

Knowledge of Mathematics and its teaching expressed by teachers when discussing external assessment

José Carlos Rodrigues Junior

Secretaria de Estado de Educação de Minas Gerais
Pirapora, MG — Brasil

✉ josecarlosfacic@gmail.com

ORCID [0000-0002-2256-2851](https://orcid.org/0000-0002-2256-2851)


Gilberto Januario


Universidade Federal de Ouro Preto
Universidade Estadual de Montes Claros
Ouro Preto, MG — Brasil

✉ gilberto.januario@unimontes.br

ORCID [0000-0003-0024-2096](https://orcid.org/0000-0003-0024-2096)



2238-0345 

10.37001/ripem.v15i1.4366 

Received • 02/10/2024

Approved • 20/11/2024

Published • 01/01/2025

Editor • Gilberto Januario 

Abstract: The aim of the study was to *discuss the knowledge that teachers express about the curriculum and external assessment of Mathematics, as expressed in the set of Proeb (Program for the Evaluation of the Public Basic Education Network) documents*. This is a case study, in which the analysis was based on the statements made by teachers when they took part in continuing training, the emphasis of which was on the curriculum and the relationship with the results of the external Mathematics assessments in the 9th year of Proeb. The theoretical framework refers to knowledge for teaching, considering *knowledge of the content, knowledge of the approach to the content and knowledge of planning*. The results show that the teachers correctly associate the descriptors assessed with the prescribed skills, but due to the demand for activities and the lack of time, they express difficulties in reading and interpreting the results of the assessments. They also reported students' difficulty in memorizing formulas and concepts, justifying the low assertiveness of certain items because they were unable to understand the teaching strategies and concepts involved, including difficulty in interpreting the statements.

Keywords: Curriculum. Mathematical Knowledge. Proeb. External Assessments.

Conocimientos sobre Matemáticas y su enseñanza expresados por los docentes al discutir la evaluación externa

Resumen: El estudio tuvo como objetivo *discutir los conocimientos que los docentes expresan sobre el currículo y la evaluación externa de Matemáticas, expresados en el conjunto de documentos del Proeb*. Se trata de un estudio de caso, en el cual el análisis se realizó a partir de las enunciaciones de docentes al participar en la educación continua, cuyo énfasis fue el currículo y la relación con los resultados de las evaluaciones externas de Matemática en el 9no del Proeb. El marco teórico se refiere a conocimientos para la enseñanza, considerando *conocimientos de contenidos, conocimientos sobre el enfoque de contenidos y conocimientos sobre planificación*. Los resultados muestran que los docentes asocian correctamente los descriptores evaluados con las habilidades prescritas, pero debido a la exigencia de actividades y la falta de tiempo expresan dificultades para leer e interpretar los resultados de las evaluaciones. También relataron dificultad de los estudiantes en la memorización de fórmulas y conceptos, justificando la baja asertividad de ciertos ítems, ya que no lograron comprender las estrategias de enseñanza y los conceptos involucrados, incluida la dificultad en la interpretación de enunciados.

Palabras clave: Curriculum. Conocimiento Matemático. Proeb. Evaluación Externa.

Conhecimentos da Matemática e seu ensino manifestados por professores ao discutir avaliação externa

Resumo: O estudo objetivou *discutir o conhecimento que os professores manifestam sobre currículo e avaliação externa de Matemática, expresso no conjunto de documentos do Proeb*. Trata-se de um estudo de caso, em que a análise foi realizada a partir das enunciações de professores ao participarem de uma formação continuada, cuja ênfase foi o currículo e a relação com os resultados das avaliações externas de Matemática no 9º do Proeb. O referencial teórico reporta-se aos conhecimentos para o ensino, considerando o *conhecimento do conteúdo, conhecimento sobre a abordagem do conteúdo e conhecimento sobre planejamento*. Os resultados mostram que os professores associam corretamente os descritores avaliados com as habilidades prescritas, mas em função da demanda de atividades e a falta de tempo, expressam possuir dificuldades com a leitura e interpretação dos resultados das avaliações. Também relataram a dificuldade dos estudantes em memorizar fórmulas e conceitos, justificando a baixa assertividade de determinados itens, por não conseguirem compreender as estratégias de ensino e os conceitos envolvidos, incluindo a dificuldade na interpretação dos enunciados.

Palavras-chave: Currículo. Conhecimento Matemático. Proeb. Avaliação Externa.

1 Contextualizing the study

In Brazil, external evaluations became more relevant in the 1990s. One of the factors that contributed to their conception and implementation was the federal government's initiative to create the Sistema de Avaliação da Educação Básica (Basic Education Assessment System — SAEB) (Machado, 2012). Since then, other external assessment systems have begun to emerge, such as the *Sistema Mineiro de Avaliação e Equidade da Educação Pública* (Minas Gerais Public Education Assessment and Equity System — Simave), which incorporates the *Programa de Avaliação da Rede Pública da Educação Básica* (Public Basic Education Network Assessment Program — Proeb), established by Resolution 14 of February 3, 2000 (Minas Gerais, 2000a), and reissued by Resolution 104 of July 2000 (Minas Gerais, 2000b). The Literacy Assessment Program — Proalfa, was incorporated with its creation in 2006 (Minas Gerais, 2012a). We recognize Proeb and Proalfa as external evaluations because we understand that they are prepared by professionals specialized in this field of consultancy (Werle, 2010), as people external to the environment where these evaluations are implemented (Machado, 2012) and are also considered large-scale due to their scope and level (Werle, 2010).

Proeb assesses students in the 5th grad of Elementary School, 9th grade of Middle School and the 3th grade of High School. In this paper, the focus is on the editions of the Mathematics assessment, its results and reports, referring to the 9th grade, which include curricular prescriptions for the entire stage of the Middle School.

The Simave¹ platform provides a collection of materials that can help teachers, from reflecting on the results to possible implications for teaching practices. Among the materials available are magazines aimed at managers, teachers and the community as a whole, with information on the assessments that have taken place, including detailed results. Another document in the collection is the reference matrix, considered to be a cross-section of the curriculum (Minas Gerais, 2021a), which incorporates descriptors organized by themes or topics, serving as a reference for the preparation of the items that make up the assessment tests (Minas Gerais, 2022b).

We believe that all of these materials can help teachers expand what they know and

¹ Simave platform: <https://avaliacaoemonitoramentosimave.caeddigital.net/#!/pagina-inicial>

build knowledge. For example, the pedagogical magazines, aimed at math teachers, present items that made up the tests, accompanied by possible answers from the students, with discussions that seek to justify what motivated the students to mark the correct answer or the distractors. Such approaches in these documents can broaden teachers' knowledge of how students reason, solve problems and form concepts in mathematics.

The results per descriptor, based on the percentage of correct answers, can reveal to teachers, in relation to the students, trends in the development of skills and in the learning of content implicit in them. This has implications not only for teachers' knowledge and their assumptions about student learning, but also for the development of the mathematics curriculum in terms of planning and carrying out lessons, a practice that requires reading and interpreting teaching guidelines in curriculum materials and evaluating, selecting and implementing tasks. In this way, it is important for mathematics teachers to be familiar with the collection of documents that make up the results of the assessments, to attribute meaning and significance to what the results present in terms of justifications that can imply teaching practices, as well as implying professional teaching knowledge about mathematics and its teaching based on large-scale external assessments.

