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Repercussions of mathematical modelling education within the scope of a professional master's degree for the practice of Basic Education teachers

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Abstract: In this paper we infer the repercussions of a mathematical modelling education within the scope of a professional master's degree for the practice of Basic Education teachers. Our theoretical support is modelling as a pedagogical alternative for teaching Mathematics and directions related to teacher's education, following three axes: learning about, learning through and teaching using modelling. The qualitative analysis of the responses from fourteen teachers to a questionnaire at the end of a Mathematical Modelling course was structured via a Tree of Idea Associations from which three categories referring to aspects of the repercussion of the course were highlighted: theoretical/conceptual aspects; didactic/pedagogical aspects; personal/social aspects. These categories revealed that the theoretical/conceptual, as well as the didactic/pedagogical aspects were directly related to the directions adopted in the discipline; the personal/social aspects were directed towards the treatment of teacher educators within the scope of the discipline.

Keywords: Mathematics Education. Mathematical Modelling. Teacher's Continuing Education.

Repercusiones de la formación en modelación matemática en el ámbito de una maestría profesional para la práctica de docentes de la Educación Básica

Resumen: En este artículo inferimos las repercusiones de la formación en modelación matemática en el ámbito de una maestría profesional para la práctica de los docentes de Educación Básica. Nuestro aporte teórico es la modelación como alternativa pedagógica para la enseñanza de las Matemáticas y direcciones relacionadas con la formación docente, siguiendo tres ejes: aprender sobre, aprender a través y enseñar mediante la modelación. El análisis cualitativo de las respuestas de catorce profesores a un cuestionario al final de un curso de Modelación Matemática se estructuró mediante un árbol asociativo de ideas del que se destacaron tres categorías referentes a aspectos de la repercusión del curso: aspectos teóricos/conceptuales; aspectos didácticos/pedagógicos; aspectos personales/sociales. Estas categorías revelaron que los aspectos teóricos/conceptual, así como didáctico/pedagógico, estaban directamente relacionados con las direcciones adoptadas en la disciplina; los aspectos personales/sociales estuvieron dirigidos al tratamiento de los formadores de docentes en el ámbito de la disciplina.

Palabras clave: Educación Matemática. Modelación Matemática. Formación Continua Docente.

Repercussões de uma formação em modelagem matemática no âmbito de

um mestrado profissional para a prática de professores da Educação Básica

Resumo: Neste artigo inferimos sobre as repercussões de uma formação em modelagem matemática no âmbito de um mestrado profissional para a prática de professores da Educação Básica. Nosso aporte teórico é a modelagem como alternativa pedagógica para ensinar Matemática e encaminhamentos relativos à formação de professores, seguindo três eixos: aprender sobre aprender por meio e ensinar usando modelagem. A análise qualitativa de respostas de quatorze professores a um questionário no final de uma disciplina de Modelagem Matemática foi estruturada via árvore de associação de ideias das quais três categorias referentes a aspectos da repercussão da disciplina foram evidenciadas: aspectos teóricos/conceituais; aspectos didáticos/pedagógicos; aspectos pessoais/sociais. Essas categorias revelaram que os aspectos teóricos/conceituais, bem como os didáticos/pedagógicos estiveram diretamente relacionados aos encaminhamentos adotados na disciplina; já os pessoais/sociais se direcionaram para o tratamento das professoras-formadoras no âmbito da disciplina.

Palavras-chave: Educação Matemática. Modelagem Matemática. Formação Continuada de Professores.

1 Introduction

With the aim of putting Mathematics into practice and envisioning the formation of responsible citizens, mathematical modelling addresses both formative and humanistic aspects. The first kind aims to make the learning of mathematical theories and methods feasible, in accordance with their rules and definitions (Swan, Turner, Yoon & Muller, 2007); the second aims to provide experiences in which Mathematics can be used to analyze, argue, and make decisions, intending to foster critical and creative education, as well as serve as a tool for social and cultural transformation (Maass, Sorge, Romero-Ariza, Hesse & Straser, 2022).

This proposal has been capturing the attention of Basic Education teachers, who tirelessly seek alternatives that can provide a Mathematics education promoting student engagement while also making sense—both in terms of facilitating the understanding of Mathematics and teaching students how to apply it to real-world problem-solving situations. This approach to Mathematics contrasts with the traditional one, which focuses on the teacher introducing concepts and having students practice exercises similar to those given as examples.

Mathematical modelling is based on the principle of sharing the responsibility for learning with the students, granting them greater autonomy (Almeida, Silva & Vertuan, 2012; Melo & Bisognin, 2021). In this regard, it aligns with the teacher's role described by Tardif and Lessard (2014), who emphasize interactivity as the primary focus of the teacher's work. Interactivity is a key aspect of modelling, as by giving students the freedom to make their own choices, the teacher's role is to guide them in taking advantage of learning opportunities, which occur through conversations, questions, suggestions—essentially, interaction (Bassanezi, 2002).

In this sense, educational initiatives in modelling are necessary to ensure that teachers can act with confidence, providing guidance on how to prepare for lessons, approach concepts, and guide students (Niss & Blum, 2020). Although modelling is addressed in some initial teacher education courses, breaking away from established practices is not easy, even when there is a willingness to do so. The workload in the classroom hinders the search for alternative practices, and there is a lack of time and space to engage in dialogue with peers, study new possibilities, and plan differentiated practices.

In this way, it is essential to provide educational opportunities that offer both theoretical and practical support, with space for dialogue with peers and guidance from expert teachers who, in addition to teaching what modelling is, create the conditions for in-service teachers to incorporate it into their practice based on studies, planning, and reflections.

There are several configurations that education in mathematical modelling can take: communities of practice (Braz, 2017), extension courses (Forner, 2018), collaborative groups (Mutti, 2020), among others. However, we highlight professional master's programs, which have proven to be valuable options for providing modelling experiences to teachers involved in professional development, both in terms of research linked to practice and the practice itself.

