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Phenomenological significance of experiences of mathematics degree students when developing mathematical modeling activities with digital technologies

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Abstract: When working on mathematical modeling with the aid of digital technologies, we aim to investigate the subject, his/her cognitive and critical-social aspects, which are constituted by their experiences. So, we asked ourselves: What experiences do students have when developing a mathematical modeling activity with the help of digital technologies? This question requires attention to the manifestations of the experiences involved in the mathematical learning process. We took a phenomenological approach to research to present the description, analysis, and interpretation of the categories emerging from the units of meaning. The analysis showed that experiences must be viewed from the individual to the group, explaining the movement of knowledge constitution from the perspective of the person interacting with others, rather than someone who is merely immersed in a group. There is also the possibility of opening up a region of inquiry in the field of mathematics education with implications for the classroom that address students' experiences with mathematics and digital technologies.

Keywords: Mathematics Education. Phenomenology. Constitution of Knowledge.

Significancia fenomenológica de las experiencias de estudiantes de la carrera de matemáticas al desarrollar actividades de modelación matemática con tecnologías digitales

Resumen: Al trabajar la modelización matemática con la ayuda de las tecnologías digitales, buscamos investigar al sujeto, sus aspectos cognitivos y crítico-sociales, constituidos por sus experiencias. Por ello nos preguntamos ¿Qué experiencias tienen los estudiantes al desarrollar una actividad de modelización matemática con ayuda de las tecnologías digitales? Esta pregunta requiere prestar atención a las manifestaciones de las experiencias implicadas en el proceso de aprendizaje matemático. Adoptamos un enfoque fenomenológico de investigación para presentar la descripción, análisis e interpretación de las categorías emergentes de las unidades de significado. Los análisis mostraron que es necesario mirar las experiencias desde lo individual hacia lo grupal, explicando el movimiento de la constitución del conocimiento desde la perspectiva de quien está con el otro y no de quien está meramente inmerso en un grupo.

Existe también la posibilidad de abrir una región de indagación dentro del campo de la educación matemática con implicaciones para el aula que aborden las experiencias de los alumnos con las matemáticas y las tecnologías digitales.

Palabras clave: Educación Matemática. Fenomenologìa. Constitución del Conocimiento.

Significação fenomenológica de vivências de estudantes de licenciatura em matemática ao desenvolverem atividades de modelagem matemática com tecnologias digitais

Resumo: Ao trabalhar a modelagem matemática com auxílio de tecnologias digitais, buscamos investigar o sujeito, seus aspectos cognitivos e crítico-sociais, constituídos pelas vivências. Assim, interrogamos: Que vivências dos estudantes se mostram ao desenvolverem uma atividade de modelagem matemática auxiliada por tecnologias digitais? Essa interrogação exige atenção às manifestações das vivências envolvidas no processo de aprendizagem matemática. Assumimos uma postura fenomenológica de pesquisa, de modo a apresentar a descrição, análise e interpretação das categorias emergentes das unidades de significado. As análises evidenciaram que as vivências precisam ser visadas do indivíduo para o grupo, explicitando o movimento de constituição do conhecimento pela perspectiva da pessoa que está com o outro e não de alguém meramente imerso em um grupo. Emerge, ainda, a possibilidade de abrir uma região de inquérito no âmbito da educação matemática com implicações para a sala de aula que abordem as vivências dos estudantes com matemática e tecnologias digitais.

Palavras-chave: Educação Matemática. Fenomenologia. Constituição do Conhecimento.

1 Introduction

This article was developed within the scope of an inquiry region that investigates mathematical knowledge and the lived body in the performance of an activity that aims at "[...] the production of mathematical knowledge, carried out by our own body and respective ways of living in the world together with others" (Bicudo & Klüber, 2021, p. 52). We understand one's own body (*der eigene Leib*, from a first-person perspective) or lived body (*Leib*) as a totality "[...] that exposes itself as intentional carnality, moving in the world spatially/temporally, to act toward what it perceives to be requesting action" (Bicudo, 2009, p. 152).

The philosophical perspectives of the aspects we seek to elucidate in this research align with the investigations developed by the research groups Phenomenology in Mathematics Education (Fenomenologia em Educação Matemática - FEM) and Phenomenological Research in Mathematics Education (Investigação Fenomenológica na Educação Matemática - IFEM), which present as research focus the philosophical understandings of mathematical knowledge and the lived body, as well as the various aspects of mathematics education.

According to Bicudo and Klüber (2021), reflecting on how mathematical knowledge is constituted in the lived body is to understand that "[...] the realization of the act takes place in the experience lived by one's own body, that is, in the body understood in its carnal, psychic, and spiritual totality" (p. 63). These dimensions constitute the experiences we record in consciousness acts (Ales Bello, 2006). In other words, these experiences "[...] occur in the very act of perception carried out by the subject, a lived body that, intentionally, attentively addresses what it seeks to know about what it sees" (Bicudo & Klüber, 2021, p. 63).

