Learning sources used by Mozambican undergraduate students in mathematics education

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Abstract: This research aimed to characterize the learning sources and the relationships that 26 students enrolled in the 1st to the 4th year of the mathematics teaching degree course at Rovuma University – Cabo Delgado Extension, Mozambique, established with these sources to learn mathematics. These students participated by completing an online form that required them to list the learning sources and describe how each contributed to their mathematics learning. The information was analyzed using content analysis procedures, and, based on the learning configuration, three categories emerged into which eight sources were allocated: human (teachers, peers, and relatives), textual (books, articles, and handouts), and technological-digital (video lessons and digital games). The "human" category was the most recurrent, and the relationship established was dialogical; with the others, it was dogmatic. The teacher was the primary source, and the other sources were used as alternatives due to the teacher's ineffectiveness.

Keywords: Mathematics Learning. Illeris' Model of Mearning. Learning Configuration. Mathematics Teaching.

Fontes de aprendizagem mobilizadas por licenciandos moçambicanos em ensino de matemática

Resumo: Esta pesquisa procurou caracterizar as fontes de aprendizagem e as relações que 26 estudantes matriculados do 1º ao 4º ano do curso de Licenciatura em Ensino de Matemática da Universidade Rovuma – Extensão de Cabo Delgado, Moçambique, estabeleciam com tais fontes para aprenderem matemática. Esses estudantes participaram preenchendo um formulário *on-line* que requeria a menção das fontes e a descrição de como elas colaboravam com a aprendizagem da matemática. As informações foram analisadas à luz dos procedimentos da análise de conteúdo e, com amparo do conceito de configuração de aprendizagem, emergiram três categorias em que foram alocadas oito fontes, nomeadamente: humanos (professores, colegas e parentes), textuais (livros, artigos e apostilas) e tecnológico-digitais (videoaulas e jogos digitais). A categoria denominada humanos foi a mais recorrente e a relação estabelecida foi dialógica e com as demais a relação foi dogmática. O professor foi a fonte primordial e primaz e as demais fontes foram alternativas em decorrência da ineficácia docente.

Fuentes de aprendizaje utilizados por estudiantes universitarios mozambiqueños en educación matemática

Resumen: Esta investigación tuvo como objetivo caracterizar las fuentes de aprendizaje y las relaciones que 26 estudiantes matriculados del 1.º al 4.º año de la carrera de Profesorado de Matemáticas en la Universidad Rovuma – Extensión Cabo Delgado, Mozambique, establecieron con estas fuentes para aprender matemáticas. Estos estudiantes participaron completando un formulario en línea que requería mencionar las fuentes y describir cómo contribuyeron al aprendizaje de las matemáticas. La información se analizó mediante procedimientos de análisis de contenido y, con base en la configuración del aprendizaje, emergieron tres categorías en las que se asignaron ocho fuentes: humanas (profesores, compañeros y familiares), textuales (libros, artículos y folletos) y tecnológico-digitales (lecciones de video y juegos digitales). La categoría denominada "humana" fue la más recurrente, y la relación establecida fue dialógica; con las demás, la relación fue dogmática. El profesor fue la fuente primaria y principal, y las otras fuentes fueron alternativas debido a la ineficacia del profesor.

Palabras clave: Aprendizaje en Matemáticas. Modelo de Aprendizaje de Illeris. Configuración del Aprendizaje. Enseñanza de las Matemáticas.

1 Introduction

Learning is a process based on the learner's interaction with the environment, through practice or other forms of experience. Such a statement can be found in Schunk (2012) and Illeris (2009), who indicate that interactions with various sources besides the teacher can foster learning. In fact, the existence of self-taught learners, that is, those who engage in self-regulated learning and minimize or dispense with dependence on teachers, attests to this diversity of sources (Adam *et al.*, 2017).

Several studies point to some sources of mathematics learning, among which we highlight: the internet, cited by Samuels and Williams (2010) and Adam *et al.* (2017); digital videos, indicated by Amaral (2013), Cardoso, Kato, and Oliveira (2014), and Cardoso, Oliveira, and Kato (2015); digital platforms with their applications and games, researched by Chang and Yang (2016) and Adam *et al.* (2017); articles and other texts related to mathematics, commented by Adam *et al.* (2017). Each of these studies features a specific source in student learning, rather than a broad set of sources that encourage students to learn mathematics.

If on the one hand there are several sources through which students can independently learn or self-learn mathematics, making them active or protagonists in the classroom, our teaching experience in the mathematics teaching degree course has revealed that students depend excessively on teachers for this learning process - such that they expect teachers to play the leading role, transmiting content to be drilled or sometimes presenting doubts or questions that generate discussions/debates. This teaching and learning procedure is considered teacher-centered pedagogy, "favorable to mechanized and routine learning, not leading to the development of higher competencies" (Oliveira, Negreiros, & Neves, 2015, p. 1032). Therefore, teacher exposure, favorable to uncritical memorization and mechanization (Ponte, 2017), seems to resist other forms of teaching and other sources of learning.

Inspired by Hasibuan, Saragih, and Amry (2019), who note that no single source meets the various needs of the learning process, in this investigation, we sought to identify a general

set of 'external' sources or environments that contribute to student learning. In addition to the survey, we sought to characterize those sources and examine the relationships students established with them during the mathematics learning process. To this end, we formulated the following research question: What sources do students in the mathematics teaching degree course resort to, and what relationships do they establish with these sources to support their mathematics learning?

