

Licentiate Degree in Mathematics and the knowledge specific to teaching: discussions of the VIII FPMat

Ênio Freire de Paula

Instituto Federal de São Paulo
Presidente Epitácio, SP — Brasil

✉ eniodepaula@ifsp.edu.br

id 0000-0003-0395-4689


Eliane Matesco Cristóvão


Universidade Federal de Itajubá
Itajubá, MG — Brasil

✉ limatesco@unifei.edu.br

id 0000-0002-3070-1030




2238-0345 

10.37001/ripen.v14i4.4235 

Received • 28/06/2024

Approved • 14/07/2024

Published • 01/09/2024

Editor • Gilberto Januario 

Abstract: In this article, we present an expansion of the reflections that permeated the process of constructing the triggering text of the Discussion Group *Licentiate Degree in Mathematics and the knowledge specific to teaching* (DG 01) of the VIII National Forum for the Initial Training of Teachers who Teach Mathematics (FPMat in the acronym in Portuguese) held in the city of Teresina, Piauí, from November 30 to December 02, 2023. To this end, we started from the processes that culminated in the construction of the text that triggered the discussions of DG 01, resulting from the contributions of 14 Regional Directorates of the Brazilian Society of Mathematics Education (SBEM in the acronym in Portuguese) and then we present the eight points of convergence and the four proposals to be forwarded to the SBEM National Executive Board, collectively constructed within the scope of DG 01 and problematized with the others present at the Final Plenary of the event.

Keywords: Initial Formation. Mathematics Teachers. Knowledge. Teaching.

La licenciatura en enseñanza de las Matemáticas y los conocimientos propios de la docencia: discusiones del VIII FPMat

Resumen: Este artículo amplía las reflexiones que permearon el proceso de construcción del texto disparador del Grupo de Discusión *La licenciatura en enseñanza de las matemáticas y los conocimientos propios de la docencia* (GD 01) del VIII Fórum Nacional de Formação Inicial de Professores que Ensinam Matemática (FPMat), que se realizó en la ciudad de Teresina, estado de Piauí, Brasil, del 30 de noviembre al 2 de diciembre de 2023. Para tal fin, se parte de los procesos que culminaron en la construcción del texto disparador de las discusiones del GD 01, originado a partir de las contribuciones de las 14 direcciones regionales de la Sociedade Brasileira de Educação Matemática (SBEM). A continuación, se presentan los ocho puntos de convergencia y las cuatro propuestas de orientación dirigidas a la dirección nacional ejecutiva de la SBEM, que se construyeron colectivamente en el GD 01 y se problematizaron con todos los presentes en la sesión plenaria final del evento.

Palabras clave: Formación Inicial. Profesores de Matemáticas. Conocimientos. Enseñanza.

Licenciatura em Matemática e os conhecimentos próprios da docência: discussões do VIII FPMat

Resumo: Nesse artigo, apresentamos uma ampliação das reflexões que permearam o processo de construção do texto disparador do Grupo de Discussão *Licenciatura em Matemática e os conhecimentos próprios da docência* (GD 01) do VIII Fórum Nacional de Formação Inicial de Professores que Ensinam Matemática (FPMat), realizado na cidade de Teresina, no Piauí, de

30 de novembro a 02 de dezembro de 2023. Para tanto, partimos dos processos que culminaram na construção do texto disparador das discussões do GD 01, o qual foi oriundo das contribuições de 14 Diretorias Regionais da Sociedade Brasileira de Educação Matemática (SBEM). Em seguida, apresentamos os oito pontos de convergência e as quatro propostas de encaminhamento à Diretoria Nacional Executiva da SBEM, construídos coletivamente no âmbito do GD 01 e problematizados com os demais presentes na ocasião da Plenária Final do evento.

Palavras-chave: Formação Inicial. Professores de Matemática. Conhecimentos. Ensino.

1 Introduction

The VIII National Forum for the Initial Training of Teachers who Teach Mathematics (FPMat in the acronym in Portuguese), held in Teresina, Piauí, from November 30 to December 2, 2023, had as its general theme *National Policies for Training Teachers who teach Mathematics: reflections, challenges and propositions* and was divided into five Thematic Discussion Groups (DG). In this text, we present an expansion of the ideas presented for sharing in the Proceedings of the event. The aim is to share, with the community of mathematics educators, the understandings that converged in the context of the Discussion Group *Licentiate Degree in Mathematics and the knowledge specific to teaching* (DG 01) and which were discussed in the final plenary of the event. Therefore, in addition to the first reflective exercises, constructed from summaries of the discussions that took place in the Forums of 14 Regional Directorates (RD) of the Brazilian Society of Mathematics Education (SBEM in the acronym in Portuguese), focusing on the theme of GD 01, we bring, in full, the points of convergence between the discussions and forwarding proposals directed to the National Executive Board of SBEM.