Under this phenomenological understanding, this article aims to explain students' experiences manifested while developing a mathematical modeling activity with digital technologies in a school subject of the mathematics degree course. This search was guided by the question: What experiences do students have when developing a mathematical modeling activity aided by digital technologies? This question demands attention to the manifestations of experiences because people evoke them when addressing any phenomenon without noticing them.

In line with Bicudo (2011), we adopted a qualitative approach from a phenomenological perspective, seeking to clarify the question since, from the phenomenological perspective of research, we always start with a question; that is, we ask ourselves what the question is questioning. Thus, "the questioning behaves as if it were a backdrop where the researcher's questions find their ground, making sense" (Bicudo, 2011, p. 23). With it, we glance at what our region of inquiry asks for and pursue how we can produce the data, as well as define the significant subjects of the research.

The movement of looking at and reflecting on our region of inquiry consists of the "[...] search for significant subjects who can speak of experiences, lived in their daily lives, concerning the person being questioned, and of important texts and works by significant authors who in a mediated way speak of the person being questioned" (Bicudo, 2011, p. 42). Furthermore, it is important to clarify that our investigation also aligns with Klüber's (2023) procedures when he investigated a child's experiences when solving a geometry problem.

Finally, it is worth highlighting that the phenomenological procedure requires "[...] working with senses and meanings that do not occur in themselves, but that are constituted and revealed in different ways [...]" (Bicudo, 2011, p. 41). When we face the research question and seek meanings and significance that can explain it, we consider it pertinent to expose, albeit briefly, the relevant aspects of our region of inquiry. To this end, we present understandings concerning mathematical modeling and, subsequently, phenomenology, digital technologies, and experiences.

2 Understandings about mathematical modeling in mathematics education

Mathematical modeling[1] in the context of mathematics education has been the object of investigation by several researchers such as Burak (1987, 1992, 2004, 2017, 2020), Barbosa (2001, 2003, 2008), Klüber (2007, 2012, 2013, 2016), Menezes (2021), and Klüber, Tambarussi and Mutti (2022), who discuss the potential and challenges faced when using modeling in mathematics teaching. Although some of these works were published more than two decades ago, the authors' understandings concerning mathematical modeling remain.

Such research highlights a growing movement in research in modeling, as well as its use in the classroom context, which may be related to the possibility that it provides a more critical and reflective look at mathematical and non-mathematical aspects, in addition to making teaching more dynamic and meaningful (Burak, 2004, 2017) to "[...] portray the participant's perspective, their opinions, suggestions, and understandings" (Burak, 2020, p. 24). Burak conceives mathematical modeling as an opportunity for students to develop "[...] complex skills of observing, exploring, conjecturing, proving, recording procedures [...] among others" (Burak, 2020, p. 29).

Barbosa (2001, 2003) asserts that modeling can be considered a learning environment in which teaching focuses on mathematically investigating situations from other realities to favor students' interest in acting in society by studying what makes sense to them. Thus, modeling activities can be taken "as a way of challenging the ideology of certainty and placing

1 The terms mathematical modeling and modeling will be used throughout the text as synonyms.

critical lenses on the applications of mathematics" (Barbosa, 2003, p. 68). Considering the teaching and learning of mathematics in this context "[...] carries with it the vision of a mathematics not restricted to its own context, but capable of building relationships between what is learned inside and outside school, a mathematics built on the interaction of man with the world, a mathematics with history" (Burak, 2020, p. 25).

Menezes (2021) reports that when approaching mathematical modeling as a pedagogical strategy, it is essential to explore "[...] students' proposals for approaching a problem situation, drawing their attention to characteristics of the problem situation not previously observed, and challenging students to present more than one solution, to test different paths" (Menezes, 2021, p. 66). In this context, students can investigate, reflect, and problematize these situations to propose solutions and alternatives critically and decide on their viability (Ramos; Franchi, 2024), "[...] to form a student who can transpose the knowledge acquired in a non-fragmented way or simply closed in on itself" (Melo; Bisognin, 2021, p. 25-26).

In addition to each author's understanding, we realize that teaching from those authors' modeling perspective allows students to investigate using themes from their realities and interests, which may motivate them to understand mathematical concepts critically and reflectively. In this way, they can integrate them with other knowledge and the context of the society in which they live, enabling them to express themselves about such context, among other things. Ramos and Franchi (2024, p. 5) corroborate this by saying: "In modeling, it is important to have an interdisciplinary stance since, to solve the investigated problems, knowledge from different areas may be necessary."