We believe that by answering this question, students can be inspired to trigger interventional actions that will expand the list of sources and improve the possibilities for relationships established with such sources for mathematics learning.

In the next section, we clarify what we understand by learning: the learning sources – especially in mathematics – and the concept of learning configuration.

2 Theoretical framework

Learning is a consensually important and complex process that has been conceived from various theoretical points of view throughout the ages, leading to theoretical blurring in its conceptualization (Illeris, 2009; Schunk, 2012).

Schunk (2012) points out differences in the causes, processes, and consequences of learning and adds that, epistemologically and philosophically, two tendencies are considered disjoint from conceiving learning, both of which establish the origin, nature, limits, and methods of knowledge.

For Illeris (2009) and Schunk (2012), some questions and trends can contribute to mark such inconsistencies, among them we relate the questions: "How can we know? How can we learn anything new? What is the source of knowledge?" (Schunk, 2012, p. 5) and rationalist (cognitivist) and empiricist (connectionist and/or behavioralist) tendencies.

As for the questions, we will, little by little, present evidence to answer them in this article. Regarding trends, the rationalist situates learning in the mind or in the cognitive structure. Illeris (2009) and Schunk (2012) assume such a position that we can complement with Schunk's (2012, p. 5) statement: "Knowledge derives from reason without resorting to the senses", and that the core of this trend is in the processing of information, in "its construction, acquisition, organization, coding, rehearsal, storage in memory, and retrieval or non-recovery of memory" (Schunk, 2012, p. 22).

In contrast, the empiricist (connectionist and/or behavioralist) tendency situates learning in experience, senses, or interaction with others, considering external or environmental factors and observable phenomena (Illeris, 2009; Schunk, 2012).

Schunk (2012) points out that these theoretical concepts and delimitations have implications for teaching practice, i.e., behavioral theories prescribe that teachers must organize the environment so that students can respond adequately to stimuli, and cognitive theories emphasize making learning meaningful and taking into account students' perceptions of themselves and their learning environments.

Recognizing the importance of the various approaches to learning, especially cognitive and behavioral approaches, Illeris (2009) proposed a comprehensive learning model, covering the broad field of learning in a structured way, including: the bases (psychological, biological, and social); the internal conditions (dispositions, age, and subjective situations); the external situations (place of learning, society, and objective situations); the applications (pedagogical and learning policy) that influence and are influenced by this process.

The Illeris model is composed, in an inclusive and integrated way, of external processes of the learner's interaction with the environment and of individual internal processes of acquisition. Interaction is the general basis and provides the impulses that trigger the learning process, and acquisition is specific and individual (Illeris, 2009).

According to the author, the processes of interaction and acquisition are related through a transitive communication that spans three dimensions of learning: environment, content, and incentive. Illeris (2009) represented the interactions between the learning dimensions using double arrows: the interaction between the environment and the learner with a vertical double arrow, and the interaction between acquisition (content—incentive) with a horizontal double arrow.
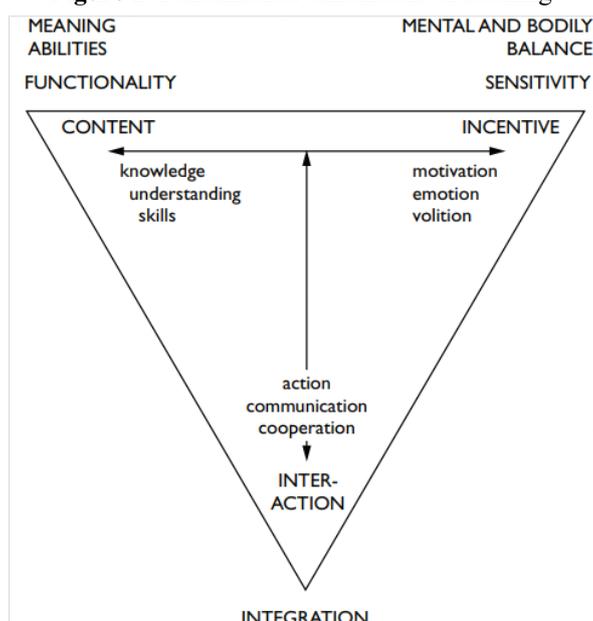
Figures 1 and 2 below represent the learning processes and dimensions established by Illeris (2009).

Figure 1: Fundamental learning processes
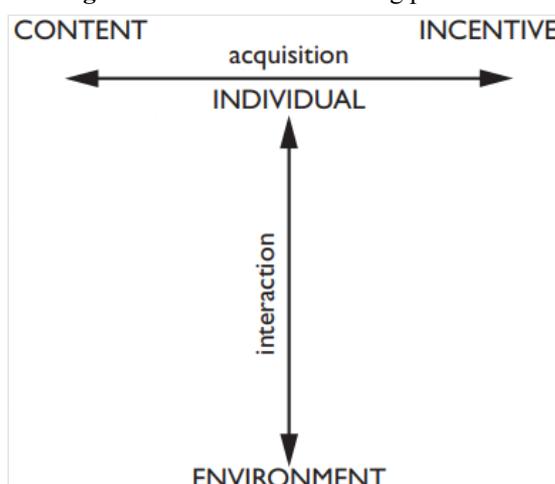
Source: Illeris (2009, p. 9)

Figure 2: Fundamental dimensions of learning

Source: Illeris (2009, p. 10)

The author explains that, in the dimensions of environment, actions, communication,

and cooperation, favoring occurs through interaction with the learner; the beginning of the learning process is the learner, which occurs in the form of perception, transmission, experience, imitation, activity, and participation. The interaction between the learner and the environment at this time is due to contact with learning sources, which can be understood as components of the learning environment that encourage students and collaborate with their learning.