With this in mind, Costa (2019) argues that institutions need to take proper ownership of the results and develop meaningful pedagogical work, i.e. that the results of external assessments provide teachers with guidelines for planning pedagogical practice (Souza & Bonamino, 2012). By analysing the materials made available by Simave and relating them to the results and considerations presented by Proeb, with implications for promoting and adapting lesson plans and teaching strategies, teachers can re-signify what they know and build knowledge of the content, the curriculum proposals and the way students organize mathematical thinking. Thus, the participation of school management in fostering an understanding of the results, with the possibility of organizing continuing training in order to detail them, as well as the fact that teaching staff know and understand enough to guide teachers in planning and implementing lessons, makes it easier for teachers to explore the results and take them into account when planning their teaching practices.

In this sense, the study reported in this paper² set out to *discuss the knowledge that teachers express about the curriculum and external assessment of mathematics, as expressed in the set of Proeb documents*. The study is part of a larger research project carried out by the Curriculum in Mathematics Education Research Group (GPCEEM).

2 Knowledge of the curriculum and external assessment

Knowledge of the curriculum enables teachers to create the conditions for students to construct their learning. With this in mind, Shulman (1986, 1987) presented categories of basic teaching knowledge, referring to the knowledge that teachers need to mobilize or build in order to plan and develop teaching processes. Among these categories, *curriculum knowledge* refers to knowledge of curriculum materials and teaching programs — including content, its organization, selection and approach; objectives; methodological strategies; and assessment. Subsequently, Shulman (1987) incorporated two other categories, entitled *content knowledge* and *pedagogical knowledge*, associating them with *knowledge of the curriculum*, considering them to be necessary knowledge for teachers.

Based on the categories of *content knowledge* and *pedagogical content knowledge* (Shulman, 1986), and considering the knowledge inherent to teachers who teach Mathematics,

² This paper is part of the master's thesis developed in the Postgraduate Program in Education at the State University of Montes Claros, organized in multipaper format, written by the first author and supervised by the second.

Ball and collaborators (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008) developed the *Mathematical Knowledge for Teaching* (MKT) model, organized into six domains, one of which is *Content and Curriculum Knowledge* (CCK).

Based on Ball, Hill and Bass' (2005) and Ball, Thames and Phelps' (2008) characterization of this dimension, we understand that knowledge of the Mathematics curriculum comprises the principles and objectives that operate the organization and selection of content, as well as the approach and methodological strategies relating to concepts, procedures and attitudes. It also includes knowledge about reading and interpreting teaching guidelines in curriculum documents and materials, as well as evaluating and selecting tasks when planning and implementing teaching practices.

In his studies on professional teaching knowledge, Ponte (2012) presents *knowledge of the curriculum*, referring to knowledge of the contents of mathematics and the integrated relationship of these contents with those of other subjects, observing how they are worked on, taking into account the objectives, organization and layout in the teaching process, paying attention to those that demand more time and attention. For the author, *knowledge of the curriculum* includes the need for teachers to know the profile of their students, how they learn, what their main difficulties are and in what social and cultural context they are involved.

By considering some models of knowledge that teachers need to construct or mobilize in order to develop the curriculum — including Shulman (1986, 1987) and Ball, Thames and Phelps (2008) — Remillard and Kim (2017) propose the *Knowledge of Curriculum Embedded Mathematics* (KCEM) model, which focuses on teachers' knowledge as it relates to curriculum materials.

In their studies, Remillard and Kim (2017) mention other authors who address professional knowledge in Mathematics, such as Collopy (2003), referring to the importance of teachers' knowledge to interpret and put into practice curriculum resources; Ebby and Sirinides (2015), on teachers' knowledge when engaging in the formative assessment process, including the interpretation of students' responses to mathematics tasks; Baumert et al. (2010), in relation to their research using statistical methods, revealing that the correlation indices between *pedagogical content knowledge* and *content knowledge* increase as a function of teachers' experience, and when compared, *pedagogical content knowledge* has greater predictive power in terms of students' progress and learning and, furthermore, *content knowledge* can facilitate the adjustment of the material worked on with the proposed curriculum. Thus, Baumert et al. (2010) encourage teachers, whenever possible, to expand their knowledge, for example, through continuing education courses.

Similarly, Davis and Krajcik (2005) also present contributions in the field of curriculum knowledge in the light of Shulman's categories (1986, 1987), as Fuentes and Ma (2018) also discuss, emphasizing the relevance of educational curriculum materials to teachers' knowledge, in the sense that these materials support both teachers and students in their learning.

When we consider what the aforementioned authors present as the *knowledge of the curriculum* necessary for teachers to teach mathematics, we understand that this knowledge involves the ability to know and relate the contents, as well as to evaluate their approach in the curriculum materials, including the ability to readjust the proposed activities according to the cognitive demands of their students. This includes knowledge of student learning. We understand that *knowledge of the curriculum* is manifested in the relationship that teachers establish with curriculum guidance documents, curriculum materials and other resources that support teaching and learning processes. This knowledge involves the students' knowledge of the teaching strategies to be used with them, including the use of digital technologies and the

adoption of assessment instruments in line with the teaching objectives.

Incorporated into knowledge of the curriculum is the teacher's competence in getting to know their students in order to anticipate their main difficulties; the procedures used that can lead them into error; what can enhance the development of different reasoning and mathematical thinking; and to identify and problematize their limitations and issues relating to subjectivity, racism, gender, social and cultural contexts, among many other issues that are urgently needed to be considered by the curriculum.

Therefore, knowledge of the curriculum also includes other knowledge needed to observe, implement and monitor student assessment processes. In the study presented here, we have included external assessments as the basis of professional teaching knowledge in mathematics.

Knowledge of external assessments, as a dimension of curriculum knowledge, whether at municipal, state or national level, involves what teachers need to know in order to read and interpret their processes and results, in partnership with the teaching team and support from school management. In this understanding, knowledge of external assessment involves teachers' ability to know what proficiency is, not in the sense of knowing how to calculate it, but understanding that there is proficiency per student and that this result has its importance; knowing that there is average proficiency for the class, the school and even regionally and statewide; that based on proficiency, students are categorized in terms of possible learning; that the test taken by the students is made up of items, and that these were drawn up based on a reference matrix made available by the Department of Education or the competent body. We think that this knowledge can also be improved or built up in training sessions, which could take place during collective meetings.

In this context, it is also part of the knowledge of external evaluation to know that there are magazines and/or reports available, with information that addresses the social context; to know which prescribed contents were flagged with low, medium or high assertiveness; to know how to analyze and interpret the pedagogical considerations presented by the evaluation system, including understanding the proposals of external evaluation from the perspective of learning. In addition to all this knowledge, a suggestion would be for the teacher to look back at the teaching strategies already used, observing the results analyzed, in order to understand what was successful or unsuccessful, assessing whether there is a need to develop or use other approaches and strategies.

