 2021 pandemic, right? Same as 2022, how it was, how it changed, isn't that right? Or was it supposed to be about cars, like it was from 2021? In 2021, traffic was lighter and today it is heavier, hence the pollution.*
> Teacher: *I selected a one-year difference so you can compare. Has it gotten better, worse, is it the same? What caused this? How can I ask a question that refers to this? So, try to start writing the question, and you can call me, and I'll help you like I did with Ra's group.*
> Mi: *How are we going to do it? I was thinking about what the teacher said. Let's start with cars. The pollution was like that. Let's write, Ro. Are you the one who is going to write? So, put it like this…*
> Mi: *Air pollution has changed a lot between 2021 and 2022. In 2021 there was less pollution, very different because of the pandemic.*
> Mi: *So, the pandemic, let me see… Teacher, we are going to ask a question as if we were asking...*
> Mi: *So, we can put it like this: 1- Were there many changes between 2021 and 2022?*
> Ma: *It changed a lot because of cars, it decreased because there were no cars on the streets.*
> Mi: *Regarding air pollution, it decreased a lot in 2021, it was very low. Got it, Ma? In 2022, it increased more, do you understand? Ro put it like this: In 2021, pollution dropped a lot, because of cars, motorcycles, and trucks. They were no longer on the streets, but they were parked.*
> Mi: *Why, in 2022, why has pollution increased? Because the pandemic started to weaken, cars started to return and it kept increasing.*
> Mi: *The pandemic was getting better. As cars started running again, pollution increased.*
> Ro: 2- *Why is pollution returning so much in 2022?*

We perceived that students *thought aloud*, *identified* the different contributions, *advocated,* and *reformulated* until they found a satisfactory issue.

As a development of the activities, having discussed the issues presented together in the classroom, they concluded that the increase in car circulation led to increased pollution. They were *challenged* to propose alternatives that could reduce the number of cars on the streets. Although not explicitly written, there was a new problem formulated (Stage 2 - Problematization). To decide what type of transport the group would present as an alternative, new research was necessary (Stage 3 - Investigative research to choose possible solutions).

The following excerpt refers to the transition moment between Stage 3 (Investigative research to choose possible solutions) and Stage 4 (Group choice of the proposed solution to be investigated). While the other groups quickly decided on a proposed solution, this group took a long time to decide. The students talked to the teacher about the alternatives they were researching. To help them decide on the group's choice, she *got in contact* with and sought to know their ideas, *identified* the opinions of each participant, and encouraged reflection on all of them.

> Teacher*: Let's go. What was the solution proposed by the group?*
> Mi: *We were thinking about the train.*
> Teacher*: Train is a solution. It does not use polluting fuel, it is electric.*
> Mi: *We were thinking about the train to talk about it. Ma talked about the plane too.*
> Teacher: *That's great, I'd love to fly, but how do I get from the airport to my house? Would you need another car?*
> Mi: *Ro talked about motorcycles. I spoke about motorcycles.*
> Teacher*: Motorcycles and scooters are the same thing. Do they pollute less than cars?*
> Mi: *I don't think so.*
> Teacher: *Ro, were you the one who put the scooter in? Why do you think the scooter is a good option?*
> Ro: *I think it pollutes less than the car.*
> Teacher: *But are you still going to research this?*
> Mi: *We must research.*
> Me: *I've just checked; scooters pollute 16 times more than cars. I put the scooter.*
> Teacher: *Mi, it pollutes 16 times more, is that it?*
> Mu, Ro, Ma [laugh and answer]: *Yes. Scratch that one. Put an X here and this one is off the list.*
> Teacher: *Does this option remain here?*
> Mi: *Scooter, no...*

We noticed that student Ro proposed, without much thought, the scooter as an option. He *thought aloud* and presented his opinion. The teacher *challenged* students to justify this proposal. After research, they *evaluated* and *identified* its unfeasibility. Based on the data obtained, all groups *advocated* against it.

The group continued its discussions and considered the electric car as an alternative. At a certain point, Ma *perceived* the need to compare the data and *advocated.*

> Ma: *First, we can research like this: how much does a car pollute? Then, we compare it with the train and research: how much does a train pollute?*
> Ma: *Just put there how much an electric car pollutes.*
> Mi: *An electric car produces 25g of* CO_2.
> Mu: *The electric car has less impact on the environment.*
> Mi: *We have to compare, understand? Because, like, since yesterday we've been wanting to compare this. So, we have to compare. Whether the train pollutes more or the car pollutes more.*

There was some confusion at this point about the type of comparison they would have to make to defend the train as the alternative initially thought of by the group, and the teacher intervened.