Regarding the articulation between mathematical modeling and digital technologies, it is important to highlight that students' research development is linked to the teacher's ability to develop activities related to the curriculum components simultaneously with students' areas of interest. By associating modeling with digital technologies, Allevato (2005) sought to analyze how students establish relationships when carrying out activities in the classroom using pencil and paper compared to activities mediated by digital technologies. The author argues that constructing knowledge in an environment that incorporates computers allows students to develop new ways of thinking based on actions that involve simulation, experimentation, and visualization, which Malheiros (2004) and Borssoi (2013) corroborate by saying that activities involving mathematical modeling and digital technologies instigate and challenge students by giving them greater responsibility, encouraging the use of technology as an incentive for collaborative work. At an international level, Galbraith and Fisher (2021) reviewed the use of technologies in mathematical modeling contexts. Despite asserting the relevance of this integration and its potential for a specific group of students, they understand that the process does not occur uniformly for everyone.

Incorporating technology becomes essential when considering the familiarity that most young people demonstrate with various digital resources. That said, Greefrath and Siller (2017) explain that modeling activities are susceptible to digital tools because students can use them in several ways to research, calculate, build, visualize, and experiment through simulations. Silva, Araki, and Borssoi (2018) state that mathematical modeling activities can provide students with a leading role during an investigation. Technology can be used as a stimulus strategy, given most students' proximity and dexterity with digital tools. However, the conception of technology, whether as a master, servant, partner, or extension of oneself, impacts the way of working in mathematics education (Galbraith & Fisher, 2021).

The different conceptions or theoretical perspectives of mathematical modeling presented usually describe cognitive and critical-social visions and, with them, advance the work with digital technologies. However, there is a relative lack of understanding in a philosophical sense and of the subject in its entirety, seen as a human person. Discussions about the human person were only touched upon by Klüber (2007, 2012), who has focused on studying philosophical questions aligned with phenomenological thought and, later deepened by Klüber *et al.* (2022), presenting understandings pertinent to the philosophical problem of representation and its implications for mathematical modeling.

In this study, Klüber *et al.* (2022) discuss the idea of reality, an aspect present in the modeling concepts presented previously. The authors adopt a phenomenological view when looking at reality. They present a perspective that opposes the empiricist view that conceives it as articulated with everyday life and independent of the subject (Klüber *et al.*, 2022). They understand that it is not possible to describe reality through mathematics directly "[...] since there is no possibility of mathematics being a mere representation of reality (*Vorstellung*), that is, it does not refer to a previous image or one imprinted in the mind about the object" (Klüber *et al.*, 2022, p. 311).

In this philosophical perspective, aligned with phenomenological thought, several possibilities are presented to explain, through mathematics, so-called real situations since "[...] this explanation aims to expose mathematical aspects that do not designate real objects, but associate mathematical idealities with the aspects that are targeted in this situation" (Klüber *et al.*, 2022, p. 313). In this sense, the authors understand that,

> [...] there is a movement of the body (*Leib*) and of thinking in the search to associate mathematical ideals with the aspects that appear in the studied situation. This is where the axis of the teacher's work lies when teaching mathematics with modeling to obtain a model or even for the mathematical explanation of the situation, whether about the mathematical knowledge and any other knowledge students already have or the presentation of new knowledge (Klüber *et al.*, 2022, p. 313).

When the teacher works with modeling, he/she needs to understand the mathematical aspects present in the situation studied to visualize what he/she wants to explain with mathematics and how to associate it with other aspects present in the situation (Klüber *et al.*, 2022). To do this, it is necessary to "[...] overcome the idea that mathematics will emerge from a simple situation because mathematics emerges from the movement of associating aspects that are not immediately given [...]" (Klüber *et al.*, 2022, p. 315). This movement can occur through perception, and its meaning is realized between experiences and how they manifest themselves to us (Ales Bello, 2006). According to Klüber (2023, p. 171), "It is by intertwining experiences that the constitution of knowledge occurs for the subject. This brief discussion about experiences in a phenomenological sense is resumed below in its different aspects.

3 Phenomenology, digital technologies, and experiences

Research on digital technologies (DT) within the scope of phenomenology has had a special focus, mainly regarding perception and reflection, because the relationship between digital technologies and human experience is configured as an interaction between the human being and the world mediated by these technologies.

DTs have played a significant role in the way we perceive, interact, and understand the life-world, as "the cybernetic world, under the understanding of the incorporation of digital technological resources into the life-world, is thus forged by technologies" (Rosa, 2023, p. 192).

By considering digital technologies phenomenologically, we are prompted to examine the nature of lived experience mediated by these tools, focusing on how they manifest themselves in individuals' lives and how their bodies experience them. It is by being one's own body that the subject relates to the world around him/her and others, "it is the vehicle of being in/with the world and speaks of its material totality and its intentionality revealed in the ways of being, which are shown in movements of life, for life, and with life" (Rosa, 2023, p. 192).