Considering what we have discussed, when planning mathematics teaching practices — an activity that involves reading and interpreting teaching guidelines in documents and curricular materials; evaluating, selecting and implementing tasks; choosing teaching approaches and strategies — it would be appropriate for teachers to build and mobilize knowledge related to large-scale external assessments in order to create opportunities for students to construct meaningful and significant learning, an aspect that helps achieve teaching objectives. Thus, knowing and discussing what teachers know about external assessment or what the set of documents can provide in terms of teaching knowledge, seems to be relevant not only for research in Mathematics Education, but also for thinking about and implementing training processes for teachers.

3 Methodological Design

Taking Larry's (2022) ideas as a starting point, research is the creative use of existing knowledge, based on organized and systematized work, the collection of data and important documents, as well as their analysis and interpretation, in order to produce new information,

conceptions, concepts and understandings. With this in mind, research carried out in the field of Mathematics Education has produced knowledge that helps to explain, understand and improve perceptions and practices in the teaching and learning of Mathematics.

As we are guided by the knowledge produced in the teaching experience and based on the ideas of Larry (2022), the repertoire of the study presented here is supported by theoretical and methodological references, conducted under the focus of the qualitative approach, as it considers that there is an involvement of people when they express their ideas, conceptions, practices and meanings in the perception of a phenomenon from a certain context (Gamboa, 2018). Since the study aims to *discuss the knowledge that teachers express about the curriculum and external assessment of Mathematics, expressed in the set of Proeb documents*, we identified that the case study is the method that best fits the proposal, as it “provides the in-depth knowledge of a delimited reality that the results achieved can allow and formulate hypotheses for the direction of other research” (Triviños, 1987, p. 111).

The *case study* was the most appropriate because it allowed us to observe and shed light on the discussions about the teachers' knowledge of mathematics as it relates to the Proeb external assessment, specifically in the 9th grade of elementary school. In order to make the observations possible, we considered a group of teachers who teach mathematics. Of these, all have a degree in Mathematics; 15 are female; 12 have permanent positions and 1 is studying for a Master's degree in Education. They all work in state schools linked to the Pirapora Regional Teaching Superintendency.

Due to the different dates, shifts and working hours of the teachers, the invitation to the state schools was made in person through the managers (Director or Vice-Director); at the municipal school, the invitation was made via e-mail to a civil servant who works at the Municipal Education Department. Along with the invitation, an organization was presented, stipulating that there would be five weekly remote meetings, via Google Meet, on Thursdays, from 7pm to 9pm, during the period April 11 to May 9, 2024, with all meetings being recorded.

The night shift was chosen because most of the teachers work during the day; the remote format was chosen to enable teachers who lived outside the municipality of Pirapora to take part. Each teacher who accepted the invitation was given a Free and Informed Consent Form, informing them of the title of the research, as well as the objectives, risks, procedures, benefits and other essential information that provided clarification and safety for the participants.

At the first meeting, the results of the assertiveness of the 9th grade Mathematics descriptors from the Proeb assessment, which took place in the 2014, 2016, 2018, 2019, 2021 and 2022 editions, were presented, revealing their average percentage of correct answers per school and for the municipality of Pirapora, considering the data from the schools in the Pirapora Regional Superintendency of Education and the state of Minas Gerais, which were made available by the Minas Gerais Undersecretariat for the Development of Basic Education via the *Sistema Eletrônico do Serviço de Informação ao Cidadão* (Electronic System for Citizen Information Service — e-SIC), as shown by the service protocol 01260003025202361. Data was presented for each edition, without revealing the names of the schools; the average percentage of correct answers for each descriptor, for the schools, for the Pirapora Regional School District and for the state of Minas Gerais; the lowest, median and highest percentages were observed; and what these results might imply for everyday classroom life, considering a history of six editions of the assessment, covering an interval of approximately nine years.

During the meetings, the percentages were shown in the form of tables and graphs, with visual formatting designed to give teachers a better understanding of the results when looking at the data. On the surface in the first meeting, but intensively in the others, the knowledge

needed and expressed by teachers and students to solve the items that made up the tests in previous editions was discussed. The items discussed were taken from the magazines available on the Simave portal, which are aimed at teachers who teach mathematics and contain information for teachers to use when planning and developing teaching strategies.

In the five training meetings, 19 teachers agreed to take part, and for personal reasons, not all of them were able to attend all the meetings. In the first and fourth meetings, 17 teachers attended; in the second and fifth meetings, 15 teachers took part; and in the third meeting, 19 teachers.

The recording of each meeting was transcribed, followed by the textualization of the teachers' utterances. Afterwards, we selected excerpts referring to the participants' knowledge of mathematics and its teaching, incorporated into the external assessments, which we will present and analyse, organized into three categories: *knowledge of the content*, *knowledge of the approach to the content* and *knowledge of planning*.

4 Analysis

For the discussions with the teachers³ on the results of the percentage of correct answers per descriptor in the Proeb external assessments, associations were made with the corresponding skills and examples of items presented and discussed in the pedagogical magazines addressed to math teachers. Thus, using formative guidelines made up of open questions, it was possible to develop discussions from which evidence emerged of the knowledge of mathematics and its teaching that is necessary for students to be able to construct their learning, as well as answer the items in the assessments.

Not all the teachers who took part in the meetings spoke or wrote in the chat. For this reason, the excerpts from the statements in the following sections are from those teachers who participated and were involved in the discussions.

4.1 Knowledge of the content

This section presents excerpts from the utterances and records of the participants as they discuss examples of items taken from a set of pedagogical magazines addressed to mathematics teachers and made available on the Simave platform. The statements show a set of knowledge of mathematics content expressed by the teachers. The criterion for choosing the items for discussion was based on the relationship with descriptors that had the lowest average percentage of correct answers in the 2016 to 2022 editions of the assessment, including those with average answers, since it was not possible to have a broad discussion on all the descriptors due to the time available for teachers to participate. Discussions were also held on the descriptors that have had the highest percentage of correct answers, for example, those related to the topic of *Information Processing*.

During the training, each item discussed was associated with the descriptors in the reference matrix, as mentioned by teacher Sophie-Marie and the other participants when they commented on “Descriptor D4”, “Descriptor D7”, “Descriptor D43”, “Descriptor D61”, “Descriptor D29” and “Descriptor D4”⁴. These expressions portray the teachers' perceptions

³ The real names of the professors are not disclosed. In the case of the women, we have chosen to use the pseudonyms Hypatia (of Alexandria), Katherine (Johnson), Maria Laura, Maryam (Mirzakhani), Sophie-Marie, Emmy Noether, Mary Ellen Rudin and Jo Boaler; and in the case of the men, we have chosen to use Galileo, Isaac and Leonhard Euler. These names were chosen in analogy to the people who stood out for their contributions to mathematics.

⁴ In relation to the assessment items, the examples of descriptors discussed were: D04 — Classify triangles by means of their properties; D07 — Recognize angles as a change of direction or turn, identifying right angles and non-right angles; D29 — Use the calculation of the area of two-dimensional figures in problem solving; D43 — Recognize fractions as a representation

based on their knowledge of the reference matrix and the item being assessed, as well as the content incorporated into them.