> Ma: *We were comparing the electric car with the train.*
> Teacher: *It's not an electric car, it's a normal car.*
> Mi: *Yes. Oh, that's right, it's a normal car.*

The intervention was intended to clarify that they had to compare the alternative proposed by the group with the type of car that was in circulation, the car with combustion engines. The student *identified* it.

The teacher *perceived* the group's referral to the train as an alternative and encouraged participants to seek data that allows for comparison (Stage 5 - Investigation and data collection on the solution proposal made by the group), *challenging* them to compare the number of passengers per trip on the train and by car.

> Teacher: *Do you know how many people fit in the car and on the train?*
> Mi: *Many.*
> Teacher: *It's not many, we need numbers to be able to compare.*
> Mi: *Just a minute*. [The student stopped and searched for information on the internet]
> Mu: *There is room for five in the car.*
> Teacher: *Look, write this down, please, guys! Write it down.*
> Teacher: *How many people fit in a car?*
> Ma: *Five.*
> Teacher: *So, I can say that a train takes how many cars off the street?*
> Mi: *Several.*

Then, Mi *thought aloud***,** without much reflection. But the teacher *challenged* him to establish parameters for comparison.

> Teacher: *There are not several, you need the quantity, how do you compare with several?*
> Ma: *Five to seven. Teacher, five to seven people fit inside the car, it depends because some cars are big.*
> Mi: *One good thing, the train is good.*
> Mi*: Which means of transport pollute the most?*
> Ma: *Cars, buses, trucks, and motor vehicles are the biggest cause of pollution in the world's cities.*
> Ma: *They should put the train running in the middle of the city. A train has a capacity for 250 passengers, and to carry the same number of people, it would take three hours with 50 cars, and generally, the traffic does not have that capacity. That's why rail transport is viable* [mentioning information obtained from the internet].
> Mi: *Of 250* [passengers].
> Mu: *No, everything.*
> Mi: *It takes 50 cars or three buses off the streets.*
> Mi: *Let's call the teacher. Teacher, did you know that?*

In the previous dialogue, we see moments where students *thought aloud*, giving opinions (like when Mi said the train was good). But they kept looking for information. Ma found the information and presented it to the group. When they *identified* that the information was found, the students wanted to present it to the teacher to inform her about the data found and not simply to validate it. The teacher is seen as a group member, not an evaluator.

Starting Stage 6 (Record the data and the justifications for the choice), students consulted their notes, compared the information obtained with what they know about the city, with numerical data, and with the calculations they made based on them, and weighted the group's choice. The concern in presenting the correct information was noticeable. In the following dialogue, we notice that students *evaluated* the information and *reformulated* the record on the Santo André railway network and the names of the stations. One of them *thought*

aloud about the names of the stations and compared them with the information obtained, which left him intrigued.

> Mi: *Mu, what are you writing about? About the tracks, right?! We've already written this, we have to write another type of thing.*
> Mu: *Yes, I'm going to put Santo André here, it's 174.8 km and not 140 km.*
> Mi: *How long?*
> Mu: *174 km.*
> Ma: *What did he say that I didn't understand?*
> Mu: *That Santo André has 174.8 kilometers of tracks.*
> Ma*: Ah, I understand.*
> Mu: *How many stations does the city of Santo André have? Mayor Celso Daniel…*
> Mu: *Mayor Saladino, Pirelli, Utinga. I'll put this here, in our city we have four train stations.*
> Mi: *Didn't you think it was strange that it said Pirelli there?*
> Ma: *It's a station. It's not a tire brand.*
> Mi: *Where did you see this, are you sure?*

After this questioning, the students returned to their records, asked about each item that they thought had to be written, and returned to the main question, which was precisely the solution proposed by the group: replacing cars with combustion engines with trains. They mentioned the size of the railway network and the number of train stations, leaving student Mu in doubt as to whether using the train would be viable. He *evaluated* the number of stations in the city, *perceived* that the number of stations was insufficient, and *advocated* by saying that "*there should be more stations*." For the student, it would not be possible for everyone to reach the stations, due to the distance and the size of the city. He *perceived* that, perhaps alone, the group's choice does not meet the need for transporting people without using cars.