From this perspective, the construction of identity and self-image becomes increasingly influenced by DTs, raising questions about the authenticity and integrity of personal experience. Given the above, DTs highlight our lives, they are part of them; therefore, they can no longer be used as tools to speed up processes or be considered a means to an end but rather as constituent resources of the being as a body (Rosa & Caldeira, 2018).

One's own body is the means through which experiences are lived, interpreted, and integrated into human experience. "Each experience is shown through our own body, which reveals its intentionality and reveals itself as "being-with-DTs," which conceives the idea of 'being,' which is a verb, which is movement, and which identifies and updates itself with the cybernetic world" (Rosa, 2023, p. 197-198). In other words, "DTs are the means by which the 'being' perceives itself and reveals itself by showing itself" (Rosa, 2022, p. 38).

"Experiences are the very structure by which each person is conscious, being a physical body (*Körper*), but also a living body or one's own body (*Leib*)" (Klüber, 2023, p. 173). According to Ales Bello (2006), experiences have three spheres or dimensions: body or perceptive experiences, psyche or psychological experiences, and spirit or spiritual experiences, all "united" in one's own body or lived body. Psychological experiences refer to reactions, impulses; and spiritual experiences are reflective, evaluative, decisive acts. Bodily acts, or perceptive experiences, are given by experiences related to bodily sensations, that is, by our instincts (Bicudo & Klüber, 2021).

Knowledge is conceived as resulting from the movement of the subject's lived experiences. These lived experiences, mediated by sensory perception and interaction with the world, are continually interpreted, assimilated, and transformed by the subject because, being movement, they will be evoked and manifested "[...] in a distinct way, always retrospective compared to their moment of occurrence" (Klüber, 2023, p. 173). This brief incursion into the relationship between knowledge, embodiment, and technologies is revisited in the following sections, where we will analyze and interpret students' lived experiences while developing the activity.

4 Methods of proceeding and producing data

The research presented seeks to understand students' experiences when developing a mathematical modeling activity with the help of digital technologies. To do this, we adopt a phenomenological perspective, with a close look at what is shown about the phenomenon investigated in a reduction movement.

In phenomenology, the phenomenon is defined as "[...] what is shown in the act of intuition carried out by an individually contextualized subject, who looks towards what is shown attentively and who perceives what is shown [...]" (Bicudo, 2011, p. 30). Thus, we carry out a reduction movement, of "putting in parentheses" any and all prior beliefs about the phenomenon studied. This movement is characterized by "not accepting established positions on the phenomenon as a starting point, logical sequence, and validation. However, it starts from the phenomenon itself [...] in an incessant search to understand the meanings of what is shown" (Klüber, 2012, p. 55).

When seeking meanings of the phenomenon revealed in front of the question: What experiences are revealed when students develop a mathematical modeling activity using digital technologies? We direct our gaze to manifestations such as looks, physiognomies, movements of the body, and ways of communicating, that is, the different forms of expression the significant subjects of the research present. This procedure could be confused with the typical observation in positivist research and qualitative approaches. However, we assume the pair seeing/seen instead of the pair observer/observed since, phenomenologically, we do not admit the separation between subject and object (Bicudo, 2011).

The experiences to be explained occurred during a mathematical modeling activity developed during four class hours in the *Technologies Applied to Mathematics Teaching* subject. The authors of this paper taught the course, which is part of the curriculum of a teaching degree in mathematics. This activity asked students to model a friction cart[2] to study aspects related to the content of functions with the help of digital technologies. Brito and Almeida (2005, p. 80-81) state that "mathematical modeling activities can help students understand the dynamic aspect of the concept of function, thinking of it as a relationship between variables." Furthermore, the help of technology provides better visualization of this relationship through graphic constructions and curve adjustments, which can help understand concepts and the meanings that functions can assume.

To mediate the investigation, we asked students two questions: What (approximate) relationships can we establish between the speed reached by the cart and time? What would be the (approximate) speed reached by the cart in five seconds? It is worth mentioning that this activity was inspired by Silva, Borssoi, and Almeida (2015), who present a semiotic analysis of mathematical modeling activities mediated by the use of technology, one of which is the study of the path taken by a friction cart through the software of video analysis *Tracker.* That software allows users to analyze videos through image frames, making it possible to select sets of frames to be analyzed and the variables of interest regarding the phenomenon under study (Bezerra Júnior, Oliveira, Lenz, & Saavedra, 2012).

Given the two questions presented for the investigation of the activity, we divided it into two moments. At first, we asked the students to divide into groups and reflect on the questions without the help of the cart and the technological tool, which would be the materials used to support the investigation. This moment was intended to discuss initial hypotheses regarding the question.

In the second moment, we asked the students to continue the investigation using the friction cart and the technological tool as support. Students reported not knowing the *Tracker* software, so we presented the main features and let the groups explore the tool autonomously. This moment allowed students to reflect on what they had initially discussed to demonstrate their lack of (understanding) of the movement prior to using the cart.