To this end, we chose, at the outset, to briefly discuss the meanings of the DG theme itself, based on research and publications carried out and/or guided by members of the Training of Teachers who Teach Mathematics Working Group (WG07). Next, we list the RDs that sent their syntheses, contributing to the ideas presented and the synthesis itself, divided into four thematic axes: (1) Understandings regarding the DG theme; (2) Problems raised; (3) Main referrals from Regional Forums and (4) Guiding questions for discussions at the National Forum. The thematic axes were organized with a view to highlighting convergences, but also particularities highlighted by the RD, as the understandings were quite varied. Based on the questions presented in Axis 4, we listed those that guided the discussions in the National Forum's DG 01. Finally, we list the eight points of convergence and the four proposals for direction presented in the final plenary.

2 The concept of [Mathematics] knowledge specific to teaching

The term *knowledge specific to teaching* was suggested by a group of researchers, members of SBEM's WG07, who have been developing research focusing on the Licentiate Degree in Mathematics (LM). Since 2018, SBEM's WG07 has been promoting the development of large-scale interinstitutional research, with a view to providing support for debates on the training of teachers who teach mathematics (TTM). This theme meets the problematizing objectives listed for the discussions in DG 01: "(i) the knowledge of mathematics teachers; (ii) constituent elements of their Professional Identity; (iii) aspects of the teaching profession; (iv) new learning environments and (v) urgent challenges of the Licentiate Degree in Mathematics in the current contexts" (FPMat, 2023).

Among the research developed within the scope of SBEM's WG07, we highlight the

research published by Zaidan *et. al* (2021), whose general objective was to map the pedagogical projects of Brazilian LM courses in operation in 2019, which were aligned with Resolution CNE/CP n. 02/2015. The study also described and analyzed the training paths. The results showed that, among other aspects, the majority of courses are organized into groups that, although including studies regarding the theoretical perspectives of the fields of Mathematics, Mathematics Education and Education, do so in a fragmented manner (Zaidan *et al.*, 2021). Furthermore, among the results, the authors point out that 40% of the course time is dedicated exclusively to the study of Academic Mathematics (Moreira; David, 2005) with few spaces that promote the relationship with teaching from the teacher's point of view, from a professional perspective. In other words, without a dialogue with the demands of teaching practice in Basic Education (Zaidan *et al.*, 2021).

Based on this result, the team of researchers understood that the training model was questioned, but that the place of Mathematics in the model was not yet. For this reason, the question: *What is the mathematics knowledge specific to teaching?* was consolidated as a central issue for the development of another project, entitled *A curriculum for the Licentiate Degree in Mathematics from the point of view of Mathematics Education — the necessary renewal of the training of Mathematics teachers, with a view to the teaching in basic education profession*. This second project culminated in the production of an essay in which the concept of *mathematical knowledge specific to teaching* is presented, that is

it places the demands required by teaching practice at the center of the training process, that is, taken as an object of study (curricular component) of the LM subjects. Therefore, this practice is not only the arrival point for training, but also the starting point for the study of mathematics. Here, school mathematics, closely linked to the professional teaching practice, would guide the entire structure of the Pedagogical Project of the Licentiate Degree in Mathematics Course (Cristovão *et. al.*, 2024).

Within the scope of DG 01, we intentionally chose not to focus exclusively on mathematical knowledge, in order to expand the potential of the discussions. This way, everyone who focuses on research into teacher training could discuss this knowledge from different points of view, based on their own understandings. Based on this assumption, below we present the RDs that sent their summaries and our reflective exercise to write the initial synthesis of the ideas presented in each of the Regional Forums held.

3 The RD's collaborations

To create the triggering text for the DG 01 discussions, we received the summaries sent by 14 RDs¹. This general synthesis is divided into the four thematic axes presented in the introduction. Next, we articulate the points relating to the four thematic axes that we list for reflection: (1) Understandings regarding the DG theme; (2) Issues raised; (3) Main directions and (4) Questions guiding the discussions.

3.1 Axis 1: Understandings regarding the DG theme

The dynamics of the DGs of each RD, as we already imagined, although they kept approximations, were not similar. In some of them, when starting discussions on the DG theme, the coordinators and debaters listed problematizations and/or made quick explanations

¹ Acre, Bahia, Ceará, Distrito Federal, Goiás, Minas Gerais, Mato Grosso, Paraná, Rio Grande do Sul, Rondônia, Roraima, Santa Catarina, São Paulo and Tocantins.

regarding the five problematizing intentions present in the DG syllabus.

The RD/CE DG organization itself is an example of this multiplicity. The discussions at this Regional Forum took place in two rounds, the first of which was institutional in nature, involving six institutions² and the second of state nature, an opportunity in which the discussions were socialized.