To investigate the repercussions of a mathematical modelling course, offered as part of a professional master's program in Mathematics Teaching, on the practice of in-service teachers in Basic Education engaged in professional development, we analyzed a questionnaire completed by 14 teachers at the end of the course. The purpose of using the questionnaire was to give voice to the teachers, highlighting their perceptions of their experience in the course, including the practices it prompted, while understanding that in-service teachers possess knowledge that can contribute to both their own development and that of their peers (Borges & Cyrino, 2019).

The teachers' responses were analyzed from a qualitative perspective, as we believe this approach is better suited to addressing their discursive productions, which relate to their participation in the course and their reflections on the repercussions of this participation on their practice. We used the "Tree of Idea Associations" (Spink, 2013) to structure and highlight the results.

Initially, we discuss teacher education in mathematical modelling, explaining the course organization. Then, we present the participants, the research context, and the questionnaire as the data collection instrument, outlining the analysis procedures. Finally, we provide our analysis of the teachers' responses, followed by our considerations based on inferences and connections we made.

2 Teacher education in modelling

The problematization of situations that may be part of students' daily lives has been addressed through different teaching methodologies, such as Exploratory Teaching, Problem Solving, and Mathematical Modelling. This approach is discussed in the context of teacher education, as it reveals important contributions to pedagogical practice.

In research conducted by Braga and Cyrino (2022), aspects of the professional vision of Pedagogy students were highlighted when they analyzed the actions of a Mathematics teacher developing a lesson from the perspective of Exploratory Teaching. Based on episodes of lessons recorded on video as part of a multimedia case for the problematization of area and perimeter, the pre-service teachers highlighted key actions present in the lesson supported by Exploratory Teaching, such as lesson organization and management, the establishment of connections between theory and practice, communication and argumentation that mediate mathematical activity, and knowledge of mathematical content. The promotion of professional knowledge through the approach undertaken via Exploratory Teaching emerged as a way to provide pre-service teachers with a projection for their future teaching practice.

Marins, Teixeira, and Savioli (2021), supported by Exploratory Teaching of Mathematics, revealed the mobilization and development of professional knowledge in participants of the *Programa Institucional de Bolsa de Iniciação à Docência* (PIBID). The

researchers conducted a formative process with undergraduate Mathematics students and Basic Education teachers, involving activities related to "teaching practice in moments of planning, teaching, and reflection" (Marins *et al*., 2021, p. 169). In general, the actions present in Exploratory Teaching contributed to the mobilization and development of the professional knowledge of the future teachers, with an emphasis on the selection of a challenging task, the anticipation of possible solutions, the search for knowledge on the mathematical content, the explanation of the lesson dynamics, and the use of manipulable materials.

In the research by Costa and Allevato (2019), whose data were produced with future Mathematics teachers in twelve meetings of 4 hours each, actions involving Problem Solving were experienced by the participants as problem solvers. This allowed the pre-service teachers to identify possibilities for working with this methodology through problems related to students' everyday lives. To this end, the problems were considered as a starting point for teaching, learning, and formalizing a particular mathematical content. In this way, the pre-service teachers understood that learning mathematics can occur in "a movement from the concrete (a real-world problem that serves as an instance of the mathematical concept or technique) to the abstract (a symbolic representation of a class of problems and techniques for operating with these symbols)" (Schroeder & Lester, 1989, p. 33).

The previously mentioned studies revealed innovations in initial teacher education that allowed students to experience practices where problematization became a central aspect in the discussions held in the Pedagogy and Mathematics (Education) courses. These are contemporary actions that enable pedagogical practices to be envisioned within teacher education, to later be implemented in teaching. However, teachers who did not have initial education experiences like the ones presented here lack continuous professional development to incorporate these methodologies into their lessons. Professional master's programs have proven to be a fruitful space for establishing this professional development.

Professional master's programs enable an understanding of "teacher education" that aligns with the perspective advocated by Cyrino (2013, p. 81) regarding knowledge-emancipation, which can "occur if we provide theoretical and conceptual contexts immersed in various practices, encouraging habits of conversing, investigating, questioning, reflecting, and relating theory and practice in an interactive process."

This perspective departs from, as explained by Borges and Cyrino (2019), a model of teacher education that exclusively prioritizes academic aspects and specific knowledge in each area, to the detriment of those generated directly in the environments where teachers act or will act. In this perspective, the school is also considered a formative space. Professional master's programs, therefore, constitute "spaces for education oriented towards problematization, reflection, and research on teaching practice [...]" which "[...] do not imply the abandonment or suppression of other practices and discourses already established in teaching and schools" (Losano & Fiorentini, 2021, p. 1243).

In the research reported by Losano and Fiorentini (2021), a teacher's immersion in the Professional Master's program led to changes in their school practice, where discourses from Mathematics Education became natural, and the implementation of methodologies prioritizing problematization became more frequent.

Especially for our purposes, we believe that the context of professional master's programs is conducive to teacher education in mathematical modelling, as they are often sought by teachers who aim to address needs not met during their initial teacher education. These programs provide an opportunity for teachers to explore new practices, pedagogical possibilities, and to rethink their teaching in Basic Education.

Moreover, by serving teachers who are actively teaching in the classroom, professional master's programs can provide them with experiences in modelling both as modellers and as teachers within their own practices (Tortola, Silva & Dalto, 2023). These experiences, supported by theoretical reflections and guided by an expert professional, with space to engage in dialogue with peers, can not only encourage teachers to integrate modelling into their practices but also provoke changes in their ways of teaching mathematics, addressing the formative aspirations outlined by Cyrino (2013) and, particularly, by Klüber and Tambarussi (2017).

Sousa and Almeida (2021) argue that education in modelling should provide teachers with confidence in its use and the boldness to break existing teaching paradigms. This includes learning about modelling and how it fits, or does not fit, within curriculum guidelines, the school, and other formative spaces (Almeida *et al.*, 2012); as well as gaining experience with modelling in formulating and solving problems and using it in classroom teaching of mathematics (Galbraith, 2012; Silva, Cruz & Omodei, 2024).

Based on these assumptions, we structured a Mathematical Modelling course guided by the three formative axes proposed by Almeida and Silva (2015): learning about modelling, learning through modelling, and teaching using mathematical modelling.

