Sociedade Brasileira de Educação Matemática

RIPEM

Connections between Didactic Decisions and Didactic Transposition in the Practice of Mathematics Teachers

Wuallison Firmino dos Santos
Instituto Federal de Educação, Ciência e Tecnologia da Paraíba
Patos, PB — Brasil
✉ wuallison.firmino@ifpb.edu.br
0000-0002-2354-6896

Marcus Bessa de Menezes
Universidade Federal de Pernambuco
Campina Grande, PB — Brasil
✉ marcus.bmenezes@ufpe.br
0000-0003-0850-1793

2238-0345 ISSN

10.37001/ripem.v15i1.4347 doi

Received • 17/10/2024
Approved • 09/11/2024
Published • 02/03/2025

Editor • Gilberto Januario

Abstract: This study analyzes the relationship between a mathematics teacher's didactic decisions and the process of didactic transposition in a first-year high school class. The methodological approach is qualitative, with data collected through classroom observations and interviews. Grounded in French-influenced Mathematics Didactics, particularly in the teacher's level of activity, the study explores how external, epistemic, and didactic history factors shape the teacher's choices and the process of didactic transposition. The results show that teaching undergoes continuous modifications, impacted by unforeseen circumstances and classroom interactions. The conclusions highlight that the teacher's didactic decisions and didactic transposition complement each other, contributing to the organization of school knowledge.

Keywords: Didactic Choices. Didactic Phenomena. Numerical Sets.

Conexiones entre las Decisiones Didácticas y la Transposición Didáctica en la Práctica del Profesor de Matemáticas

Resumen: Este estudio analiza las relaciones entre las decisiones didácticas de un profesor de matemáticas y el proceso de transposición didáctica en una clase de primer año de la educación secundaria. El enfoque metodológico es cualitativo, con la producción de datos realizada mediante observación de clases y entrevistas. Basado en la Didáctica de la Matemática de influencia francesa, especialmente en los niveles de actividad del profesor, investiga cómo los factores externos, epistémicos y de historia didáctica influyen en las decisiones del profesor y en los procesos de transposición didáctica. Los resultados muestran que la enseñanza se modifica continuamente, siendo impactada por imprevistos e interacciones en el aula. Las conclusiones destacan que las decisiones didácticas del profesor y la transposición didáctica se complementan, contribuyendo a la organización del saber escolar.

Palabras clave: Elecciones Didácticas. Fenómenos Didácticos. Conjuntos Numéricos.

Conexões entre Decisões Didáticas e Transposição Didática na Atuação do Professor de Matemática

Resumo: Este estudo analisa as relações entre as decisões didáticas de um professor de Matemática e o processo de transposição didática em uma turma da 1ª série do Ensino Médio. A abordagem metodológica é qualitativa, com a produção de dados realizada por meio de observação de aulas e entrevista. Fundamentado na Didática da Matemática de influência francesa, particularmente nos níveis de atividade do professor, investiga como fatores externos, epistêmicos e de história didática influenciam nas escolhas do professor e nos processos da transposição didática. Os resultados mostram que existem modificações no ensino de forma contínua, sendo este processo impactado por imprevistos e interações em sala de aula. As conclusões destacam que as decisões didáticas do professor e a transposição didática se complementam, o que contribui para a organização do saber escolar.

Palavras-chave: Escolhas Didáticas. Fenômenos Didáticos. Conjuntos Numéricos.

1 Introduction

The French-influenced didactics of mathematics has been an important theoretical contribution to Brazilian research in the past three decades. As the field of study of didactic phenomena consolidates, mathematics teachers' reflections on their pedagogical practices promote research that shows promising results in understanding teaching and learning processes.

In 1986, Brousseau highlighted the environment as an essential part of the didactic system consisting of the triad: teacher, student, and knowledge. The author emphasized that students' interaction with the environment is essential for the autonomous construction of knowledge. This environment goes beyond material resources and includes the actions and decisions of the teacher, who tries to adapt knowledge to the classroom context and create favorable conditions for learning.

This adaptation implies the transformation of knowledge to make it accessible in the school context. Chevallard and Joshua (1991) state that the mathematical knowledge taught in the classroom is distant from the original sources, revealing a difference between scientific knowledge and school knowledge. Such transformations occur when scientific knowledge becomes school knowledge, known as teaching objects. When it arrives in the classroom, this knowledge is used by the teacher and students in a process that Chevallard calls didactic transposition.

Didactic transposition involves modifications that make theoretical knowledge accessible to students through didacticization. This process requires the teacher to make continuous decisions about how to present the knowledge in an understandable way, considering the distance between scientific knowledge and school knowledge.