More specific to content knowledge, teacher Hipátia and the other teachers related the descriptors of the Reference Matrix (Minas Gerais, 2022a) to the skills prescribed in the Minas Gerais Reference Curriculum — CRMG (Minas Gerais, 2019a), by commenting, for example, on “the EF06MA19 skill”, “the EF08MA09A skill”, “the EF06MA55MG skill” and “the EF08MA31MG skill”. These expressions, taken from a larger statement, show knowledge of the content that teachers express when they establish the relationship between an item in the assessment and the descriptor in the 9th grade reference matrix and then with the skill prescribed in the CRMG.

This knowledge, in the sense of identifying the content, is relevant because it allows teachers, when analyzing the results of the assessments, to identify the themes and concepts that need more attention when planning and developing teaching strategies, since “teachers have special responsibilities in relation to content knowledge, since it serves as the primary source of understanding of the specific subjects they teach” (Shulman, 1987, p. 208). In this way, we understand the complexity demanded of teachers when analyzing the results of external assessments, and the analysis of these results goes beyond looking at proficiency rates or percentages of students by performance standard.

Rather than identifying descriptors and skills, in the training meetings, the teachers expressed what types of knowledge were desirable for students to be able to solve certain items. For example, in order to *calculate the numerical value of an algebraic expression*⁵, from an item projected on the screen, teacher Mary Ellen reports that students must know the “*basic operations such as adding, subtracting and multiplying*” (Figure 1).

In fact, the students have no way of solving an expression without using the fundamental operations. However, in this same item, teacher Maryam adds that “*students have to know how to perform operations with whole numbers*”. In these discussions, we see that, for the teachers, students need to understand more than the fundamental operations; they need to mobilize knowledge relating to operations with integers and the corresponding properties.

Figure 1: Example item

(M090272G5) Look at the algebraic expression in the table below.

$$x^2 + 2 \cdot x \cdot y + 2 \cdot x \cdot z + z^2$$

The value of this expression for $x = 1$, $y = -2$ e $z = -3$ is

A) - 18
B) - 14
C) 0
D) 20

Source: School Magazine: Mathematics [Simave] (Minas Gerais, 2021c, p. 31)

Even so, teacher Maria Laura comments that “*a difficulty in addition to working with*

associated with different meanings; and D61 — Use a system of 1st degree polynomial equations with two unknowns in problem solving. The examples of CRMG skills involved in the discussion were: EF06MA19 — Identify characteristics of triangles and classify them in relation to the measures of sides and angles; EF08MA09A - Solve, with and without the use of technology, problems that can be represented by 2nd degree polynomial equations of the $ax^2 = b$ type; EF06MA55MG — Identify angle as a change in direction; and EF08MA31MG — Solve a system of first degree equations.

⁵ Descriptor D62, identified with the average percentage of correct answers of schools in Pirapora (MG) below 41%, in the 2018, 2019, 2021 and 2022 editions of the assessment. Schools that had at least 80% of their students taking part in the assessments were considered.

integers is the order of operations”, referring to students' difficulties when solving this type of item. Maria Laura points out that it is not enough to know how to perform the fundamental operations and master the sign rules. For this teacher, students need to understand these processes, including their order. This shows that teachers are aware of the essential mathematical knowledge that students need to mobilize. It also shows how important it is for teachers to be able to explain what justifies the student following a certain order of operations, which will involve interventions and clarifying doubts in the class. It is therefore important for teachers to understand not only how processes happen, but why they happen and what the meaning is (Shulman, 1986, 1987; Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008; Lima, Bianchini & Gomes, 2018).

Broadening the discussions in the context of students' difficulties in relation to mathematics content, the way teachers report the division operation stands out. In the assessment items that were discussed, the teachers expressed situations in which the student needs to know division in order to complete the resolution. In this circumstance, teacher Mary Ellen commented: *“Today our students are arriving in sixth grade, seventh grade and even high school without knowing how to solve the four basic operations. They don't know the multiplication table”*, stating that not knowing how to divide is a reality for students in basic education.

In the discussion, other teachers agreed, as in the case of Sophie-Marie when she commented: *“In the diagnostic assessments I applied, the division operations in which the divisor was a digit, most of the students left blank, or wrote ‘I don't know’, ‘I've never seen’. They had no idea about this operation, most of them left it blank”*. In the discussions, it was not discussed whether the teachers noticed any patterns in the procedures that have led students to make mistakes or not know how to perform the division operation. However, as this is knowledge that is necessary to carry out recurring tasks, in this case we can rely on Ball and colleagues (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008) when they identified in their studies that this would be common knowledge of the content, leaving it up to the teachers to resort to different strategies for teaching this operation, in an attempt to mitigate this gap.

Thus, when asked about *the necessary knowledge that math teachers need to know in order to help students solve an item*, teacher Mary Ellen mentions “*content mastery*”, which, according to Shulman's (1986) studies, we can interpret as content knowledge. In the same vein, teacher Katherine commented: *“as a math teacher, I have to have knowledge of mathematics, basic knowledge at least, and in-depth knowledge”*, referring to what Ball and colleagues propose, called specialized content knowledge, i.e. *“teaching requires knowledge beyond that which is taught to students”* (Ball, Thames & Phelps, 2008, p. 400).

In this sense, we also reflect the considerations of the National Mathematics Advisory Panel (2008, p. 37) *apud* Baumert *et al.* (2010): “teachers should know in detail and from a more advanced perspective the mathematical content they are responsible for teaching... both before and after the level they have been assigned to teach” (p. 136). This reflects the importance of the training and knowledge of teachers who teach mathematics, because we understand that, in addition to knowledge promoting security in the teaching process, this can be a factor that enables teachers to have other perspectives from the teaching of the division operation to the planning and implementation of practical lessons, revealing the meaning of the applicability of this knowledge in the real world for students and reverberating in better results in learning and, consequently, in the percentages of correctness of the descriptors of the external assessments.

Naturally, knowing the content implies the teaching process and makes the learning

process more meaningful for the student, and the relationship between the content and the themes set out in the syllabus and issues of interest to the students can become more evident. However, the way in which the content is approached is also relevant, as we will analyze in the next section.

4.2 Knowledge of the content approach

The way in which content is approached in teaching practices is also the focus of discussion and reflection at training meetings. By getting to know the students and giving them due importance in terms of their interests, values, tastes, ways of behaving and, above all, ways of learning, Ponte (2012) argues that these attributes are decisive conditions for teacher performance. In this line of thought, the teachers expressed concern about the knowledge needed when approaching the content and carrying out the tasks with the students, referring to the fact that the students had difficulties with *interpretation* and also the fact that they forgot the *formulas* and *rules* needed in the resolution process. Thus, when approaching content, weaknesses in *interpretation* are something that can make it difficult for students to construct their learning, and in this sense Professor Leonhard Euler comments:

I think students have a lot of difficulty interpreting what an unknown is, because when we explain to them about unknowns, about the letters, they say “wow, but ‘x’ wasn’t worth that amount, so now it’s worth another amount”. So the students have this difficulty in associating that the unknown can have different values, depending on the problem situation to be solved. (Enunciation by Prof. Leonhard Euler, 2024).

