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Critical Financial Education in Mathematics Classes: an Analysis from the Perspective of the Registers of Semiotic Representation

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Abstract: This article aims to present teaching and learning situations that provide students with favorable conditions to assign meaning and significance to mathematical objects, bearing in mind the mobilization/production of mathematical knowledge, considering the Theory of Registers of Semiotic Representation. To this end, a sequence of activities was designed and carried out in a first-year high school class at a public school in the state of Minas Gerais. The sequence took as its starting point contributions from the Theory of Registers of Semiotic Representation for the processes of teaching and learning Mathematics, enabling discussions and reflections in the context of Critical Financial Education. The results indicate that school practices aligned with students' realities tend to foster better engagement for the benefit of building mathematical knowledge and contribute to the formation of a critical and autonomous individual, capable of playing an active role in society by responding to social contradictions.

Keywords: Critical Financial Education. Financial Math. Teaching and Learning Mathematics. Records of Semiotic Representation.

Educación Financiera Crítica en las clases de Matemáticas: un análisis bajo los aspectos de los Registros de Representación Semiótica

Resumen: Este artículo tiene como objetivo presentar situaciones de enseñanza y aprendizaje que proporcionen a los alumnos condiciones favorables para asignar sentido y significado a los objetos matemáticos, teniendo en cuenta la movilización/producción de conocimientos matemáticos, a la luz de la Teoría de los Registros de Representación Semiótica. Para ello, se diseñó y desarrolló una secuencia de actividades en un grupo de primer año de la Educación Media Superior en una escuela pública de la red estatal de Minas Gerais. La secuencia tuvo como punto de partida las contribuciones de la Teoría de los Registros de Representación Semiótica para los procesos de enseñanza y aprendizaje de la Matemática, posibilitando discusiones y reflexiones en el contexto de la Educación Financiera Crítica. Los resultados indican que las prácticas escolares alineadas con la realidad de los estudiantes tienden a fomentar en ellos un mayor compromiso en beneficio de la construcción del conocimiento matemático y contribuyen a la formación de un sujeto crítico y autónomo, capaz de desempeñar un papel activo en la sociedad y de reaccionar ante las contradicciones sociales.

Palabras clave: Educación Financiera crítica. Matemática Financiera. Enseñar y Aprender Matemáticas. Registros de Representación Semiótica.

Educação Financeira Crítica nas aulas de Matemática: uma análise sob aspectos dos Registros de Representação Semiótica

Resumo: Este artigo tem por objetivo apresentar situações de ensino e aprendizagem que proporcionam aos alunos condições favoráveis para atribuir sentido e significado aos objetos matemáticos, tendo em vista a mobilização/produção de conhecimentos matemáticos, à luz da Teoria dos Registros de Representação Semiótica. Para tanto, foi elaborada e desenvolvida uma sequência de atividades em uma turma do 1º ano do Ensino Médio de uma escola pública da rede estadual de Minas Gerais. A sequência teve como ponto de partida contribuições da Teoria dos Registros de Representação Semiótica para os processos de ensino e de aprendizagem de Matemática, possibilitando discussões e reflexões no contexto da Educação Financeira Crítica. Os resultados apontam que práticas escolares alinhadas à realidade dos discentes tendem a despertar neles um melhor engajamento em benefício da construção do conhecimento matemático e contribuem para formação de um sujeito crítico, autônomo, capaz de desempenhar um papel ativo na sociedade, reagindo a contradições sociais.

Palavras-chave: Educação Financeira Crítica. Matemática Financeira. Ensino e aprendizagem de Matemática. Registros de Representação Semiótica.

1 Introduction

The objective of this article is to present teaching and learning situations that provide students with favorable conditions to assign meaning and significance to mathematical objects, bearing in mind the mobilization/production of mathematical knowledge, in light of Raymond Duval's Theory of Registers of Semiotic Representation.

This theory was established in 1995 by Raymond Duval[1]. His work—*Sémiosis et pensée humaine* (Duval, 1995)—is a milestone in TRRS (Teoria dos Registros de Representação Semiótica), and his research gained visibility in several countries, including Brazil, strongly influencing studies in Mathematics Education.

Among the scholars of this theory in Brazil, we highlight the researcher Méricles Thadeu Moretti, a professor in the Programa de Pós-Graduação em Educação Científica e Tecnológica and in the Department of Mathematics at the Universidade Federal de Santa Catarina, where he has published and supervised numerous investigations at the Master's and Doctoral levels on this subject. Among them, we may mention Simonetti and Moretti (2021). We also highlight the book *As contribuições da Teoria das Representações Semióticas para o ensino e pesquisa na Educação Matemática*, organized by Celia Finck Brandt and Méricles Thadeu Moretti, which presents a collection of articles involving TRRS (Brandt & Moretti, 2014). Other researchers, such as Silva, Vidal, and Carvalho Filho (2023), have also developed studies involving TRRS.

Regarding Financial Education (FE), it has been discussed in the academic realm of Mathematics Education (ME) and generally aligns with the assumptions of Critical Mathematics Education (CME). According to Batista (2021), CME pursues a democratic practice in the teaching and learning process so that students, beyond solving exercises, can reflect and act critically by means of Mathematics in situations that somehow involve it.

The CME movement emerged in 1980 and, according to Borba (2017), developed through exponents such as Marilyn Frankenstein and Arthur Powell (in the United States),

[1] A philosopher and psychologist by training, his studies in the field of Cognitive Psychology — conducted at the Mathematics Education Research Institute (IREM) in Strasbourg (France) — have made a significant contribution to research in Mathematics Education (Duval, 2009).

