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Movements Related to Spatial Abilities in an Animated Scenario Construction in GeoGebra

Camila Maria Koftun
Secretária de Estado da Educação
Porto União, SC — Brasil
camila.m.k@hotmail.com
0000-0002-1883-9571

Maria Ivete Basniak
Universidade Estadual do Paraná
União da Vitória, PR — Brasil
maria.basniak@unespar.edu.br
0000-0001-5172-981X

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Abstract: Spatial abilities make up spatial thinking and associated with movement help us think. With this, the objective is to investigate how and which movements associated with spatial abilities are used during the construction of an Animated Scenario in GeoGebra. These are constructions that, involving mathematical elements and movement, make it possible to discuss mathematical concepts and representations. In this case, elementary school students built the *House* Animated Scenario, involving flat and spatial geometric objects. Recordings of constructions and mathematical discussions were analyzed and it was identified that students used movements such as *rotating, repositioning, enlarging/reducing* with GeoGebra tools. These movements, related to the spatial abilities of *mental rotation, perspective, mental construction* and *spatial judgment*, helped students to validate strategies, correct errors and identify the position of geometric objects in the house. The use of these movements was necessary for the construction, especially as it involves three-dimensional geometric objects, of which it is not possible to view all sides without moving them.

Keywords: Spatial Thinking, Movements in GeoGebra, Geometric Objects, House Animated Scenario.

Movimientos Relacionados con Habilidades Espaciales en una Construcción de Escena Animada en GeoGebra

Resumen: Las habilidades espaciales conforman el pensamiento espacial y asociadas al movimiento nos ayudan a pensar. Con esto, el objetivo es investigar cómo y qué movimientos asociados a habilidades espaciales se utilizan durante la construcción de un Escenario Animado en GeoGebra. Son construcciones que, involucrando elementos matemáticos y movimiento, permiten discutir conceptos y representaciones matemáticas. En este caso, estudiantes de primaria construyeron el Escenario Animado de la *Casa*, involucrando objetos geométricos planos y espaciales. Se analizaron grabaciones de construcciones y discusiones matemáticas y se identificó que los estudiantes utilizaron movimientos como *rotar, reposicionar, ampliar/reducir* con herramientas GeoGebra. Estos movimientos, relacionados con las habilidades espaciales de *rotación mental, perspectiva, construcción mental* y *juicio espacial*, ayudaron a los estudiantes a validar estrategias, corregir errores e identificar la posición de objetos geométricos en la casa. El uso de estos movimientos fue necesario para la construcción, sobre todo porque se trata de objetos geométricos tridimensionales, de los cuales no es posible ver todos los lados sin moverlos.

Palabras clave: Pensamiento Espacial, Movimiento en GeoGebra, Objetos Geométricos, Escenario Animado Casa.

Movimentos relacionados às Habilidades Espaciais em uma construção de Cenário Animado no GeoGebra

Resumo: As habilidades espaciais compõem o pensamento espacial e associadas ao movimento nos ajudam a pensar. Com isso, objetiva-se investigar como e quais movimentos associados às habilidades espaciais são utilizados durante a construção de um Cenário Animado no GeoGebra. Tratam-se de construções que, envolvendo elementos matemáticos e movimento, possibilitam discutir conceitos e representações matemáticas. Neste caso, estudantes do Ensino Fundamental construíram o Cenário Animado *Casa*, envolvendo objetos geométricos planos e espaciais. Analisou-se gravações das construções e discussões matemáticas e identificou-se que, os estudantes utilizaram movimentos como *rotacionar, reposicionar, ampliar/reduzir* com ferramentas do GeoGebra. Esses movimentos, relacionados às habilidades espaciais de *rotação mental, perspectiva, construção mental* e *julgamento espacial*, auxiliaram os estudantes a validar estratégias, corrigir erros e identificar a posição de objetos geométricos na casa. O emprego desses movimentos foi necessário para a construção, especialmente por envolver objetos geométricos tridimensionais, nos quais não é possível visualizar todos os lados sem movimentá-los.

Palavras-chave: Pensamento Espacial. Movimento no GeoGebra. Objetos Geométricos. Cenário Animado Casa.

1 Introduction[1]

As it involves the study of shapes, location and arrangements in space, Geometry is a Mathematics field that presents relationships with spatial thinking (Brasil, 2018). In turn, spatial thinking helps to speak, think and act about space, time, people and other elements, so that some movements help in this process, as spatial thinking and the movements associated with it are the basis to form other types of thinking (Tversky, 2019). Gestures and mental movements are types of movements which are important for spatial thinking (Tversky, 2019).

Mental movements are those performed on an object represented in mind, which allow that object to *rotate*, *reflect*, *enlarge* and *reduce*, for example. Movements like these are related to spatial skills, which can be identified in people who have an ability to solve problems involving space. Therefore, spatial skills make up spatial thinking.

Dynamism of GeoGebra software makes it possible to perform movements, such as those related to spatial skills, on mathematical elements constructed in its environment, especially when using the 3D viewing window: it allows working with spatial and flat geometric objects, and the movements help to explore the characteristics of representations.

According to Notare and Basso (2016, p. 2), "one of the main contributions of digital technologies to mathematics education was to make it possible to 'concretize' mathematical objects on the computer screen". This possibility allows us to develop new ways of thinking about mathematics by manipulating and changing properties of the object represented with the software.

A type of construction in GeoGebra that involves movement and allows you to discuss mathematical concepts and representations during its development are Animated Scenarios (AS). In ASs, mathematical elements are related to software tools or commands that provide movement for construction objects, ultimately constituting an animated context/scene (Bueno;

[1] This paper is part of the master's dissertation defended in the Postgraduate Program in Mathematics Education at the State University of Paraná, organized in a multipaper format, written by the first author and supervised by the second one.

Basniak, 2020). Motion must be part of the finished AS, but it also appears during the construction process, when mathematical objects are manipulated to fit the scene that is to be constructed (Koftun; Basniak, 2024). Furthermore, the final scene may involve characters, situations from everyday life or the imagination.

In this work, we built the AS *House* from flat and spatial geometric objects and investigated how and which movements associated with spatial skills are used during the construction of this AS in GeoGebra, which was developed by a class of 7th grade Elementary School students[2].

2 Spatial Thinking and Movements

A thought starts with an idea or a problem. To move forward, you transform the initial thought into a new one, and you do this process several times, until you find a solution to the problem or mature your idea (Tversky, 2019).

When we are given the task of assembling a piece of furniture from a manual, our thoughts come to the instructions: in this case, a sequence of actions on the pieces transforms them, step by step, into the complete piece of furniture. The same logic is presented by Tversky (2019, p. 85) about thought, as if they were "actions on ideas that transform them into something else".

We understand that these actions are related to the willingness to act in a situation, transforming it with the help of movements. Movements are great allies of the thinking process, both physical movements, which are gestures that express actions based on ideas (Tversky, 2019) and mental movements, which are those that can be performed on objects constructed in mind, such as rotating, reflecting, moving, adding or removing a part of the imaginary object, among other modifications.

According to Tversky (2019), movement is present in all the space around us, although we are not always aware of it. In any conversation with a colleague, people often gesture, pointing to a direction or an object to complement their speech. In nature, flowers move towards the sun and open and close. In the artistic world, actors and dancers perform expressive movements; and in the educational environment, students and teachers also use movement in the classroom, whether to represent a mathematical symbol by drawing it in the air during an explanation or using mental movements that can help solve a problem, when it becomes difficult to represent what you are thinking.