Margolinas (1993, 2004) has contributed to the understanding of these decisions by studying teaching activity, especially teachers' choices in teaching situations. His work aims to advance the understanding of didactic decisions by linking them to the process of didactic transposition discussed by Chevallard.

In general, Margolinas' studies have influenced various studies aimed at analyzing didactic decisions. These decisions are the choices that the teacher makes in his or her actions to promote the learning of specific knowledge. They can be planned or adjusted during classroom interactions. In addition, they are fundamental in guiding the didactic process and determining how knowledge is presented to students (Bessot & Bittar, 2019).

By combining the studies of Brousseau, Margolinas, and Chevallard, we can see that these decisions are inextricably linked to didactic implementation. When teachers adapt knowledge to make it accessible, they make decisions that consider both the nature of the knowledge and the way it is presented in the classroom. These decisions reflect not only immediate choices, but also the ways in which the teacher mobilizes knowledge in the school context.

We are therefore interested in the relationship between the teacher's choices and the process of didactic transposition, with the aim of establishing an interface between the two for a deeper theoretical understanding, to contribute to the analysis of the didactic relationships established between the teacher, the students and the mathematical knowledge. Our focus is on understanding how didactic choices and didactic transposition are related when analyzing teaching practice. Thus, we ask: What are the direct and indirect implications between the teacher's didactic decisions and didactic transposition?

Although many studies have investigated didactic decisions (Margolinas, 2002; Bessot & Bittar, 2019; Lima, 2011; Espíndola & Trgalová, 2015) and didactic transposition (Chevallard, 2002; Chevallard & Joshua, 1991; Brito Menezes, 2006; Bessa de Menezes, 2010; Santos, 2019), there is still a lack of studies investigating the theoretical links between these two phenomena.

This study is not limited to isolated notes, but seeks to understand how these phenomena are related and how theoretical elements of one influence the other. To this end, we examined a situation in which numerical sets were introduced in a 1st grade high school classroom to identify signs of this interface. Next, we briefly discuss the phenomena that are the objects of study, pointing out the theoretical elements that constitute an analysis of the relationships between them.

2 The teacher's teaching decisions

The role of the teacher as a professional is broad and dynamic. They plan lessons, implement them, and, based on the results, reflect on the actions taken to promote student learning. In the process, teachers make decisions based on their beliefs, training, convictions, and experiences that influence their choices. Instructional decisions are intended not only to create a learning environment, but also to serve as an opportunity for reflection on teaching practice. Decisions are made before, during, and after teaching in an ongoing process.

Margolinas (2002, 2004) bases the analysis of these decisions on the field of mathematics didactics, extending the concept of structuring the environment proposed by Brousseau (1986). In his studies on the Theory of Didactic Situations (TSD), Brousseau (1998) emphasizes that learning takes place by adapting to an environment that presents contradictions, difficulties and imbalances, in line with Piaget's constructivist approach. However, Brousseau goes further by stating that the environment is intentionally organized by the teacher with the aim of facilitating student learning.

In this context, learning results from the student's interaction with an antagonistic environment made up of activities proposed by the teacher, prior knowledge, available materials, and interactions with peers. Based on this conception, Brousseau proposes a model for structuring the environment that defines five positions for the student and two for the teacher, one of which is related to lesson planning and the other to the conduct of classroom interactions.

Margolinas (2002, 2004) refines this theoretical model[1] by delimiting the levels of teacher activity and expanding the teaching role into different positions. The model offers a view of the teacher's interactions and choices on multiple levels.

[1] The structuring of the milieu proposed by Margolinas is considered more of a heuristic than a model. In this paper, we use the term *model* to refer to this heuristic.

> **Level +3** - Values and conceptions of teaching/learning - Educational project: educational values, conceptions of learning and teaching.
>
> **Level +2** - Theme construction - Overall didactic construction in which the lesson takes place: notions to be studied and learning to be achieved.
>
> **Level +1** - Lesson project - Specific didactic project on the lesson observed: objectives and work planning.
>
> **Level 0** - Teaching situation - Carrying out the lesson, interacting with the students, making decisions in action.
>
> **Level -1** - Observation of student activity - Perception of student activity, regulation of the work assigned to the students (Margolinas, Coulange & Bessot, 2005, p. 11, emphasis added).

This model allows for a characterization of the teacher's knowledge in various situations, in which the teacher's activities are explored and experienced at different levels, but most of the time simultaneously. These levels show teachers' knowledge in various contexts and indicate that their actions are explored and experienced at different levels, often simultaneously. In addition, they correspond to the positions the teacher acquires throughout their practice and can be used to describe their teaching decisions. *Level +3* (N+3) reflects a broad view of the teaching and learning of mathematics, while *level +2* (N+2) involves the organization of the teaching of a mathematical object. At *level + 1* (N+1), the focus is on planning, mobilizing global knowledge about students and their difficulties. At *level 0* (N0), the lesson itself takes place, with immediate decisions based on the students' representations. At *level* - 1 (N - 1), the teacher observes and identifies student errors and/or difficulties.