To revisit scenes from the subjects' experiences, we used video recording as a resource during the two moments in which the mathematical modeling activity was developed. As previously described, the students were divided into groups, each of which were video recorded. For the data analysis procedures, we chose only one group, as "the number of subjects can be defined by the repetition, in the descriptions, of important aspects of the phenomenon being questioned" (Bicudo, 2011, p. 56). Minayo (2017, p. 4) highlights that the "sample of a qualitative research must be linked to the dimension of the object (or question) which, in turn,

2 It's a toy car that works by friction. When forced to turn its wheels backwards against a surface, a spring accumulates elastic potential energy, and when the toy is released, it moves forward on its own until it runs out of energy and stops.

is linked to the choice of the group." Furthermore, the audio produced by this group was more intelligible than the others and could be analyzed without the need for special treatment.

Therefore, the data came from the footage, totaling approximately one hour of video between the two moments, which was transcribed, analyzed, and organized into significant scenes. The scenes can be described as "[...] peculiar excerpts when investigating subjects in their total acts of understanding, expression, and communication of their experiences" (Bicudo, 2011, p. 101). "A scene, by tying together several meanings in its movement, allows us to practice perceiving several dimensions that its text cannot account for" (Bicudo, 2011, p. 107). Presenting conventional data from a scene in an article is impossible because it would require attaching the video recordings, so we described what happened as significant scenes. In other words, the video recording does not carry ready data in phenomenology. Data are produced in the interstice between the gaze of the researcher who interrogates and the lived experience of the significant subjects.

Given the impossibility of presenting conventional scene data, we will exemplify a transcription: "C2M2: when reflecting on the approximate speed reached by the cart in 5s, they looked at the graph in software but could not identify it. They considered experimenting by releasing the cart and timing the seconds. They intuited that at time 5s, the speed would be zero, as the cart would have already stopped. They seemed satisfied with the result, expressing happiness."

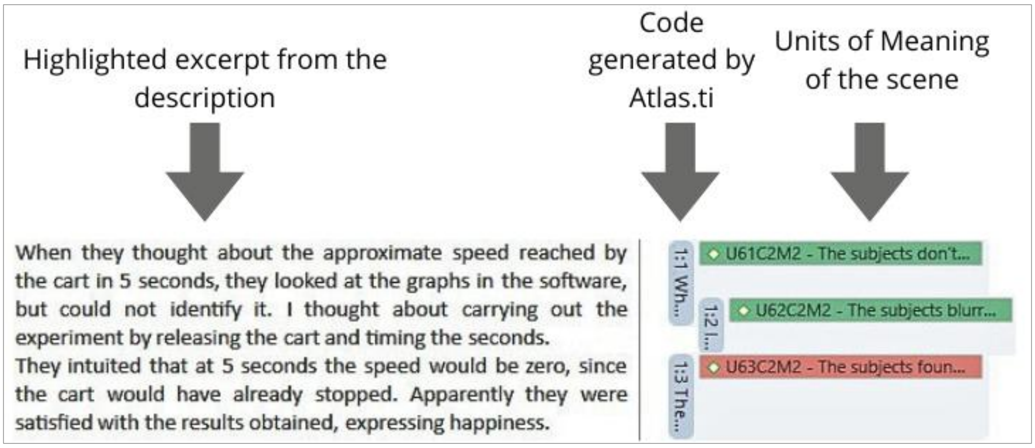
Then, we begin the analyses of the scenes, reading the descriptions repeatedly and carefully to identify the aspects concerning the person being questioned, that is, carrying out the ideographic analysis and thus constituting the units of meaning (UM). Ideographic analysis refers to "using ideograms, that is, expressions of ideas through symbols" (Bicudo, 2011, p. 58). This analysis seeks to make visible the ideas expressed in the descriptions of the scenes, from which units of meaning that "result from the careful reading of the descriptions in their entirety in which the researcher searches for the meaning of the experiences lived by the subject" are revealed (Brito, 2018, p. 99).

It is worth highlighting that the UMs are not given as such in the descriptions; they are articulated by the researcher when searching for meanings in the text described regarding what is being questioned. That is, the researcher carries out careful readings of the written text "[...] intending to highlight what is important in what is being said regarding the question" (Bicudo, 2011, p. 26). During this process, we used the *Atlas.ti*[3] software, which helped us organize analyses in qualitative research and provides support. However, the phenomenological reductions were conducted by the researcher (Klüber, 2014).

The units of meaning were divided into two moments in the activity developed. This movement led to the articulations on the excerpts, that is, it led to the establishment of the units of meaning within each scene, according to the moments, called units of meaning of the scene (UMScene), in this case, in each moment. The process of constituting these units of meaning of the scene was inspired by Brito (2018), who presents the scenes and subjects' expressions in each one's context and their meanings articulated by the researcher when inquiring based on the research question. Figure 1 illustrates a highlighted excerpt from the scene descriptions and the units of meaning that were articulated in this excerpt.