Learning about mathematical modelling refers to actions aimed at a theoretical and epistemic understanding of modelling, which includes knowledge of different conceptions and perspectives and their implications for practice, as well as knowledge of how to do it. This is typically described through procedures and actions organized in non-linear cycles and schemes, which contribute to guiding the implementation of modelling practice.

Learning through mathematical modelling refers to actions related to the development of practices as modellers, in which teachers involved in professional development need to formulate and solve problems. In other words, it involves actions that place teachers in the role of students and provide genuine experiences where they can engage in the process of modelling. Evidently, the teacher working in the classroom cannot detach themselves from their "being a teacher" (Cyrino, 2013), and thus discussions about how their students would approach or how they could use such activities in the classroom may arise.

Teaching using mathematical modelling involves actions that challenge the teacher in developing modelling practices in the classroom. Aiming to understand how to guide students toward learning mathematics, the teacher involved in professional development focuses on actions that can help students engage in the modelling process. Through these actions, the teacher is encouraged to reflect on their role in practice, supported by insights from an experienced professional and peers.

Articulating the learning of mathematical modelling with teaching using mathematical modelling, based on these three axes, constitutes, according to Almeida and Silva (2015), a robust perspective for teacher education in mathematical modelling, as it brings teaching and learning hand in hand (Borromeo Ferri, 2018).

3 Methodological aspects

The professional master's program in Mathematics Teaching at a university in Paraná is intended for teachers who hold a degree in Mathematics or are qualified to teach in elementary school. In this program, teachers involved in ongoing professional development can take courses organized into two groups: pedagogical and content specific.

The course "Mathematical Modelling in the Perspective of Teaching" is of a pedagogical

nature and has been offered since 2016, typically in the first semester of each year, with a duration of 45 hours. The syllabus focuses on exploring different perspectives of Mathematical Modelling in mathematics teaching.

Considering the syllabus and understanding that the three formative axes can address one of the objectives of the aforementioned professional master's program (enabling reflection on teaching practices in order to (re)orient them based on necessary aspects), the course has been organized to allow teachers in continuing education to study theoretical texts, develop modelling activities as modellers, and plan and implement modelling practices in Basic Education.

In 2021, the course took place from March 26th to July 16th and was taught by two instructor-professors. Fourteen teachers participated that year, all of whom held a degree in Mathematics (Education). Of these teachers, twelve reported having had contact with mathematical modelling before the master's course, but none had implemented it in the classroom. To plan and implement the practices, the teachers worked in pairs or groups of three, resulting in six practices being planned and implemented.

The teachers were informed about their participation as part of the course design structure, the results of which were part of the research project approved in the CNPq/2021 Universal Call. After being informed about the data to be used, all participants signed an informed consent form. These teachers are referred to in the paper as P1, P2, ..., P14.

In the last class meeting on July 16th, the teachers completed a Google Forms questionnaire with 10 questions: 1) How do you characterize Mathematical Modelling? 2) Do you feel prepared to use Mathematical Modelling now? Justify. 3) Describe your actions in developing the radar activity. 4) Describe your actions in developing the activity of the third familiarization moment. 5) Describe your actions in planning and implementing the modelling activity in the classroom with students. 6) Comment on the organization of the course, specifically on the three axes: learning about, learning through, and teaching using modelling. 7) In your opinion, how did the course "Mathematical Modelling in the Perspective of Teaching" contribute to your professional development? 8) Do you intend to develop other mathematical modelling activities with students? Why? 9) Share your impressions of the course, the instructors, and anything else you think is important to share. 10) Do you have any suggestions for changes to the course? If so, what are they?

In this paper, with the aim of investigating the repercussions of a Mathematical Modelling course offered as part of a professional master's program in Mathematics Teaching on the practices of in-service teachers in Basic Education engaged in professional development, we focus our analysis on the responses of fourteen teachers to the mentioned questionnaire.

The analysis is supported by a qualitative approach based on the Tree of Idea Associations (Spink, 2013). This strategy visualizes argumentative construction (or co-construction) and aims to make the chain of repertoires in the segments most illustrative of the phenomena under study visible. In our research, this strategy was used to highlight the repercussions of the design of a Mathematical Modelling course on teachers involved in continuing education.

4 Analytical movement

Considering the teachers' responses to the questions, we undertook an analytical process that began with a preliminary reading aimed at identifying indications of the course's repercussions on teacher education. This reading revealed patterns that emerged into categories

related to theoretical/conceptual aspects; didactic/pedagogical aspects; and affective aspects, considering both personal and social approaches.

A revisit of the responses was necessary to select relevant excerpts that revealed signs of these categories. Using the *text color* tool in the word processor, we employed blue to highlight excerpts related to theoretical/conceptual aspects, red for excerpts related to didactic/pedagogical aspects, and pink for excerpts related to personal/social aspects. Table 1 illustrates the highlights from the responses of teachers P1, ..., P5 for question 2.

Table 1: Excerpt from the analytical movement with *text color* highlights for the responses to question 2

Teacher	Highlighted excerpts
P1	[...], I feel more confident in applying it in my classes as a teacher. I have built a new perspective on modelling, both in relation to the procedures and in how to approach this topic, starting from the students' curiosities in their speeches.
P2	Prepared with some insecurity due to the wide variety of content that can be assumed.
P3	Sim, because I hadn't worked remotely with the students before, but the experience I had during an application for the course left me better prepared for any other ways of applying the methodology.
P4	Yes. The way the course was taught, all the content covered, made me feel confident and prepared.
P5	Yes, as the classes went on, the familiarization with mathematical modelling through the teacher's guidance became more concise. Of course, I still have much to learn, but what's most interesting is that I no longer see myself practicing the same style of teaching. Every example I experienced during this semester in the course sparked new ideas in me. Listening to my colleagues during their presentations was also very good because it was possible to observe the different ways of approaching the same theme. Anyway, I hope to continue on this path.

Source: Research Data

The same highlighting approach used for question 2 was applied to the responses of the fourteen teachers to all ten questions. This process led to the structure of our analysis, which is based on three emerging categories: theoretical/conceptual aspects, didactic/pedagogical aspects, and personal/social aspects, in dialogue with the literature on mathematical modelling. To reference the responses, we used the teachers' codes and the question number. For example, P3Q1 refers to the response of P3 to question Q1.