Paulus Gerdes and John Volmink (in Africa), Munir Fasheh (in Palestine), Ubiratan D'Ambrosio (in Brazil), and Ole Skovsmose and Stieg Mellin-Olsen (in Europe). Not all of them used the nomenclature "Critical Mathematics Education" to name part of their work oriented toward that approach, and certainly there are other individuals, in other parts of the world, who develop practices that fit within this movement.

For Borba (2017), CME is a movement concerned with the political aspects of Mathematics Education, bringing forth in its genesis the following questions: a) Whose interests does Mathematics Education serve? b) For whom should ME be directed? c) How can we prevent the processes analyzed by ME from perpetuating harmful prejudices against oppressed groups such as Black, Indigenous, and female workers?

Skovsmose (2017) reiterates that this movement discusses the importance of the teacher-student relationship in order to establish partnerships, understanding Critical Education in line with Paulo Freire as regards that relationship, in connection with what is termed Emancipatory Pedagogy. In that context, the teacher, while teaching through dialogue with students, also learns and becomes jointly responsible for a process in which everyone grows. Skovsmose (2017) brings to the center of the Mathematics Education debate questions related to power: "Coming from Denmark, my perspective is to be in the vicinity of power centers, but at the same time in the midst of highly technological development; this naturally influences my conception of Critical Education" (Skovsmose, 2017, p. 101).

Skovsmose (2021) emphasizes that CME is not reduced to a subarea of ME and is not occupied with methodologies, pedagogical techniques, or programmatic content. According to Silva (2021), CME expresses concerns in the field of ME related to concepts such as democracy, justice and social injustice, power relations, equity, racism, inclusion, prior knowledge of students and teachers, mathemacy[2], among others. These may be addressed in any area, topic, or school setting: "there is no specific discipline or space for Critical Mathematics Education to take place" (Silva, 2021, p. 9).

Kistemann Jr., Coutinho, and Pessoa (2021) affirm, based on Skovsmose (2001), that Critical Mathematics Education encompasses a model that must respond to social contradictions and question existing models, going beyond Mathematics itself. D'Ambrosio (2021) notes that, upon delving into the assumptions of ME and CME, teachers sometimes have the feeling they are not teaching Mathematics and may ask, "But what does this have to do with Mathematics Education? And I answer: It has everything to do with it" (D'Ambrosio, 2021, p. 9).

In line with the CME proposed by Skovsmose (2001), another field of knowledge is growing: Critical Financial Education (CFE), chiefly advocated by Marco Aurélio Kistemann Jr., a professor and researcher in the Department of Mathematics at the Universidade Federal de Juiz de Fora (UFJF).

CFE is widely discussed in the sphere of Mathematics Education and is concerned with disseminating Financial Education (FE) information in order to contribute to forming/building citizens who can reflect on the model of society in which we are immersed, debate and intervene in political and economic issues, understand the necessity of making sustainable financial decisions, and struggle for fair wages and dignified living conditions for all. That is, defending measures that can help minimize poverty and social inequalities that plague society on both national and international levels.

[2] *Mathemacy* can be interpreted by focusing on its social dimension, thereby formulating a possible conception of Mathematics Education for citizenship (Skovsmose, 2021).

In the view of Kistemann Jr., Coutinho, and Pessoa (2021), CME and FE converge insofar as their underpinnings and concerns are directed toward shaping a critical, autonomous individual who knows how to make day-to-day decisions aiming to improve their personal, social, and economic life responsibly and in accordance with the principles of ethics and citizenship existing in society.

Just as CME can be approached in any area or knowledge field, the same holds for FE. It should be highlighted that CME and FE are not limited to the exercise paradigm described by Skovsmose (2021), since carrying out calculation procedures without reflecting on them does not foster critical and reflective development of the learner. By way of illustration, consider a problem situation taken from the book *Um convite à Educação Matemática Crítica* ("An invitation to Critical Mathematics Education"), by Ole Skovsmose (2021):

> Caring for a madman costs the state 4 marks[3] a day. Caring for a cripple, 4.5 marks. For an epileptic, 3.5 marks. The average is 4 marks a day and the number of patients is 300,000. How much would be saved if these individuals were eliminated? (Skovsmose, 2021, p. 15).

The mathematical solution for this situation is readily found by multiplying the average cost (4) by the number of patients (300,000). However, when we apply the assumptions of CME and FE, it is crucial to emphasize that problem situations of this nature must be avoided in the classroom, since the reflections they prompt do not align with the parameters of ethics and citizenship for training a citizen in line with the values upheld by society. It is unacceptable to agree that eliminating people would be a viable solution for the economic and financial sector.

Likewise, the principles of CME and FE do not endorse problem situations that encourage the accumulation and/or saving of goods and financial resources without taking into account the sustainability of the planet (through actions stemming from excessive consumerism) and the companies that engage in mass production (those that do not account for environmental responsibility nor show concern about various modes of production, storage, transport, or product quality), among other factors. It is necessary to envision a conception of teaching and learning Mathematics and Financial Education that also takes into account the environment in which people live, their cultures, and each individual's particularities.

In this regard, we concur with the studies by Batista, Crisóstomo, and Macêdo (2022a, 2022b), as they consider investigations into Financial Education — viewed as a research topic in Mathematics Education — highly relevant. This approach enables the contextualization and application of mathematical content in real situations and also pertains to citizenship education by fostering a more conscious, reflective stance in situations involving money management.