> Ma: *Did you put everything there? The calculations and what was written?*
> Mu: *The car calculations, the rail calculations.*
> Mi: *We've already done the math on the tracks, the cars, it's all done.*
> Mu: *There are not enough train stations. If a person lives far away, they must use a car or automobiles.*
> Ma: *Okay, there is no calculation.*
> Mu: *Three, four stations only; if she lives far away, how is she going to walk so much to get to the train station? There should be more train stations.*
> Mi: *Show it to the teacher. Place the period after Utinga.*

In Stage 7 (Plenary session to present the groups' solutions and critically reflect on the topic), each group presented its investigations and conclusions. To this end, the teacher adopted the dynamic of interviewing the groups, seeking to ensure that all aspects researched and the responses to the problems raised were presented to the class and enabled reflections on the topics researched. She established a dialogue with the groups to help them present their data, *reflect* on them, and *advocate*. Students in group 3 *advocated* in front of the whole class, presented the positive and negative points of the solution chosen by them and *evaluated* its viability. They could not conclude whether the group's option was appropriate. They *identified* that the train could replace cars, but that this solution presented difficulties, as more stations and more investments would be necessary, which, according to information from the group itself, would be high.

> Teacher: *Now we have group 3 here and I would like to ask what is your option to solve the problem?*

Mu: *Our option is the train.*
Ma: We discovered that the train can carry 250 people and can take 50 cars off the streets.
Mi: *And we also thought about the kilometer of railway and thinking about this train solution as it is quite expensive, perhaps it is not a very good solution, because the railway line costs six million per kilometer, so it is not a good solution as it is quite expensive.*
Mu: *I'll talk about the negative point, there aren't many train stations in the city, so we need to improve this, because when people live far away, they'll have to use a car that pollutes the air, so there's no point in using a car that pollutes the air and going by train, because it will pollute the air.*
Teacher: *Likewise...*
Mu: *Likewise...*
Teacher: *I wanted to ask Mi. Mi, how many train stations are there in our city?*
Mi: *There are four.*
Teacher: *There are four train stations. Are these four stations enough to make all the residents of our city use the train?*
Teacher: *We have four train stations in our city, Prefeito Saladino, Prefeito Celso Daniel, Utinga, and Capuava. Each station is in a different neighborhood; some are closer, such as Utinga from Prefeito Saladino, which are not distant stations, but they transport you to a [different] neighborhood. As Mu said, they don't transport you very far. Is there any option, for example, so that when I get off at Prefeito Celso Daniel station, in the center of Santo André, I can get home without using a car? Let's assume your house is here in the neighborhood. Is it nearby? Is it far? Would that be okay? Would that be a solution?*
Mi: *It would be a problem, because it's far away and the car, like, takes you home faster, so it wouldn't be a very good solution.*
Teacher: *Wouldn't that be a very good solution?*
Mi: *Yes.*
Mu: *And the train, as it doesn't have many stations, you sometimes have to walk a lot to get where you want, because, like, your house is there and the train is here, then it will pass like this, then like this, like, it will take a while for it to arrive.* [The student refers to making a transfer]
Teacher: *I understand, it goes through other places first, is that it?*
Ma: *The train ticket is cheaper than buying a car, and you have to take it home to be able to drive.*
Teacher: *And what's the problem with taking it home? I didn't understand this question.*
Ma: *Because many people cannot afford to buy a car.*
Mi: *The train ticket is much cheaper, costing four to five reais.*
Mu: *As the price of gasoline has increased, many people are having difficulty, and during the pandemic, it is also difficult, we must use the train more to avoid polluting the air too much; it is cheaper than buying a car and going there to fill up, and you walk more.*

It is possible to see from the dialogue that the students evaluated the viability of the solution proposed by them, validating or not the solution for specific conditions of reality (Stage 7 - Critical reflection on the topic).

Teacher: *Does the group conclude that the possibility that you researched is that the train is a viable solution for our city?*
Mi: *Not much, but why...*
Teacher: *We must give a yes or no* answer.
Mi: *Yeah...*
Teacher: *Does it solve the problem of cars?*
Mi: *Yes.*
Teacher: *Does it solve the commute from home to work?*