The content dimension refers to what is learned: knowledge, understanding, skills, opinions, insights, meanings, capabilities, attitudes, values, forms of behavior, methods, strategies, etc. (Illeris, 2009).

Finally, the incentive dimension provides and directs the mental energy necessary for the learning process. It comprises elements such as feelings, emotions, motivation, and volition. Its ultimate function is to ensure the learner's continuous mental balance and, thus, "simultaneously develop a personal sensitivity" (Illeris, 2009, p. 10).

According to Illeris (2009), the dimensions of content and incentive are internal and mutually influence each other, and, based on Figures 1 and 2, we understand that the author generally articulated two major learning paradigms, assuming the sequence: stimulus — elaboration and acquisition — response.

This view corroborates current interpretations related to learning and didactic perspectives. Thus, the origin of stimuli that trigger the learning of knowledge surpasses the sources of the didactic relationship of the usual model of a standard classroom, characterized by uniting three "objects": the teacher, the teaching, and the knowing (taught) (Chevallard, 2013), which forms a didactic-pedagogical triangle (Arruda & Passos, 2017) and represents a relationship with knowing (Charlot, 2000) in formal education (Arruda & Passos, 2015).

The relationship with knowing, however, is "the subject's relationship with the world, with himself, and with others" (Charlot, 2000, p. 77). Therefore, it is essentially a relationship between the subject and the world, if we understand that the subject and others with whom they relate are part of it. Thus, the world (Charlot, 2000) is equivalent to the dimension of the environment (Illeris, 2009), which is full of multiple sources of actions, communication, and cooperation that favor learning processes. Take, for example, the case of mathematics.

Mathematics is considered and recognized as the "mother" or "basis" of science, technology, intellectual development, and also as an index of the evolution of civilization (Chang & Yang, 2016). According to Van Den Heuvel-Panhuizen and Drijvers (2020), Hans Freudenthal classified mathematics as a human activity. In turn, Hasibuan, Saragih, and Amry (2019) listed it as a means to think clearly and logically, solve everyday problems, recognize patterns, generalize experiences, be creative, and increase cultural knowledge. In contrast, mathematics is considered too difficult (Chang & Yang, 2016) and is therefore feared.

For most students, learning mathematics is not fun, but a nightmare (Ali, 2011). This scenario, in turn, serves as the starting point for searches and proposals of pedagogical-didactic alternatives aimed at supporting or complementing mathematics learning, one of which is the inclusion of learning sources in the teaching and learning processes.

The sources of learning that contribute to mathematics learning have increased significantly in recent years, especially with the emergence of digital technologies, as reported by Samuels and Williams (2010) and Adam *et al.* (2017). Also, in this argumentative movement, Hasibuan, Saragih, and Amry (2019, p. 244) conclude that "the use of learning materials provides good benefits in learning" and, among other benefits, list improvements in students' mathematical skills, the ability to understand mathematical concepts, and self-learning.

Maulana *et al.* (2021) identified that, in addition to the teacher and peers, students resort to printed texts (books/manuals, notebooks), audiovisual and/or technological material (videos, internet), etc. Regarding the internet, Adams *et al.* (2017) highlight its role in online and self-regulated learning.

Digital technologies have been gaining ground among students' preferences and cultivating students' interest in learning mathematics. Chang and Yang (2016), Adam *et al.* (2017), and Hasibuan, Saragih, and Amry (2019) provide information on this. According to Samuels and Williams (2010, p. 26), "adapting to contemporary students' preferences has led to a shift from textbooks to more accessible sources, such as webpages, brochures, and downloadable video clips." These sources have provided some cases of learning independence, self-study, and self-learning (Hasibuan, Saragih, & Amry, 2019).

Learning independence is different from self-study. While learning independence is related to personal interest (formulating questions and seeking answers through the explanation of others: teachers, friends, colleagues, etc.), self-study is related to autonomy and self-regulated learning (Adam *et al.*, 2017) or self-learning: involvement in activities and learning process without the services and explanations of teachers, friends, colleagues, etc. (Hasibuan, Saragih, & Amry, 2019).

Several researchers have shown the contribution of digital-technological sources in self-study and online learning of mathematics, namely digital videos (Amaral, 2013; Cardoso, Kato & Oliveira, 2014; Cardoso, Oliveira & Kato, 2015; Adam *et al.*, 2017), digital platforms (digital applications and games) (Chang & Yang, 2016; Adam *et al.*, 2017), and articles and other mathematics texts (Adam *et al.*, 2017; Maulana *et al.*, 2021).

Regarding digital videos, Amaral (2013) says: "Currently, the videos available on the internet, of small clippings of classes, are the most sought after by students, who watch them when they have any conceptual doubt" (p. 39). Cardoso, Kato, and Oliveira (2014) found that *YouTube* videos (from the researched channel) are more frequently searched during school evaluations, providing support for studies. These authors describe that the contribution of the videos goes beyond what the students observe in them, since "the students do not restrict their representations about the contents only in what the videos present, but rather interconnect such information with their own ideas" (Cardoso, Oliveira, & Kato, 2015, p. 36).