> The teaching practice that seeks to bridge the gap between mathematical content and reality through a critical and reflective perspective underscores the need to articulate mathematics with contextualized topics, which can be enhanced by addressing specific themes in Financial Education, especially those related to the current economic scenario. (Batista, Crisóstomo & Macêdo, 2022b, p. 195)

Mathematics classes, starting from students' everyday situation s— such as an electricity bill — where knowledge objects like function concepts, domain, codomain, growth, and decay are explored. These are approached using contributions from the Theory of Registers

[3] "Mark" refers to the *Deutsche Mark*, a German currency. It was the official currency of the Federal Republic of Germany from 1949 to 2002. Available at: https://en.wikipedia.org/wiki/Deutsche_Mark. Accessed on June 18, 2023.

of Semiotic Representation (TRSR) in its various representational registers — including natural language; numerical; algebraic; tabular; and graphical — that enable students to mobilize/build their knowledge for the benefit of life in society.

The study was conducted with a first-year full-time high school class (EMTI) at a public state school in the municipality of Montes Claros, Minas Gerais (MG). The results of the intervention are presented subsequently.

It is important to note that this research was submitted to the Research Ethics Committee (CEP) of the Universidade Estadual de Montes Claros (UNIMONTES) and approved under the substantiated ruling no. 5.580.777, dated August 13, 2022, and the Certificate of Presentation for Ethical Consideration (CAAE) no. 61180122.6.0000.5146.

In addition to the introduction, which presents the topic and orients the reader regarding the research objectives, the text is organized into sections, each of which addresses the following: considerations on Financial Mathematics and Financial Education; aspects of the Registers of Semiotic Representation in learning Mathematics; methodological procedures; the sequence of activities carried out, students' written work, and the discussion of results. Finally, concluding remarks are presented.

2 Considerations on Financial Mathematics and Financial Education

In the BNCC (Brazilian National Common Curricular Base) for the area of Mathematics (Brasil, 2018), FE (Financial Education) is mentioned within certain skills as previously indicated. Some of these skills may be linked to Financial Mathematics concepts; others may not. Almouloud and Coutinho (2020) and Chiappetta and Silva (2021) point out that teachers and students commonly conflate FE with Financial Mathematics (FM). In this context, those authors present FE and FM from the perspective of three focuses, namely: purpose, emphasis, and objective. For FE, the purpose involves understanding information, training, and guidance on financial concepts and products; the emphasis is on knowing values and gaining the capacity to recognize situations of opportunity and risk; and the objective is to form individuals and societies that are conscious of consumption. As for FM, they define it as a branch of Mathematics whose purpose is to improve one's understanding of the practice of calculation or procedures involving time-valued financial products. Its emphasis is on models that allow one to assess and compare the value of money at various points in time, with the objective of forming individuals and societies conscious in the handling of currency in their studies and analyses.

According to Kistemann Jr., Coutinho, and Pessoa (2021), FE is not necessarily linked to FM, but depending on the teacher's approach, FM is a fundamental tool for its implementation as well as for informed decision-making.

In Kistemann Jr. view (2020), schools must exercise caution when incorporating Financial Education into their pedagogical proposals so as not to disseminate a conception that serves the interests of agencies and entities solely seeking profit without genuine concern for people's real needs—such as the fight against hunger, poverty, and social injustices, among other issues that primarily characterize social ills.

It is essential to emphasize that teaching and learning Financial Mathematics does not ensure understanding of Financial Education, since FE relates to the approach, depth, and scope that the teacher provides in fostering discussions that lead students to grasp its concepts. Thus, FM activities can become FE activities depending on the teacher's approach, depth, and scope, as illustrated in the following example:

> Calculate the interest earned on a principal of R$ 10,000.00 invested at a simple annual interest rate of 10% for five months. As we can see, this problem does not involve a context, so the student need only apply their mathematical knowledge. We do not consider this question to be about Financial Education but strictly about Financial Mathematics; however, in classroom practice, depending on the approach, depth, and scope the teacher employs, it can be turned into an FE activity (Kistemann Jr., Coutinho & Pessoa, 2021, p. 38).

A study presented by Santos and Pessoa (2021) analyzed collections of Mathematics textbooks from the 2014 National Textbook and Teaching Material Program (PNLD) and classified various Financial Mathematics activities as Financial Education. Their rationale is rooted in the guidelines found in the teacher's manual: "it is important to emphasize that the activity in question is regarded as FE because it contains the clearly stated guidance in the teacher's manual" (Santos & Pessoa, 2021, p. 121).

The authors highlight an activity taken from a 4th-grade book in the 2014 series "A Conquista da Matemática," which features a list of school supplies parents should buy for their 4th-grade children at a fictitious school, along with a price list for those items at a stationery store. Two questions are posed to the students, asking them to perform calculation procedures. The list includes the following materials and quantities: 6 black pencils; 2 erasers; 2 sharpeners; 3 notebooks of 100 sheets; 1 box of colored pencils with 12 colors; 1 set of gouache paints; 1 box of colored pencils with 6 colors; 1 drawing pad; 4 sheets of white construction paper; 100 sheets of printer paper; and 2 elastic folders.

The price list shows the following items, quantities, and prices: sharpener, 1 real (each); black pencil, 3 reais (package of 6 units); 100-sheet notebook, 5 reais (each); drawing pad, 3 reais (each); 6-color gouache set, 13 reais; construction paper, 1 real each; 24-color box of pencils, 5 reais (each); printer paper, 4 reais (package of 100 sheets); 12-color gouache set, 20 reais; eraser, 1 real (each); 12-color pencils, 4 reais (each); and elastic folder, 2 reais (each).

The task proposed: "Mrs. Nair purchased all the school supplies for Camila from this store, in the quantities indicated on the list. A) Do you think this purchase exceeded or stayed under 60 reais? B) Show your calculations in your notebook to find out how much Mrs. Nair paid for these materials."