5 Analyses

In the category highlighting *theoretical/conceptual aspects*, the teachers revealed that they recognized the formative axes they were subjected to and described actions on how to develop modelling both as teachers and modellers, referring to the phases of the activity. These aspects were most frequently revealed in responses to Q1, where teachers were asked to characterize mathematical modelling; in responses to Q3, Q4, and Q5, where they described actions they carried out for the development of modelling activities, whether within the subject or in implementing the practice; and in responses to Q6, where they signaled recognition of the formative axes.

In fact, we assumed from the planning stage of the course that integrating theory and practice could provide "the participants the opportunity to learn about the content and to have the experience of a teaching method they can use in their lessons at the same time" (Borromeo Ferri, 2018, p. 4). The responses to P3Q6 and P4Q6 revealed the repercussion of the course on

understanding modelling practices according to actions as both teachers and students.

- [...] so being presented in axes that involved both theory and practice, whether as a student or teacher, greatly helped everyone gain a better understanding of modelling. (P3Q6)
- [...] learning about what it is, what it's for... learning through, for example, performing mathematical modelling activities, which is important for putting all theoretical knowledge into practice, and teaching using mathematical modelling is already putting into practice everything you've learned, while learning together. (P4Q6)

In this way, incorporating the understanding of what modelling is can, in continuous professional development, help teachers "overcome the strictly empiricist and pragmatic view of the teacher's practice in relation to modelling, moving towards a realm in which it is accepted that the 'how to do' is imbued with theory" (Almeida *et al.*, 2012, p. 24). By engaging with theoretical approaches, teachers become aware that there is a way to develop a modelling activity:

- Modelling has a step-by-step process that must be followed in order to be applied correctly: familiarization, mathematization, solution, interpretation of results, and validation. (P1Q1)
- A learning strategy that, through a real-life situation, follows stages, but not in a systemic manner. (P10Q1)

The indication of stages in the development of modelling activities is discussed in the literature, forming cycles that signal how (or should signal) the development of a mathematical modelling activity takes place. However, as Borromeo Ferri (2018) asserts, modellers may not follow the cycle linearly, moving back and forth through the procedures as many times as they deem necessary, constituting what the author calls individual modelling routes. The response from P1Q1, however, seemed to reveal that, for her, the stages of the cycle are rigid: "[...] has a step-by-step process that must be followed in order to be applied correctly." This suggests that additional discussions are needed, in which the teachers, especially P1, would highlight the non-linearity of the modelling cycle.

However, for P10, it was clear that the approach occurs "[...] not in a systemic manner [...]." This emphasis was also present in the responses from P9 and P3 to Q5, where they highlighted the need to explore more than one resolution to reach a solution for a problem they investigated, as emphasized in the excerpts:

- We collected data together, created a problem, and *solved it in multiple ways until we found the one that was as clear as possible*. (P9Q5, emphasis added)
- [...], however, when we discussed the activity with the class, some points were raised that would require us *to reconsider our model*. (P3Q5, emphasis added)

In the case of the response P3Q5, revisiting the resolution became necessary after communicating the results to classmates in the Mathematical Modelling course. Through communication, "an argumentation that can convince the modellers themselves and those to whom the results are accessible that the proposed solution is reasonable and consistent" is developed (Almeida *et al.*, 2012, p. 19). Indeed, during the communication, due to questions

from classmates, it was necessary to reconsider the mathematical model.

The mention of a mathematical model appeared in 24 responses from the teachers. The mathematical structuring of the situation under study is both relevant and necessary due to the nature of the modelling activity, in which mathematization occurs—"a representation of aspects of an extra-mathematical domain by means of some mathematical entities and the relations between them" (Niss & Blum, 2020, p. 6). This representation had a repercussion, manifesting in two characterizations: model as a means and model as an end.

There are teachers who understood the mathematical model as a support for argumentation and the communication of results (Almeida *et al.*, 2012). In this sense, "the use of mathematical models depends on the knowledge of facts and phenomena, the recognizable behavior of real objects and systems, usually expressed by laws, mostly empirically derived" (D'Ambrosio, 2009, p. 92). In this regard, it serves as a means to study a phenomenon, a real-world problem situation, as the following excerpts clarify:

- From an educational point of view, Modelling is an activity that involves real data, in which students are asked to reflect on the subject, justifying their answers through mathematical models. (P3Q1)
- The next step was to think of a mathematical model that would address this situation; several were created, and we chose one. After performing the calculations, we proceeded with validating the data, where we were able to confirm that our model was correct or close to it. (P13Q3)

However, depending on how the modelling activity is conducted, it can be considered a "dynamic process used for obtaining and validating mathematical models" (Bassanezi, 2002, p. 24), such that the mathematical representation can, in a way, be recognized as the end. This characterization of the model was present in the responses from the teachers involved in professional development regarding their understanding of modelling, as well as when describing actions in which modelling activities were developed:

- It involves taking a real-life problem and developing and solving it, creating a mathematical model from it. (P12Q1)
- The construction of a solution, referred to as a mathematical model, aimed at solving real-world context problems. (P8Q1)
- The mathematical model was established through a table and the creation of a graph in Excel and GeoGebra [...]. (P11Q5)

We conjecture that the teachers' responses grouped in the category of *theoretical/conceptual aspects* allowed us to highlight the repercussions of the course related to the understanding of what modelling is, as well as the way a modelling activity is developed, where practical and theoretical actions were intertwined, as shown in the excerpt from P7Q2:

- [...] because, through the activities developed during the course and the 3 proposed moments, I was able to experience mathematical modelling, understanding the processes and stages, as well as the concepts and approaches in the application of modelling activities. (P7Q2)

By recognizing and adopting mathematical modelling as a pedagogical alternative, one of the objectives is to support "the learning of mathematics, by offering motivation for its study,

as well as interpretation, meaning, proper understanding, and sustainable retention of its concepts, results, methods, and theories" (Niss & Blum, 2020, p. 28). Although we understand that motivation is also linked to the internal conditions of the learner, it can be stimulated by external factors, such as the teacher's actions.