Digital applications and games are described as sources that aggregate "elements that maximize interactions, movements, fantasy, colors, and sounds, and instigate the new generation to remain immersed in a parallel world for hours, in a way, acquiring learning in the broadest sense" (Silva Brito & Camargo Sant'Ana, 2020, p. 420). Specifically, they stand out for triggering interest in mathematics and self-study, as well as for providing self-learning, self-correction, and increased learning effectiveness at their own pace, inside and outside the school space (Chang & Yang, 2016). Abreu and Andrade (2023) also found that the digital game used in their research enabled the observation of other ways of learning and applying mathematics and promoted the development of logical reasoning.

In these descriptions, we understand that digital games cover the basic components of learning: students' affective, psychomotor, and cognitive abilities. In addition, they can be used by the teacher as teaching resources and by the student as a source of self-study and self-learning of mathematical concepts.

The recognition of multiple learning sources gave rise to the concept of learning configuration, which refers to "all learning possibilities and environments, physical or virtual, whether formal, informal or non-formal" (Arruda & Passos, 2015, p. 11). The learning

configuration is a relationship of knowing, similar to the didactic-pedagogical triangle but broader, encompassing multiple sources of learning in addition to the teacher. Thus, its structure replaces T (teacher) by S (source of knowledge), as shown in Figure 3.

Figure 3: Learning setup
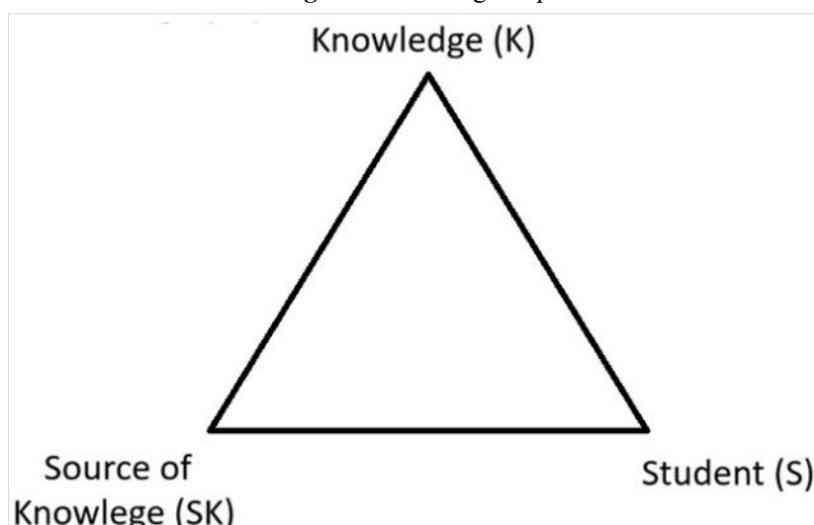
Source: The author/Adapted from Arruda & Passos (2015, p. 11)

In Figure 3, S (apprentice) is the subject who learns, K (knowing) is knowledge/information (subjective or objective) that is shared with the subject, and S (source of knowing) is the point (objective or subjective) of origin of the knowing, which can be people, real, mental, or sensory objects, a digital platform, an activity, etc. (Arruda & Passos, 2015). Therefore, the source of knowing can be any object in the environment, including those in the classroom, as long as it favors learning. Hasibuan, Saragih, and Amry (2019) corroborate the learning configuration, stating that "there is no single source of learning that can meet all types of needs in the learning process" (p. 244).

Adam *et al.* (2017) and Hasibuan, Saragih, and Amry (2019) present examples and results from their research on self-taught individuals who minimize or dispense with dependence on teacher services. Ribeiro *et al.* (2016) describe the existence of active methodological strategies, such as Peer instruction; Oliveira *et al.* (2018) state that in Team-Based Learning (TBL) there is active group and collaborative learning; Barbosa and Moura (2013, p. 55), in turn, conclude that, in active learning, "the student interacts with the subject under study – listening, speaking, asking, discussing, doing, and teaching". In both Peer instruction and TBL, the source of learning is the colleague, from whom one listens and with whom one discusses or does through teaching and learning. These methodologies are dialogical, and each member of the group is a source of learning for the other.

These methodological strategies oppose expository, dogmatic teaching, in which the teacher plays a central role and transmits content, and the student assumes a peripheral role as a mere receiver, although several active and dialogical methodological strategies flow into the didactic relationship: teacher, student, and knowing. In addition, both strategies emerged from student learning problems associated with ineffective teaching (Ribeiro *et al.*, 2016; Oliveira *et al.*, 2018).

Below, we discuss the methodological procedures adopted in this investigation.

3 Methodological procedures

This qualitative and descriptive research sought to characterize the sources of learning and the relationships that students (S — in the learning configuration) of a teaching degree course established with them to learn mathematics.

We assume, as Flick (2014, p. 5) indicates, that qualitative research deals with the "classification and interpretation of linguistic (or visual) material to make statements about implicit and explicit dimensions and structures of meaning construction in the material and what is represented in it". This leads us to consider that the analytical process in this research approach requires extracting the explicit and latent meanings of what is being researched. As a result, this research aimed to identify the sources of mathematics learning and to classify and interpret them, taking into account the dimensions of the learning model exposed by Illeris (2009), presented in Figure 2.