Based on Leonhard Euler's statement, we reflect that, when approaching the learning of certain content that requires the meaning of the concepts of unknown and variable, this approach can be implemented by thinking of ways that can mitigate students' difficulties when making use of the skill of interpretation.

In this way, when observing students' difficulty in understanding that, at any given moment or in any given situation, variables can take on different values, unlike unknowns, this is a sign for teachers when approaching, for example, the teaching of algebra content: the exercise of interpretation, being able to vary examples and explore implications for misinterpretations.

However, difficulties with interpretation go beyond the context of an unknown, as reported by teacher Katherine: *“I think the students' biggest difficulty with this problem is knowing how to read and understand what is being asked of them”*, referring to the students' difficulty in solving problems using 2nd degree polynomial equations, and she adds: *“the student has to understand the problem, interpret it”*, in cases where the student needs to solve a problem using systems of equations.

This knowledge, discussed by the teachers, emerges from the knowledge built up by living with the students, as also observed by Ball and collaborators (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008) in their studies, for whom it is important to know and listen to the students, predict their difficulties and know how they understand mathematics, as well as their conceptions and misconceptions. In this understanding, Professor Galileo collaborates by commenting on the mobilization of prior knowledge when approaching some content:

I believe it's very important to know the student's prior knowledge, even at university there is this, to know what their prior knowledge is on the subject, so that we can move forward in what is proposed, in the skills, in the descriptors. So that we can teach in a clearer and simpler way, it would be to contextualize, or take it to practical knowledge, take it to a situation that is palpable, that the student can identify in

their life, in their day-to-day life. (Enunciation by Prof. Galileu, 2024).

With this in mind, Professor Galileo stresses the importance of teachers getting to know their students, including the profile of the class and the objectivity of contextualization. In this way, Galileo emphasizes the importance of not forgetting to develop teaching strategies that are related to the students' reality, so that they can interact with something that makes sense to them. This statement also draws our attention to the fact that, even before constructing certain types of learning, it is necessary to identify whether the students have the necessary knowledge, which will promote meaning for what will be constructed, without disregarding a critical process.

Still on the content approach, Sophie-Marie comments that *“students forget what they've been taught and they don't have the habit of studying. When you remind them, they say, I've already learned it, it's easy, but either they don't study or they forget easily. They think everything can be looked up.”* In this context, it is pertinent to consider, in the approach to content, the exploration of knowledge that students may have forgotten, whether they refer to fundamental operations, or reading and interpretation or other prior knowledge, identified by teachers and which may compromise learning and imply low results in external assessments.

In one of the meetings, the teachers were shown excerpts from the analysis considered most important, taken from Souza's dissertation (2015). The study looked at the percentages of correct answers for the descriptors in the Prova Brasil Mathematics assessment for the 9th grade, which took place in 2011. Souza (2015) reports the contents that had low assertiveness, such as, for example, *metric relations in the right triangle, to solve problems; solve problems involving notions of volume, calculate the numerical value of an algebraic expression; greater difficulty in topics such as Space and Shape, Quantities and Measures and Numbers and Operations*. The author reports that in the topic of *Information Processing*, no percentages with low assertiveness were identified. The situation of the results presented by Souza (2015) is similar to what was discussed regarding the results of the schools located in the municipality of Pirapora and those located in Minas Gerais as a whole. When discussing the fact that the students did not, in general, show low assertiveness in the descriptors relating to the topic of *Information Processing*, both in the Prova Brasil assessment in 2011 and in Minas Gerais, on the occasion of the Proeb assessments, teacher Maria Laura made the following comment:

Students have difficulty with everything related to memorization. Memorizing formulas, 2nd degree equations, metric relations in the right triangle, etc. We noticed that they had difficulty with everything related to memorizing formulas and sequences. And why didn't they have this difficulty with information processing? Because it doesn't require a formula, it's more interpretation, it doesn't have that prerequisite that is so demanded. You work out a formula with the students, I get to the 3rd year of high school, they can't remember a Bhaskara formula and ask you to put it on the board. That's why I believe that content that requires formulas, knowledge, memorization, they have this difficulty. (Enunciation by Profa. Maria Laura, 2024).

With regard to Maria Laura's comments, the Minas Gerais Reference Curriculum - CRMG (Minas Gerais, 2021b) recommends that students build their learning of mathematics content on the basis of logical thinking, creativity and intuition. Also according to the document, “researchers [...] state that by learning concepts, students are able to access the knowledge that is compressed in their brains, facilitating its application and/or expanding their knowledge” (Minas Gerais, 2021b, p. 147). The document also makes it clear that once students understand the solving strategies and concepts involved, this is memorized and, when necessary, can be mobilized to solve a problem. This is what the teachers mean when they

associate the low assertiveness of certain test items with the fact that the student has not built up an understanding of problem-solving procedures and formed the underlying concepts, as also commented by teacher Sophie-Marie: *“the students don't bother to memorize the concepts, they can't memorize anything, and when they are asked to memorize, to study, they answer that they search on Google”*. We infer that, in the teacher's statement, there is an understanding that students do not see the need to form concepts, understand expressions (formulas) and solving procedures, since they can use digital resources.

Observing the comments of some teachers when they associate the need to memorize concepts and formulas with the results of items, referring to content that involves the need for various procedures, such as calculating areas, perimeters, knowing how to transform units of measurement, they show what Ball, Thames and Phelps (2008) called specialized content knowledge. For these authors, this knowledge is associated with the need for teachers to create the conditions for students to develop fluency with extended mathematical knowledge, so that they are subsequently able to develop unusual mathematical ideas and procedures. To do this, according to Ball, Thames and Phelps (2008), teachers need to have unpacked mathematical knowledge, since making it possible for students to learn certain characteristics of specific content is part of the act of teaching.

Thus, in order to mitigate students' difficulties in the scenario involving teaching and learning processes, the teacher's common knowledge and specialized knowledge of the content is necessary when designing and developing lesson plans, the focus of analysis in the next section.

4.3 Knowledge of planning

As mentioned, one of the meetings presented the results of the percentage of correct answers for each Proeb math descriptor, from the 2014 to 2022 editions of the assessment. In addition to discussing, problematizing and disseminating the results, one of the intentions was to identify whether the teachers would analyze the data from the assessments, and whether the results would imply the elaboration, revision or adaptation of planning, as evidenced in the statements. After the presentation, the teachers were asked *if they analyze the results of the external assessments every year, considering the results of proficiency and the percentage of correct answers per descriptor*. There were 15 teachers present at this meeting and, of these, teachers Leonhard Euler, Edilene Magalhães and Cristina Nobre replied in the chat that they *“don't analyze”*; teachers Hipátia and Mary Ellen also replied in the chat that they *“do analyze”*. Professor Sophie-Marie commented that:

These descriptors, which the students had low assertiveness in, are because in addition to the student having to learn the content and have the knowledge, as you said, of the numerical value, it's ... of systems, it's not because they haven't seen it, it's just that solving the questions requires real knowledge of the content, it involves operations with numbers with negative and positive signs, and today's students don't want to study, and there are some who mark anything in the test. So, as well as us teachers having to work on the content, encouraging them to get into the habit of doing the activities, doing the exercises in order to retain and learn, we also have to work on their awareness of the responsibility that this test entails. There are students who mark anything on the test, don't even think about what they're marking, and then the results are like that. This shows how unconcerned the students are. (Enunciation by Profa. Sophie-Marie, 2024).