The RD/GO DG coordinator started the debate with the quote: *Those who can do, those who cannot, teach*³. Next, the coordinator presented the knowledge categories of Shulman (1986) and also some knowledge models aimed at teaching Mathematics, such as the MKT (Ball; Thames; Phelps, 2008) and MTSK (Carrillo-Yañez *et. al.*, 2018). He also used a situation in which a teacher, with various backgrounds, did not know how to explain to a child why the inverting and multiplying algorithm for dividing fractions works. Based on Ball, Thames and Phelps (2008) explained that

teaching may require a specialized form of pure subject matter knowledge — "pure" because it is not mixed with knowledge of students or pedagogy and is therefore distinct from the knowledge of pedagogical content identified by Shulman and his colleagues and "specialized" because it is not needed or used in contexts other than mathematics teaching [RD/GO DG Synthesis, 2023]

Based on Carrillo-Yañez *et. al.* (2018), the coordinator also explained what this specialized knowledge includes. The participants considered *teaching identity as an objective, pre-established characteristic, as a common place for Mathematics teachers, a static model, possible to be achieved through training courses and/or professional practice*. And then, the coordinator presented authors who defend the singularity of identity and the importance of seeing the person as a teacher and considering the personal within the social and professional traits of the profession.

From a similar perspective, the RD/CE DG, when highlighting the classifications of general and specific teaching knowledge/knowledge for Mathematics teaching, in addition to the authors mentioned above, brought studies by Maurice Tardif, Clermont Gauthier, Demerval Saviani and Selma Garrido Pimenta. The RD/MT DG also considered the contributions of Paulo Freire.

In the RD/TO DG, the dynamic chosen for discussion involved the presentation of three works and the discussions began with the construction of a word cloud, based on the answers to the questions: *"What is teaching knowledge? What does it take to be good Mathematics teachers?"*. The most frequent words were *"teaching, knowledge, competence, skill, attitude, preparation, experience, practical reflection, student, teaching and assessment methodologies"*. Some participants reported their experiences, highlighting the importance of contact with teachers working in Basic Education for teacher training, while others said they did not have this contact in their initial training.

The RD/MG GD brought as its central problematization the motto *"What Mathematics predominates in LM"* with the aim of promoting discussions regarding the specificities of the training processes of mathematics teachers. *"It is worth highlighting the breadth of the geographic distribution of the members in this group (people from Minas, northerners, people from Pará), which was made up of Mathematics graduates, primary school teachers,*

² Universidade Regional do Cariri (URCA), Universidade Estadual do Ceará (UECE), Universidade Federal do Cariri (UFCA), Universidade Estadual do Vale do Acaraú (UVA), Universidade Federal do Ceará (UFC) e Instituto, Federal de Educação, Ciência e Tecnologia do Ceará (IFCE).

³ From this excerpt, we will use *italics* to indicate excerpts from the synthesis sent by the DGs of the different RD.

postgraduate students and other participants from related areas with training, for example, in Engineering and History". After presenting the objectives of the DG, the coordinator allocated most of the time to discussions, starting with a provocation regarding the nature of mathematical knowledge specific to teaching; that is, assuming that what characterizes a profession is something very specific, which requires knowledge specific to that profession, a fact that differentiates teaching from other professions.

In the RD/AC DG, the coordinator commented on the knowledge and knowledge of mathematics teachers, highlighting the great debate about what is necessary for the initial training. The phrase *"The teacher knows a lot of mathematics, but cannot understand what he is teaching, reflecting the concern around what to teach and how to teach it"* was *problematized*". The coordinator highlighted authors who address mathematics "to" and "to teach". *"The mathematics "to" teach is that which comes from the mathematical disciplinary field and the mathematics "to teach" are related to the tools for teaching mathematics"*.

The RD/PR DG did not directly discuss the concept, but presented ideas that are close to the understandings presented by the majority of the RD when highlighting the need to rethink

the view that we commonly attribute to subjects, that we must be careful not to look at them as isolated subjects, but always reflect on how it can contribute to teacher training as a whole. It also reports that the recent reformulations proposed (Resolutions of 2015 and 2019) were much discussed and thought of as a way of articulating the desired training and proposed by the resolutions, in an integrated and collective way, with teachers who work on different fronts, talking between themselves (pure mathematics, applied mathematics, mathematics education and education). It is also important to note that the subjects proposed in these resolutions must not only be allocated to a single subject, under the responsibility of a single teacher, but must be the roles of everyone, regardless of their area of activity (pure mathematics, applied mathematics, mathematics education and education). [Synthesis of the RD/PR DG, 2023]

Still from this perspective, the RD/CE DG indicates that concerns about *"[...] teachers [we] want to train and what training we want and what we offer these professionals [...]"* permeates the

need to take the discussion about teaching knowledge to licentiate courses, to the student and teaching body and, above all, to the Structuring Teaching Centers (NDE in Portuguese), responsible for updating the curriculum of these training programs. Only by reflecting on the desired graduate profile, which includes the knowledge expected of a mathematics teacher, will a powerful curricular reformulation take place. [Synthesis of the RD/CE DG, 2023, highlights present in the original]

The RD/SP DG articulated the problematizations of the theme through guiding syntheses constructed by the invited debaters, an opportunity in which reflections were made regarding (i) the processes aimed at continued training (enhancers of actions also in initial training) and (ii) of the articulation between the reflective knowledge of the Mathematics teacher and Financial Education. From the discussions, the group understood, among the crucial challenges for LM, *"[...] the mobilization of knowledge specific to teaching, the incorporation of extension into curricula and the promotion of projects that allow future teachers to act as protagonists in the construction of their knowledge about teaching [...]"*. This is stated in its synthesis, *"[...] These challenges reflect the search for more effective and collaborative training, aiming to improve mathematics education and teacher practice"*. It is worth

highlighting that the organizers of Forum Paulista published a thematic issue⁴ in *Revista de Educação Matemática* (REMat), a publication from RD/SP, socializing with the community the perspectives discussed and their directions.