Therefore, movements help us make decisions related to different types of thinking. There are a multitude of classes that we can mention, such as social, computational, creative, design, abstract, mathematical thinking and others, but "spatial thinking is the core of our existence", and its relationship with movement deserves emphasis (Tversky, 2019, p. 59).

Spatial thinking is developed in our everyday experience through observation of space, and is the basis of how we speak, think and act about space, but also about time, emotions, social relationships and much more (Tversky, 2019). According to Tversky (2019), the movements associated with spatial thinking are considered the foundation for other types of thinking.

Many decisions and comparisons made in social, geographical, cultural and political spheres, for example, are also influenced by spatial thinking. If we need to cross the street when a car is approaching, we think about whether there is enough time to travel the distance to the

[2] In Brazil, Elementary School 7th grade is intended for children aged 12.

other side or whether the driver will slow down, a judgment that is part spatial and part social, and the mistake can be costly (Tversky, 2019).

The relationship between spatial thinking and other thoughts is highlighted in neuroscientific experiments mentioned by Tversky (2019), which indicate that the brain regions responsible for orienting and moving us through space, that is, related to spatial thinking, ended up, over time, acquiring other functions, such as *representing temporal*, *conceptual*, *individual* and *idea information*. Then, other types of thinking have a basis in spatial thinking, especially abstract ones. As the author indicates, we emphasize that it is just a base, and not its entire structure.

We are not perfect in spatial thinking or any other type of thinking, considering its complexity and breadth, and so we often make mistakes in spatial decisions (Tversky, 2019). On the other hand, some people find it easier than others to perform actions related to spatial thinking, which generally involve movements made over objects mentally. A set of tasks involving these movements is used as a test to measure a person's spatial thinking skills, or spatial abilities (SA) (Tversky, 2019).

For Tversky (2019), people have this type of ability when they are predisposed to deal with situations about space, in which movements related to the *mental rotation* of an object, changes in the observation *perspective*, *spatial judgment* about distance and position, in addition to changes in shape, size and other properties of objects *constructed in mind* are regularly present. The following section presents more details about these SA.

3 Movements in Spatial Thinking Skills

Just as there are different types of thinking, the skills about thoughts are also diverse: musical, athletic, verbal, visual, spatial and other skills (Tversky, 2019). SA makes up spatial thinking and are related to different ways of acting on ideas and representations in space. We have already seen that *actions on ideas* are how Tversky (2019) considers the process of thinking.

Even people who are skilled at SA need to practice being stand out. Sport is a good example: "to be an elite jumper, shortstop or quarterback[3], you need special physical characteristics, talent and *training*" (Tversky, 2019, p. 98, our emphasis). Thus, even those who have difficulties with spatial notions can develop some type of SA through practicing tasks that stimulate them, as they are not innate to the subject (Tversky, 2019).

These tasks require different strategies/thoughts, which may require different SA, so that the subject may be good at one but not another or may be good at all or none (Tversky, 2019). To try out an example of these tasks, look at Figure 1 below and decide whether the pairs of images are identical or mirrored[4].

Figure 1: F, R, 5

[3] Shortstop is the name of a player's position in baseball, and quarterback is the position of a player in American football.

[4] After solving the task, watch the animation of the figure's elements in the following video: https://youtu.be/s1WGt1V9IKA

Source: Tversky (2019, p. 87).

To solve this problem, you need to rotate the letters and the number as many times as necessary until you can compare them and identify that Rs and 5s are mirrored, and Fs are not. This task exercises the mental rotation of objects, which is a visual-spatial action compared to observing something that is rotating in space or in different orientations (Tversky, 2019). This is one of the main SA and is often related to the implementation of others. Then, tasks involving *mental rotation* serve as tests to measure this SA, just as other tasks involving *perspective*, *mental construction* and *spatial judgment* are used to measure SA in these respective branches of spatial thinking. There are other types of SA, but we focused this study on these four, which are related to movements performed mentally.

The number of attempts and the time the subject takes to complete these tasks are used as an indication of difficulty or ease with SA (Tversky, 2019). On the other hand, according to Tversky (2019), it is not necessary to undergo specific tests to experience *mental rotation*, as we use it in everyday tasks, such as putting a key in a lock, assembling puzzles, or when we recognize objects that are not in their usual positions. The same goes for other SA.

To dictate the position of objects and recognize them in space, it is sometimes necessary to take a different spatial *perspective*. Our mind allows us to assume internal *perspectives*, contrary to the one we are in, when we put ourselves in the place of another person or even an object (Tversky, 2019). It is also possible to assume an external *perspective* when we are outside, observing, analyzing or seeking the appropriate position for an object.

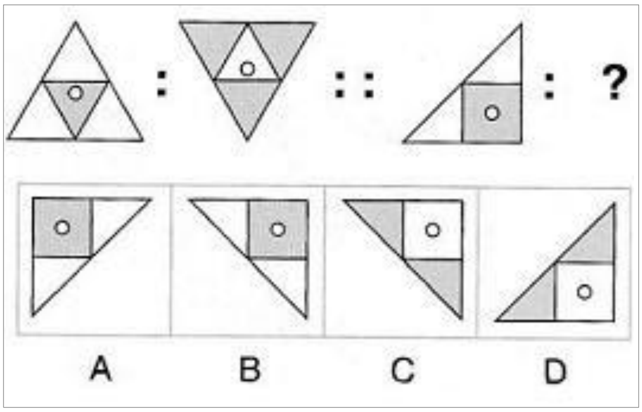
In a test involving the SA perspective described by Tversky (2019), the subject is asked to read a text that describes an environment with objects and people, and from there the subject needs to answer where what was, in relation to different reference points. Participants reported that, to answer the questions, they imagined themselves in the place of people and objects and thus performed the task without difficulty.

According to the author, this type of change of perspective (internal) is considered an egocentric structure, which is useful for maintaining control of where one is and where one moves. On the other hand, it is not an efficient way when the perspective assumed is external: in this case, “to think about space in a more general way, we need to take the ego out of space and form an allocentric representation” (Tversky, 2019, p. 68).

Before we can mentally rotate an object or change its position, a representation must be built in mind. *Mental construction* makes it possible to represent an object in mind based on something we have seen, or that has been described through language. From there, imagination allows you to change the shapes, sizes and properties of the object, rotate, reflect, add a part and determine where it is and what it does (Tversky, 2019). Therefore, in addition to creating objects, *mental construction* also allows modifications to be made to them.

As examples of tasks involving SA of *mental construction*, we can mention analogies with geometric figures (Figure 2), in which a series of transformations on the representation must be used to reach the solution.

Figure 2: Example of geometric analogy



Source: Quizizz (2021).

People can perform these manipulations with greater or lesser ease, depending on the sequence of actions employed. For some of them, it may be easier to rotate first; while for others, starting by changing the color of the triangles is necessary.

To solve these and other tasks, people gather information that seems relevant and try to understand the relationship between them. This is also linked to another SA, *spatial judgment*, which allows comparisons to be made and influences decisions about distance (near, far), size (small, large) and direction (right, left, front and back) in space (Tversky, 2019). Thereunto, the mind uses some mechanisms to guide *spatial judgment*, such as reference points, different perspectives of an environment, rotation and alignment, but these do not always help to infer the correct answer (Tversky, 2019).

In tests involving *spatial judgment*, Tversky (2019) cites the comparison between maps of the same place. Participants should indicate the distance between cities and their location, for example, which is east or west. Most participants ended up answering the tests incorrectly because, supported by other SA and the perception of each individual, the mind seems to rotate the representations so that they appear more aligned, it also estimates that the distance between places belonging to the same group is smaller than when compared to other groups, which can lead to mistakes (Tversky, 2019). This is an example of the influence of spatial thinking on other thoughts.