Mi: *No.*

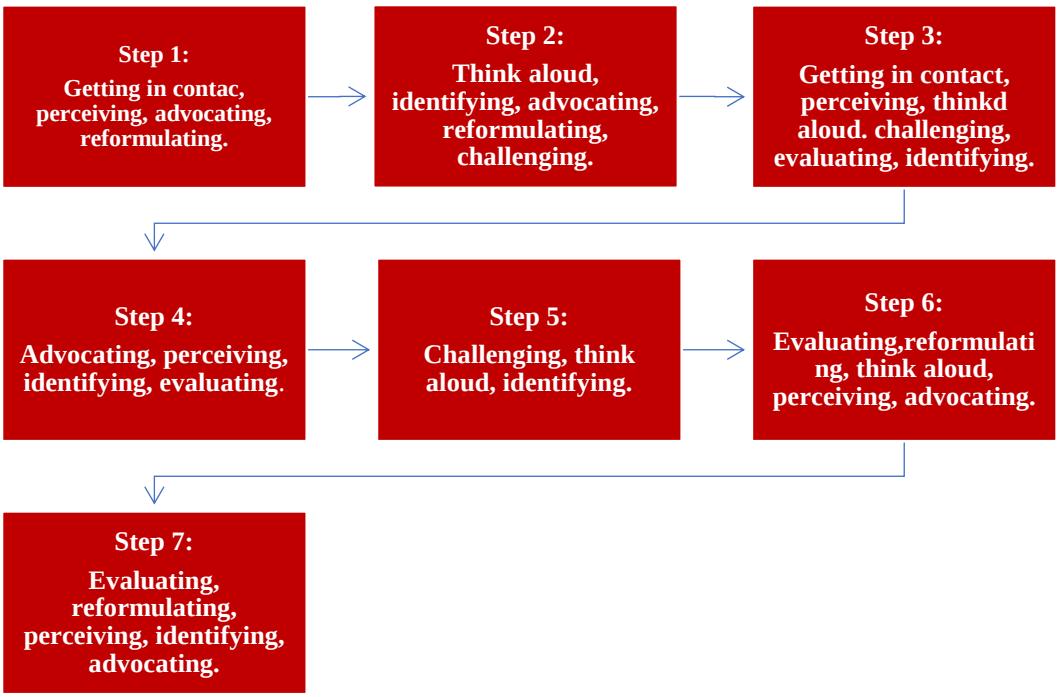
The change in students' attitudes throughout the process is noticeable. Initially, they *advocated* based on personal opinions, without criteria or data to confirm them. Throughout the investigation, students, challenged to support their claims with data and arguments, often *perceived* that their first impressions were not always correct. They began to consider the information, the opinions of their classmates, *identified,* and *reformulated* their perceptions.

Below, we present a global analysis of the dialogues highlighted and characterized, also considering, in addition to the IC model, other references regarding dialogue in pedagogical practices.

6 Global analyses of the dialogues highlighted

To highlight and characterize the dialogues presented above, we based ourselves on the IC model presented by Alrø and Skovsmose (2010). We use dialogic acts (*thinking aloud*, *getting in contact*, *perceiving*, *identifying*, *evaluating*, *challenging*, *reformulating*, and *advocating*) to highlight the qualities of these dialogues in relation to learning. We noticed that all these dialogical acts occurred at some point and appeared in the speeches of the students and the teacher, evidencing the quality of communication. We also noted that dialogical acts took place at all stages of the project. Figure 1 presents those dialogical acts that were identified in each stage, as already described and analyzed in the text.

Figure 1: Dialogical acts present by stage



Source: Research data (2022)

We interpret that the presence of dialogical acts is related to the teacher's intervention, which required, at times, the emergence of some of these acts. Always attentive to the work of the groups and open to listening, she identified moments when it was necessary to clarify, encourage participation, stimulate problematization, and challenge people to seek information,

to justify their positions and responses. Finally, in the plenary session, she sought strategies to ensure that all group results were presented and discussed, enabling students to reflect and take a stand in a respectful manner.

Thus, a dialogue was established when carrying out the investigation, and it was possible to have a more well-founded view of the problem and the solution. The foundation was based on obtaining data and analyses carried out based on dialogues in groups, often using mathematical tools. Alrø and Skovsmose (2010, p. 133) state that "participating in dialogue is also a form of action and production of meaning through the use of language. To dialogue means to act in cooperation."

We identified, throughout the project, through established communication, the elements pointed out by Freire (1993) to characterize dialogue: love for the world and for men, modesty, hope, faith, trust, and true thinking. In the search for proposals to improve air quality, students demonstrate love for the world and for people by trying to transform the place they live into a better and healthier environment. They demonstrated true thinking by presenting their ideas about reality to the group and putting them up for discussion, listening to and considering the opinions of others. They acted with confidence, substantiating their proposals, demonstrating the faith, humbleness and hope of those who try to transform reality and are not content to just wait –but, at the same time, they revealed that they were aware of their limitations.

In the data collected from group 3, we found particularities that are in line with Freire's ideas (1993, p. 67) when he states that "Dialogue precedes action in planning, provides data for problematization, and underpins and supports reflection, leading people to transformative action." In the Air Pollution project, students showed, through dialogue, that they were looking for changes in their environment and to do so, they organized their ideas, investigated, and based themselves on this information to raise possibilities for improvement in their city, i.e., "To achieve the objectives of transformation, dialogue implies responsibility, direction, determination, discipline, objectives" (p. 67). It was possible to visualize these elements throughout the project.