As the Federal District (DF) PEM Training Forum took place in November 2022, it did not align with the themes proposed by FPMat. Therefore, RD/DF sent a synthesis with notes regarding the LM courses. From this synthesis, we highlight aspects relating to the problems and directions.

3.2 Axis 2: Issues raised

In the RD/GO DG, there was a discussion about the use of artificial intelligence in classrooms, especially ChatGPT, highlighting the concern with ethics when carrying out research. In its report, after the presentation of the knowledge models, it was asked whether the initial and/or continuing training courses enable the acquisition/connection of this knowledge, the record of the RD/GO DG was

Of the entities present, the UFCAT professor mentioned that the contents are taught in specific subjects; pedagogical; internships and optional subjects; the MTSK model is distributed across this list of subjects. At UEG, a professor pointed out that this model is not specifically worked on, but there is a distribution of its nuances in specific disciplines; pedagogical; internships and free core subjects, which are those chosen outside the Mathematics course matrix. [RD/GO DG Synthesis, 2023]

In other words, it was not possible to verify the presence of this concern with the knowledge specific to teaching in specific mathematics disciplines, as analyzed by Zaidan *et al* (2021). This seems to be a problem raised by the DG of RD/GO.

The RD/TO DG coordinator highlighted the power struggle between academic mathematics and the mathematics required for teaching, both in training and within the school itself, which often takes academic mathematics as the starting point of teaching. He then presented the challenges of LM in current contexts and commented on the three stages of becoming a teacher: recruitment; training; work in the classroom. In the final synthesis, he listed as the main challenges

To promote recruitment or attractiveness policy for good students to pursue a career in teaching. Transformation of degree curricular structures with a focus on the professional activities of mathematics teachers in basic education schools. To promote more activities that put undergraduate students into effective work in the classroom. [RD/TO DG Synthesis, 2023]

Discussions regarding spaces of power struggle or *territories in dispute* in the initial mathematics teacher training curriculum (RD/SP) are coherent with investigations that signal the concerns of the area of Mathematics Education in promoting social justice, equity and of alterity, occur in a field of ideological and political struggle (Cyrino, 2006, 2017; Souza, Teixeira, Baldino & Cabral, 1995).

Questions regarding the need to overcome the format of technical rationality (3+1 format) still present in LM also arose among participants in several DGs from various RDs (AC, MG, RR and SP). For the DG of RD/MG, for example

⁴ The special issue entitled VIII Paulista Forum for the Training of Teachers who Teach Mathematics is composed of seven articles. Each of the DGs sent a synthesis text of the activities carried out. The access link is: <https://www.revistasbemsp.com.br/index.php/REMat-SP/issue/view/24>.

The licentiate degrees are still characterized by a strong mark of what several authors have called “three plus one” insofar as it separates “specific knowledge”, understood as academic mathematics, mathematics specific to the professional practice of professional mathematicians and “pedagogical knowledge”, mistakenly understood as knowledge in the areas of mathematics education and education and others, that is, we train mathematics teachers without paying due attention to what will be the core of their professional identity, their professional development and their future professional work. [RD/MG DG Synthesis, 2023]

In the same direction, the participants of the RD/AC and RD/RR DGs highlighted the need to adapt the curricula to current changes in teaching, as possible trajectories for overcoming experiences of bachelor's degree valorization in LM.

Many problems were raised by the RD/RO DG, in the form of questions. We chose to highlight here some of these issues, whose discussions were in more depth. The first signals that “[...] *speeches based on not wanting to be a teacher have a bias from the society, of non-acceptance, of not being well-regarded*”. The participants also highlighted that “[...] *in the training process, attempts are made to implement subjects, offer reflections, reformulate curricula, but the reflection of this process is not perceived in the classroom*”; in other words, there is a timid relationship between the profession and training. As a school problem, which also affects the profession, one of the participants highlights that

mathematics professors are one of the most demanding in terms of external assessments, so teaching work is dictated by and through this assessment process. Mathematics teaching is not to train critical, diverse students who understand broader, non-content-oriented mathematics, it is just to train mathematical skills, content and the like. [RD/RO DG Synthesis, 2023]

Furthermore, participants highlight the “[...] *difficulty for teachers to be able to participate in training, since the departments have their own structure and demand, but this is something that needs to be reevaluated*”. Another problem listed by the RD/RO DG is related to the “[...] *number of training courses in the EaD network*”.