SAs, whatever they may be, are internal to the subject. Situations proposed by third parties, such as block games, puzzles, sensory experiences or spatial tasks can stimulate the development or mobilization of such SA, but there is no specific action that guarantees this development.

Tversky (2019) states that these provocative tasks should be proposed to the subjects from childhood, as SAs are fundamental for many professions, daily activities and for personal development. In addition to playing games with children, parents and teachers “can enrich experiences with spatial conversations”, using words such as front, back, inside, outside, parallel, perpendicular, diagonal, area and others (Tversky, 2019, p. 103).

Furthermore, training a type of SA can help the performance of other skills that were not directly stimulated (Tversky, 2019). It has not yet been possible to understand and determine the multiple SAs that exist, despite numerous attempts that have occurred, according to the author. Then we focus on the discussion of only four SA, in which we identify relationships with the study field of mathematics, especially when it is worked on using the GeoGebra software involving the construction of ASs.

4 Movements in GeoGebra

Spatial thinking and SA are not conditioned by Mathematics. We identified the breadth of this theme, which can be used to study phenomena in areas that share some similarity with spatial thinking, and according to Tversky (2019, p. 88), "mathematical thinking has a layer of spatial thinking".

Geometry, as a Mathematics study field, is associated with spatial thinking, as it works with geometric shapes, distances, sizes and other properties of two-dimensional and three-dimensional space.

In this work, we address geometry content using GeoGebra, because this software has tools and commands that allow you to *move*, *rotate*, *enlarge* and *reduce* mathematical objects constructed there. Therefore, we relate these executed movements by the software to mental movements associated with SAs mentioned in the previous section.

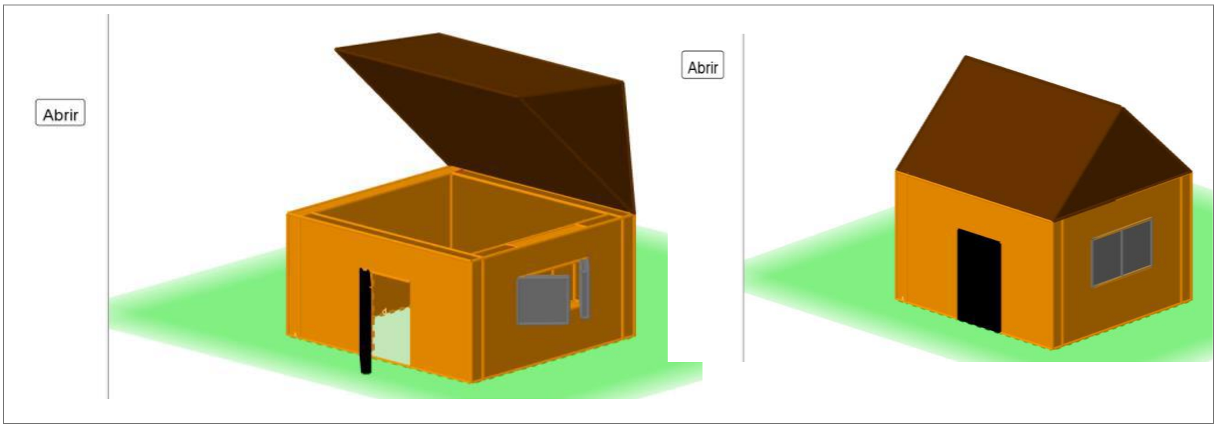
Subjects can perform constructions, rotations and changes of perspective in their minds, but with support of GeoGebra tools, they have the possibility of externalizing these actions, as "a way of placing the mind in the world" (Tversky, 2019, p. 89).

These movements can appear during development stages of constructions in GeoGebra, as is the case with the construction of the Animated Scene *House* discussed below.

5 Context and Methodological Guidelines

AS *House* planning (Figure 3) and others involved a team of academics who had already been working with this type of construction in other research and in an extension project. All ASs developed in this context involved structuring content of Geometry.

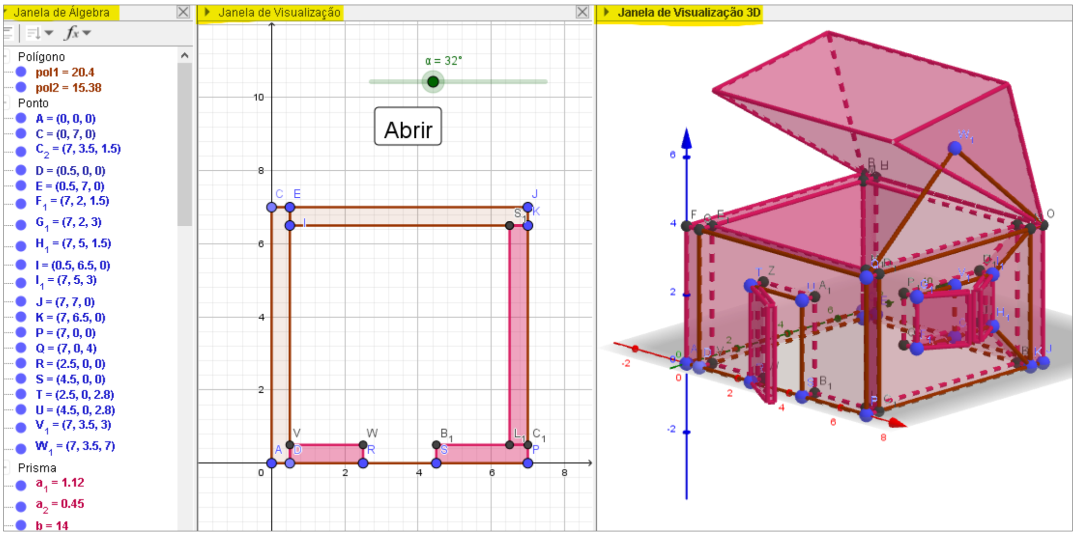
Figure 3: AS *House* finished, open and closed



Source: Research data.

AS *House* involved mathematical elements that provided discussions on geometric concepts and representations. Then, from this construction, the objective was for students to be able to recognize and differentiate characteristics of flat and spatial geometric objects, particularly polygons and polyhedrons. The union of these geometric objects in different sizes and shapes gives rise to the house representation built in the GeoGebra 3D viewing window, also articulating the 2D viewing window and algebra window (Figure 4).

Figure 4: Construction of AS *House* in different GeoGebra windows



Source: Research data.

In this context, to investigate how and which movements associated with spatial skills are used during the construction of this AS in GeoGebra, we defined as subjects of empirical research a class of Basic Education students who carried out this construction. Information about the meetings held with students is below.

5.1 Participating students and data selection

The class that participated in the proposal to build the AS was the 7th grade of a full-time school in the public network of the State of Paraná. During the period (May to July 2022) in which the proposal was developed, classes in the education network took place in person, after the period of emergency remote teaching resulting from the Covid-19 pandemic, which began in 2020.

During the meetings, the 28 students in the class were divided into two computer labs (L1 and L2), belonging to the university that shares the building with the school. The division was made so that everyone had access to a computer to carry out the construction, favoring discussions in the classroom carried out by the Assistant Team (AT) that worked in these meetings, also for the subsequent data analysis. Despite the separate environments, the same planning was followed for meetings with students.

In each computer lab there was an AT composed of three members each, in which one of them led the discussions and the class guidelines, and the others answered specific questions from the students and helped in stages of the AS construction, supported by a script[5] prepared in advance.