In practical terms, we characterize the sources of student learning based on the information of the students participating in the investigation, and the interpretation is based on the comparison and inference of this information, supported by the theoretical framework on learning (Illeris, 2009; Schunk, 2012) and the configuration of learning (Arruda & Passos, 2015).

The participants in this research were students of the mathematics teaching degree course at Rovuma University — Cabo Delgado Extension, who consented to participate via an invitation in the header of the online form used as the data collection instrument. Of the 102 students in this course, 29 from the 1st year, 25 from the 2nd year, 23 from the 3rd year, and 25 from the 4th year, 34 students joined the invitation, but eight of them were not considered eligible because they did not provide the requested information correctly. Thus, for the analysis, we focused on 26 students from four classes: 4 in the 1st year, 8 in the 2nd, 5 in the 3rd, and 9 in the 4th.

Due to the commitment to preserve the identity of the participants expressed in the invitation, they were identified as follows: Sn-t, where S is the initial of the word student, n ($n = 1, 2, 3, ...$) is the sequence of the participant in the respective class and t ($t = I, II, III, IV$) is the class (1st, 2nd, 3rd, or 4th year). Thus, S2-III is student 2 of the 3rd year.

The information was collected from October 20 to November 10, 2023, and included the completion of an online form that asked students to point out the learning resource and explain how it contributed to their learning by answering the following: (i) From which sources do you learn mathematics?, and (ii) Describe in detail how each of the sources helps you learn mathematics. Therefore, the questions were open, and the sources of mathematics learning emerged from the data collection process.

The process of data analysis and the production of results consisted of applying the procedures of content analysis (Schreier, 2014; Bardin, 2016) to their answers, henceforth, the *corpus*. We adopted content analysis because it allows us to describe the meaning of data, while offering the possibility of reducing information, systematizing, and increasing flexibility (Schreier, 2014; Bardin, 2016).

Content analysis basically involves three stages: coding, categorization, and interpretation (Schreier, 2014), or pre-analysis, exploration of the material, and treatment of the results, inference, and interpretation (Bardin, 2016). By comparing the descriptions of the stages for each author, we understand that the pre-analysis precedes the coding, and that the other stages of Bardin (2016) correspond to those proposed by Schreier (2014). Thus, the analytical exercise carried out essentially followed the stages proposed by Bardin (2016), being

broad and inclusive without ignoring the precepts of Schreier (2014).

In practical terms, during the pre-analysis, we did fluctuating readings of the students' material (students' answers in the form), defined participation criteria, selected the material to be considered for the analysis, and formally prepared it, making spelling corrections to make the excerpts understandable.

In the exploration phase of the material, we fragmented and codified the *corpus* information, in an exercise of identifying the learning sources and the relationships that students indicated, always in a recursive process of reading and interpreting what was exposed in the *corpus*. In this stage, we codify learning sources and synonymous relationships in the same way. For example, we identified the Book and the Manual with the same code.

In the treatment of the results, we categorized, classifying and grouping the learning sources according to their designations and relationships, using the same codes, and described them based on their characteristics and didactic relationships with the student. The categories were emerging, and the descriptions of the learning sources and the relationships manifested by the students were supported by excerpts recorded in the form of their answers and by graphs of learning networks built with the *Gephi*© software. The graphs consisted of arcs between students and the learning sources they assisted (graph nodes), forming the union. The arcs indicate the link between the student and the source of learning. Thus, the graphs allowed us to visualize the network of learning sources with which students related and how often they accessed them.

Subsequently, the inference and interpretation consisted of relating the learning sources to the students as apprentices, in a process of comparison and discussion, based on the concepts and characteristics of the learning described by Illeris (2009) and Schunk (2012) and the contribution of the sources in the mathematics learning (Amaral, 2013; Cardoso, Kato & Oliveira, 2014; Cardoso, Oliveira & Kato, 2015; Adam *et al.*, 2017), without neglecting other concepts about learning presented in a previous section of this article.

4 Students' relationships with learning sources

The analytical process carried out on the *corpus* yielded three categories, to which we allocated eight sources. In this section, we present and describe these sources, the relationships students manifested, and, finally, we discuss their influence on student learning.

As already indicated in the summary, three categories of learning sources emerged from this interpretative movement: human, textual, and technological-digital, in which we could allocate the eight sources highlighted by the deponents: in the first — teachers, colleagues, and relatives; in the second — books, articles, and Handouts; in the third — video classes and digital games, arranged in this order according to the frequency in the demonstrations.

In the human category, there are people and their influence on student learning, which can occur through direct or synchronous interaction, through which knowings are acquired and/or clarified. In the textual category, we have the sources in the form of texts, regardless of their physical nature (real or virtual). The influence of these sources on student learning is predominantly for clarifying doubts, but also includes the acquisition of new content or procedures. In the technological-digital category, dynamic sources are accessed and consumed exclusively via digital devices, such as computers, tablets, and smartphones. In addition to clarifying doubts and acquiring new content and procedures, they have the effect of "relaxing the mind" and help develop logical reasoning, as reported by the undergraduates.

In Chart 1 below, we present and describe each of the eight sources. In the first column,

we listed the sources; in the second, we described them based on what the students recorded in their answers; and in the third column, we included examples of manifestations to clarify the reader about our allocations and interpretations.