There is no isolation between these levels; they interact dynamically. As Coulange (2012, p. 19) exemplifies, at N0, the teacher deals with "[...] the result of his observations of the students' mathematical activity in a learning situation, but also with elements of his lesson plan, drawn up at the higher level + 1". These interactions reveal the teacher's different positions, which reflect the learning involved in teaching practice, influenced by the decisions that need to be made, as Lima (2011, p. 365) points out: "[...] regardless of the influence the teacher suffers from factors of diverse origins, their conceptions about the nature of teaching and learning certainly play an important role in their didactic decisions".

Bessot and Bittar (2019) identify three types of factors that influence teaching decisions: external, epistemic and historical-didactic factors. External factors refer to conditions that the teacher does not control, such as unforeseen meetings. Epistemic factors involve the teacher's personal relationship with the object to be taught, the subject, and the teaching and learning processes. History-didactic factors relate to the shared history between the teacher and the students, considering memories of previous interactions.

There is also a distinction between immediate and planned decisions. This distinction is the result of the teacher's awareness of the possible choices. The former is called micro-decisions and the latter macro-decisions, according to Espindola, Silva and Brito Júnior (2020). The analysis of didactic decisions and the factors that influence them is a fertile object of study for the Didactics of Mathematics, establishing, for example, connections with didactic transposition, discussed briefly below.

3 Didactic transposition: changes in knowledge

Introduced by Verret in 1975, the concept of didactic transposition was developed to study the vulgarization of knowledge to make it accessible and suitable for teaching and learning. However, Yves Chevallard deepened the study of this phenomenon by analyzing the path of knowledge from its scientific production to its entry into the school. He describes, “Knowledge content, having been designated as knowledge to be taught, then undergoes a series of adaptive transformations that make it suitable to take its place among the 'objects of teaching'" (Chevallard & Joshua, 1991, p. 39).

This process of transformation takes place in two phases: external and internal. External didactic transposition involves the transformation of scientific knowledge into knowledge to be taught before it reaches the school context. This work is carried out by the noosphere, defined by Chevallard (2002) as an invisible institution made up of teachers, didactics, technicians from public institutions - such as the Ministry of Education in Brazil - and other actors involved in the educational scenario.

Chevallard and Joshua (1991, p. 33) consider the noosphere to be "[...] the operational center of the transposition process, since it allows knowledge to pass from one institution to another".

The transposition of knowledge involves a long journey that starts in the academy, goes through the production and communication of reference knowledge, and finally reaches the school through curricula and textbooks, for example. The noosphere acts as an intermediary between the educational system and society, adapting knowledge at each stage. As Chevallard and Joshua (1991, p. 16) note, "The knowledge to be taught and the knowledge taught are necessarily different from scientific knowledge".

The second stage, called internal didactic transposition, refers to the transition from the knowledge to be taught to the knowledge actually taught. It is at this stage that the teacher's work becomes visible, although he or she also participates in the external didactic transposition, for example by proposing curricula. In planning, the concept of prepared knowledge emerges, described by Ravel (2003) as the knowledge present in the lesson plan, shaped by the teacher's expectations of the students.

In the practice of teaching, what is planned is often modified in the light of the conditions of teaching, which gives rise to the knowledge that is actually taught. The teacher, with his or her subjectivity, develops ways of teaching that generate differences between what was planned and what was taught, often because of their oral verbalization, which transforms knowledge into a new text.

This new text, adapted by the teacher, is not limited to official prescriptions, but is temporized according to the relationship the teacher has with mathematical knowledge (Câmara dos Santos, 1997). Thus, understanding didactic choices, especially at the N+1, N0 and N1 levels, implies considering the factors that guide the teacher's choices during internal didactic transposition.

Based on the theoretical discussions presented, the next topic describes the methodological choices adopted to study the relationship between didactic decisions and didactic transposition.

4 Methodological aspects

Our methodological choices are determined by the guiding question of this study, as well as our conceptions of the subject and the objectives set. We chose to characterize it as qualitative research (Stake, 2011), since it focuses on understanding didactic phenomena, considering the relevance of the meanings of interactions in the didactic system.

In this sense, we used data from the Master's research conducted in Campina Grande, on the campus of the Federal Institute of Education, Science, and Technology of Paraíba (IFPB). To achieve our objectives, we selected Mathematics Teacher P as a participant whose didactic action will be studied to understand his didactic choices and decisions in the classroom context.