Such activities are considered Financial Mathematics since there is a single correct answer upon carrying out the calculations. However, the authors categorized it as FE because in the teacher's manual, the following recommendations are made:

> Ask students if their families research product prices before buying them and whether they consider this action important. Take this opportunity to talk with them about the need to consider not only the product's price but also the cost-benefit ratio, the availability of money, and the real necessity of the purchase. Point out that sometimes an offer may look appealing but be unnecessary, such as a 12-color gouache set, among other important factors to consider when shopping. (Giovanni Jr. & Castrucci, 2014)

In the view of Santos and Pessoa (2021), FE goes beyond performing calculation procedures. Students should be prompted to consider and assess their behaviors, attitudes, and habits alongside their families, so that the transfer of classroom concepts provides them with the conditions to use these concepts in daily life. In this sense, FE plays an important role in guiding people toward making decisions conscientiously and responsibly.

It is crucial to highlight that in FE there is no purely right or wrong answer. The outcome depends on each person's context, financial resources, the necessity of purchasing certain products, and other factors. Similarly, in financial decision-making, there is no single correct answer; rather, there is the choice that best meets each individual's circumstances, applying available alternatives in a way that makes one feel comfortable and without regrets later. The educator must be aware of this and be deeply committed to social responsibility.

According to Batista (2021),

> Financial Education is a topic of considerable importance, and its teaching depends on a properly literate faculty, one that knows and masters Financial Mathematics concepts and applies the assumptions of Critical Mathematics in the classroom, leading students to appropriate these mathematical concepts with a view to the exercise of citizenship. (Batista, 2021, p. 56)

In that context, it is deemed relevant for the teacher to be familiar with theories and teaching-learning strategies that facilitate learning, so that students can take ownership of mathematical concepts and use them for a life in society. In that direction, this paper sought to employ aspects of the Registers of Semiotic Representation in Mathematics learning, as discussed in the next section.

3 Aspects of the Registers of Semiotic Representation in Mathematics Learning

The Theory of Registers of Semiotic Representation (TRSR), established by Raymond Duval in 1995, is widely addressed in the field of Mathematics Education and exerts influence on its learning processes. Drawing on this theory, Duval (2009) underscores that a student learns Mathematics when they manage to assign sense and meaning to the object under study. The author refines the theory with a cognitivist focus, seeking to explain how students mobilize Mathematics learning so that comprehension and assimilation of the knowledge objects can be consolidated.

With respect to Mathematics learning, Duval (2009) points out that its construction/assimilation process takes place through semiotic representations. Access to a mathematical object is only possible via its representations. Therefore, the scholar notes that TRSR is a semiocognitive theory, focusing on the processes of Mathematics learning.

For Duval (2009), Mathematics learning provides a privileged field of study for analyzing fundamental cognitive activities such as conceptualization, reasoning, problem solving, and text comprehension. A hallmark of Mathematics learning is that these cognitive activities require the use of expression and representation systems that go beyond natural language or images—varied systems including the different ways of writing numbers; symbolic notations for objects; algebraic and logical scripts parallel to natural language that can express relationships and operations; geometric figures; perspective representations; Cartesian graphs; networks; diagrams; schemata; among others.

Duval (2009) explains that a single number can be represented in several ways—for example, decimal notation, fractional notation, and exponential notation. However,

> It is necessary to distinguish the operational meaning assigned to the signifier from the number represented. That operational meaning is not the same for $0{,}25$, for $\frac{1}{4}$, and for $25\,.\,10^{-2}$. This is because the addition procedures used for each of the following three sums are not the same: "$(0{,}25 + 0{,}25 = 0{,}5;)$", "$(\frac{1}{4} + \frac{1}{4} = \frac{1}{2};)$" and

> "$(25.10^{-2} + 25.10^{-2} = 50.10^{-2})$". Each of these three signifiers "$(0{,}25, \frac{1}{4}, 25.10^{-2})$" has a different operational meaning but represents the same number. (Duval, 2009, p. 60).

Mathematical understanding requires that students be able to differentiate an object from its representation, carrying out treatments of the representation register in use, even naturally effecting the necessary conversion among them. Treatment is the transformation that occurs within a single representation register. For example, given the function defined by $f(x) = 2x + 1$, determining the value of x when $f(x) = 0$ involves performing the operation $2x + 1 = 0$ to find $x = -\frac{1}{2}$. In this procedure, treatment is what has taken place. Conversion is the transformation from one representation into another belonging to a different register—for example, sketching the graph of the function $f(x) = 2x + 1$, which would be a representation in another register.

According to Bassoi and Peccin (2014, p. 192), "from a pedagogical standpoint, Duval (2009) states that teachers explore only treatments in the classroom, but it is in the conversion among different registers of semiotic representation that mathematical learning takes place."

Drawing on TRSR in this study, the researchers sought to employ treatments and conversions as parameters for analyzing the students' written work produced during the development of the proposed tasks in the activity sequence. The following sections present the methodological path used to gather and analyze the data in this study.

4 Methodological Procedures

This investigation fits into the participant research methodology. According to Brandão (2006), a research project is considered "participant" not because social actors take part as supporting characters, but rather because it is planned, carried out, and unfolds through their active and increasing participation. The teacher's interventions—by the first author of this article—as well as the students' participation and engagement were key in obtaining data.

Regarding the approach to the research problem, this is a qualitative study. For its execution, an activity sequence was designed and carried out with a first-year full-time high school class (EMTI) at a public state school in the municipality of Montes Claros, MG, under the responsibility of the first author, who also served as the teacher in that class. She taught five class hours (ch/a) per week: one ch/a of Mathematics Remediation; two ch/a of Experimental Practice; and two ch/a of Tutoring—components of the EMTI in 2022. These classes were used to implement the activity sequence, the main instrument for data collection in this research.