Before creating AS *House*, students built two other scenarios, all involving Geometry structuring content to familiarize themselves with some GeoGebra commands and the dynamics of the meetings. First, the finished AS was presented to students so that they would know what

[5] The construction script for AS *House* and other ASs built by students during the research interventions can be accessed in the appendices of master's dissertation by Koftun (in press).

would be built in that meeting, and the first steps of construction were shown in detail on a projected screen. From there, students continued their construction based on the previous steps or exploring the software tools and mathematical objects involved.

The discussions in this work focus on the construction process carried out by L1 students, because in this environment, the person responsible for leading the discussions was the first author of this work (Researcher). The remaining AT members is identified as A1 and A2.

Four meetings of approximately 100 minutes were needed with the class to build and discuss the AS *House.* Data was collected through recordings of screens of each computer used by the students, audio and video recordings of L1, and written records by students and the researcher. For screen recordings, Apowersoft was used in the internet browser, a free online tool that has the function of recording activities performed on the screen computer, in addition to recording audio.

Technical problems occurred during data collection, such as recordings of screen computer that were lost because they turned off unexpectedly, without it being possible to save the recording or the student's progress in the construction. Furthermore, we were unable to record audio from all computers due to limited external microphones, and as a result, some of the computer screen recordings were left without audio, in addition to other video recordings that were corrupted. This reduced the amount of material to be analyzed and was the first criterion for selecting the data collected.

Although each student had a computer available to carry out the construction, they were instructed that they could exchange ideas with their colleagues about development of their stages. Then, there were students who chose to produce a construction in pairs.

Presentation of the analyses follows the steps established for the AS construction, and the excerpts consist of clippings from the students' computer screens, conversation transcripts and discussions that took place during classes, and written records from the students. These data were selected considering the moments of construction in which it was possible to identify influences of the movements used, or the absence thereof.

It is possible to access the excerpts of the screen recordings when you click on the images that represent them throughout the text; in this case, a new tab will open in which a video of the situation can be played: you must be connected to the internet to access it. The use of videos seems more appropriate to us than a sequence of images that seeks to represent the students' actions on the software, considering that movements performed in the stages of AS construction are important elements for the analysis. Audios were removed from the videos to preserve the students' identification, and we also adopted fictitious names for students for the same reason.

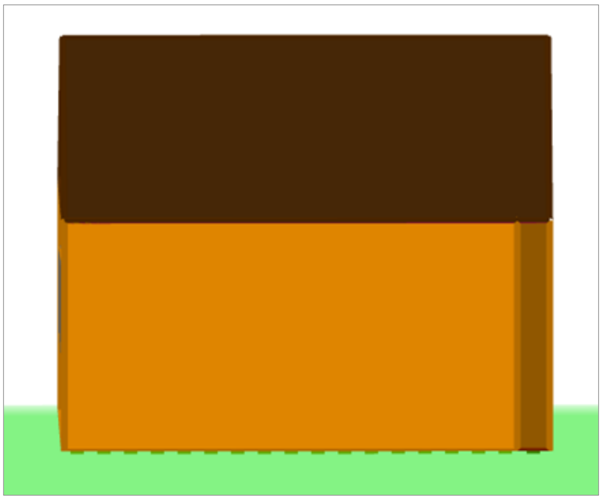
We report and analyze, below, the constructions developed by the students: *Lucas, Maria, Ana, Paulo, Beatriz, Caroline* and the pairs *André and Gustavo, Diego and Eduardo*. We sought to identify, in their actions and speeches, indicators of movements and actions related to the SAs of *mental construction*, *mental rotation*, *perspective* and *spatial judgment*.

6 Results and Discussion

1st stage: Presentation of Animated Scenery *House*

Before starting to develop AS *House*, the final construction was presented to students to identify what was being represented in GeoGebra. However, the position of the object adopted at that moment (Figure 5) influenced the students' responses.

Figure 5: Rear view of the house



Source: Research data

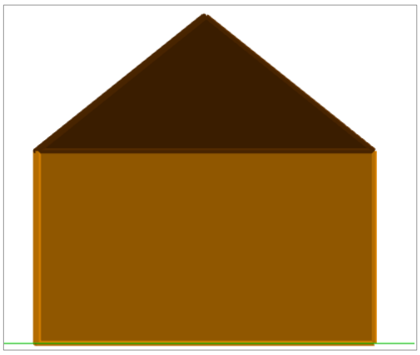
The following excerpt presents the students' statements when they observed the representation in figure 5.

Researcher:	*This is the next construction we are going to do, what do you think it is?*
Gustavo:	*A door.*
Diego:	*A box.*
Lucas:	*A chest.*
Eduardo:	*A box that looks like a chest.*
Beatriz:	*A cube.*
Eduardo:	*A chocolate bar.*

In this position, the students did not identify that the construction was a house and mentioned other objects like the shape presented. The position adopted was intentional, so that students would realize that three-dimensional objects need to be observed from different positions to identify characteristics that are part of their identity. Especially geometric objects, when they are represented in different positions than usual, end up not being recognized in some situations (Machado, Bortolossi & Almeida Junior, 2019).

After that, the position of the house was changed again, showing the side (Figure 6). At this point, the rotation movement of the AS until it reached this position was not presented, so that the students would not identify all the elements that make up the AS and characterize it as a house, since this would be discussed later.

Figure 6: Side view of the house



Source: Research data.

Students were again asked about the object represented, as can be read in the excerpt below.

Researcher:	*And now, is it different?*
All students:	*A house!*
Researcher:	*What do you identify here that looks like a house?*
Lucas and Eduardo:	*The roof and the wall.*
Researcher:	*Is there anything else missing from the house?*
Eduardo:	*Door and window.*
Researcher:	*And in which GeoGebra window is this construction?*
Lucas:	*3D*
Researcher:	*And how do we see if there are any more of these elements built?*
Beatriz:	*Rotating the house.*
Lucas:	*Take the mouse and do this, rotate the plane* [perform the movement on the computer].
Researcher:	*Then A1 will rotate for you to see if there is anything else hidden.*
Lucas:	*There's nothing... Wow, a window and a door!*
Eduardo:	*Wow!*
Beatriz:	*Wow! I thought it was just like that, the house.*

When students observe the triangle above the rectangle, they say that it represents a house, and they relate these geometric shapes to a roof and a wall. As they already know a house, that is, they already have the *mental construction* of this object, they can notice that the elements presented resemble the characteristics of a house.

Based on questions, students mention other elements that are part of a house, but do not appear in this representation, such as doors and windows. Since it is a three-dimensional object and is in the GeoGebra 3D window, students stated that it was possible to *rotate* the object, which was necessary to recognize its particularities and modify the perspective from which it is observed.

At this stage, the house was rotated, and students were able to observe it from different *perspectives* (Figure 3), when they were able to identify elements that were not visible before. *Eduardo* and *Beatriz* were surprised when they realized that the complete house involves other geometric objects besides the triangle and rectangle they observed previously.

The approach to spatial geometric objects in software enables a different visualization based on movements. If the house were observed from the front, from the side, and then from other positions, without considering the *rotational* movement, it would be possible to recognize the elements that compose it, but we would have no notion of depth, height, slope and the distance between its elements. This often happens when we look at static images in a book (Machado *et al*., 2019).

The *rotation* movement, which changed *perspectives*, was essential for identifying the characteristics of the house and knowing what they should build in the next stages.

2nd stage: First Wall Construction and Mathematical Discussions

Students were encouraged to suggest how to begin the construction of the AS presented based on questions that sought to relate the construction with mathematical elements that we consider necessary to shape the house, as can be read in the following excerpt.