In this case, teacher Sophie-Marie said that as well as creating the conditions for their students to learn, they need to make an extra effort to make them take the assessments (solving the items) seriously, so that they don't mark the alternatives without reading and interpreting,

in other words, randomly. With this in mind, teacher Kelly Chamone commented in the chat: “no, many times we don't analyze because it's just data, because many don't do it for the knowledge, but rather play at random”, referring to the fact that students don't take external assessments seriously. In the same vein, teacher Cristiane also commented in the chat: “exactly Kelly”, agreeing with the students' lack of seriousness. Still on the subject of the teachers' analysis of the results, Emmy Noether commented:

No, partly. Last year I started doing these analyses and as it's a very detailed analysis, very time-consuming, I think it could have come as a more succinct report for us teachers, directed at the school and more aligned. You have to keep looking for the results and this search, with the great demand for activities that exists today, makes it impossible, so I stopped analyzing. (Enunciation by Profa. Emmy Noether, 2024).

Then teacher Emmy Noether added: “It needs to be more visual, the way it was shown in this training is more visual, and being more visual, you can understand it more easily”. That said, the teacher pointed out the need to revise the formatting of the data that is made available by the Minas Gerais State Department of Education to schools, in other words, for each school to have access to its data in a personalized way, because the way it is currently made available is complex. Professor Maria Laura then added:

In fact, we analyze because we are all required to do so. But I agree with Professor Emmy Noether that the way these results are made available could be more succinct, including the way they were presented in this training. The way it was presented here made it easier to identify those descriptors that need to be worked on more. I think that this way, presented in this training, is an easier way to work, because we don't have much time, so it would have to be more objective. (Enunciation by Profa. Maria Laura, 2024).

Of the 15 teachers present, almost four answered that they don't analyze; two answered that they do; and three gave their opinions. The other seven teachers did not comment. We understand that not all teachers are able to analyze the results because they find the current way in which the Department makes the data available difficult or complex and, consequently, their lesson plans and teaching strategies may not be revised or adapted based on what the data has revealed about students' assertiveness when answering the test items. When you ask *teachers if they use the same teaching strategies every year, regardless of whether they analyze the results or not? Is the way you teach the same every year? If not, which ones? Do you take into account the results of external assessments?* Teacher Maria Laura commented:

We try to understand the class. I think the strategy sometimes depends on the class we're working with, depending on their profile, their previous knowledge, observing how far we can develop a certain strategy with the students, evaluating whether it needs to be changed, also taking into account the feedback the students give us on the strategy that was used, and if it didn't work, we make the change. (Enunciation by Profa. Maria Laura, 2024).

The teacher's statement shows how important it is for the teacher to know the profile of the class they are working with, because with the clues they get, they will have elements to organize or reorganize their lesson planning. Following on from Maria Laura's comments, Sophie-Marie adds that “*the same content doesn't work the same way in one classroom as it does in another, it depends on the class*”, pointing out that each class needs careful attention from the teacher when implementing the plans. In these discussions, the teachers' concern also emerged about the importance of retaking certain content, which they considered relevant when drawing up the plans, due to students forgetting it. In this context, Professor Leonhard Euler

commented:

We have to retake all the content constantly. Mathematics, in a way, can't go on without always revisiting the content from previous years. When they get to secondary school, there's content that you're going to explain to students that's from 6th grade, 7th grade, that the student doesn't remember or hasn't seen or has seen for a long time and forgotten. That's why we have to constantly revisit certain contents. (Enunciation by Prof. Leonhard Euler, 2024).

Addressing the retaking of content in planning implies mitigating possible doubts, learning gaps, as well as collaborating with the student's interest in learning new content, while also considering strategies that are different from the usual ones. In this way, teacher Katherine comments that teachers should “*work with different activities, use the computer room, games, crossword puzzles, different activities to see if they can achieve the objective, that is, that the students can learn to solve*”, that is, by meeting the proposed objectives there is an indication that the planning has fulfilled its purpose. According to the teachers' statements, in a non-fragmented way, we note that the planning to be developed is in line with the proposals of Pontes (2012), when considering the organization of classes and preparation of tasks for students to carry out, the forms and ways of organizing activities and learning assessment.

5 Considerations

The training meetings with the mathematics teachers brought moments of reflection and learning, providing the opportunity to answer the objective: *to discuss the teachers' knowledge of the mathematics curriculum and external assessment, as expressed in the set of Proeb documents*. In this way, the study aimed to identify what knowledge teachers express about the curriculum and external assessment, which is evident in the set of Proeb documents.

As part of the category of *knowledge about how to approach the content*, one of the pieces of knowledge expressed by teachers refers to the difficulty students have in correctly reading and interpreting task statements. That's why we think it's important for teachers to take care when approaching content, providing a broad understanding of what is being read and discussed, as well as understanding when solving tasks based on the information presented in the statement. Thus, it is part of teaching practice to continue to develop with them the exercise of interpreting what is being discussed and, if possible, contextualizing it, giving meaning to terms and expanding vocabulary, since if students don't know how to read and interpret correctly, there is a greater chance that they won't be able to solve the tasks or even the external assessment items.

The teachers revealed knowledge based on the following points: the fact that students don't understand the concepts, methods and strategies for solving the tasks, because they think they can look them up on Google at any time; the lack of commitment on the part of the students when solving the external assessment items, marking the alternatives at random; and the fact that some teachers are unmotivated or ignore the analysis of the external assessment results reports when planning their teaching practices, due to the excess of routine demands. These issues indicate that the fact that some students have not built up an understanding of the concepts, strategies or methods for solving them is compromising their ability to solve the proposed tasks and, consequently, some of the items in the external assessments and, for this reason, they end up marking the alternatives in the tests at random, which is why, from the statements made by some teachers, we can see that some may not see the point in analyzing the results, considering also the fact that they find them difficult to analyze and interpret.

In relation to demotivation, and being a teacher who works in Basic Education, we infer

that the following aspects may be motivating teachers in relation to their demands:

- Draw up, revise and implement the planning for all the weekly lessons and possibly, using their own resources, pay for the purchase of materials to implement the teaching practices;
- Prepare tasks and be attentive to creating opportunities for parallel recovery, for each set of tasks developed, for students who have not built up the expected learning, for those who have not yet mastered writing and reading, and also considering those students with learning difficulties;
- Individually correct the tasks solved by the students, observing the errors and successes of each one, and identifying if there are any patterns in the errors in the solutions or answers, in the hope of using this data to revise the teaching strategies;
- Being attentive to assessment processes, not forgetting to prioritize the qualitative over the quantitative;
- Planning, organizing and developing events for students in accordance with the proposed school calendar, published annually by the State Department of Education and approved by the school board;
- Dealing with adverse situations of indiscipline;
- Lidar com problemas de relação interpessoal dos estudantes;
- Set aside time to read resolutions, memos and other published documents that refer to teachers' duties;
- Participate in continuing training with a view to expanding what they know and how this affects teaching practices and the learning process of their students;
- Meet with peers to evaluate and select curriculum materials in the context of the Programa Nacional do Livro e do Material Didático (National Textbook and Teaching Materials Program — PNLD);
- Filling in the digital diary in good time to record attendance and grades, as well as closing it every two or three months;
- Participate in Module 2 collective meetings in the after-school hours or on Saturdays, when there is a need to make up the number of school days.