[3] Software licensed from the State University of Western Paraná (Unioeste) within the Postgraduate Program in Science Education and Mathematics Education (PPGECEM) scope.

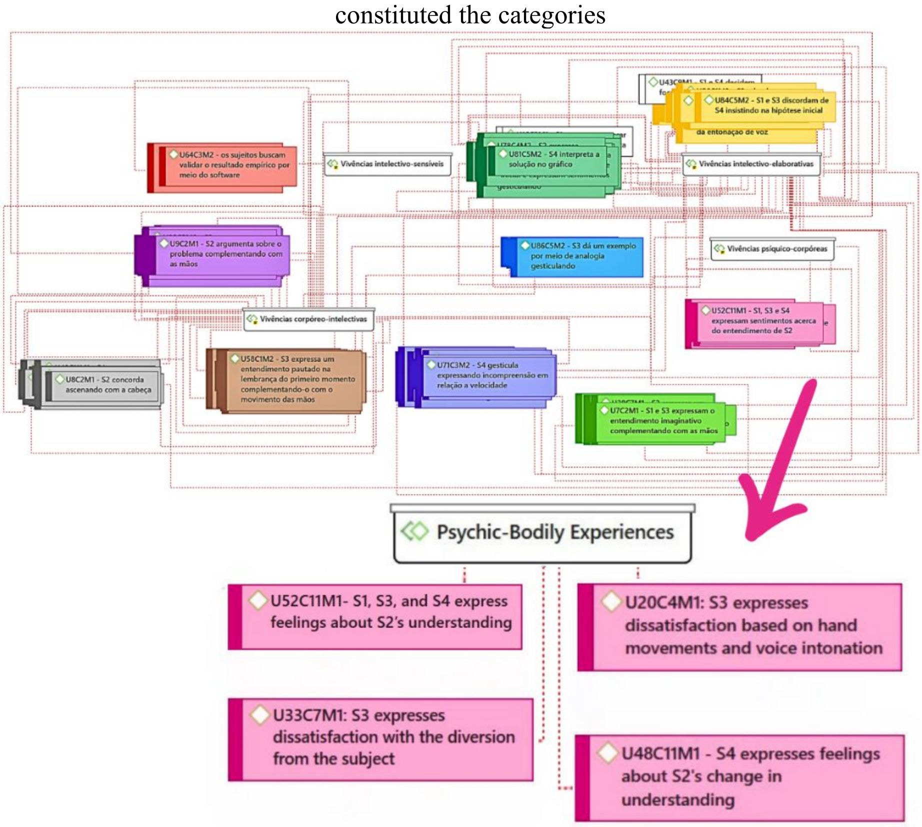
Figure 1: Example of the highlighted excerpt from the descriptions, the code generated by *Atlas.ti* and the units of meaning of the scene



Source: Prepared by the authors with the help of *Atlas.ti* (2024)

After constituting the units of meaning of the scene, we aimed to identify their convergences to explain the structure of the investigated phenomenon. This movement describes nomothetic analysis, which is "successive reductions that seek the transcendence of individual aspects of ideographic analysis, that is, they seek the great convergences or nucleus of ideas that reveal the structure of the phenomenon" (Brito, 2018, p. 101). These converging ideas culminated in open categories that emerged through a careful movement of analysis and reading, seeking to understand individual and convergent meanings. In this analysis movement, we used the *Atlas.ti* software network tool, which helped visualize and group all the UMScenes, as illustrated in Figure 2.

Figure 2: Example of the movement of convergence of the units of meaning carried out in *Atlas.ti*, which constituted the categories



Source: Prepared by the authors with the help of *Atlas.ti* (2024)

With the organized categories, we began summarizing and describing them. According to Giorgi (2010, p. 394), this description "[...] means attributing a linguistic expression to the object of a given act, exactly as it appears within this act." The categories' descriptions are presented, followed by their interpretations, carried out through a hermeneutic movement, in which the aim is to explain what was understood and expressed by the subject. In other words, hermeneutics consists of "an interpretation that aims to show the meaning perceived by the researcher in the subject's words" (Paulo, Amaral, & Santiago, 2010, p. 82). This movement of description and interpretation of categories is exposed in the following section.

5 Analysis of experiences

Although Husserlian transcendental reduction indicates three major dimensions of experiences, spiritual, psychic, and perceptive, our investigation will present those that manifested themselves in the course of students' actions, that is, in the movement between experiences. Therefore, when questioning what experiences students have when developing mathematical modeling activities with digital technologies, four categories emerged: psychic-bodily experiences, intellectual-sensitive experiences, bodily-intellective experiences, and intellectual-elaborative experiences. These categories focus on pairs of experiences since meaning always occurs between experiences (Ales Bello, 2006) and not in isolation.