Teacher P holds a degree in mathematics, a master's degree in applied mathematics, and a Ph.D. in petroleum science and engineering. At the time of the data collection, he was teaching a first-year high school class in the Integrated Mining Technical Course, consisting of 35 students. The class is the context analyzed in our reflections.

For this study, we adopted two methodological procedures: direct observation of classes and interviews.

- During the observation, we followed and recorded the class in loco for three weeks in 2018 in a module on numerical sets with a focus on natural numbers. The video recording began in the second week, to minimize interference in the classroom routine and to avoid strongly influencing the actions of the teacher and the students;
- In the interview, we opted for a semi-structured approach to understand the teacher's conceptions and the didactic implications of her actions, aspects that are relevant for the analysis of the internal didactic transposition and her didactic choices.

As this was a classroom study, we considered the decision factors mentioned by Bessot and Bittar (2019) and discussed above, which may directly or indirectly influence teachers' decisions. We divided the analysis into three categories: external factors, epistemic factors, and history teaching.

The analysis articulates the observed elements with discussions on didactic choices and didactic implementation, enriched by transcribed excerpts from the lessons and the interview. It is based on the factors mentioned above and partially integrates the levels of teacher activity proposed by Margolinas (2004), allowing a deeper understanding of teaching practice.

Finally, we describe and discuss significant situations identified in the teaching of natural numbers, which guided the analysis and stood out in the reflections of the study.

5 Discussion and results.

To reflect on the relationships between the didactic phenomena considered in this study, we elucidated the decision-making factors described by Bessot and Bittar (2019), highlighting aspects inherent to didactic transposition as a continuous process. This phenomenon manifests itself from macro decisions, related to the selection and choice of teaching objects according to the teacher's intentions, to micro decisions, occurring in the interaction and adaptation of the mathematics teacher in the classroom.

Before beginning the description and analysis of the data, it is important to point out that this work does not aim to evaluate the quality of the teacher's teaching or to make value judgments. We understand that the classroom is a dynamic environment, comparable to a living organism, in which actions and reactions are triggered and have an impact on teaching practice. In this context, the teacher conducts his/her lesson with didactic and pedagogical purposes, whether they follow the initial planning, in a space that involves a diversity of students and limited time in sessions.

As explained in the methodological section, this study is based on data collected at two points: lesson observation, the primary source of analysis, and interviews with the teacher, which complement and deepen some situations analyzed.

Based on the three decision-making factors presented by Bessot and Bittar (2019): external factors, epistemic factors, and didactic history, we sought to identify the elements that link didactic decisions to the dynamics of didactic transposition.

5.1 External factors

External factors influence teachers' decisions because they are beyond their control and affect their actions. These factors include both institutional constraints in the school context and unforeseen circumstances.

In what follows, we analyze how didactic choices and didactic implementation are related, based on a critical perspective and on the theoretical framework adopted. At the beginning of the lesson, a dialogue takes place between the teacher and the students about the ownership of the textbook, as described in the table below.

Chart 1: Excerpt from teacher *P*'s lesson

Q: Do you have the books yet? **Students (A)**: No! **P**: Well then, I'll bring it, I'll prepare some material, I'm waiting for the book to arrive, no problem. Next week, I'll bring (*sic*) the material that works with this material here; let's not wait any longer for the book, okay? So, guys, let's get started, all right! In today's lesson, the part that works specifically on number sets.

Source: Santos (2019)

This dialog suggests that the professor makes an immediate decision, redirecting his actions in the absence of the professors. By indicating that he will bring material for the next lesson, it is assumed that he will prepare a list of exercises on the content.

The decision is influenced by an external circumstantial factor: the delay in delivering the books to the students. In the interview, the professor mentions that *this is a problem that comes from the federal government project and usually arrives a little late*, reporting that the situation has lasted for around three months. Despite his initial plan to use the book, the professor adapted the lesson, telling the class that he would prepare other material to make up for the unavailability of the book.

This episode highlights aspects of didactic transposition, especially regarding the transition from scientific knowledge to the knowledge to be taught - the stage known as external didactic transposition. This process directly influences classroom practice, affecting internal didactic transposition, in which students deal with knowledge that, although related to scientific knowledge, is adjusted to the needs of the school.

The transformation of knowledge goes a long way: it starts with academic production, goes through the work of researchers and reaches schools through curricula and didactic books.

Another relevant point is the interaction between different levels of professors' activities, with an emphasis on level +1. At this level, the professor reflects on immediate decisions during the lesson and simultaneously plans the next sessions, setting objectives such as the application of exercises. This dynamic reinforces the observations of Margolinas (2002) and Lima (2011) about the interactive and non-linear nature of teaching activity.