The proposed tasks in the activity sequence were completed between October 24 and 31, 2022, over six class hours. All students received the printed materials and had access to them only during class time, in the presence of the teacher-researcher. Observations and notes of the students' comments were made during these classes. All notes taken by the students were collected as physical records for later analysis.

The tasks in the activity sequence had the following specific objectives: (i) to identify functions, starting with an electricity bill, in their natural language, algebraic, and graphical representations, converting these representations from one to another; (ii) to identify the domain, image, growth, and decay of these functions; and (iii) to engage in discussions within the context of Critical Financial Education.

Eighteen students who accepted the invitation and were present during the six class

hours in the period indicated took part in these activities. They were briefed on the research objectives and informed that their identities would be protected, in line with the ethical principles used in human-subjects research. Thus, the activity sequence served as the primary data collection instrument. All the proposed tasks were carried out in the classroom, in the presence of the teacher-researcher, with her mediation whenever necessary.

During the activities, the teacher-researcher carefully observed the students' participation, their engagement with the activities, and their interactions with each other and with the teacher. Whenever questions arose, they were immediately resolved by the teacher or by other students. To preserve the participants' anonymity and track the process by which each student constructed knowledge, their work was numbered randomly from 1 to 18. Therefore, they are referred to as students A1, A2, A3, …, A17, A18. The results are presented in the following section.

5 The Activity Sequence and the Students' Written Work

The activity sequence was implemented over six 50-minute class hours. The first class hour took place in the video room and was devoted to a preliminary exploration of the topic. For that, students watched the video "CONTA DE LUZ, aquilo que não te contaram" ("Electricity Bill: What They Didn't Tell You")[4]. They then discussed the importance of paying taxes; where the tax money we pay goes; the consequences for clients who steal electricity by making illegal "taps" into power lines; how power companies prevent incurring losses from those thefts; and other related questions..

The teaching methodology used in that class aligns with the investigative scenarios proposed by Skovsmose (2021). In such scenarios, the teacher cannot fully predict the direction the lesson may take: "the paths of investigative scenarios are not clearly marked" (Skovsmose, 2021, p. 47). According to the author, when an investigative scenario is introduced to the students, a multitude of possibilities opens up, and research-based activities can be used, since "this is quite different from what one usually sees in traditional Mathematics instruction, with its sets of ready-made exercises" (Skovsmose, 2021, p. 46).

Following this line of thought, before moving on to Task 1 proposed in the activity sequence, the teacher-researcher deemed it advisable to plan an extra class (class 2) so that some of the issues raised in class 1 could be addressed in more depth. Consequently, the next class took place in the school's computer lab and utilized internet research as a pedagogical resource.

The students watched the video "Educação Fiscal e Cidadania – Tributos: Que história é essa?" ["Tax Education and Citizenship—Taxes: What's This About?"], then conducted research, took notes, and shared their findings with their classmates and the teacher-researcher. The class was guided by the following questions: 1) What are taxes? 2) Why do we pay taxes? 3) Where does the tax money we pay go? 4) In your opinion, is paying taxes important? Why? 5) Were you aware that taxes, fees, and contributions are included in the products and services we consume on a daily basis? Comment.

The discussions and reflections offered by that class helped students gain insights into the importance and purpose of levying taxes, although they also stated that they do not "think it's good" to pay so many taxes.

[4] Available at: https://www.youtube.com/watch?v=LS46YMbaRhc. Acessed on 18 jun. 2023.

Based on the students' dialogues and written notes (presented below), it may be inferred that they understand the purpose of collecting taxes, but regard it as a burden on families with lower incomes and also question the necessity of so many taxes that we are obliged to pay, in addition to the hardships faced especially by poorer families.

> In fact, the collection of taxes is necessary for the operation of public services (A5, 2022). It is through raising government levies (taxes, fees, and social contributions) that the state's public machinery is maintained (A9, 2022). These levies are the source of revenue that should return to the population through the provision of public services, including health, education, transportation, road and highway improvements, food, and public buildings (A1, 2022). Essential products such as rice, beans, water, and other items indispensable for survival should be exempt from taxes (A7, 2022). The poorest should not be so burdened by high tax rates, which are mainly embedded in staple goods (A2, 2022). The rich should pay more taxes. Rice, beans, vegetables, and other items that make up the staple foods of the poorer population should be exempt from taxes (A7, 2022). I do not find it so important for people with low incomes to pay taxes at the same level as rich people (A14, 2022). I was aware that taxes are included in the products we consume, but I believe many of them are unnecessary (A10, 2022). Regarding taxes included in products, I think it is a fair way to charge, but in certain circumstances, they exceed reasonable limits on some items (A17, 2022). It is important to pay taxes to help our country, as this can ensure the provision of services and fulfillment of its obligations to society (A6, 2022).

The participants' written records corroborate Kistemann Jr., Coutinho, and Pessoa (2021) by showing that lessons situated within the context of Financial Education (FE) and Critical Mathematics Education (CME) open up spaces for broad discussions. These discussions can awaken in students a political consciousness, leading them to consider citizens' economic and social inclusion and to pursue social justice.

According to Skovsmose (2017), a student's main activity is not merely attending classes but rather generating and engaging in activities that foster interaction with teachers and peers, so as to play an active role in identifying and fighting against social disparities. Drawing on Giroux (1989), Skovsmose (2017) emphasizes that schools must be defended as spaces that educate students to be critical citizens who are capable of challenging and believing that their actions will make a difference in society. Therefore, students must be presented with forms of knowledge that give them both the conviction and the opportunity to strive for a quality of life that includes all the benefits of being human.