We understand that this set of demands, among others, are necessary actions for the proper development of school practices and teaching and learning processes; demands or actions that also require the health and willingness of teachers so that they can be carried out simultaneously and in an articulated manner.

We can see that the knowledge expressed by teachers can serve as inspiration when planning different and innovative strategies, with the intention of motivating students to build their learning. Although it is already done by many, it would be pertinent to incorporate into teaching practice, whenever possible, the habit of making students aware of having a critical and serious eye when solving the proposed tasks, as well as the items in the external assessments, including the reasons for this seriousness. However, it would be interesting to develop this practice throughout the school year and not just on the eve of the assessments.

When looking at examples of tasks and items, knowledge was shown in order to identify their relationship with the descriptors and skills prescribed in the *Minas Gerais Reference Curriculum*, revealing that the teachers are aware of and can relate to the prescriptive materials.

This relationship is necessary for them to be able to analyze the results of the assessments and associate them, where possible, with lesson planning. However, there were no comments associated with curricular materials and other teaching resources, such as the importance of choosing the materials evaluated and distributed by the *Programa Nacional do Livro e do Material Didático* (National Textbook and Teaching Materials Program — PNLD), as well as the importance of using these materials in teaching and learning processes.

Although the teachers showed that they were familiar with the Reference Matrix for the 9th year of elementary school, they did not discuss, for example, their relationship with Simave's pedagogical magazines, which are aimed at teachers, including those who teach mathematics, and what these magazines can contribute about why students make certain mistakes.

The teachers' statements also showed that the demands and working conditions of teachers are factors that make it difficult for them to analyze the results of the evaluations and incorporate them into their teaching plans, since the reports are considered extensive and require time to analyze.

With regard to the demands and working conditions, and observing the Minas Gerais state decree 45.841 of December 26, 2011 on the accumulation of up to two teaching positions, there are employees with extensive workloads who may not have the time to analyze the results. However, organizing and guaranteeing time for teachers to study can help teachers gain a broad and specialized understanding of the results of external assessments. A suggestion for educational institutions that have not yet joined, and considering Resolution No. 4. 968 of February 23, 2024, which stipulates that a timetable of activities must be drawn up setting out the days, times and activities to be carried out, would be to include in the timetable, throughout the school year, training sessions for teachers, for example: a broad understanding of the results by looking at the percentages of correct answers to the descriptors and the grouping of students by performance standard; the correlation of the percentages of correct answers with the percentages of other curricular components assessed in the quarterly assessments, seeking to carry out joint actions such as reading and interpreting texts and questions; moments for teachers to get together by area, to share, think about and develop joint strategic actions; opportunities to study and reflect on the materials available on the Simave portal, such as pedagogical magazines; moments of training with teachers on the use of digital teaching objects, including the use of the computer lab and the digital resources available⁶.

We understand that possible actions could be developed by the Minas Gerais State Department of Education, such as the feasibility of making personalized data from external evaluations available by school. If this is not possible, a proposal would be to hold training sessions with principals and teams from the Regional Education Superintendencies or school principals, in order to provide more clarity in the dissemination and understanding, interpretation and usefulness of the results by the management team, teachers and pedagogical staff, allowing more effective and evident actions to be taken.

The teachers showed that they know their students, especially when they mentioned their difficulties. For this knowledge, in the *content knowledge* category, the difficulty of students trying to perform division operations with one digit in the divisor was addressed. This indicates that, when working with tasks, with any test item that requires the student to be able to perform the *division* operation, the result may not be as expected, which is why we

⁶ Logical and electrical networks of state schools in Minas Gerais undergo restructuring, available at this link; accessed on Sept. 22nd, 2023.

recommend that teachers continue to review the need to consider content from previous years, reviewing with different approaches and strategies. This can enable teachers to get to know their students better and promote conditions to overcome these weaknesses.

We understand that the objective set for the study reported in the paper has been met, as it reveals and discusses pertinent knowledge about the curriculum and external evaluation; giving visibility to the potentialities in the exercise of teaching; as well as highlighting the weaknesses that need to be discussed and reduced, as they are still immersed in the teaching and learning processes and can imply learning practices and external evaluations.

References

- Adams, L. (2021). [What is research?](#) *Global Scientific Journals*, 9(12), 387-407.
- Ball, D. L.; Hill, H. C. & Bass, H. (2005). [Knowing Mathematics for teaching: who knows mathematics well enough to teach third grade, and how can we decide?](#) *American Educator*, 29(3), 14-17, 20-22, 43-46.
- Ball, D. L.; Thames, M. H. & Phelps, G. (2008). [Content knowledge for teaching: what makes it special?](#) *Journal of Teacher Education*, 59(5), 389-407.
- Baumert, J.; Kunter, M.; Blum, W.; Brunner, M.; Voss, T.; Jordan, A.; Klusmann, U.; Krauss, S.; Neubrand, M. & Tsai, Y. (2010). [Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress.](#) *American Educational Research Journal*, 47(1), 133-180.
- Bonamino, A. & Souza, S. Z. (2012). [Três gerações de avaliação da Educação Básica no Brasil: interfaces com o currículo da/na escola.](#) *Educação e Pesquisa*, 38(2), 373-388.
- Brasil. (2011). [Lei n. 12.527, de 18 de novembro de 2011.](#) Regula o acesso a informações previsto no inciso XXXIII do art. 5º, no inciso II do § 3º do art. 37 e no § 2º do art. 216 da Constituição Federal; altera a Lei n. 8.112, de 11 de dezembro de 1990; revoga a Lei n. 11.111, de 5 de maio de 2005, e dispositivos da Lei no 8.159, de 8 de janeiro de 1991; e dá outras providências. Brasília, DF: Diário Oficial da União, 18 nov.
- Brasil. (2012). Decreto n. 7.724, de 16 de maio de 2012. [Regulamenta a Lei nº 12.527, de 18 de novembro de 2011, que dispõe sobre o acesso a informações previsto no inciso XXXIII do caput do art. 5º, no inciso II do § 3º do art. 37 e no § 2º do art. 216 da Constituição.](#) Brasília, DF: Diário Oficial da União, 16 maio.
- Collopy, R. (2003). [Curriculum materials as a professional development tool: how a Mathematics textbook affected two teachers' learning.](#) *The Elementary School Journal*, 103(3), 287-311.
- Costa, D. G. (2019). [Baixo desempenho em Matemática e práticas de ensino: inquietações necessárias, explicações possíveis.](#) 147f. Dissertação (Mestrado em Gestão e Avaliação em Educação Pública). Universidade Federal de Juiz de Fora. Juiz de Fora, MG.
- Davis, E. A. & Krajcik, J. S. (2005). [Designing Educative Curriculum Materials to promote teacher learning.](#) *Educational Researcher*, 34(3), 3-14, 2005.
- Ebby, C. B. & Sirinides, P. M. (2015) [Conceptualizing teachers' capacity for learning trajectory-oriented formative assessment in Mathematics.](#) In: J. Middleton; J. Cai & S. Hwang. (Ed.). *Large-scale studies in Mathematics Education* (pp. 159-176). New York: Springer.
- Fuentes, S. Q. & Ma, J. (2018). [Promoting teacher learning: a framework for evaluating the](#)