Chart 1: Sources of mathematics learning

Sources		Descriptions	Examples
Humans	Teachers	The person responsible for teaching in the classroom. Teachers are described as indispensable and primordial, with the function of explaining, mediating, facilitating, guiding, and/or validating student production.	*During a class on a specific topic, the teacher has contributed to my learning by providing explanations and practical examples of the subject* (S2-IV). *I learn mathematics through the teacher, to the extent that he explains (summarizing the theme to remember later), gives examples, and evaluates me* (S3-I).
	Colleagues	Students of the same course, current or former, with whom the student develops discussions, exchanges experiences, when flows other forms of knowledge, improves the understanding of the content and clarifies doubts arising from classes.	*Group studies with colleagues are one of the most important means for our learning, because during the studies, there are different points of view and thus a clear idea about a certain exercise or question is reached. Sometimes, during class, when a student detects a doubt, but does not have the opportunity to share it, when they leave, they can immediately clarify their doubts with a peer, if they have any. Explaining to colleagues helps us to enrich our knowledge; to the extent that we explain to someone, we better understand the subject* (S1-II).
	Relatives	The students' brothers, who explain to clarify doubts coming from the classroom.	*[...] my licensed brother helps me by clarifying the doubts or explanation of a certain subject* (S1-II).
Textual	Books	The main source of consultation, in which they carry out readings to enrich knowledge, deepen content, obtain procedures and solved exercises. In addition, the books present detailed information, several procedures related to the same content, and also allow the discussion and clarification of doubts.	*By reading books, I can understand the teacher's explanations better* (S1-I). *From the book, I can acquire knowledge, many works I do on such a basis; in addition, I solve exercises* (S4-III). *In specific circumstances, I learn from the resource of manuals. These contribute to my learning. It is known that there is a lot of diversity in the ways in which the same content is explained in different manuals. Some of them present clear procedures, that guide me in learning* (S2-IV).

	Articles	Scientific texts published in journals or specialized scientific journals used by students to study and clarify school contents.	*With the articles, I can say that I learn mathematics by looking at the different contexts that several teachers bring to a subject. I learned, for example, a very simple way to memorize the multiplication table and to extract roots from a number in a very simple way, without having to use much theory. I learned how to use Gauss Jordan's method in an article that talked about systems of equations, because in the books, it was too complex and I did not understand well how to proceed. As articles, they are much more didactic than the advanced calculus books that are necessary to lay the foundations for understanding more complex subjects. In general, the articles make it easier for me to see things differently, as they present ideas that a teacher uses to explain them to students in a very simple way* (S7-II).
	Handouts	"Collection of classes or lectures"[1] described as a material with many explanations and organized.	*There is a lot of explanation and organization* (S5-IV).
Technological-digital	Video classes	Moving images filmed in teaching environments and made available on *YouTube*. They are described as synthetic, facilitating, or explanatory of doubts and exercises, allowing visualizing, commenting, and receiveing feedback (if in doubt).	*They are very important to our learning, as they help us to understand the subjects in different ways. Much more when it comes to a work-to-present, often those topics that are new to us students, but with the YouTube videos feature, the student can understand that topic* (S1-II). *YouTube contributes significantly to learning, as I watch exercise resolution video lessons, thus facilitating my understanding of certain content* (S2-IV).
	Digital games	Digital mathematical games are described as playful sources of leisure and training of the mind for the development of logical reasoning.	*I use the mathematical games on the Playstore to learn more by having fun with friends or at times when I feel like just need to rest my mind, so I play some games that have to do with logical reasoning to create a habit in my mind* (S8-II).

Source: The authors

The descriptions and examples inserted in Chart 1 reveal that the relationships between the student and the source can be dialogical or dogmatic in terms of communication orientation. The dialogical didactic relations are bidirectional; the source supports the acquisition of knowledge in an interactive environment where the student can ask questions or challenge knowledge and obtain clarifications, both synchronously and asynchronously. These relationships occur with the sources of the human category, including the teachers responsible for the video lessons watched, who provide feedback after the student comments.

Dogmatic relationships are unidirectional, with knowledge flowing from the source to the student. It is not dogmatism in the sense of knowing how to present it as an absolute truth, but as a communication that does not allow the student to raise doubts about the provisional source of the information. This relationship occurs between the student and the sources present in the textual and technological-digital categories.

In the graphs that follow, we show the relationships between the student (red node) and

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[1]Houaiss electronic dictionary in Portuguese 3.0.

the learning sources (green node). Figure 4 shows the types of relationships (dogmatic or dialogical). Dogmatic relationships are represented by an edge connecting the student to the source of learning, and dialogical relationships by two edges. The relationship between colleagues is represented by red edges connecting students.