In the third class session, students were arranged into groups of five. Each student received the printed material (a sequence of activities with three tasks) and a residential electricity bill from the municipality of Montes Claros.

In Task 1, the starting point was an electricity bill from the Minas Gerais Power Company (CEMIG). Students observed the taxes charged; additional tariffs; the cost per kilowatt-hour (kWh), both with and without taxes; examined how the total payment amount on the electric bill is calculated; discussed the interest charged for late payments; and raised other issues. They held discussions, took individual notes, and at the end of class, shared their ideas with the other groups.

It is important to mention that in Minas Gerais, the tax rate for the Imposto sobre Circulação de Mercadorias e Serviços (ICMS) [Tax on the Circulation of Goods and Services] on electricity bills ranges between 18% and 30% and must be levied on consumption rather than on the total amount owed. Students performed percentage calculations to see whether there were any irregularities in the charges on the bills provided in class.

Beginning with item k in Task 1 (Figure 1), the questions posed were: *k) Is there any consistent pattern between the monthly kWh consumption and the total amount to be paid? Comment. l) Is it possible to represent this pattern using a mathematical statement? Which statement? m) Does this pattern represent a function? Comment. n) If this pattern is a function, what is its domain? Justify your answer. o) If this pattern is a function, what is its codomain? Justify your answer. p) If this pattern is a function, what is the range of that function? Justify your answer. q) If this pattern represents a function, is it increasing or decreasing? Justify your answer.*

To answer these items, students talked among the members of their own groups and with the other teams, while the teacher-researcher provided any necessary mediation. Within each group, their dialogue was evident: *"How do you find the domain of a function?" "And the codomain?" "I'm having trouble constructing this mathematical statement. Right away, you can tell it's an increasing function, right? Because the higher the kWh consumption, the higher the amount to be paid."*

It is noteworthy that the dialogue established among the groups played a key role in the students' mobilization/construction of knowledge. The discussions were collective, while the written work was completed individually, given that each student had an electricity bill with unique data. See the written record from student A9 (Figure 1). Observe that the student's mobilization/construction of knowledge emerged from his interest in using his reasoning skills to craft answers based on the electricity bill data and on his prior knowledge.

Figure 1: Written record by student A9

k) Is there any regularity between the monthly consumption in kW/h and the total amount to be paid? Comment.
Yes

l) Is it possible to represent this regularity with a mathematical expression? Which one?
Yes. $\boldsymbol{f(x) = 0,95x + e}$

m) Does this regularity represent a function? Comment.
Yes, a first-degree (linear) function.

n) If this regularity is a function, what is its domain? Justify your answer.
Any value greater than zero, because x represents the value corresponding to kW [kWh].

o) If this regularity is a function, what is its codomain? Justify your answer.
The codomain is the real numbers.

p) If this regularity represents a function, is it increasing or decreasing? Justify your answer.
The codomain will be the real numbers.

q) If this regularity represents a function, is it increasing or decreasing? Justify your answer.
It is an increasing function, because the more kW [kWh] is consumed, the higher the amount to be paid.

(Transcription of A9's responses, 2022)

Source: research data (2022).

Notice that the student's thought process and knowledge construction were prompted by his willingness to think critically and formulate answers based on the electricity bill data and his own prior knowledge. Of the 18 students who completed this task, all struggled to provide

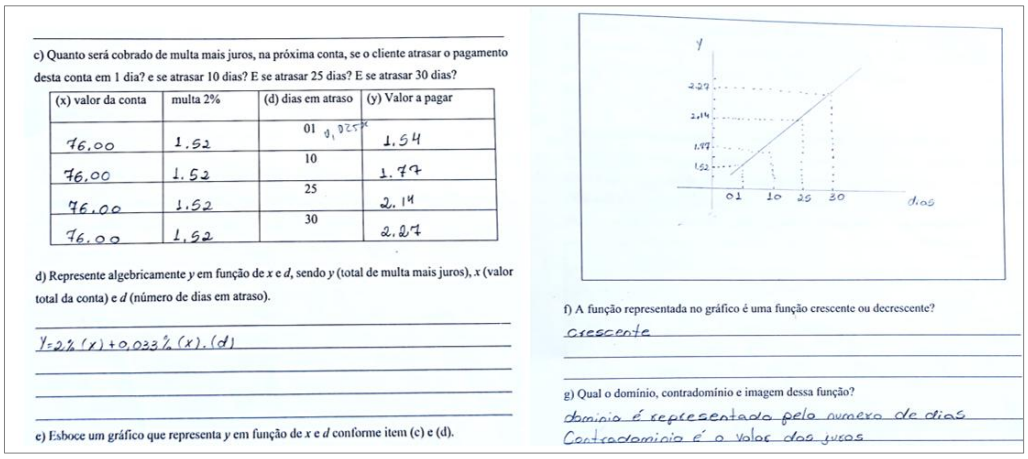
a clear written explanation of what "domain" and "codomain" of a function mean, requiring a later review of these concepts to ensure deeper comprehension.

Only four students (about 22.2%) spontaneously used an algebraic representation to write out the mathematical expression in item l. This observation aligns with Duval's (2009) findings that learners consider it difficult to move between and coordinate different representational registers, even though such transitions are essential for mathematical learning. Indeed, the more naturally these transitions occur, the greater a student's mathematical proficiency.