In the RD/DF DG, participants commented on the

fear [of teachers at all levels] of getting lost in the decentralization of the process of building knowledge and learning. The need to lose the fear of discovering how to do it, which leads us to the necessary investment in scientific research with the aim of active methodologies in the initial training of teachers who teach mathematics. [RD/DF DG Synthesis, 2023]

The concerns listed in the synthesis of the RD/DF DG also had an impact on the synthesis of the RD/RR DG, by recording the positions of students and teachers regarding points that were not necessarily converging. The first deals with the *gap between the mathematics studied at the undergraduate level and the mathematics to be taught in Basic Education school*. While LM students report their difficulties in understanding the content covered at university (despite highlighting that they were good in Mathematics while they were in high school), teachers working at LM “[...] *state that there is already an effort on the part of the teaching staff to overcome this gap*”. When discussing teaching knowledge, there was a consensus among students and teachers on the need for both trainers and teachers to have knowledge of the mathematical field and focus on expanding knowledge related to teaching-learning processes.

Part of the discussions held in the RD/PR DG is related to the training concern present in LM by highlighting “[...] *the fact that some mathematics professionals are unable to develop*

pure mathematics subjects in a different way and related to Basic Education, due to the amount of content present in the syllabus". In this context, the challenges imposed by the school were also reported, among which was the "[...] *OCR (online class registration) in basic education, and creating platforms for teaching and student activity*".

From the synthesis of the RD/BA DG, still regarding the challenges, we highlight three problematizations that emerged from the discussions. The first deals with the importance of cultural, political and social influences in the context of initial training. Subsequently, the recognition of the multiplicity of roles assumed by the teacher (author and actor) as a critical political trainer, committed to the quality of teaching. Finally, the concern with building/planning actions that minimize the gaps in mathematical knowledge increased by the COVID-19 pandemic context. The questions outline the importance of the university space articulating partnerships with Basic Education institutions in actions (present and future) to mobilize the construction of learning environments that value the disruption of mathematical pedagogical practices based exclusively on the exercise paradigm (Skovsmose, 2000).

3.3 Axis 3: Main directions

Although in the DG syntheses of some RDs the directions were not explicitly demarcated, since the dynamics of the records were different, it was possible to identify the intentions directed to the VIII FPMat present in all of them.

Of the directions listed by the RD/BA DG, we highlight: (i) the promotion of institutionalization policies for the Institutional Teaching Initiation Scholarship Program (Pibid) and the Pedagogical Residency Program (PRP); (ii) the construction of living laboratories that consider local and regional contexts and (iii) the planning of actions closely related to mental health (anxiety, depression and others of emotional nature) in the space of the LM and also in school environments.

Several DGs highlighted the importance of Pibid and Pedagogical Residency (BA, GO, DF, PR among others). The RD/GO synthesis highlighted Pibid and PRP, defending these programs as State actions and not Government actions. RD/DF DG participants highlighted the need to think about Project Pedagogy in Degrees in the context of the PRP. Along the same lines, the RD/PR DG points to the "need to create learning environments that help future teachers and that impact the teacher's professional identity and knowledge: PIBID, PR, internship, tenure grants, etc. Such environments would aim to improve teaching and restructure curricula".

In the RD/TO DG, several directions were made, but, at the end of the text, syntheses were presented, which we took as a reference for this text. For the participants, LM courses "[...] need to take as a reference the teaching activities in basic education that mathematics teachers develop instead of taking academic mathematics as a reference. Teaching knowledge and professional knowledge can guide curricular reformulations". By defending that the LM course has its own identity, they signal that there is a need for "[...] making a commitment and having a curricular organization that focuses on the work of the Basic Education Mathematics teacher, whether on content, practices and real situations that are experienced in schools". To this end, it is necessary to seek a collaboration regime between the University and Basic Education Schools, in order to build a continuing training policy for Basic Education professionals. It was also highlighted that the

need for a greater approximation between the initial training proposal and the beginning of a teaching career in basic education, considering the needs and obstacles that beginning teachers face in this process

of their professional development, and that the quality of this approximation or experience of real practices classroom activities or other activities specific to schools can contribute to preventing teaching from giving up in the first two years of a career. [RD/TO DG Synthesis, 2023]

The case of evasion was also present in the DG discussions of several RDs (among them RR, SC and TO), as well as signaling the need for changes in the pedagogical practices of trainers and basic education teachers (CE, MT and SP for example) and the challenges of fostering the interest of young people in studying LM (MT, SC, TO).