- educative features of mathematics curriculum materials. *Journal of Mathematics Teacher Education*, 21(4), 351-385.
- Gamboa, S. S. (2018). *Pesquisa em Educação: métodos e epistemologias* (3 ed. rev., atual. e ampl.). Chapecó, SC: Argos.
- Lima, G. L.; Bianchini, B. L. & Gomes, E. (2018). [Conhecimentos docentes e o Modelo Didático da Matemática em Contexto: reflexões iniciais](#). *Educação Matemática Debate*, 2(4), 116-135.
- Machado, C. (2012). [Avaliação Externa e Gestão Escolar: reflexões sobre o uso dos resultados](#). *Revista @mbienteeducação*, 5(1), 70-82.
- Minas Gerais. (2012b). [Decreto n. 45.969, de 24 de maio de 2012](#). Regulamenta o acesso à informação no âmbito do Poder Executivo. Belo Horizonte, MG: Diário do Executivo, 28 jul.
- Minas Gerais. Conselho Estadual de Educação. (2019b). [Resolução n. 470, de 27 de junho de 2019](#). Institui e orienta a implementação do Currículo Referência de Minas Gerais da Educação Infantil e do Ensino Fundamental nas escolas do Sistema de Ensino de Minas Gerais. Belo Horizonte, MG: Diário do Executivo, 30 jul.
- Minas Gerais. Conselho Estadual de Educação. (2021b). [Resolução n. 481, de 1 de agosto de 2021](#). Institui e orienta a implementação do Currículo Referência de Minas Gerais nas escolas de Educação Básica do Sistema de Ensino do Estado de Minas Gerais. Belo Horizonte, MG: Diário do Executivo, 3 ago.
- Minas Gerais. Secretaria de Estado de Educação. (2000a). [Resolução n. 14, de 3 de fevereiro de 2000](#). Institui o Sistema Mineiro de Avaliação da Educação Pública (SIMAVE) e cria o Programa da Rede Pública de Educação Básica — Proeb. Belo Horizonte, MG: Diário do Executivo, 4 fev.
- Minas Gerais. Secretaria de Estado de Educação. (2000b). [Resolução n. 104, de 14 de julho de 2000](#). Reedita, com alterações, a Resolução n. 14, de 3 de fevereiro de 2000, que instituiu o Sistema Mineiro de Avaliação da Educação Pública — Simave e cria o Programa de Avaliação da Rede Pública de Educação Básica — Proeb. Belo Horizonte, MG: Diário do Executivo, 19 jul.
- Minas Gerais. Secretaria de Estado de Educação. (2011). [Decreto n. 45.841, de 26 de dezembro de 2011](#). Dispõe sobre o processo de acumulação de cargos, funções ou empregos públicos no âmbito da administração pública direta e indireta do Poder Executivo e dá outras providências. Belo Horizonte, MG: Diário do Executivo, 27 dez.
- Minas Gerais. Secretaria de Estado de Educação. (2019a). [Currículo Referência de Minas Gerais: Educação Infantil e Ensino Fundamental](#). Belo Horizonte, MG: SEE.
- Minas Gerais. Secretaria de Estado de Educação. (2024a). [Resolução n. 4.968, de 23 de fevereiro de 2024](#). Estabelece normas para o cumprimento da carga horária destinada às atividades extraclasse pelo Professor de Educação Básica das escolas da Rede Estadual de Ensino de Minas Gerais. Belo Horizonte, MG: Diário do Executivo, 24 fev.
- Minas Gerais. Secretaria de Estado de Educação. Sistema Mineiro de Avaliação e Equidade da Educação Pública. (2012a). [Revista da Gestão Escolar](#) (v. 2). Belo Horizonte, MG; Juiz de Fora, MG: SEE; UFJF.
- Minas Gerais. Secretaria de Estado de Educação. Sistema Mineiro de Avaliação e Equidade da

- Educação Pública. (2021a). *Revista da Rede* (v. 2). Belo Horizonte, MG; Juiz de Fora, MG: SEE; UFJF.
- Minas Gerais. Secretaria de Estado de Educação. Sistema Mineiro de Avaliação e Equidade da Educação Pública. *Revista da Escola: Matemática*, v. 1. Juiz de Fora: SEE/UFJF, 2022b.
- Minas Gerais. Secretaria de Estado de Educação. Sistema Mineiro de Avaliação e Equidade da Educação Pública. (2021c). *Revista da Escola: Matemática* (v. 1). Belo Horizonte, MG; Juiz de Fora, MG: SEE; UFJF.
- Minas Gerais. Secretaria de Estado de Educação. Sistema Mineiro de Avaliação e Equidade da Educação Pública. (2022a). *Sumário Executivo* (v. 6). Belo Horizonte, MG; Juiz de Fora, MG: SEE; UFJF.
- Ponte, J. P. (2012). Estudando o conhecimento e o desenvolvimento profissional do professor de Matemática. In: N. Planas. (Coord.). *Teoría, crítica y práctica de la Educación Matemática* (pp. 83-98). Barcelona: Grao.
- Remillard, J. T. & Kim, O. (2017). *Knowledge of curriculum embedded mathematics: exploring a critical domain of teaching*. *Educational Studies in Mathematics*, 96, 65-81.
- Sacristán, J. G. (2013). O que significa o currículo? In: J. G. Sacristán. (Org). *Saberes e incertezas sobre o currículo* (Tradução de E. F. F. Rosa, pp.15-35). Porto Alegre, RS: Penso.
- Shulman, L. S. (1986). *Those who understand: knowledge growth in teaching*. *Educational Researcher*, 15(2), 4-14.
- Shulman, L. S. (1987). *Knowledge and teaching: foundations of the new reform*. *Harvard Educational Review*, 57(1), 1-23.
- Souza, A. O. (2015). *Um olhar sobre a Prova Brasil: análise dos resultados em Matemática*. 77f. Dissertação (Mestrado em Educação, Cultura e Comunicação em Periferias Urbanas). Universidade do Estado do Rio de Janeiro. Duque de Caxias, RJ.
- Trivinhos, A. N. S. (1987). *Introdução à Pesquisa em Ciências Sociais: a pesquisa qualitativa em Educação*. São Paulo, SP: Atlas.
- Werle, F. O. C. (2010). Sistema de avaliação da educação no Brasil: abordagem por níveis de segmentação. In: F. O. C. Werle (Org). *Avaliação em larga escala: foco na escola* (pp. 21-36). Brasília, DF: Liber Livros.