The category of psychic-bodily experiences was named as such because it refers to the two dimensions that stood out most clearly, as it highlights feelings expressed by students, mainly dissatisfaction with the understanding of their colleagues. In contrast, there are manifestations of positive feelings about the change in understanding of one particular subject, accompanied by bodily and gestural movements. These aspects emerge in the units "U52C11M1: S1, S3, and S4 express feelings about S2's understanding," "U20C4M1: S3 expresses dissatisfaction based on hand movements and voice intonation," and "U33C7M1: S3 expresses dissatisfaction with the diversion from the subject."

In mathematical modeling activities in mathematics education, group work is certainly typified by all Brazilian authors. Ferreira and Jacobini (2010) state that group work is not an easy task for students, as they need to discuss and make decisions collectively, even with differences of ideas. In contrast, the authors describe that collective work can enhance collaborative action related to students' assistance to each other. However, they do not focus on the ways of being in the group. In this sense, in the hermeneutic movement of interpretation of this category, we understand that the feelings students expressed are closely linked to the perspective of the collective work in which they were involved, as they may manifest (dis)satisfaction, or any other psychic-bodily experiences, for example, discouragement, associated with apathy in the discussion. Therefore, it is possible to state that when developing collective activities, different experiences can manifest depending on the group's individual and relational aspects. For this reason, group work may not always be a catalyst for progress; it can also be an inhibitor, depending on how each individual understands and manages the feelings expressed by the other group members.

The group we investigated was formed by emotional proximity, as we understood from our classroom experience with them. Thus, they could freely express this experience with each other. If they did not have this proximity, this type of manifestation could be inhibited or concealed by the subjects, preventing the advancement of understanding of the topic and the mathematics in question. The change in feeling regarding the colleague's understanding is a manifestation before the object of knowledge they were discussing, which highlights the intersubjective role of the relationships established for what they were debating. (Dis)satisfaction is a particular experience that emerges from the group's emotional bond, and the fact that there is conviction provides feedback of acceptance and recognition of each others' ideas.

The category of intellectual-sensitive experiences concerns those lived experiences that indicate thinking about objects targeted here and now; that is, they can be seen and handled. As exemplified in units "U64C3M2: subjects seek to validate the empirical result through software," "U63C2M2: subjects find the possible result through experimentation without the help of the software."

Please note that we are not talking about objects of knowledge but objects of the phenomenal world. In other words, it is because it addresses the objects experienced, those that refer to what is seen-in-the-act-of-seeing, i.e., they do not refer to the object itself but to the articulation of what is seen and its ways of manifesting itself. These experiences arise from the close relationship between what is perceptually shown and what is thought about what was shown.

Here, the students' reactions to the results are evident, based on the data they obtained, which they call empirical, in the development of the activity, with or without the help of the software.

The curiosity shown by students when comparing empirical data during the activity indicates an internal need to confirm what they have just lived. When testing and developing other simulations using the software, they demonstrate the resumption of what they understood about the issues involved during the activity.

Considering that the activity involved a physical phenomenon of the cart's movement, the need for the subjects to revisit the lived experience emerges in an attempt to derive an empirical understanding. However, by focusing on the cart's movement and developing and reassessing the meanings, they oscillate between thinking about and seeing to be thought. This fact could give the idea that the physical object communicates knowledge; however, this oscillation between thinking and seeing is done by the subject, acting in the constitution of knowledge and the communication that he/she establishes with others.

The coming and going of the physical object and its movement show that the experiment does not communicate knowledge. In this coming and going, knowledge is constituted. By looking deeper into a dialectic between intellectual and perceptive (sensitive) experiences, we can affirm that each return to the sensitive object allowed us to think about new aspects of what they were trying to understand, the speed of the cart and that something was lost when the experiment became out of focus. Therefore, thinking mathematically about movement does not arise from the movement itself but from the sense of measurement given to the movement and which is not immediately understood. It is necessary to look from another perspective, that is, with mathematics, with the necessary sense of measurement requested by the phenomenon, now in a phenomenological sense, called into question.

The bodily-intellectual experiences manifest themselves only through the body, without elaborate written, drawn, or spoken language, requiring inferences to be seen. It speaks more about the subject's expression about something than about what this something means to the subject. In this category, (mis)understandings, doubts, communication about the correspondence of ideas expressed as (mis)agreement, and the idea about the concept in the face of the discussions held by the group are evident. These aspects are manifested through bodily movements, such as manual gestures and facial expressions, sometimes isolated, sometimes simultaneous, as highlighted in the units "U39C8M1: S1 explains the difference between two concepts based on memory complemented with hand movements," "U22C5M1:

S4 expresses doubt based on facial expressions," and "U87C5M2: S4 presents an understanding through facial and gestural expressions."