The teacher's actions in the development of a modelling activity may differ from those typically used in the classroom, where there is a need to shift from a lecture-based approach followed by exercises to situations that are essentially investigative (Almeida *et al.*, 2012).

As a result of the teachers' actions in response to modelling activities, there was a recurring recognition in the responses to the survey questions of the importance of confidence, organization, attention to students' records, questioning, and the conceptualization of the content taught or to be taught within the scope of the course. This led us to highlight the category of *didactic/pedagogical aspects*.

In organizing the course with the three formative axes in mind, we primarily focused on teachers experiencing modelling both theoretically and practically. This is because we believe that, to prepare teaching professionals to implement modelling practices, experience is essential. This approach is also supported by Borromeo Ferri (2018, p. 4), who states that her educational courses give "the participants the opportunity to learn about the content and to have the experience of a teaching method they can use in their lessons at the same time." The following excerpts highlight the importance of experience in teacher education:

- [...] the experience I had during the application [of a modelling activity] as part of the course made me better prepared for any other ways of applying the methodology. (P3Q2)
- [...] Now, having gone through this experience, I feel more confident in applying this approach in my classes. (P10Q7)

Such responses align with Larrosa's (2019, p. 23) statement that "One does not write about experience, but from it." By experiencing the development of a mathematical modelling activity, the teacher may feel better prepared (P3Q2) or even more confident (P10Q7) in implementing pedagogical practices where this approach is present, potentially even radically changing the way they view and plan their lessons:

- [...] I no longer see myself practicing the same teaching style. (P5Q2)
- I believe I will become a better and more critical teacher in my lessons and the activities I apply with my students. (P6Q7)

However, this shift in understanding of practice takes shape as the teacher becomes familiar with mathematical modelling in the role of a guide. Familiarization creates a natural environment for the teacher to articulate three perspectives developed at the core of a modelling activity: "definition, investigation, and resolution" (Almeida *et al.*, 2012, p. 26). This understanding of familiarity and the naturalization of practice in the classroom was indicated in P6's response:

- [...] although it is not easy, I believe that as I engage in more modelling activities with the students, it will become easier for both me and the students. (P6Q2)

Becoming familiarized through lived experiences was a positive repercussion of the

course in teacher education. Therefore, we can highlight that the confidence to face the challenge of implementing classroom practices, as well as embracing potential changes in lesson planning, was established:

- Knowledge produces confidence. As I developed the activities during the course, I gained more knowledge about mathematical modelling and, consequently, more confidence in using it in the classroom. (P8Q2)
- [...] providing greater security and confidence to develop and implement other activities in different classes. (P7Q7)
- [...] I feel more confident applying it in my lessons as a teacher [...]. (P1Q2)

The experience, in a way, served as a means for the teacher to face their challenges, encouraging them to adopt new practices, supporting new experiences that could help them plan lessons with modelling, and even anticipate actions to prepare for implementation in other situations, as revealed in P5Q4's response:

- For the planning, I first considered which class I could apply the activity to. Since we were in a remote learning situation and some classes had low participation, this was an important factor in my decision. Therefore, the class with the most student participation was an 8th-grade class, which is why the activity was planned for them. After defining the class, the scheduled content was the study of monomials, so I researched and looked for ideas. [...] The professor, my advisor, suggested sending a form prior to the lesson so that the students would familiarize themselves with the topic, in line with the flipped classroom approach, a model used in Hybrid Teaching. [...] With all this in hand, I created four problem-solving situations to apply to the 8th-grade students, who were divided into groups based on their responses to the form. The details of the activity development [...] involved letting the students speak, that is, allowing them to express their opinions on what they would like to study. Furthermore, while they were working in groups, I tried to intervene without "giving" them the answer. (P5Q4)

For Almeida and Silva (2015, p. 15), a modelling activity "provides support for the teacher to reflect on their practice and conduct activities according to the school context in which they are situated." In the context experienced by P5, the choice of the most participative class, the creation of a form for students to fill out, as well as the organization of the groups based on their responses to the form, were supported by the invitation from the course instructor and the remote context in which P5 was immersed. The remote context became "an option to be considered when Mathematics teachers and students are unable to interact in person" (Menezes, Braga, Santos & Bairral, 2022, p. 19). The remote context was also considered by P8, where different structural organizational methods were implemented, supporting their work with the inclusion of multiple technological resources:

- Organization of digital platforms (forms, WhatsApp groups, Jamboard rooms) to begin the development of the activity. (P8Q4)

The organization of students into groups, either in person or remotely as mentioned by P8, was a recurring practice in modelling activities developed in the course and planned by the teachers in education. This is because "Mathematical Modelling in the classroom can be seen as an essentially cooperative activity" (Almeida *et al.*, 2012, p. 33). The focus on organizing students into groups was evident in the teachers' responses when they referred to the classroom:

- A mathematics problem situation, where students need to participate actively, is essentially cooperative, involving group work in an investigative process. (P6Q1)
- The class was divided into groups through a draw, with possible changes later based on affinities, resulting in seven groups of four students and one group of five students [...]. (P7Q4)

The teachers also expressed concerns about the students working in groups, organizing their written records to continue solving the modelling activities, as well as sharing the data with their classmates. This concern was reflected in the excerpt from P7's response:

- On the scheduled date, during the class, and with their notes in hand, the groups presented the data they had collected to their classmates, promoting a collective discussion and comparing the results obtained. To conclude, they created a bar chart for each group based on the collected data. Later, these charts were compiled into a single exhibition by the teachers, providing an opportunity to observe and compare the data in a broader context, visualizing which yeast option would be the best for the investigated situation, and synthesizing the activity. (P7Q4)

However, ensuring that students recorded their data for later consultation or even sharing with their peers gave the teachers the role of a guide. A guide is someone who points out paths, asks questions, encourages reflection, does not accept what is inadequate, suggests procedures, does not provide ready-made answers, does not accept that "anything goes", and does not relinquish their authority as a teacher (Almeida *et al.*, 2012). The actions of a guide were acknowledged by the teachers in education, in responses to Q4, which related to planning and implementing practice. Mentions of actions such as guiding, gathering, prompting, monitoring, and questioning were present in their answers, as evidenced by the excerpts transcribed below:

- But I remained the *guide* for the activities developed by them, questioning and assisting when necessary. (P1Q4, emphasis added)
- After making the cakes and taking notes, we *gathered* on Meet, individually with each group, for a discussion and guidance of the activity, *prompting* the students to reflect. (P7Q4, emphasis added)
- We were prepared for *questions*, as well as possible solutions, even the wrong ones, which helped us a lot during the application lessons. (P9Q4, emphasis added)

We can conjecture that the teachers' endeavors, to some extent, confronted the findings of Tortola *et al*. (2023, p. 179), who state that "there are difficulties in assuming the role of a guide, as teachers are still deeply ingrained with the idea that they must be the holders of knowledge." However, it was evident in P9Q4 that the teachers planned the activity collaboratively and were prepared for questions, as well as possible solutions, even incorrect ones. The anticipation made by P9 and her group was a way to structure how to handle the situation. This approach aligned with the indications of Tardif and Lessard (2014, p. 63), who state that "the order of interactions fundamentally depends on [the teacher's] own initiative."

From the planning, the approaches for teaching/conceptualizing the content could be anticipated, where the goal of enabling "students to use their mathematical knowledge to solve real problems" (Galbraith, 2012, p. 13) being emphasized, as reflected in the responses from three teachers:

- In the planning, we had already chosen the content based on the school's curriculum plan, and

from that, we determined the themes and the problem. (P3Q4)

- Where students are encouraged to work with their prior knowledge, apply it, and, at times, develop new mathematical knowledge through it. (P6Q8)
- [...] it is a different way of working with mathematics, where students produce their mathematical knowledge by solving problems from their daily lives or areas of interest. (P1Q7)

Revisiting prior knowledge or even addressing new content to be investigated were actions taken by the teachers based on themes they believed to be of interest to the students, understanding that modelling structures and creates part of the reality, dependent on the knowledge, intentions, and interests of those solving the problem (Niss & Blum, 2020), as evidenced in the following excerpts from their responses:

- For the topic, we wanted to choose something that would capture the students' attention for the activity, so we decided to work with social media. (P3Q4)
- The idea of investigating the mathematical concepts involved in the growth of a cake came from a student in the eighth-grade class, where one member of the group was teaching. Thus, the planning for the implementation of the activity began. (P7Q4)

However, the literature already points out that it is not enough to simply start with a theme of interest to the students; it is necessary that they are encouraged to respond to the problem, as well as motivated to research information that may be necessary. Responses from teachers to the questionnaire revealed that the teaching approaches adopted seemed to lead to both the interest and the mobilization of investigative actions being incorporated into the mathematical approach to the situation:

- [...] allows students to reframe mathematics through research and various solutions. (P12Q6)
- I believe that the use of modelling makes students more motivated, as the mathematical content gains meaning, since they are able to draw connections between theory and practice, thus valuing mathematics, in addition to creating more critical individuals with better arguments. (P13Q8)
- I noticed during the application of the activity that their engagement was greater compared to classes where the content presentation precedes the development of the lessons. (P12Q8)

Taking into account the points raised in the responses of the teachers grouped in the category of *didactic/pedagogical aspects*, we understand that repercussions related to limitations and obstacles concerning confidence and security were overcome through the experience of developing modelling activities within the scope of the course, as well as the planning and implementation of practices in Basic Education. These experiences also addressed actions related to the organization of students into groups, the request for records during the development of the activity, the necessary questioning to instigate reflection and guide the activity, as well as the understanding of considerations about the students' knowledge and the conceptualization of "new" knowledge.

The third category highlights *personal/social aspects*, revealing repercussions that led the teacher in education to reflect on the course they took, specifically in terms of lesson organization and learning, their participation and involvement in activities, working with peers, relationships with the instructors, the relevance of mathematical modelling in the classroom, and the implications of its use both for the students' view of mathematics and for their teaching

practice — planning, didactics, and student engagement in the lessons.

Thus, the personal and social aspects, which constitute the dual focus of this category, complement each other in such a way that the reflections, although of a personal nature, primarily resulted from social relationships established with the instructors responsible for the course, with peers—teachers also in education—and with their students, as they experienced the development of a modelling practice in the classroom.

The responses from the teachers in education indicated repercussions tied to these ideas. When revealing their impressions about the organization of the course, they highlighted the importance of the role played by the instructors, as shown in the excerpts:

- [...] the way [the instructor] conducted the course, the constructive suggestions and criticism — everything added a lot. (P6Q9)
- [...] the instructors were always attentive, even outside of class hours, in addressing doubts and contributing to the better development of the activities. (P8Q9)
- [...] it was in conversations with the instructors that I got the idea to work with the operation of the parking meters here in the city. (P5Q4)

The teachers in education also highlighted the importance of how the instructors organized the course, according to the three formative axes, to familiarize them with modelling:

- […] very well organized, we learned everything about modelling with the definition and execution of activities by the professor or together, we did some activities and only then applied them to the students. (P6Q6)

Being indicated as a great challenge or a challenging activity, some teachers reported a certain apprehension regarding modelling at the beginning of the course, and the way it was organized contributed to overcoming this apprehension.

- [...] I was a little scared because I had the feeling that I wouldn't be able to carry out an activity with this proposal; it's hard to explain this feeling, but I believe that new things have this effect on us. The way the course is organized gradually places us in front of modelling without us realizing it, or it's such an engaging approach that the fear fades away without us even noticing it. (P5Q6)
- [...] the way the course was presented is very important for those who are beginning to study the subject and also for those who already know it, as it provides a different perspective both as a teacher and as a student. (P3Q9)

Regarding the dynamics of the classes, the teachers in education highlighted the feasibility and encouragement of collaborative work among peers, promoting interaction, discussion, and reflection on the activities developed. This approach allowed them to feel comfortable expressing their ideas (P7Q9), which they found important for their personal experiences, as indicated by P10 in response to Q9. The teachers in education rated their experiences of group work positively, where they presented their ideas and listened to those of their colleagues, as evidenced by the following excerpts:

- [...] it was good to listen to colleagues during their presentations, as well as to present to the

class, where we received various contributions for implementation in the classroom. (P1Q4)

- [...] especially during the discussions, we had different perspectives between the class and the instructor-professors, which led to a very good experience with the exchange of ideas and thoughts about the activities. (P3Q7)

Another point that stood out in the teachers in education's responses regarding group work is related to the experience of guiding a modelling activity with a group in a class (P7Q2), which was new for P7. According to P13, in response to Q4: "this was the modelling that taught me the most, considering the practice and that we could have assisting colleagues for the exchange of ideas."