Although the textbook was already available to the professor, he stressed the need to prepare material for the next lesson, highlighting the process of didactic choices in internal didactic transposition.

The professor is a central agent in internal didactic transposition (Bessa de Menezes, 2010; Santos, 2019). Even with the use of didactic textbooks, he or she creates a new text, personalized by the relationship he or she has with the knowledge in question. These choices generate unique knowledge, as each professor shapes their expectations according to the specificities of their class and the knowledge taught.

Given the nature of the master's research (Santos, 2019), it was not possible to assess how replacing the material influenced the way the content was presented, or even understood by the students. However, this situation raises the possibility of investigating whether decisions to change the material could result in a lesson with a different text to the one provided in the didactic.

5.2 Epistemic factors

The analysis of professors' actions in relation to epistemic factors is conducted in terms of micro-decisions, i.e., + decisions made by the professor in interaction with the students. These interactions are shaped by mathematical knowledge (rapport au savoir). Knowledge in the didactic system is essential because the relationship established between professors and students reveals and defines factors that influence the learning of certain content, as Brito de Menezes (2006) points out.

In this context, we present two questions that guide the analysis of the professors' immediate decisions at level 0, as well as their interface with the didactic transposition. The first question asks whether the professor's decision to include historical contexts of mathematics when approaching natural numbers was planned or was the result of an immediate response to interaction with students. The second question explores the justifications the professor used to re-explore the binary system and relate it to sets of numbers. These questions aim to understand the motivations and implications of the didactic choices made.

Below is an excerpt that led us to consider the first question as intrinsic to epistemic factors.

Chart 2: Excerpt from the lesson taught by professor *P*

Q: For example, there are Roman numerals, remember? Which are also symbols, but we have what? Indo-Arabic! That is the one we use, right? It's... which is also called a numeral, in honor of Al-khwarizmi. Therefore, guys, look at... when you went shopping in ancient times, as it didn't have numerical writing, what did you do? Do you remember how to count? Does anyone remember how they counted? Did you have a computer back then? When you weren't born... Did you have a computer, a calculator? There was no calculator, so how did people count? How did you count? **Students**: On your fingers [...] Then in the morning he would put the cattle out to pasture... what did he do to avoid losing the animal? When the animal passed, he'd put a little mark on the ground, wouldn't he? Do you remember that he would put a little mark on the ground? With every animal, a scratch; with every scratch, an animal... There was a two-way correspondence: little animal, little animal. Guys, there's one thing, though: if there was a big gale, it would cover the little stripes, and he wouldn't know if it was the number of animals he'd put out to graze or the

number of animals he'd collected at the end of the afternoon. When it wasn't the little stripes, do you remember the other way?
Student: Pedro
Q: What was it?
Pupils: Pedro!

Source: Santos (2019)

In the excerpt above, we observe that the professor's choice to include a historical perspective, although possibly planned, was reinforced by the interaction with the students, especially in the face of a humorous comment: *To complicate our lives...*. The professor took the opportunity to engage the class, adapting his approach in real time.

This situation reflects the professor's relationship with knowledge and with the students. As Brito de Menezes (2006) points out, every relationship with knowledge is also an epistemological relationship. The inclusion of historical elements in the teaching of natural numbers can generate different learning outcomes, depending on the approach chosen by the professor.

According to the National Common Core Curriculum (Brazil, 2018), number sets should be explored in greater depth in secondary school, allowing students to understand the need to expand these sets throughout history. In elementary school, the content is often approached in a fragmented and simplified way, without sufficiently exploring the justifications for the creation of new sets. As Souza (2017, p. 13) notes, "[...] to meet the needs and challenges of man and science, new categories of numbers have emerged and joined the existing ones". In addition, the author goes on to conclude that "[...] with the passage of time, out of practicality, the need arose to group them, forming structures with common characteristics and properties, which constitute the numerical sets" (Souza, 2017, p. 13).

In high school, however, numerical sets are often presented as a review, with no historical or conceptual depth. The historical approach adopted by professor *P* highlights the value of these construction processes for understanding concepts, reinforcing the importance of the historical context in school mathematics.

Bessot and Bittar (2019) emphasize that the professor is an epistemological subject whose decisions directly affect the cognitive processes involved in teaching and learning. By exploring historical elements and connecting primitive counting with natural numbers, professors actively position themselves in the face of students' knowledge and learning.