> For many students at different educational levels, changing the form of a representation is a difficult operation and sometimes even impossible. It is as if most students' grasp of a topic remains limited to whatever form of representation was originally used. (Duval, 2009, p. 34)

During the fourth class, Task 2 prompted students to discuss late-payment interest fees on electricity bills. They also had to use various semiotic representation registers—such as decimal, percentage, tabular, algebraic, and graphical—to handle the required calculations. This task forced them to work across multiple representation registers and switch between them. In Figure 2 and Figure 3, we see an example from student A17. Starting from the tabular register, she constructed the mathematical expression for item d and then plotted the graph for item e. Clearly, drawing on different representation registers can be a valuable teaching and learning strategy for Mathematics.

Figure 2: Student A17's written records

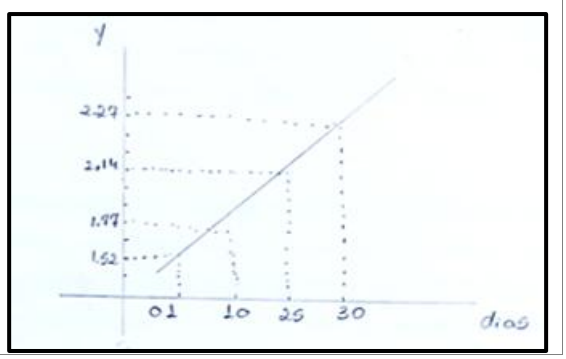


Source: Research data (2022).

Figure 3: Records by student A17

How much will be charged in penalties plus interest on the next bill if the customer delays payment of this bill by 1 day? And if payment is delayed by 10 days? What if it is delayed by 25 days? And 30 days?

(*x*) Bill amount	Penalty 2%	(*d*) Days late	(*y*) Amount to pay
76,00	1,52	01	1,54
76,00	1,52	10	1,77
76,00	1,52	25	2,14



76,00	1,52	30	2,27

d) Represent it algebraically as a function of *x* and *d*, where *y* is (the total of the penalty plus interest), *x* is (the total amount of the bill), and *d* is (the number of days late).

$$y = 2\%(x) + 0,033\%(x).(d)$$

e) Sketch a graph that represents *y* as a function of *x* and *d*, according to items (c) and (d).

f) Is the function represented in the graph increasing or decreasing?
increasing
g) What are the domain, codomain, and range of this function?
The domain is represented by the number of days late. The codomain is the amount of interest..
(Transcription of A17's record, 2022.)

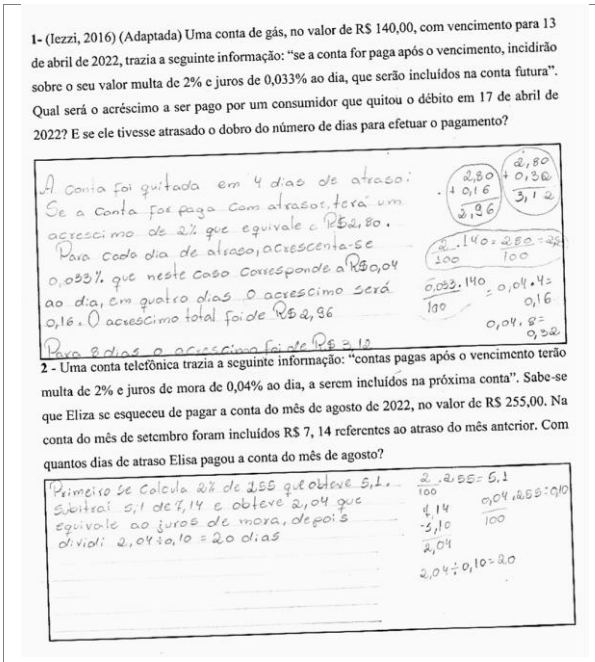
Source: Research Data (2022).

In this task, the oral discussions helped students understand Financial Education. Their calculation procedures showed that, whenever possible, families should plan to pay monthly bills—such as electricity, water, and internet—by the due date. Otherwise, as shown in the table in item c, they may incur additional interest and late fees.

According to Skovsmose (2017), Mathematics should not be used to exert power over people. Making sound decisions calls for a critical analysis of the situation, because there are often other factors to be weighed in financial matters—for example, if on the bill's due date a family does not have the funds to pay, or if they must use the money set aside for that purpose to cover essential expenses like food or medicine. It is important to bear in mind that financial decision-making depends on many variables and on each person's specific financial circumstances. In this sense, school-based Financial Education should guide people toward the most suitable decisions given their realities.

During the fifth class, students were assigned Task 3, which consisted of solving two Financial Mathematics problems adapted from a high school textbook (Figure 4).

Figure 4: Student A17's written records



1- (Iezzi, 2016) (Adapted) A gas bill of R$140.00, due on April 13, 2022, contained the following information: "if the bill is paid after the due date, a 2% penalty and daily interest of 0.033% will be charged, which will be included in the future bill." What additional amount must be paid by a consumer who settled the debt on April 17, 2022? And what if they had delayed twice as many days to make the payment?
The bill was settled 4 days late. If the bill was paid late, there is a 2% surcharge, which amounts to R$2.80. For each day of delay, 0.033% is added, which in this case corresponds to R$0.04 per day; over four days, that addition is R$0.16. The total extra amount was R$2.96. For 8 days late, the extra amount was R$3.12
(Transcription of A17's record, 2022.)

2 - A phone bill included the following statement: "bills paid after the due date will incur a 2% penalty and daily interest of 0.04%, which will be included in the next bill." It is known that Eliza forgot to pay the August 2022 bill, amounting to R$255.00. On the September bill, R$7.14 was added due to the previous month's delay. How many days late did Eliza pay the August bill?
First, 2% of 2.55 was calculated, which yielded 5.1. Then 5.1 was subtracted from 7.14, resulting in 2.04, which corresponds to the late interest. Next, 2.04% ÷ 10 = 20 days. (Transcription of A17's record, 2022.)