Unlike the other categories, this does not communicate meanings that can be more widely explored in terms of explicit language but indicates that other spiritual experiences participate in bodily expressions. Agreeing or disagreeing is how spiritual and psychological experiences are manifested, as they involve evaluating what was said, remembering meanings, establishing a judgment, and making a decision. These ostensive elements, i.e., what is visible, palpable, arise from the very body that participates in the constitution of knowledge.

These experiences communicate meanings of movement to the subjects themselves, as the body makes the movement that would be of the intentional object that is thought, in this case, of the friction-cart-in-motion. Therefore, understanding the phenomenon involves perceptive experiences, allowing us to think not only about the object, as if it were a reflection from the outside in, but also thinking-as-the-object, with the body. This level of understanding proves necessary and inseparable for the cognition of objects that are not mathematical.

The body is a way of being aware of the objects that are studied and reveals that knowledge, whether correct or not in conceptual terms, is strictly linked to the way of expressing oneself bodily, highlighting certainties based on memories or consolidated beliefs, i.e., the lived experience is lived again, activating its meanings for the phenomenon studied.

The category of intellectual-elaborative experiences lies between what is experienced as a memory and the initial understanding of ideas, moving on to more developed levels, such as imagination, projection through hypotheses, reflection, analogy, etc.

These experiences highlight students' lack of understanding of the development of the activity and possible solutions. Sometimes they persist in the initial hypothesis, sometimes they disagree based on their experience or their own idea about the concept in light of the discussions carried out by the group. In this case, they rethink their understandings and analogies to identify and collectively assume or deny the same argument. Subjects share and add ideas and express themselves through gestures, voice intonation, and with the help of materials such as paper and pen.

The manifestations analyzed through gestures or spoken expressions appear as understandings based on analogies of the subjects correlated to the movement of interpretation, whether they are graphic of the initial hypotheses or with technological resources. The aforementioned analogies emerge from questions, discussions, and reflections, which converge towards a collective consensus, according to the units of meaning "U50C11M1: S2 understands and agrees with colleagues, expressing understanding through pen and paper," "U71C3M2: S4 gestures expressing incomprehension regarding speed," "U56C1M2: S2 recalls that he disagreed with his colleagues' understanding, arguing through facial expressions, gestures, and voice intonation."

All actions and acts, combined with the moment these subjects find themselves, give rise to the possibility of mastering the context. The learning reported here appears to have evoked experiences from studies carried out previously. This movement conforms to the intellectual acts concerning memory, which allow themselves to be targeted by other acts, such as reflection, judgment, and evaluation of what they did. Elaborative experiences are necessary to advance the understanding of what they were studying and, in the intersubjective alignment or confrontation, they seem to be the most important when seeking to advance the understanding or production of the meaning of mathematical idealities that do not have a sensitive correspondence with the phenomenal objects studied, at least immediately. These experiences are closely linked to abstract and creative acts in the sense of producing new ideas or more refined and comprehensive ideas about the phenomenon in question.

6 Final considerations

Students' experiences during the development of the mathematical modeling activity with digital technologies aim not only at observable events or behaviors but also at the perceptions, meanings, and intentions underlying these experiences. We considered students' emotional, cognitive, and contextual aspects of mathematical learning mediated by technology. In other words, how students manifested psychic, spiritual, and bodily aspects.

This investigation allowed us to open up another horizon of understanding about group work developed in mathematical modeling activities. In general, collective action is taken in its entirety, and experiences must be viewed from the individual to the group. In other words, even studies that scrutinize student actions did not carry out this analytical movement of thinking about the person in the context of the group since the references preconceived collective action as positive. Here, group work resonated with what subjects experienced in their movement of being with the phenomenon under study, with others, and with the available technologies.

From the point of view of experiences with digital technologies, the sense of measurement that emerged from the modeling question and the use of technological devices indicated that the materiality offered by the software regarding the precision of the measurement and the graphical developments was the aspect that required the most elaborate pairs of experiences. In other words, we understand that it would not be possible, at least within the scope of a classroom, to find correspondence between the intellectual elaboration of the hypotheses raised and the cart's speed without the digital technologies that became an extension of the living body itself. Therefore, the statement that the subject-is-with-technology shows theoretical and procedural strength, demarcating specific ways of constituting knowledge production.

The analysis carried out opens up a field of research that investigates not only the production of mathematical knowledge but its movement of the constitution in the classroom from the perspective of the person who is with the other and not of someone who is absorbed by the group or simply in a context of technological tools.

Finally, we understand that the topic is relevant, both for its novelty in the context of research in mathematical modeling and for the possibility of opening a region of inquiry with implications for the classroom that address the experiences lived by students with mathematics and digital technologies.

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