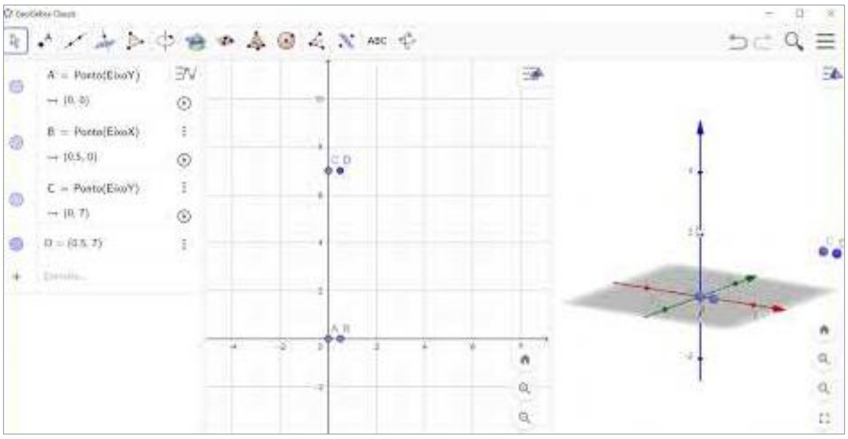
Researcher:	*Let's first think about how to build a house wall. What shape does this house wall have?* [pointing to a smooth wall]
André:	*A rectangle.*

Lucas: *A thinner rectangle.*
Researcher: *And how does it* [the rectangle] *must be?*
Lucas: *In 2D first.*
Researcher: *Why 2D first?*
Lucas: *Then we adjust it in 3D window.*

Students consider the prism that represents one of the walls of the house as the rectangle, which is a flat geometric shape: the shape of the faces of the prism. Although construction starts with the rectangle, it has different properties than the prism.

Lucas' speech expresses that he established a relationship between the 2D and 3D viewing windows, that is, he identified that the object created in one environment represented in the other, and that changes to the object can be made in both windows. His actions on the software reiterate this statement (Figure 7), because after *Lucas* constructed the points and a rectangle in 2D window, he observed the representation in 3D window, *rotating* the construction to view it in different positions, seeking the *perspective* that would allow him to better evaluate the constructed elements to continue his construction (Figure 7). Here, the movements act as a check of the action carried out by the student on what he informed the software.

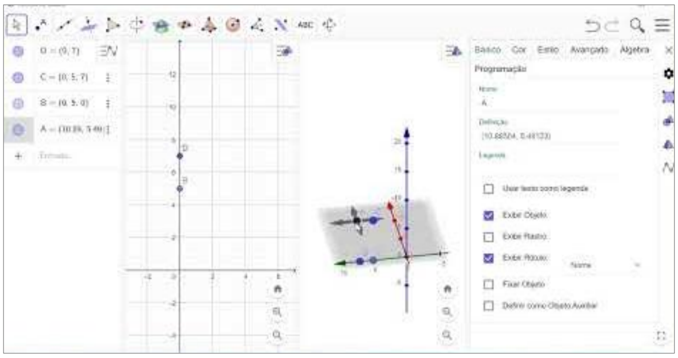
Figure 7: Movements in GeoGebra 2D and 3D window



Source: Research data.

On the other hand, *Paulo* constructed and manipulated the points in 3D window, and the *perspective* chosen at first led him to errors in his *spatial judgment*, as it gave the impression that the points were located on the same plane and aligned. When the student *rotated* the 3D window, he was able to identify that one of the points was not in the same plane as the other points, and it was necessary to make changes (Figure 8). Again, we identified that the movement helped in the construction process, enabling the correct identification of the location of points.

Figure 8: Movements in 3D window



Source: Research data.

After building the rectangle, students were asked about the representation they obtained, comparing it with the wall they intended to build.

Researcher:	*So, what appeared on your screen?*
Lucas:	*For me, a rectangle appeared in 2D and 3D… in fact, the 3D one only has one piece.*
Researcher:	*And objects in 3D generally, how many dimensions do they have?*
Gustavo:	*Width and length?*
Eduardo:	*They have 3 dimensions.*
Researcher:	*And these two objects represented* [the rectangle in two windows]*, do they have some difference in this part?*
Gustavo:	*The height?*
Researcher:	*Does the one over there* [rectangle represented in 3D window] *have height?*
Gustavo:	*I don't think so.*
Researcher:	*How do we see if this object has a third dimension? [...] Do you remember the house from the beginning?*
Eduardo:	*Rotate.*
	[A1 rotates the 3D window and students conclude that the objects are the same, at that moment].
Researcher:	*So, here, we already have the base rectangle, what's missing to become a wall?*
Lucas:	*Height.*

Lucas, Gustavo and *André* recognized that using only the rectangle suggested by them previously was not enough to build an object that represented the characteristics of a house wall. *Rotation* performed by A1, in 3D viewing window, helped this perception, showing that the constructed rectangle has no height.

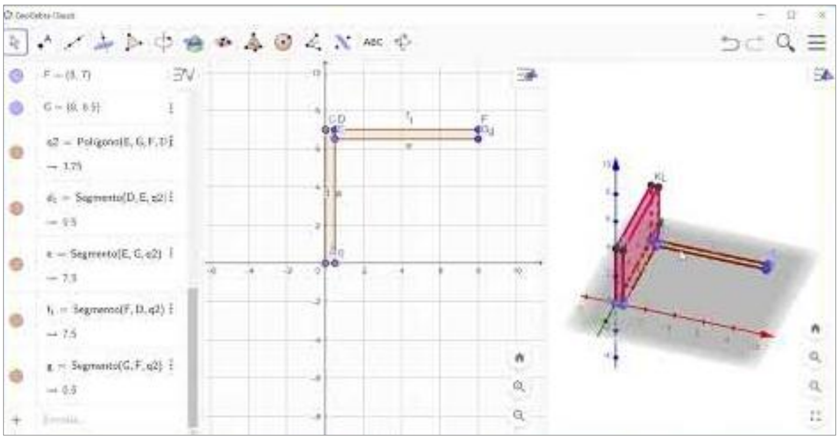
Next, the use of the extrusion tool for prism was indicated, necessary to continue the construction of the geometric object. The following excerpt begins with a student who was unable to complete this stage of construction and leads to a discussion with other students.

Beatriz:	*Teacher, mine doesn't work.*
Researcher:	*You need to create a 3D viewpoint for it to work... Why is it that if I click in 2D window it doesn't work? Does anyone know?*
André:	*Because in 2D there will be no height.*
Researcher:	*That's right!*
	[...]
Researcher:	*And now, can you see any difference between 2D and 3D object?*
All students:	*Yes.*
Lucas:	*Wow, now it is ok.*

Beatriz was trying to use the *extrusion tool for prism* in GeoGebra 2D window, which is not possible due to the characteristics of this environment, which does not allow the construction of three-dimensional objects, as pointed out by *André*. The AT helped other students who were having difficulty at this stage. After the prism was built, the first wall was finished, and with this, students were able to identify differences between 2D and 3D window representations.

The construction of the second wall was done in a similar way by students. *Lucas* again *rotates* the 3D viewing window to analyze the constructed object, and when it assumes different *perspectives*, he uses this environment as a basis for his spatial orientation (Figure 9). After rotating, it usually leaves both windows in the same *perspective* and then moves on to the next step.

Figure 9: Construction of polygon and prism by checking with movement



Source: Research data.

Spatial judgment that occurs from the *rotation* combined with change of *perspective* shows how these actions are important in the construction stages, so that the student can compare the *mental construction* of the object that he/she intends to build with representation that was effectively represented in the software, and thus, understand whether it is possible to move on to the next stage or whether changes are necessary.