It appears that the work was effectively carried out in a collaborative manner, which is why it was difficult for P5, in response to Q3, to write actions in the singular. Collaborative work, in addition to being a prerequisite for the development of mathematical modelling activities (Almeida *et al.*, 2012; Niss & Blum, 2020), aligns with the perspective of teacher education advocated by Cyrino (2013), of knowledge-emancipation. Emancipation here refers to mastering knowledge about modelling and how to develop it in the classroom (according to the first category highlighted in this research), but also to the liberation from the constraints imposed by the educational system, where the teacher finds themselves alone in the classroom with students, having to make decisions about situations they have never experienced and with no time to teach differently.

The experiences provided by the course prompted changes in the teachers in education's views on modelling, ranging from fear (P5Q6) to a desire to learn more and do more, even though challenges remain ahead (responses from P1, P5, P6, P7, P9, P13, and P14 to Q2). These changes served as motivation for some teachers in relation to their teaching practice:

- It opened my mind a bit; I'm tired of those traditional, old-fashioned, and obvious problem situations and activities. (P6Q7)
- [...] I intend to apply some of the activities that I learned in the classes. (P11Q8)
- You can be sure that I am now looking at math teaching from a different perspective. (P1Q7)

However, it was P5 who emphasized these changes the most in their responses, possibly due to the initial view expressed regarding mathematical modelling:

- [...] teaching using modelling made me reflect on my practice as a teacher. I believe my lessons, my way of looking at the students, as well as my planning, will never be the same again. It was challenging, yet rewarding. (P5Q6)
- [...] a desire to continue studying this trend in Mathematics Education, which has a lot to contribute to our education and teaching practice. (P5Q7)
- [...] I had such a happy feeling, a sense of gratitude, of stopping and thinking: wow, I'm thinking and creating my own problem situations, without a textbook, without the internet with ready-made content. I hope to continue with this same motivation and enthusiasm. (P5Q8)

However, what seems to have had the most repercussion on the teachers' reflections about their practices, prompting changes, was the different view of mathematics and its relationship with the world that, according to them, mathematical modelling can provide.

- They are able to realize that mathematics is present in situations where they might not have imagined that mathematical content could be the solution. I intend to work more from this perspective to develop these skills in my students. (P1Q7)
- [...] when the teacher teaches through modelling, it leads the student to understand problem situations from their daily life that they often hadn't noticed. They learn to reflect, reason, and use problem-solving strategies to find a solution. (P11Q6)
- [...] it develops, beyond mathematical knowledge, new perspectives on the world in students, making them more critical and engaged with the activities. (P1Q8)
- [...] the development of the [modelling] activity with the students is extremely important for them to understand that mathematics goes beyond the classroom. (P3Q8)

This new way of viewing mathematics was perceived thanks to the opportunity the teachers in education had to develop modelling activities as students and reflect on their feasibility in the classroom for their own students. According to P5, in response to Q6: "the fact of positioning ourselves as students makes us think about our future lesson plans." Moreover, as students, they realized how much modelling challenges students (P9Q7), how much it demands attention (P11Q3), and requires the student to move from a mere listener to an active participant in their own learning, bridging theory to practice (P8Q6).

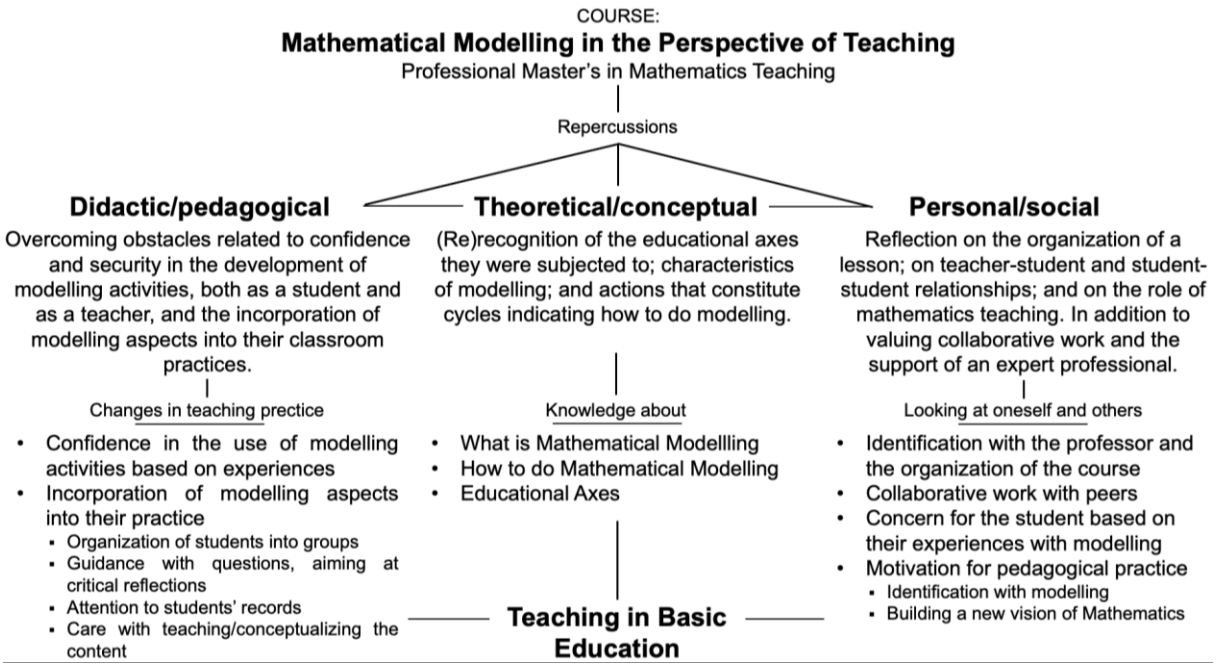
On the other hand, after the practices in which they taught using modelling, the teachers reported that it was clear how actively the students participated in these activities (P8Q8), that they "embraced" the activity and managed to carry it out well (P3Q4), with participation above average (P12Q4). This is because modelling is a teaching methodology that sparks students' interest (P4Q8). Therefore, it is an approach that not only aims to expand knowledge in Mathematics but also to "organize a way of acting and thinking" (Melo & Bisongin, 2021, p. 29).