Considering this, we can see that the professor's decisions are influenced by his pedagogical and didactic knowledge, driven by the answers or questions presented by the students. This importance becomes clear during the interview, when he highlights the value of learning about numerical sets, pointing out that this knowledge is fundamental for understanding concepts and for success in Higher Education: *You go to Higher Education, you're going to study a limit, you're going to study a derivative, you're going to study an integral, and it's all based on what? Numerical sets!*

These reflections are immersed in the process of internal didactic transposition, since it is at this stage that we see indications of the professor's relationship with the knowledge to be taught and the knowledge taught. The latter, in turn, is delineated by the professor's decisions at the time of the lesson itself, by observing what a student's response provokes in the professor and, in particular, in the knowledge produced and revealed by the professor in the choices to be made. There are also elements at higher levels that refer to the chain of themes to be explored in the students' school career.

Based on the epistemic factors, we analyzed another situation inherent in the second question, which shows connections between the professor's decisions and didactic transposition. Below is an extract from the lesson transcript, in which the professor introduces the binary system as a subset of natural numbers to the students.

Chart 3: Excerpt from the lesson taught by professor *P*

Q: Cousins... that's another kind of set. Do you know this set here (*write in the chart X = {0,1}*), or not? **A**: binary. **Q**: Ein? (*sic*) Binary, guys, binary. This set here guys, (*pointing to)* it's going to give us the decimal numbering system, the ten's (*sic*)! It will give us the decimal numbering system. But not this one (*pointing to X = {0,1}*); this one will give us the binary numbering system. Then I ask: Is this set here important? (*pointing to)*Significant, and is this one important? (*pointing to X = {0,1}*), is this one important? Is it? What is it important for? **A**: Computer language. **Q**: Computer language, guys, computer language, okay? All the information you work on in computing, what do you work on? Zero and one. It doesn't work with all these numbers here, guys (*pointing to*).

Source: Santos (2019)

This excerpt shows the professor introducing the binary system as an extension of numerical sets. The decision to explore the binary system may have been the result of classroom interaction and shows how the professor adapts the text of knowledge to the needs of the class.

The use of the binary system reflects the specific knowledge in the technical vocational school. We could ask ourselves: was working with the binary system in the teaching of numerical sets a choice made by the professor in all the 1st grade classes of the institution, or was it the result of real interactions during the lesson? In the interview, the professor pointed out that zero and one are the basis of all arithmetic. This didactic choice shows the influence of the professor's relationship with knowledge and the institution in which he works.

This exploration by the professor reveals what knowledge he considers necessary for the student to master elements of mathematical knowledge, reflecting on how the student learns in the face of examples in another numbering system, in this case the binary system. Bessa de Menezes (2010) emphasizes that the changes professors make to the knowledge to be taught are closely related to the relationship they have with that knowledge, which influences the teaching strategies they use.

The professor's decisions are articulated between levels N+1, N0, and N-1, showing how decisions can be planned or arise in response to students' reactions. The decisions made at level N+1 reflect what Ravel (2003) calls *prepared knowledge*, i.e., the knowledge that the professor plans to teach, often presented in the syllabus (course project). This knowledge, which may or may not be formally written down, guides the teaching of a specific mathematical object and is based on the predictability of what will be taught to the students. This knowledge is the result of didactic and mathematical choices, and therefore presents itself in its own way for each professor, since expectations can be different for each of them, given the particularities of the classes of students and the relationship with the knowledge to be taught.

Thus, a professor who prepares his or her course relies on the official syllabus, the available textbooks, or his or her own mathematical knowledge of the subject. However, their decisions are not limited to these elements; they also include choices about the text of knowledge, considering how it will be interpreted in the classroom, where factors such as time constraints, organization, interaction with professors, and the didactic knowledge accumulated throughout the professor's professional career come into play.

According to Ravel (2003, p. 7), this series of decisions contributes to the construction of the teaching project. It is important to note that the knowledge prepared is different from the knowledge to be taught, highlighting the dynamics of the process of internal didactic transposition, in which knowledge is adapted and transformed to suit the pedagogical and didactic context and the needs of the students.

5.3 Factors of the didactic story type

The factors of the didactic history type refer to the professor's sharing of experiences with students about a piece of knowledge in the didactic context. Teaching practice involves events that mark professors' memories, creating meaningful memories of teaching moments. Below is a clip from the transcript of a lesson in which we identified evidence of this factor.

Chart 4: Excerpt from professor *P*'s lesson

P: I'm going to do another operation, guys, look! I'm going to put (*sic*) a little one here, okay (*sic*)? I'm going to put (*sic*) itty-bitty! Two to the third. Two to the third is how much, guys? Six, isn't it? (*write 6*). Isn't that six? So, guys, I've done the fifth count, is that right (*sic*)?
Students: That's not right! No, it's not!
P: Is (*sic*) that right, guys? So, let's wake up! This mistake is very common... it's not two times three! What do you mean, "two times three"? It's two times two, times two (*write 2.2.2 =*)*on the board,* okay (*sic*)? That's our eight (writes on board 8). So, guys, this error here, even in higher education, is quite common (*erases the 6 from the board and writes 8*), ok (*sic*)? Let's be careful when doing our math! So look, guys, we've done another operation here, which is potentiating. But we can also come here and do another operation, which is radication, oh (*sic*)? (*writes on the blackboard = 3*) And what does the root of nine give? Three! Because three squared gives nine.