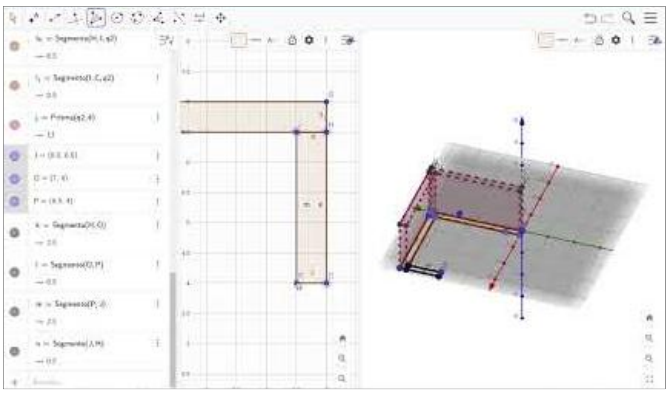
3rd stage: Students' Strategies in Building the Wall with the Door

To build the house wall that has a door, considering the concavity of the polyhedron, it was not possible to follow the same step by step, and it was intentional as a challenge to students.

Ana used the *zoom* tool, which allows you to enlarge or reduce the scale of the Cartesian plane when inserting points in 2D window (Figure 10). She considered the number of squares in the grid to measure the distance between the points, trying to leave a pre-established space for the door to open. The *zoom* tool allowed *Ana* to be precise about where she inserted the points, because as the grid is enlarged, the smaller squares become apparent, and the axes assume decimal numbers.

These actions corroborated the student's *spatial judgment*, because without the use of *zoom*, she would not be able, or it would be much more difficult to plot points directly in 2D window to identify the necessary location and build polygons of the same size, including an opening for the door between them. Then, she used the extrude tool for prism, assigning height to the objects.

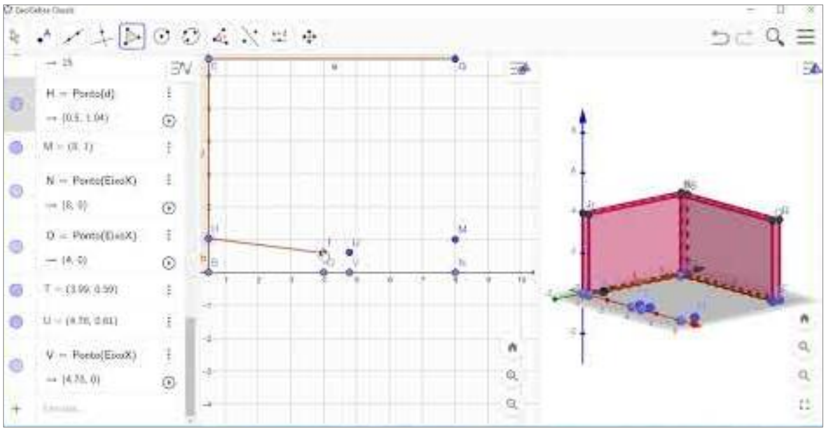
Figure 10: Using zoom in construction



Source: Research data.

The pair *Diego* and *Eduardo* and the student *Lucas* tried to represent the wall shape with the opening for the door in 2D window by inserting points. When *Lucas* performed the extrusion for the prism and *rotated* the construction, observing the object from different *perspectives*, he was able to identify that the representation returned was not what he expected (Figure 11). We identified that the student relied on the comparison between the 2D and 3D windows in most stages of the AS construction, and that this was the main source he used for decision-making, which goes beyond the simple imagination of the geometric object.

Figure 11: Initial construction of the wall with the door



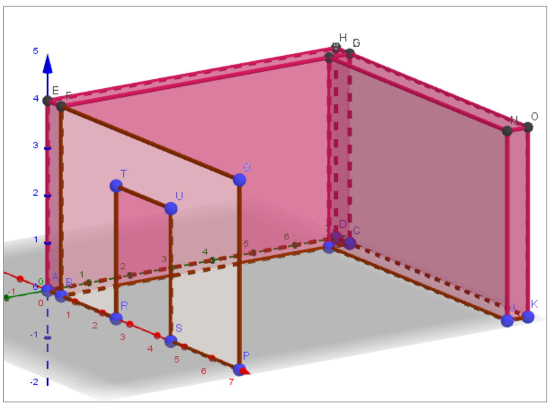
Source: Research data.

The student then tried to make changes, repositioning the vertices of the polygon in 2D window. Thereunto, he also relied on the use of the *zoom* tool. After trying, he gave up on building the wall using this strategy, as he was unable to achieve the desired three-dimensional representation and built something like what student *Ana* did.

We consider, from this, that the movements made possible by the *zoom* tool also influence and are important in the process of building the house, especially for detailed parts, to which students dedicated a lot of attention, so as not to leave the vertices misaligned and consequently the wall deformed. Thus, we point out that the *zoom* also acts as a type of perspective of the object, as it allows identifying details in the construction that would not be possible without enlargement and reduction movements. In this way, it also influences *spatial judgment* through alignment and precision in locating objects on an enlarged or reduced scale.

After some time given to students to build this wall, another possibility for building this stage was shown, according to the script (Figure 12).

Figure 12: Construction of the wall with the door by the AT



Source: Research data.

Even if students were developing other strategies, we would like to draw attention to this possibility, so that students work with the location of coordinates in space, and not only on the plane. To do so, a discussion about the wall shape was held with students.

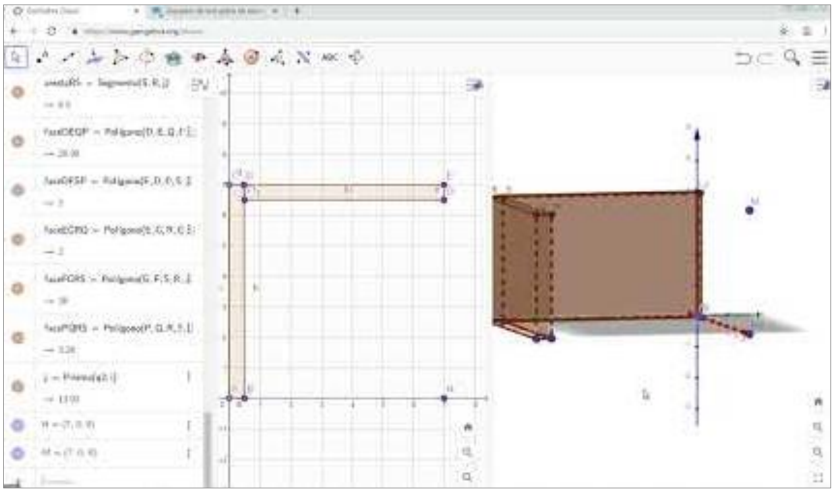
Researcher:	*Thinking about this wall with the door, what shape will it have? On the others it was a rectangle.*
Lucas:	*A rectangle with a hole in the middle.*
Researcher:	*So, if we were to draw this shape, it would be something like this, right?* [The researcher draws the shape on the board] *[...] And how are we going to build this shape in GeoGebra?*
Diego:	*Start with the points.*
Researcher:	*But where are we going to put these points?*
Lucas:	*Create two points as a base and place the ones that will make the hole in 3D.*
André:	*They are at different heights.*

The front view of the wall was presented so that students could recognize the shape we wanted to build. Following *Diego*'s suggestion to start by building points, *Lucas* realized that not all points should be built on the plane, possibly considering the tests he performed when he tried to build this wall using another strategy, previously (Figure 11). *André* identified that, when the point is not constructed on the plane, the height is modified.

In this case, to construct the points that represent the vertices of the polygon in space, it is more difficult to plot them directly in 3D window, so that they are positioned exactly in the desired place. Thus, it was indicated that the coordinates of the points should be typed into the GeoGebra input box. At this point, students had the opportunity to explore other ways of representing a point, working with ordered pairs belonging to Analytical Geometry. The first points were created step by step with students to exemplify the process, and the rest were for them to build on their own.