However, it is important to highlight that it is not just about motivating students' participation in class. Students seem to play a more crucial role in motivating the teachers in their pursuit of continuous professional development. The teachers are genuinely concerned with participation and engagement, but also with the well-being of the students, the establishment of good teacher-student and student-student relationships, and, most importantly, with learning—characteristics that are fostered through modelling, according to the teachers in education, as shown in the following excerpts:

- A strategy that, in some way, brings students closer to their teacher and peers. (P9Q1)
- To think about the activity, we sought words that could spark our students' curiosity, so they would be more eager to solve it. (P9Q4)
- There was a discussion with the students in the class in order to choose the best theme together. (P13Q4)
- It was fantastic to realize that we can develop the same activity in multiple ways, without pre-established formulas and concepts, and extend this way of thinking to the students in the classroom. (P13Q7)

To summarize, the repercussions of the mathematical modelling course, offered within a professional master's program in Mathematics Teaching, on the practice of teachers in education working in Basic Education, as revealed by these three categories, are outlined through the Tree of Idea Associations shown in Figure 1.

Figure 1: Repercussions of the course on teachers' practice



Source: Developed by the authors

The tree highlights the three aspects revealed by the responses of the teachers in education, which form its structure, supported by the trunk, grounded in teaching in Basic Education. These aspects are discussed through the "Mathematical Modelling in the Perspective of Teaching" course, offered as part of a Professional Master's program in Mathematics Teaching. The three aspects branch out into descriptions of the repercussions and topics that provide insight into the teachers' perceptions regarding their participation in the course and its repercussions on their practices in and for Basic Education.

6 Final considerations

In order to foster a discussion about teaching in Basic Education, particularly regarding the incorporation of mathematical modelling into the set of pedagogical alternatives for teaching mathematics, this research investigated the repercussions of a mathematical modelling course, offered as part of a professional master's program in Mathematics Teaching, on the practices of in-service teachers in Basic Education.

Considering that when a teacher begins a course, whether long or short-term, they aim not only to "pro-pose a path, but also to provide a way to start walking, to move forward" (Larrosa, 2019, p. 21), the progression in the journey of teachers seeking professional development in modelling through the aforementioned program is supported by a framework that encompasses three formative axes: learning about mathematical modelling, learning through mathematical modelling, and teaching using mathematical modelling (Almeida & Silva, 2015). In this framework, theoretical approaches are integrated with practices in which teachers develop modelling activities both as modellers and through the planning and implementation of activities within Basic Education subjects. As a result, teachers who have not previously engaged in pedagogical practices involving modelling are encouraged, through hands-on experience with modelling activities, to take the first step by being invited to implement a practice with their students. Indeed, the experience with mathematical modelling activities in the course, as revealed in the work of Costa and Allevato (2019), enabled teachers involved in professional development to identify opportunities for implementing classroom

practices.

In the context of the "Mathematical Modelling in the Perspective of Teaching" course, the steps were guided by instructor-professors who also facilitated the sharing of ideas among the teachers until the planning stage. With the shared experiences, more than just a single step, a path is already set that teachers can take with Basic Education students, recognizing themselves as guides.

But, in fact, is this journey surrounded by encouragement? What are the repercussions of the course on the teachers' professional development? To give voice to the teachers, a questionnaire with ten open-ended questions about their impressions of the course and the practices it prompted was completed by the 14 participants during the final meeting. The analysis of their responses revealed repercussions in three aspects: theoretical/conceptual, didactic/pedagogical, and personal/social.

The responses that supported the categories revealed that the repercussions related to the theoretical/conceptual aspects, as well as those related to the didactic/pedagogical aspects, were directly linked to the approaches adopted in the course, according to the formative axes of learning about modelling, learning through modelling, and teaching using modelling. However, the personal/social aspects focused on how the instructor-professors interacted with the teachers involved in professional development through the mediation they provided.

The repercussions related to the didactic/pedagogical aspects directly impacted changes in teaching practices and contributed to the integration of modelling into the classroom. The repercussions related to the theoretical/conceptual aspects supported teachers in developing modelling activities, both as teachers and as students, fostering knowledge about modelling and its development. Finally, the repercussions concerning the personal/social aspects encouraged self-reflection and reflection on others, revealing new possibilities for collaboration with peers, students, and instructor-professors. Together, these aspects empowered the teachers involved in professional development, giving them the courage and confidence to explore the use of modelling in the classroom and leaving them with a desire to do more.

The repercussions highlighted in our research pertain to a group of teachers who participated in a continuing education program, where aspects related to their understandings of *what modelling is and how to do modelling* were intertwined with the perspective adopted by the instructor-professors—mathematical modelling as a pedagogical alternative for teaching mathematics. In this sense, the repercussions from different perspectives on modelling, as addressed in the literature, were not discussed, constituting a limitation of our study. These perspectives could have been incorporated into the *learning about modelling* axis, in a manner similar to the lessons supported by Exploratory Teaching, as done by Braga and Cyrino (2022). In this case, one possible approach to investigating such repercussions would involve studying pedagogical practices planned and implemented according to different perspectives, allowing the teachers involved in professional development to choose one to follow.

In relation to teacher education that focuses on other methodologies beginning with the problematization of a situation, such as Exploratory Teaching and Problem Solving, the educational approaches we have implemented for Mathematical Modelling can be practically applied in other teacher education courses. The results of these applications could broaden the implementation of different trends in Mathematics Education.

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