Source: Santos (2019)

The above episode illustrates the professor's action in dealing with a common error in the calculation of potentiating by anticipating it to draw the students' attention to it. This decision reflects his previous experience with common student errors.

According to Bessot and Bittar (2019), didactic historical factors are related to the professor's memory and interactions with different students and classes. The professor's decision to anticipate the error reveals an accumulated experience throughout his career with the potentiating algorithm.

In an interview, the professor pointed out that when he taught first-through-third-year undergraduates, he observed a tremendous need for the basics. In the lesson excerpt, he mentions that the error recurs in higher education as well, suggesting familiarity with the issue of potentiating and the difficulties students have in learning this content.

According to Bessot and Bittar (2019), this experience highlights the influence of didactic history factors. In the case analyzed, we also noticed an interface between epistemic factors and didactic history. The decision to anticipate the error reflects both the professor's knowledge of teaching and his concern to manage the lesson so that students understand the mathematical knowledge involved.

For learning to take place, it's not enough to warn about mistakes; the professor has to consider the knowledge the students already have and the factors that lead them to make repeated mistakes, as the teacher himself points out. This scenario illustrates the topaz effect described by Brousseau (1998), in which the professor provides clues, but the students don't understand the mathematical object. In the example, the professor makes an error in calculating two cubes and uses questions and gestures to encourage the students to see the error.

The professor's decisions are based on his or her mathematical and didactic knowledge, which adapts to the modifications required by the didactic transposition, especially when recurring errors occur. Teaching practice is then shaped by the students' responses and the identification of difficulties during the lesson, which occurs at level 0, according to immediate decisions and planning.

These decisions are influenced by changes in the knowledge to be taught, and the knowledge planned, as Ravel (2003) points out. The professor creates mechanisms to facilitate the didactic functioning of the lesson, motivated by the need to teach content to the professors, freeing himself from the textbook. This confirms Almouloud's studies (2011, p. 195) when he states that "[...] the professor's work obviously presupposes knowledge of the object of knowledge, but also of the way in which students construct their knowledge."

Chevallard (2002) adds that the subject's personal relationship with objects or institutions manifests itself in the performance of tasks and techniques, as in the case of the potentiating operation analyzed.

6 Final considerations

This study explores the connections between didactic decisions and didactic transposition, based on an analysis of a mathematics professor's practice when dealing with numerical sets. Crucial theoretical elements were revealed that deserve attention in future research in the field of Mathematics Didactics.

The results indicate that the professor's choices reflect a convergence between didactic decisions and didactic transposition, especially considering the impact of these choices on the students' knowledge. The professor's decisions do not occur in isolation; they are interdependent and complementary, confirming the interrelationship between the levels of the professor's activity.

However, when analyzing the decision factors, we observed that there is not always a continuous interaction between all the levels proposed by Margolinas (2004). Instead, this interaction tends to occur in specific blocks, such as levels N+1, N0 and N-1, as evidenced in the analyses of this study.

Furthermore, regarding the decision factors, we identified that, although other studies do not indicate perceptible overlaps between them, the professor's knowledge of the mathematical object and the domain to which it belongs is fundamental. This is reflected in his experiences with students, particularly in how he deals with their difficulties.

In this context, the *time of knowledge* plays an important role in discussions of the relationship between professors' decisions and didactic transposition. As highlighted by Câmara dos Santos (1997), the *professors' time* and the *time of knowledge* directly influence teaching practice. The time dedicated to each piece of content reflects the professor's relationship with knowledge and how he or she manages this relationship during teaching, impacting didactic transposition and student learning.

Student errors and difficulties thus emerge as critical elements for understanding the didactic history factor in teaching decisions, especially considering internal didactic transposition.

The interrelationship between didactic decisions and didactic transposition suggests the need for a theoretical scheme that integrates these connections, offering a more holistic view of teaching practice. This scheme can explore the dimensions of external and internal transposition by organizing the professor's activity levels into blocks. The N+1 levels, a fine line that connects the external to the internal, being a critical point at which professors' knowledge undergoes the greatest transformations. The concept of prepared knowledge, described by Ravel (2003), suggests that the professor adapts the prescribed knowledge according to the specific needs of the classroom, adjusting it to better meet the demands of the students and the educational context. In future research, it would be interesting to develop models that capture this dynamic.