Based on this, the first action carried out by the pair *André* and *Gustavo* was to *rotate* the construction to adjust the *perspective* of the house's construction, positioning it on the side (Figure 13), so that they could better visualize the position in which they intended to build the points for the door: this strategy facilitates *spatial judgment*.

Figure 13: Adjusting the perspective for building the wall with the door

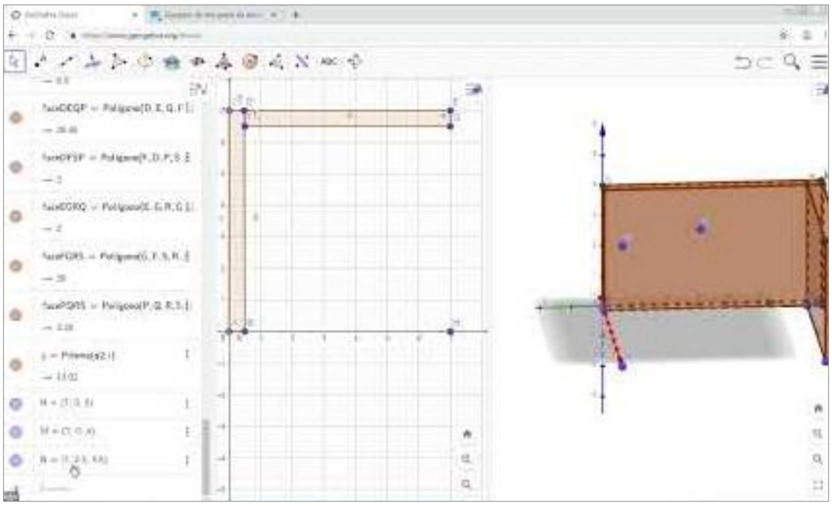


Source: Research data.

When they started typing the coordinates of the points, the students established a relationship between the representation of the algebra window and 3D window, because even before clicking *enter* to create the point, students identified that its location was not in the expected place and modified the coordinates (Figure 14).

After creating the first point, students quickly *rotated* the 3D window to check the position of the point from different perspectives (Figure 14). They realized that it would be necessary to modify the position of the point, and they did this by changing the coordinates in the algebra window (Figure 14).

Figure 14: Identifying and modifying the position of the point in 3D window



Source: Research data.

At that moment, the students copied the coordinates of the point from a colleague next to them, but possibly, when analyzing the location where the point was positioned, *Luís Gustavo* concluded: *x is red, y is green and z is blue*, referring to the representative color of the axes. The relationship established between the axes and the coordinates is another indication of association with Analytical Geometry in the investigations on the point belonging to the three-dimensional space. This observation helped him to perceive the pattern between the coordinates of the points, and he explains what he understood to his colleague *André*, as can be read in the following excerpt.

Gustavo: *The red one is the one at 7, so the other is the green one.*
André: *I'll try to put the 2.*
Gustavo: *I got it. If we put 4.5, it will be on top of that one, and we must put one else.*
André: *I didn't get it.*
Gustavo: *Here you copy the one you want to place the point on top of, just change the height.*
André: *Ok, the last one is height. Now I get it.*

Gustavo identified that, when he wanted to construct a point that was positioned on the same *line* as another point, changing only the height, the first two coordinates could be repeated, optimizing the process of creating the points (Figure 15).

The relationship established between algebraic and graphical representation of the points was an important step in improving *spatial judgment* in GeoGebra environment, so that the insertion of other points occurred without major difficulties. After inserting each point, *Gustavo* and *André* relied on the rotation of 3D window to check the position occupied.

In general, when we construct or manipulate geometric objects in GeoGebra 3D window, *rotation* movements, choosing the most appropriate *perspective* and *spatial judgment* are both important and necessary to carry out the construction and to understand how it occurs.

Figure 15: Points sequence of the door

●	H = (7, 0, 0)
●	M = (7, 0, 4)
●	N = (7, 2.5, 0)
●	O = (7, 4.5, 0)
●	T = (7, 4.5, 3)
●	U = (7, 2.5, 3)

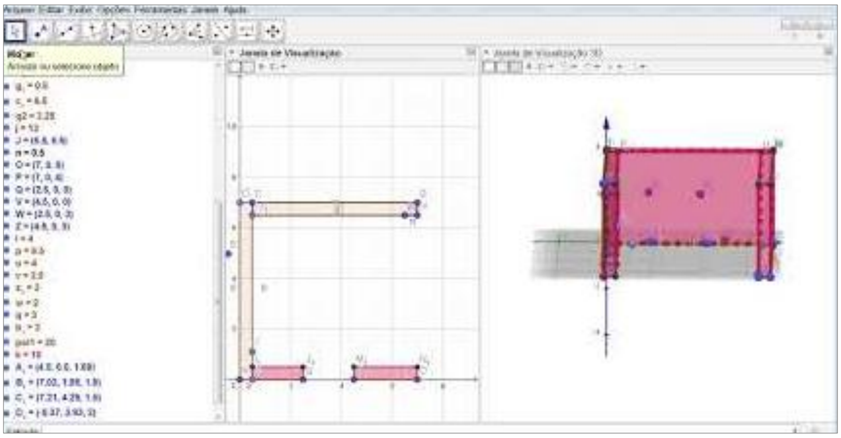
Source: Research data.

4th stage: Building the Wall with the Window

The movements made during construction by most students seem intuitive and even obvious. However, not everyone thought or felt the need to *rotate*, change the viewing *perspective* of the object or apply *zoom*, for example. *Maria* did not make any relevant changes to the position of the house and presented great difficulty in the construction process.

When building the last wall of the house, *Maria* decided to plot the points directly in 3D window. There were already two points that were created at an earlier time, so she created two more and tried to build a polygon by joining these points to form a window. However, after *Maria* started plotting the points, making several attempts, the student did not move the construction even once, and from the *perspective* from which she observed the house, she assumed that the construction was correct (Figure 16). So, when she decided to move the construction, she realized that the points were created outside the space in which the house was represented, and she had to start over (Figure 16).

Figure 16: Construction of window points



Source: Research data.

In this case, although *Maria* had the *mental construction* of the object she intended to build, the lack of *rotation* movements or change of *perspective* after each stage of construction harmed the student, as the strategies used to arrange the points could have been corrected beforehand, if she had manipulated the construction. Furthermore, she did not relate the representations of the point in the different GeoGebra windows. When moving points by manipulating them in 3D window, she didn't notice that the values of their coordinates were changed in algebra window, which would be another way of realizing that the point was not occupying an appropriate position.

Caroline and *Paulo* encountered a similar problem with plotting points in 3D window, as can be read in the following excerpt.