The use of technologies was signaled in several RDs (such as DF, TO, MT and SC, for example). In the RD/DF DG, the initial discussions of the participants

shed light on the crucial importance of technologies in the field of Didactics. [...] There was a unanimous understanding that technologies should not be isolated, but harmoniously incorporated into the educational panorama. [...] A point highlighted with enthusiasm was the diversity of technological possibilities in mathematical learning processes. We have identified a series of approaches that can be integrated into the initial training of teachers who teach mathematics. These possibilities represent a range of strategies that can enrich the way mathematical concepts are taught and assimilated. [...] The challenge is often not just the availability of resources, but the lack of adequate training to use them in an innovative way. [RD/DF DG Synthesis, 2023]

In the RD/TO DG, the use of technologies and the care in providing contact with assessments of different natures are articulated as an intentional action to encourage, even in initial training, the participation of LM students in activities that take place in the school environment, such as events (Mathematics Fairs, for example); as well as, promoting projects and actions that occur in partnership with teachers working in Basic Education. Actions in the same direction were signaled by the DG of RD/MT, when relating the use of digital platforms (such as Canva Education and Kahoot) and Learning Objects (DG of RD/SC) to aspects that promote creative learning.

The triad construction, consolidation and expansion of links between Higher Education institutions and education networks, given that initial and continuing training must be understood as part of the teaching professional development process, were highlighted in DGs of various RDs (among them are BA, MT, RS and SP). This intention is articulated with two directions proposed in the RD/RR DG, namely: (i) to provide solid training regarding learning theories and (ii) to develop “[...] *pedagogical proposals designed based on a real LM student and not an idealized student*”. Together, these directions demarcate the need to overcome the fragmentation between specific and pedagogical training, as well as the need to observe the specificities of the LM target audience, such as working students. The RD/RS DG also signaled the need for the teacher to “[...] *be attentive to identifying students' difficulties and needs, in order to modify and adapt these strategies to the students' reality and context*”.

In the RD/MG DG, the importance of considering different places and different communities was discussed. In other words, to place this mathematical knowledge in the context in which it develops, but also the need to problematize “[...] *what mathematics am I working on in teacher training? in order to make considerations about the appropriation of mathematical knowledge*”. Completing this idea, one of the participants highlights that “[...] *undergraduate licentiate students see the need to learn mathematics that they will use in practice and that, during teaching, professionals will be equipped with to deal with this mathematics*”. It was also highlighted that there is an “[...] *importance of constituting a critical mass that allows us to bring to light and make aspects, values and principles that we consider basic in the training of future mathematics teachers*”. Closing the discussions, one of the

participants highlights the

need to build within the course a decolonial perspective that brings us closer to valuing who we are as a culture, as a people that develops, to break even minimally with the ties that bind us to a Eurocentric mathematics to the way of understanding mathematics that is also imported. With this, it suggests that we stop being colonized also in terms of the way we act within the scope of mathematics degrees and seeking to promote a mathematics specific to the teaching profession. [RD/MG DG Synthesis, 2023]

In the discussions of this very diverse DG, it was also highlighted that “[...] *the relevance of addressing, within the school, issues relating to relationships, ethnicities, environmental issues and other transversal themes that are linked to other areas*”. The RD/RO DG also raised this question, suggesting an approach more focused on ethnomathematics. In the end, they suggested “[...] *the proposal that SBEM undertakes a process of intense discussion of the licentiate degree course in mathematics that should predominate in this course and the production of materials that can support work of this nature*”. Still in this perspective,

Referring to the need to build a new perspective for teacher training, DG1-AC participants highlighted the importance of considering school autonomy, the school curriculum, collective work, facing the dissonance between what the university teaches and the school work, addressing aspects of individual training and collective training (Nóvoa, Vernaund) and constitutes a reference axis for professional development, from a political, social, cultural and pedagogical perspective, and to think in an interdisciplinary way, looking at existing contexts, highlighting that teacher training is human training [RD/RO DG Synthesis, 2023].

In the discussions of the RD/PR DG, the emphasis was on the knowledge specific to teaching and who is responsible for it. In its report it is highlighted that the

Licentiate degree subjects should not be limited to dealing with the contents of Basic Education in a simplified way, but teachers must assume the responsibility that the academic training of mathematics graduates must gain depth in its plurality (pedagogical knowledge, mathematical knowledge, curricular knowledge, cultural knowledge, social knowledge, etc.). [RD/PR DG Synthesis, 2023].

The participants in this DG point out that there are practices to minimize this rupture, such as bringing the debate between undergraduate students in Mathematics and teachers working in Basic Education closer together, reducing the space between University and school.

In summary, they present a list of directions related to this topic

(i) Organization of the different Licentiate degree curricula, thinking about themes associated with teacher's knowledge. (ii) Importance of the relationship between professionals in Mathematics Education, pure Mathematics and applied Mathematics, so that aspects related to the teacher's identity and knowledge are prioritized. (iii) Importance of debating topics in mathematics degrees: citizenship, ethics, morals, inclusion and diversity. [RD/PR DG Synthesis, 2023]

As a first reflective exercise based on the syntheses shared by 14 DGs from different SBEM RDs, our challenge was to trigger possibilities for referrals to discuss together in the FPMat⁵.