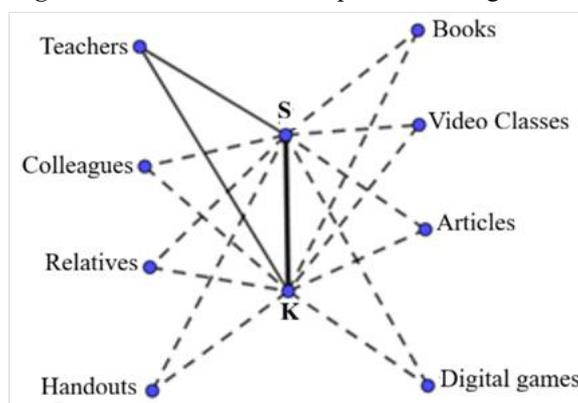**Figure 4**: Didactic relationship source of learning-student
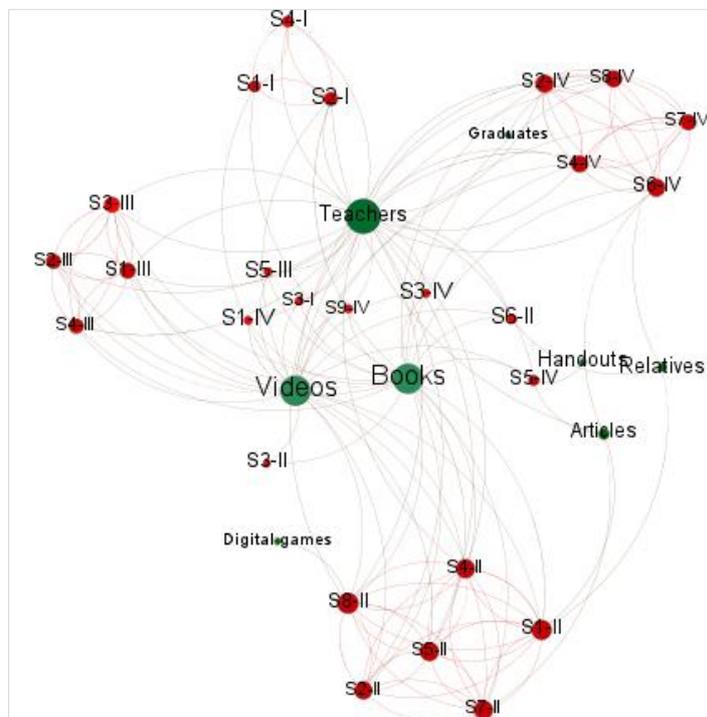
This graph shows that students rely on many sources of learning beyond the teacher. Some of these multiple relationships with learning sources occur in the same student and knowing, such that the student-knowing relationship becomes central to learning processes. Thus, the third component of the relationship with knowledge (S — source of knowing) aims to consolidate the central relationship.

Based on the didactic relationship established in the didactic-pedagogical triangle (Arruda & Passos, 2017), in Figure 5, we sketch a schema of student learning showing that a student and a knowing can form several didactic-pedagogical triangles, aggregating one or several sources of learning.

**Figure 5**: Student's relationship with learning sources

In Figure 5, the didactic relationship teacher, student and knowing, which we represent with continuous line edges, is infallible in the investigated context. The other relationships represented by dashed lines are alternative or complementary to the relationship with the teacher, especially when it does not effectively carry out student learning, and the student is active and/or self-taught. Therefore, the learning configuration aggregates and summarizes several didactic-pedagogical triangles, with variation of the source of knowing.

When analyzing the list of learning sources with which students indicated establishing relationships, we generated the graph shown in Figure 6, in which node size is proportional to the frequency of use of each source by students. To understand the dimension of peer preference, we represent peers as nodes in this graph.

Figure 6: Frequency with which learning sources are accessed
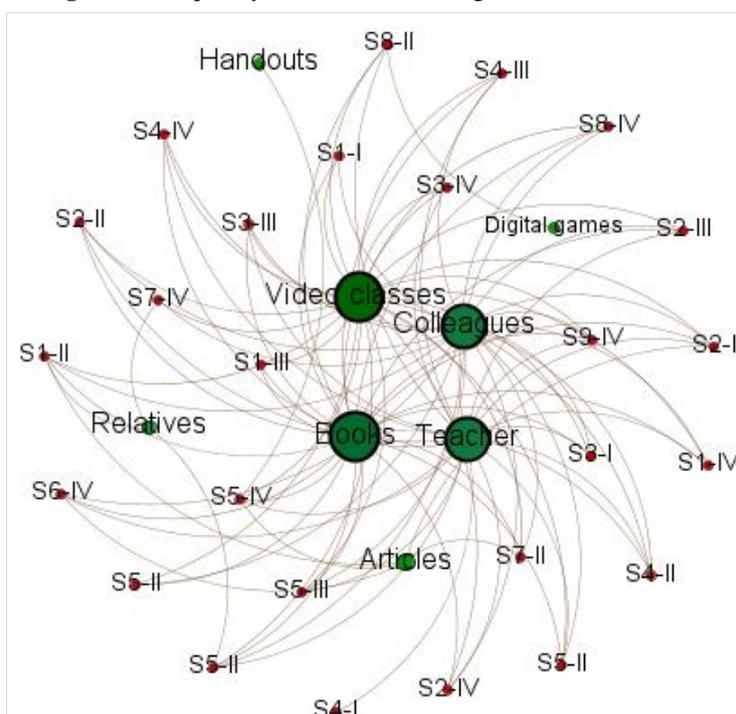
Source: The authors

The graph shows that students' most used learning sources were teachers, colleagues, books, and video classes.

The existence of external sources that intervene and collaborate with students' learning corroborates the learning field model (Illeris, 2009) and Schunk's (2012) conception of learning, which understands learning as a dialogical process between external and internal processes of interaction and acquisition. It also corroborates the learning configuration presented by Arruda and Passos (2015), who also postulate the existence of more than one learning source (Hasibuan, Saragih, & Amry, 2019).