The dialogical relationship also occurred between the teacher and the students. They saw the teacher as a member of the group. Those involved had space to speak and listen, which is in accordance with Faustino (2018, pp. 48-49): "Listening or speaking are not exclusive tasks of one of the subjects in the educational process. Both participate in the process; that is why teacher and student can speak, can listen, can dialogue."

From this dialogical environment, the search for social transformation can emerge, as stated by Freire and Shor (1986, p. 65): "Dialogue seals the relationship between cognitive subjects, who can then act critically to transform reality." This search for transformation was evident in the dialogues of the members involved in the search for information to correctly choose the proposal to improve air quality in the city.

According to Freire and Shor (1986, p. 28), dialogue can generate mobilization: "If students engage in a critical dialogue, I see this as an act of mobilization because they have decided to become human beings who investigate their own reality together." This mobilization can accompany students when they act outside of school: "If the process is liberating, students and teachers will undertake a transformation that includes the context outside the classroom" (p. 27).

Because the topic of air pollution involves environmental issues, one student said he could help in other ways and distributed seeds that he said, "*would help improve the amount of oxygen in the air*." Thus, he looked for ways to mitigate the problem by acting in his

environment to transform it –in this case, making it possible to increase the number of trees in the neighborhood.

This participation aligns with Freire and Shor's (1986, p. 60) ideas about student participation: "They really learn to participate. But what is impossible is to teach participation without participation. It is impossible to just talk about it without experiencing it. [...] Democracy is the same thing: you learn democracy by doing democracy."

Thus, we can affirm that the Air Pollution project, in which the modelling practice was included, managed to promote "the maintenance of democratic principles" (Faustino, 2018, p. 201) through dialogue, which occurred through respect, acceptance of others' ideas, presentation of one's own ideas, speaking and listening, argumentation about mathematical strategies, thinking and critical positioning that favored democracy. Students had opportunities to use dialogue, mathematical knowledge, and group work. According to Faustino, when children learn mathematics in a dialogical way, "they learn to work in groups, to help each other and to engage and resolve activities collaboratively. Therefore, through dialogic interaction, children not only learn mathematics but also learn to interact democratically" (p. 20).

7 Considerations

This article presents and analyze data from the master's research entitled *O diálogo em práticas de modelagem matemática nos anos iniciais do ensino fundamental: Suas manifestações e contribuições para a formação do estudante* [Dialogue in mathematical modelling practices in the early years of elementary school: Its manifestations and contributions to student education]. The data were collected by the researcher-teacher throughout the Air Pollution project, in which the practice of mathematical modelling from a socio-critical perspective was included. We sought to answer the guiding question: "*How can dialogue be highlighted in mathematical modelling practices in the early years of elementary school?*" The dialogues were highlighted through the characterization of communication between participants according to the IC model presented by Alrø and Skovsmose (2010). They were also identified in practice, considering the stages of project development, and analyzed globally, considering references regarding dialogue in pedagogical practices such as Faustino (2018), Freire (1993), and Freire and Shor (1986).

Air pollution is a global environmental issue that also concerns the city's population. In the modelling practice, students could collectively reflect on this situation, problematize based on their reality, investigate reliable information, propose solutions, reflect critically on the proposed alternative solutions, and decide on the viability or not of the solution in the context studied.

Searching for reliable information and analyzing data allows students not only to mathematize through the processing of information but also to relate data in a practical way to everyday events. In our case, the students suggested replacing combustion cars with trains as a means of transportation, arguing that this would reduce pollution. However, they identified that this alternative would not alone be adequate to solve the city's transportation problem.

This type of project makes it possible to approach school curriculum content in an interdisciplinary way, which can be facilitated in the initial years by the mobility and dynamism that the versatile teacher must plan and develop particular practices (Teodoro & Kato, 2021). In this practice analyzed here, we approached themes such as geography, when studying urban mobility; science, when studying the composition and emission of pollutants; oral and written language, when recording and presenting this information; history, when researching social facts in different eras; and arts, to visually represent data and results. At the end of the project,

there was a debate in which students defended their proposals, which demonstrated, through oral practices, the knowledge constructed not only to meet school requirements but also for their lives.

Classroom activities took place in a way that favored dialogue between all those involved and collective decision-making. It was essential that dialogues took place between all students and between students and the teacher to achieve the objectives related to learning curricular content and form critical and active citizens for a democratic society, as is in the interests of the CME.

We suggest a further development of this research, an investigation into dialogue in the problematization stage, focusing on the difficulties presented by students.

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