⁵ RD/SP and RD/RO listed understandings that converge and include, in our opinion, the aspects indicated in other RDs. RD/RO suggested 22 actions aimed at promoting the appreciation and training of PEM and improvements in teaching and learning

3.4 Axis 4: Questions used to guide discussions

In order to promote discussions regarding the specificities surrounding the DG theme, several RDs shared the guiding questions presented to the participants of their Regional Forums. Although in a broader view, the questions concern teacher training processes, we have been able to identify four articulating thematic intentions, namely⁶: (1) Knowledge specific to teaching; (2) Professional Identity of Mathematics Teachers and Professionalism; (3) Interrelationships between University and Basic Education Schools and (4) Valuing the teaching career. Based on the notes on the 14 RDs, we believe it is pertinent to compile them into the following questions, present in the Base Text for DG1 (promoters of discussions):

- (i) How can we outline the knowledge that is suitable for teaching? What about the mathematical knowledge specific to teaching?
- (ii) What are the spaces for discussion regarding Professional Identity in the initial and continuing training of mathematics teachers? What aspects should be considered in these contexts?
- (iii) What characteristics are expected for the relationships between training institutions and the Basic School? What are the intentions of these relationships?
- (iv) What elements can be understood as representing the appreciation of the teaching profession? Whose fight is it for this appreciation?

4 Summaries of DG 01 discussions

Based on the discussions that took place within the scope of DG 01, we shared what we called, at the time, *Understandings that maintain convergence*. The understandings are the result of collective writing, made during the DG discussions, to which all participants actively contributed. Gathered in eight points, they point out consensual signs.

Chart 1: The eight *understandings that maintain convergence*

1) The Licenciature Degree in Mathematics (LM) is understood as a teacher training course, with its own path. Therefore, we defend a movement to restructure training based on teaching-specific knowledge, that is, that takes as a starting point — not an arrival — the revisited Basic Education curriculum and values cultural, ethical, social and political aspects of mathematics, considering research results in the field of Mathematical Education.
2) The Structuring Teaching Centers, spaces of extreme relevance for discussions regarding the perspectives of mathematics teachers that we desire, need to be composed of teachers committed to teacher training, who understand the need to establish knowledge specific to teaching, that is, articulated knowledge to the work of teachers working in Basic Education, in particular, mathematical knowledge.
3) It is important that the professional profiles of trainers working in LM enhance the knowledge inherent to teaching, that is, we defend the performance of trainers who effectively develop teaching, research and extension actions related to the universes of Basic Education, teacher training and research in Mathematics Education, involving all its actors. We understand that this point is similar to other DGs of the VIII FPMat.
4) The spaces for initial training in LM need to be intertwined with the specificities of the professional

Mathematics, while RD/SP listed 20 propositions. These items are inserted as annexes in the DG 01 text prepared for the event's annals.

⁶ The ordering is alphabetical only. Due to the limits of a scientific article, we maintained the table with questions representing these articulations present in the syntheses that the RD shared with us in the text of the event's Proceedings. We ask for the readers' understanding of this decision.

identity of (future) mathematics teachers who work (will work) in Basic Education, in teacher training and in research in Mathematics Education.

5) The LM teaching staff needs to pay attention to ensuring permanence and success through articulations between Basic Education, Initial Training and professional practice, focusing on the knowledge specific to teaching.

6) We, as SBEM, must encourage and effectively build coordinated actions that centralize: (i) the needs of Basic Education, (ii) the demands of teachers working in this context and also those working in LM and (iii) personal and professional perspectives of future teachers, with a view to recognizing the relevance and protagonism of all the people involved in these (complex and dynamic) processes. This point is representative of SBEM's commitment and positioning, contrary to the fragmentation between Initial and Continuing Training.

7) It is necessary to carry out research that makes it possible to expand understanding regarding the knowledge specific to teaching arising from contemporary demands for training that are articulated with Basic Education (for example, in the different teaching modalities — including EaD and hybrid formats — and inclusion). Such research can contribute to the coordinated effort between WG07 researchers, FPMat participants, SBEM and public policies in promoting LM restructuring.

8) It is necessary to increase the effective participation of SBEM as a society (or its members as representatives/articulators) in institutional and political spaces (among which, MEC, PNLD, INEP, CAPES and CNPq, are relevant examples) related to the discussion/construction/implementation of public policies with collectively elaborated proposals aimed at training mathematics teachers.

Source: Prepared from the DG 01 discussions

These understandings denote a convergence of conceptions among researchers linked to SBEM's WG 07, which result from approximations with the research developed by these researchers (Cristovão *et. al.*, 2024; Cyrino, 2006, 2017; Moreira & David, 2005; Zaidan *et. al.*, 2021) and international references (Ball *et. al.*, 2008; Carrillo-Yañez *et. al.*, 2018).

In the highlighted points, a convergence is noticeable with the idea of mathematical knowledge specific to teaching, which places the demands required by teaching practice as the center of the training process, that is, which takes this practice as a curricular component to be studied in the licentiate degree (Cristovão *et. al.*, 2024). This demands institutionalized partnerships between university and school and contributes to the creation of a professional teaching identity (Cyrino, 2006, 2017), more committed to teaching and the social function of the school.