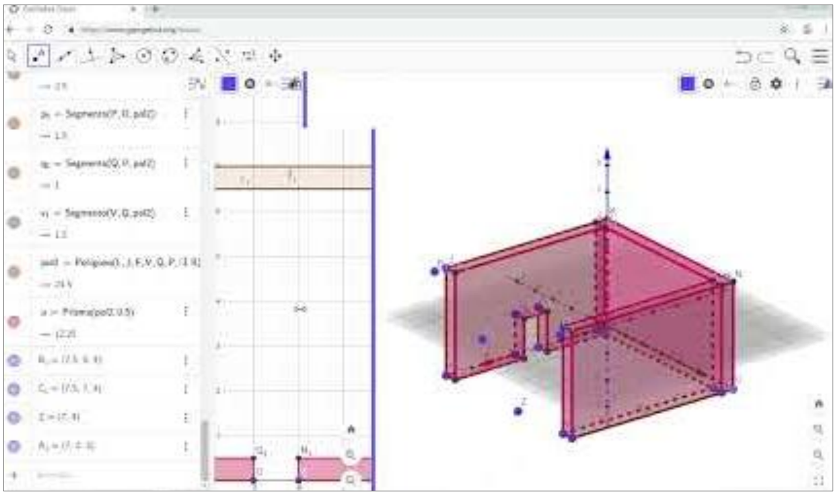
Caroline:	*Teacher, I can't make the window, it* [the point] *sticks to the floor* [on the plane].
Researcher:	*How so?*
Caroline:	*When we make the point here* [in the 3D window], *it sticks to the other wall, or when we put a good angle, it sticks to the floor.*
Researcher:	*And how do we get a point off the ground?*
Paulo:	*That's what we want to know.*
Researcher:	*What can we change in coordinates to get it off the ground?*
Paulo:	*The z goes up and down.*
Researcher:	*Exactly: when you change the z coordinate, you can lift it off the ground and up a little. That's why in this case it's better to type the coordinates in the input box, so you can already enter a point with height.*

Contrary to what *Maria* did, *Caroline*'s speech shows that she made *rotational* movements and changed the *perspective* of the house in 3D window, as she stated that she left it at *a good angle* but still had difficulty creating the points in the desired position.

After creating the point and verifying that it is necessary to change the occupied position, it is possible to move it to the right, left, up or down, as *Paulo* answers about the *z* coordinate of the point, until the desired position is achieved. Another option would be to change the coordinates directly in the algebra window, but to do so, it is necessary to identify the values to be entered, establishing a relationship between the different viewing windows. In these two possibilities of change, *spatial judgment* comes into play.

When building the last wall of the house, *Ana* compared three GeoGebra windows: algebra, 2D and 3D. The comparison was associated with the *rotation* movement of 3D window, and it was important for it to be able to position the points on desired place (Figure 17).

Figure 17: Comparison between the three GeoGebra windows



Source: Research data.

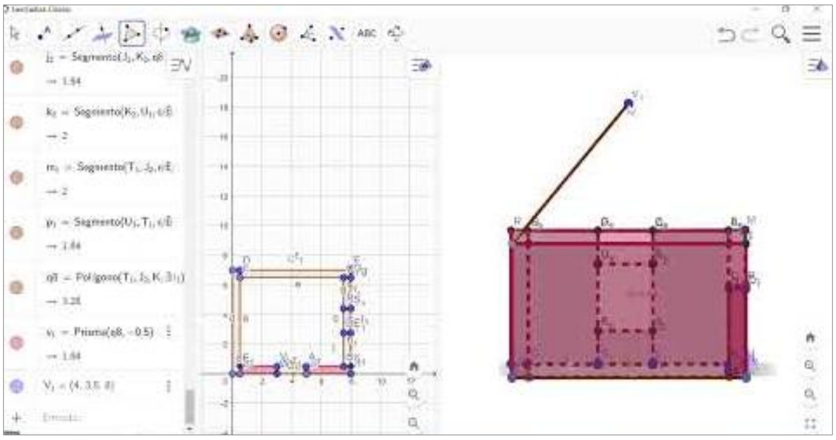
Ana recognized that, when the point is created in 2D window, it only has the *x* and *y* coordinates, and for it to be moved from the plane, it is necessary to add a value for the third coordinate*, z,* which she performed using the algebra window. After that, the representation stops appearing in 2D window and starts appearing in 3D window, where she *rotated* and changed the *perspective* to continue with the changes.

In dialogue between the pairs, it was possible to identify that, during the construction stages of this last wall of the house, the relevance of movements was noticed by them. *Gustavo*, in conversation with his partner *André*, asked, after they created points for the house window: *look at it a way to see that sometimes it's right*, *but you look at it another way, it's not*. At this point, *Gustavo* suggests changing perspective to identify whether the points were constructed in the correct place. The pair *Diego* and *Eduardo* needed to find a way to connect the points that formed the house's window without filling the inside. Then *Diego* indicated: *go there to move and take the right angle*. In this case, the right *perspective* helped the process.

5th stage: Roof construction and finishing touches

As the construction stages progressed, students improved strategies they used and began to make fewer mistakes, or when they did make them, they were able to resolve them quickly. *Lucas*, when building the roof of the house, placed a point displaced from it should have been, due to the *perspective* assumed at the time, but when he *rotated* the construction and realized what had happened, he knew how to solve the problem, stating: *hey, the number must be zero* (Figure 18). So, he made the necessary change and proceeded with the roof construction.

Figure 18: Roof construction



Source: Research data.

After the last part of the house was built, students were instructed to program the movements of the door, window, and roof as shown in the finished AS (Figure 3). Next, they had the opportunity to customize the AS, changing colors. Not all students were able to finish it.

After construction, a sheet was given to students to give feedback on the meetings, indicating *whether they enjoyed working with GeoGebra or whether they preferred to use the notebook in math classes*. Some answers can be read below.

Student 01: *In GeoGebra it is better to see because you can see the sides and shapes.*
Student 02: *In GeoGebra you can see from 6 different angles [...] and look at it shape by shape.*
Student 03: *I prefer it on paper, because in GeoGebra there are a lot of lines and points, and I get confused (my opinion).*

The majority indicated a preference for the software, except for Student 03. In addition to presenting an affirmative or negative opinion about the use of the software, the responses provided highlighted actions that refer to the movements.

7 Conclusion

The use of movements related to SA was identified in the students' actions with GeoGebra when constructing the proposed AS and brought contributions to this process.

Rotation movement was identified in almost all stages of construction and was mainly related to the SA of *mental rotation* and *perspective*. When students used this movement, they were able to identify errors in the construction and correct them, check and test their strategies on what they informed the software, comparing it with the *mental construction* they made of the object. From the change in perspective provided by rotation movement, students were able to choose the most appropriate position to build an object, or after building, they also used rotation to identify the real position in which the object was built.

GeoGebra's *zoom* tool enabled zooming in and out movements, also identified as a type of *perspective* change and associated with *spatial judgment*, since by enlarging or decreasing the scale of Cartesian plane, it becomes possible to perceive characteristics of objects that were previously inaccessible, as well as helping to determine the size, position and distance between objects.

To adjust the position of the vertices of geometric objects, repositioning movement was used on graphical representation of the points in 2D and 3D viewing windows. This movement was also related to the students' *spatial judgment* through alignment and precision in locating objects.

GeoGebra 3D window was the environment that allowed the use of more movements, considering the characteristics of this environment; unlike 2D window, in which the only movement applied was with the *zoom* tool. In several stages, the same object was compared by students in different GeoGebra windows. This also helped with the construction, as modifications to the object could be made considering its flat, spatial or algebraic representation, depending on which the student considered most viable to resolve the situation.

Therefore, the movements performed using GeoGebra tools and its dynamic environment helped students make decisions during the AS *House* construction process, realizing whether it was possible to move on to the next stage or whether they needed to make changes.

In addition to assisting in the construction process, the movements performed were important for exploring conceptual properties and representations of geometric objects involved, especially three-dimensional ones, in which it is not possible to visualize all sides without moving. Then, when comparing geometric objects in different GeoGebra windows, it was possible to recognize the characteristics that define and differentiate them.

Thus, the use of ASs as Mathematics teaching practice proved to be efficient regarding the objective proposed in this research, especially when developed in a regular Elementary School classroom context. For future research, AS constructions can be adapted to other educational contexts, diversifying the content and school level.

Acknowledgements

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