Although all sources can intervene in the content dimension, it was evident that the teacher is the main source of learning, the content provider, and incentive mobilizer to build, organize, understand, store knowings, and develop mathematical skills. In the content dimension, the teacher's contribution to student learning is through explanation and validation, which are clarifying and structuring acts.

The teaching incentive is initiated by tasks or assessments that lead students to get involved through activities that allow them to rehearse, diagnose their knowings and skills, and

store knowledge. In addition, the ineffectiveness of teaching is a driving force that leads the active student to resort to other sources of learning to clarify and/or organize knowing, in a process of independent learning (Adam *et al.,* 2017) or self-study and self-learning (Adam *et al.*, 2017; Hasibuan, Saragih, & Amry, 2019).

Tasks (exercises or presentations in the form of a seminar) play a crucial role in learning mathematics, both as prompts for seeking learning resources and as sources of individual or collective testing and discussion, which foster understanding and learning of new mathematical content or procedures.

The recourse to other sources is primarily aimed at clarifying or filling gaps arising from teaching and the preparation of seminars, which occur through observation, debate, reading, or practice. Among these sources, videos, colleagues, books, handouts, and digital games stand out, providing clarification and understanding of mathematical concepts, opportunities for self-learning, and opportunities to acquire new knowledge and develop skills (Hasibuan, Saragih, & Amry, 2019).

The relationship between student, colleagues, and knowing is the one that most explains the active nature of the student, as there are active methods and learning of the peer instruction type (Ribeiro *et al*., 2016) or team-based learning (Oliveira *et al*., 2018), characterized by the occurrence of discussions or debates about the knowing to be learned (Barbosa & Moura, 2013).

Videos, handouts, and digital games confirm the contribution of the internet in enabling students to access these learning sources (Adam *et al*., 2017; Samuels & Williams, 2010). The videos and handouts serve as sources for consultation and clarification of doubts, as well as for the preparation of seminars and works, due to their expository and organized nature. Videos, in particular, are very popular, as Cardoso, Kato, and Oliveira (2014) found.

Digital games promote the development of logical reasoning, and their use comes from students' interest and initiative (Chang & Yang, 2016; Abreu & Andrade, 2023), who also use them as sources of leisure with the potential to "rest the mind" or "reduce tension" or the comprehension of "demanding mental activities". Thus, for these students, digital games are sources of self-study and self-learning that favor the understanding of mathematical concepts in pleasant contexts.

5 Final considerations

In this research, we investigated the sources of learning that mathematics teaching undergraduates access and the relationships they establish with such sources to learn mathematics. The respondents were enrolled in the 1st to 4th years of the mathematics teaching degree course at Rovuma University, Cabo Delgado Extension, in Mozambique.

The research question we set out to answer was: What sources do students in the mathematics teaching degree course resort to, and what relationships do they establish with these sources to support their mathematics learning?

The analytical process identified eight sources of learning grouped into three categories: human (teachers, colleagues, and relatives), textual (books, articles, and handouts), and technological-digital (videos and games). These results are close to those indicated by Maulana *et al*. (2021), who found that students of the mathematics teaching degree course use colleagues, articles, books, notebooks, videos, and the teacher to solve questions of a mathematics test after taking it, indicating that these sources are part of the academic culture of the students of that course and institution.

Among the results, we see that students relate to the sources they listed in diverse ways, both in terms of preference and in terms of the essence of the bond. The diversity and multiplicity of learning sources make the student's relationship with knowledge a learning configuration (Arruda & Passos, 2015).

Humans, and therefore the sources allocated in this category, were those who most intervened in student learning, and their relationship was dialogical. From these sources, the teacher was pivotal and primordial in the school context, because he introduced the content, exposing it to the students, thus acting both as a provider of knowledge, understanding, and skills, as well as triggering interest in mathematics learning. It was evident that interest and skill development were specifically triggered by the tasks the teacher assigned to students or by their ineffectiveness, which raised doubts and questions, leading students to turn to other sources.

The intervention of other sources, especially when it resulted from the ineffectiveness of teaching work, generated new relationships with other learning sources that are complementary or alternative to the teacher-student relationship, whose central axis is the student's relationship with mathematics, that is, student learning (Arruda, Passos, & Broietti, 2021).

The students' relationship with the sources allocated in the textual and technological--digital categories was dogmatic. Although complementary, they also provided learning with new content or skills, which resulted from the search for clarification, sometimes through other procedures.

These results corroborated the lack of a single source that meets all student learning needs (Illeris, 2009; Hasibuan, Saragih, & Amry, 2019) and revealed a student dependence on physical sources other than technological-digital ones that are underused, as indicated by the aforementioned authors, and remained so in the investigated case, although videos have a considerable predominance. Moreover, these students' learning can be considered predominantly dependent on explanations from teachers, colleagues, and videos, although they are active enough to raise doubts and questions that led them to resort to and learn from other sources. Therefore, these students were predominantly independent learners rather than self-students (Adam *et al.*, 2017) or self-learners (Hasibuan, Saragih, & Amry, 2019).

The actual contribution of sources and the relationships that are established with them in mathematics learning is an object to be investigated, but it was clear that the dependence and/or preference for the teacher gives it the central role in mathematics learning, which requires proactivity to teach with zeal and, above all, to indicate and encourage students to explore other sources of mathematics learning that are underused.

Finally, we suspect that, in the coming years, artificial intelligence (AI) will become increasingly relevant as a source of learning, both for students and teachers.

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