Likewise, we collectively constructed four proposals for SBEM's DNE, which, along with the previous eight points, were presented in the final plenary of the event.

Chart 2: The four proposals constructed within the scope of DG 01 for the SBEM

(1) To nationally lead the coordination of a set of actions between Regional Directorates, mathematics teacher training institutions (especially Universities and Federal Institutes) and Basic Education schools, with the aim of building actions, proposals and materials in perspectives that socialize experiences, concerns and demands arising from this dialogical process.

(2) To nationally lead the coordination of a space for permanent discussions through coordination between the Regional Directorates, mathematics teacher training institutions (especially Universities and Federal Institutes) and Basic Education schools, with the aim of socializing actions and bringing together discussions that historically culminate in National Forums for the Initial Training of Teachers who teach Mathematics.

(3) To defend the proposal to implement financial subsidies for all LM students and also Professional Master's programs in the area of Teaching/Education/Science and Mathematics Teaching.

(4) To create mechanisms and spaces for dissemination and dialogue between SBEM and its partners, in addition to Basic Education teachers, civil society and institutional and political spaces, with the aim of demarcating its expertise in the field of initial and continuing training of mathematics teachers in its multiple contexts. The experiences of the Mathematics Fairs and the editions of the Formação Program can be considered as promising initiatives for the rapprochement between SBEM and teachers working in Basic Education.

Source: Prepared from the DG 01 discussions

The eight points of convergence revisit the problems and paths already highlighted by Zaidan *et. al.* (2021) and reinforce, at the same time as they expand, the concerns highlighted by the participants of the SBEM's DG and RD. Therefore, we understand that as teacher trainers, we are building convergent and very powerful understandings in relation to the training we want for Mathematics teachers in Basic Education. This does not mean that total consensus exists, but that SBEM has strengthened research processes and, consequently, narrowed understandings in favor of improving training. And this process can still be improved based on the indications mentioned.

Acknowledgements

We would like to thank our colleagues Profa. Dra. Vanessa Dias Moretti (UNIFESP), Profa. Dra. Fabiana Fiorezi de Marco (UFU) and Profa. Dra. Flávia Cristina de Macêdo Santana (UEFS), respectively, coordinator and vice-coordinators of the Working Group *Training of Teachers who teach Mathematics* (WG07) at SBEM, for the honorable invitation to act in the coordination of DG 01. We also thank all the colleagues present during the fruitful debates and in the collaboration for the final drafting of the convergent understandings and proposals shared here.

References

- Ball, D. L.; Thames, M. H. & Phelps, G. (2008). Content Knowledge for Teaching. *Journal of Teacher Education*, 59(5), 389-407.
- Carrillo-Yañez, J.; Climent, N.; Montes, M.; Contreras, L. C.; Flores-Medrano, E; Escudero-Ávila, D.; Vasco, D.; Rojas, N.; Flores, P.; Aguilar-González, A.; Ribeiro, M. & Muñoz-Catalán, M. C. (2018). The Mathematics Teacher's Specialised Knowledge (MTSK) Model. *Research in Mathematics Education*, 20(3), 236-253.
- Cyrino, M. C. C. T. (2006). Preparação e emancipação profissional na formação inicial do professor de matemática. In: A. M. Nacarato & M. A. V. Paiva (Org.). *A formação do professor que ensina Matemática: perspectivas e pesquisas*. (pp. 77-86.) Belo Horizonte, MG: Autêntica.
- Cyrino, M. C. C. T. (2017). Identidade Profissional de (futuros) Professores que Ensinam Matemática. *Perspectivas da Educação Matemática*, 10(24), 699-712.
- Cristovão, E. M., Ferreira, A. C., Barbosa, C. P., Coura, F. C. F., Paiva, M. A. V. & Zaidan, S. (2024). Ensaio sobre a formação matemática do futuro professor de Matemática pautada nos conhecimentos matemáticos próprios da docência. *Espaço Plural*, 19(39), 1-20.
- Moreira, P. C. & David, M. M. S. (2005). *A formação matemática do professor: licenciatura e prática docente escolar*. Belo Horizonte, MG: Autêntica.
- Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

- Skovsmose, O. (2000). Cenários para investigação. *Boletim de Educação Matemática*, 13(14), 66-91.
- Souza, A. C.; Teixeira, M. V.; Baldino, R. R. & Cabral, T. C. (1995). Novas diretrizes para a licenciatura em Matemática. *Temas e Debates*, 8(7), p. 41-65.
- Zaidan, S.; Ferreira, A. C.; Paula, E. F.; Santana, F. C. M.; Coura, F. C. F.; Pereira, P. S; Stormowski, V. (Org.) (2021). *A Licenciatura em Matemática no Brasil em 2019: análises dos projetos dos cursos que se adequaram à Resolução CNE/CP 02/2015*. Brasília, DF: SBEM.