Source: Research data (2022).

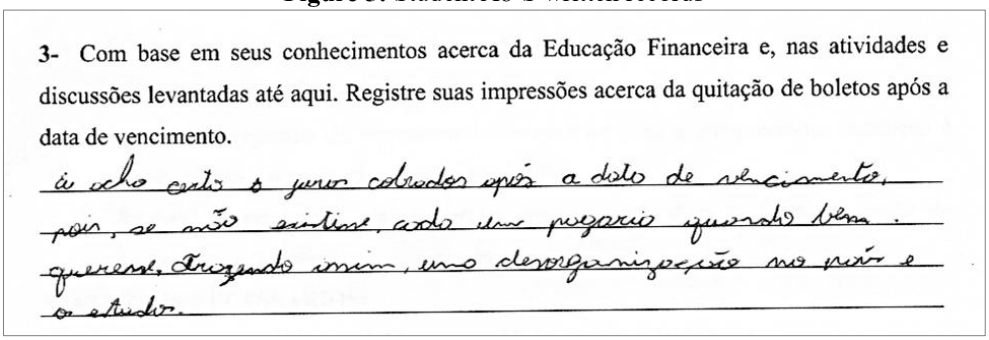
As Santos and Pessoa (2021) point out, while problems like these belong to Financial Mathematics, they can become Financial Education activities depending on the teacher's

approach and how deeply they are explored in class. For instance, teachers might ask students how families could save money on gas; whether they compare gas cylinder prices at different stores before buying; whether it is possible to reduce phone bills; or which providers offer better plans, among other questions.

To solve the proposed problems, notice that student A17 employed multiple representation registers — numerical, decimal, fractional, percentage, and, above all, natural language. She carried out treatment and conversion operations, coordinating these registers in line with each one's internal rules, which indicates mastery of the mathematical concepts in play. She effectively used natural language to detail her steps and convey her findings, demonstrating solid command of the necessary mathematical knowledge for these tasks.

During the sixth and final class of this activity sequence, students were asked to reflect on the importance of paying bills by their due date, so they would not face interest charges and penalties for late payment. As students shared their views, most agreed that it is fair for companies to charge daily late fees and penalties as a deterrent against not following the established rules, thereby avoiding mismanagement in the relevant sectors. Figure 5 and Figure 6 shows student A8's comment.

Figure 5: Student A8'S written records


3- Com base em seus conhecimentos acerca da Educação Financeira e, nas atividades e discussões levantadas até aqui. Registre suas impressões acerca da quitação de boletos após a data de vencimento.

Eu acho certo o juros cobrados após a data de vencimento, pois, se não existisse, cada um pagaria quando bem quisesse, trazendo assim, uma desorganização no país e o estado.

Source: research data (2022).

Figure 6: Record by student A8

3- Based on your knowledge of Financial Education and on the activities and discussions raised so far, please record your impressions about paying bills after their due date.
I think it's right to charge interest after the due date, because if it didn't exist, each person would pay whenever they wanted, bringing disorganization to the country and everything.
(Transcription of A8's record, 2022.)

Source: Research Data (2022).

It is worth noting that timely bill payment is much easier in a context where every family has sufficient income to meet their basic survival needs. Unfortunately, that does not reflect the reality of most Brazilian families. As Mathematics educators and Financial Education researchers, we must continue striving to ensure these rights truly reach families throughout the country.

6 Final remarks

Our aim in this article was to present teaching and learning scenarios that give students favorable conditions to attach meaning and significance to mathematical objects, considering the mobilization/production of mathematical knowledge in light of Raymond Duval's Theory of Registers of Semiotic Representation.

To achieve this, we drew on everyday situations familiar to students—an electricity bill, for example—to explore a variety of mathematical topics, including functions, domains,

codomains, and increasing/decreasing behavior, within a contextualized approach. We likewise incorporated insights from the Theory of Registers of Semiotic Representation (TRSR) across its diverse registers—natural language, numeric, algebraic, tabular, and graphical. These enabled students to activate and build knowledge, while also encouraging reflections aligned with Critical Financial Education to promote critical thinking and prepare students for active citizenship.

Based on our classroom discussions, the activity sequence we implemented, and students' written contributions, we can see that, even though they are on the brink of adulthood, these learners already grasp the rationale behind taxation. They are able to engage in debates about the need for taxes but also raise concerns regarding how taxes affect the budgets of low-income households.

From the arguments made and the texts studied, we concluded that Critical Financial Education is fundamentally important for people's lives and that schools have a pivotal role in fostering social and economic change by offering a view of Financial Education that seeks to develop critical, responsible citizens who can exercise decision-making in line with the ethical and civic principles of society.

There are indeed branches of Financial Education that emphasize entrepreneurship, just as there are individuals in certain sectors who flourish financially without formal schooling. However, you will not find doctors, engineers, or other professionals who have never attended school.

We do not endorse an approach to Financial Education limited solely to managing financial resources, promoting entrepreneurship, or encouraging the purchase of banking products. We also do not limit ourselves to teaching the buying and selling of stocks in the financial market. Similarly, we do not support approaches that merely focus on Financial Mathematics calculations without prompting deeper reflections necessary for truly understanding Financial Education.

We advocate a conception of Financial Education that addresses economic, social, and political inclusion, encourages consumers to make decisions critically, champions social justice, fights for better wages, and tackles policy issues so that everyone performing the same job receives the same pay, regardless of social class, race, or gender.

We understand that money is not an end in itself but rather a means to access what we are entitled to — healthcare, education, housing, food, the arts, cinema, and more. Mathematics Education stands alongside this effort to construct a more equitable society in which all human beings can live with dignity

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