Acknowledgements

This study was supported by the Instituto Federal de Educação, Ciência e Tecnologia (IFPB), which made it possible through the granting of leave for training, formalized by Ordinance 1085/2024 of the Rectory/IFPB, in force since July 15, 2024.

References

Almouloud, S. A. (2011). As transformações do saber científico ao saber ensinado: O caso do logaritmo. *Educar em Revista, n. Especial 1*, 191-210.

Bessa de Menezes, M. (2010). *Praxeologia do professor e do aluno: uma análise de diferenças no ensino de equações do segundo grau* (Tese de Doutorado). Universidade Federal de Pernambuco, Recife, PE.

Bessot, A. & Bittar, M. (2019). Análise de decisões de professor. *Educação Matemática Pesquisa Revista do Programa de Estudos Pós-Graduados em Educação Matemática, 21*(5), 21-29.

Brasil. Ministério da Educação. Secretaria de Educação Básica. (2018). *Base Nacional Comum Curricular: Ensino médio*. Brasília, DF.

Brito Menezes, A. P. de A. (2006). *Contrato didático e transposição didática: inter-relações entre fenômenos didáticos na iniciação à álgebra na 6ª série do ensino fundamental* (Tese de Doutorado). Universidade Federal de Pernambuco, Recife, PE.

Brousseau, G. (1986). La relation didactique: Le milieu. In *Actes de la IVème Ecole d'Eté de Didactique des Mathématiques* (pp. 54-68). Irem Paris 7.

Brousseau, G. (1998). *Théorie des situations didactiques*. Editions La pensée sauvage.

Câmara dos Santos, M. (1997). O professor e o tempo. *Tópicos Educacionais, 15*(1-2), 105-116.

Chevallard, Y. (2002). Organiser l'étude. 3. Écologie & régulation. In *Actes de la XIe École d'été de didactique des mathématiques — Revue Recherches en Didactique des Mathématiques* (pp. 41-56). Grenoble, Fr.

Chevallard, Y. & Joshua, M. A. (1991). *La transposition didactique. Du savoir savant au savoir enseigné*. Editions La pensée sauvage.

Coulange, L. (2012). *L'ordinaire dans l'enseignement des mathématiques. Les pratiques enseignantes et leurs effets sur les apprentissages des élèves* (Habilitation à Diriger des Recherches). Université Paris-Diderot - Paris VII, Paris, France.

Espindola, E. B. D. M., Silva, R. D. M. D. & Júnior, J. J. R. T. de B. (2020). Microdecisões didáticas em uma aula sobre volume de sólidos geométricos. *Educação Matemática em Revista - RS, 2*(21), 37-45.

Espindola, E. & Trgalová, J. (2015). Trabalho documental e decisões didáticas do professor de matemática: um estudo de caso. *Em Teia | Revista de Educação Matemática e Tecnológica Iberoamericana, 6*(3), 1-25.

Lima, I. M. S. (2011). Conhecimentos e concepções de professores de matemática: Análise de sequências didáticas. *Educação Matemática Pesquisa Revista do Programa de Estudos Pós-Graduados em Educação Matemática, 13*(2), 359-385.

Margolinas, C. (1993). *De l'importance du vrai et du faux dans la classe de mathématiques*. Editions La pensée sauvage.

Margolinas, C. (2002). Situations, milieux, connaissances: Analyse de l'activité du professeur. In *Actes de la 11ème Ecole d'Eté de Didactique des Mathématiques* (pp. 141-156). Editions La pensée sauvage.

Margolinas, C. (2004). *Points de vue de l'élève et du professeur. Essai de développement de la théorie des situations didactiques* (Habilitation à Diriger des Recherches). Université de Provence - Aix-Marseille I.

Margolinas, C., Coulange, L. & Bessot, A. (2005). What Can the Teacher Learn in the Classroom? *Educational Studies in Mathematics, 59*(1/3), 205-234.

Ravel, L. (2003). *Des programmes à la classe: Etude de la transposition didactique interne. Exemple de l'arithmétique en Terminale S spécialité mathématique* (Tese de Doutorado). Université Joseph-Fourier - Grenoble I.

Santos, W. F. (2019). *A transposição didática interna no ensino do conjunto dos números naturais para surdos: um estudo numa sala de aula inclusiva* (Dissertação de Mestrado). Universidade Estadual da Paraíba, Campina Grande, PB.

Souza, J. V. D. (2017). *A percepção de alunos do 1º ano do ensino médio sobre a importância e aplicações de conteúdos matemáticos relacionando a aprendizagem dos conjuntos numéricos* (Dissertação de Mestrado). Fundação Universidade Federal do Amapá, Macapá, AP.

Stake, R. E. (2011). *Pesquisa Qualitativa: Estudando como as coisas funcionam*. Penso.