Sociedade Brasileira de Educação Matemática

RIPEM

Computational Thinking in Mathematical Modeling Practices from the Perspective of Meaningful Learning Theory

Márcia Regina Kaminski
Universidade Estadual do Oeste do Paraná
Cascavel, PR — Brasil
✉ marciarkjf@gmail.com
0000-0001-5705-0322

Clodis Boscarioli
Universidade Estadual do Oeste do Paraná
Cascavel, PR — Brasil
✉ boscarioli@gmail.com
0000-0002-7110-2026

Tiago Emanuel Klüber
Universidade Estadual do Oeste do Paraná
Cascavel, PR — Brasil
✉ tiagokluber@gmail.com
0000-0003-0971-6016

2238-0345

10.37001/ripem.v15i3.4452

Received • 16/01/2025
Approved • 31/07/2025
Published • 01/09/2025

Editor • Gilberto Januario

Abstract: This article presents results that answer the research question: What aspects of Computational Thinking development emerge from the practice of modeling in Mathematics Education from the perspective of Meaningful Learning Theory? Grounded Theory was used to generate and analyze data obtained from 4th and 5th grade elementary school lessons at a municipal educational institution in Cascavel-PR. The resulting theory led to five concepts that explain the relationships between Meaningful Learning and Computational Thinking development skills in practices involving Mathematical Modeling, which were developed in interaction with the principles of Meaningful Learning Theory. This highlighted the centrality of Abstraction and the need for pedagogical practices that mobilize it when it comes to developing these processes, with Mathematical Modeling being a viable avenue for this purpose.

Keywords: Grounded Theory. Computational Thinking. Mathematical Modeling. Meaningful Learning Theory.

Pensamiento Computacional en Prácticas de Modelado Matemático desde la Perspectiva de la Teoría del Aprendizaje Significativo

Resumen: Este artículo presenta resultados que dan respuesta a la pregunta de investigación: ¿Qué aspectos del desarrollo del Pensamiento Computacional emergen de las prácticas con Modelado en Educación Matemática desde la perspectiva de la Teoría del Aprendizaje Significativo? Se adoptó la Teoría Fundamentada para la producción y análisis de datos resultantes de clases desarrolladas en los cursos de 4º y 5º año de la Enseñanza Fundamental de una institución educativa de la Red Municipal de Cascavel-PR. La teoría emergente resultó en cinco conceptos que explican las relaciones entre el Aprendizaje Significativo y las habilidades para el desarrollo del Pensamiento Computacional en prácticas con Modelado Matemático desarrollado en interacción con los principios de la Teoría del Aprendizaje Significativo, destacando la centralidad de la Abstracción y la necesidad de acciones pedagógicas. Prácticas que la movilicen, cuando el objetivo sea desarrollar estos procesos, siendo la Modelación Matemática un camino viable para tal fin.

Palabras clave: Teoría Fundamentada. Pensamiento Computacional. Modelado Matemático. Teoría del Aprendizaje Significativo.

Pensamento Computacional em Práticas de Modelagem Matemática na Perspectiva da Teoria da Aprendizagem Significativa

Resumo: Este artigo expõe resultados que respondem à indagação de pesquisa: Que aspectos do desenvolvimento do Pensamento Computacional emergem de práticas com Modelagem na Educação Matemática na perspectiva da Teoria da Aprendizagem Significativa? A *Grounded Theory* foi assumida para a produção e análise dos dados decorrentes de aulas desenvolvidas em turmas de 4º e 5º ano do Ensino Fundamental I de uma instituição pública de Cascavel-PR. A teoria emergente resultou em cinco conceitos que explicitam as relações entre a Aprendizagem Significativa e as habilidades para o desenvolvimento do Pensamento Computacional em práticas com Modelagem Matemática desenvolvidas na interação com os princípios da Teoria da Aprendizagem Significativa, evidenciando a centralidade da Abstração e a demanda de práticas pedagógicas que a mobilizem, quando o objetivo é o de desenvolver habilidades para o Pensamento Computacional e facilitar a Aprendizagem Significativa, sendo a Modelagem Matemática um caminho viável para esse fim.

Palavras-chave: *Grounded Theory*. Pensamento Computacional. Modelagem Matemática. Teoria da Aprendizagem Significativa.

1 Introduction

Among the challenges of including Computational Thinking (CT) in education, particularly in Mathematics Education (ME) is the need to clarify its relationship with other areas of knowledge and to choose pedagogical practices that favor its inclusion from this perspective (Navarro & Sousa, 2023; Valente, 2016; Vieira, Santana, & Raabe, 2017).

Based on a study of CT implementation in education reported by Kaminski, Klüber, and Boscarioli (2021) and a literature review, we identified the need for a theoretical-methodological framework to support CT development with other areas of knowledge. Based on these results, we propose the integration of CT, Mathematical Modeling (MM) in Mathematics Education, and the Theory of Meaningful Learning (TML) as a means to this end. This is not arbitrary; there is evidence that problem-solving principles are necessary for developing CT (Navarro & Sousa, 2023), and these principles are present in MM practices and TML (Burak & Aragão, 2012).

With this understanding, pedagogical practices were planned and developed with 4th and 5th grade students from a public school in Cascavel-PR during the year 2022. This field research, approved by the Human Research Ethics Committee of the State University of Western Paraná (Unioeste), under Opinion n.º 3.490.463 dated August 7, 2019, allowed us to theorize about the following question based on the students' performance during the developed practices: what aspects of CT development emerge from practices with Modeling in Mathematics Education from the perspective of TML?

In this article[1], we briefly explain how we understand CT, MM, and Meaningful Learning, and present an analysis of the data produced from the perspective of Grounded Theory (GT) (Charmaz, 2009) and the emerging theory of data in light of the articulation of the three areas studied.

[1] This article is part of a doctoral thesis defended in the Postgraduate Program in Science Education and Mathematics Education (PPGECEM) at the State University of Western Paraná (Unioeste), organized in multipaper format, written by the first author, supervised by the second author, and co-supervised by the third author (Kaminski, 2023).

2 Computational Thinking, Mathematical Modeling, and Meaningful Learning

CT is generally understood as the application of computing principles to problem solving in multiple areas (Wing, 2006). These principles involve mental processes that are subject to different interpretations regarding what is inherent to CT or unique to computing. Four processes are widely mentioned in the literature as being associated with CT: abstraction, decomposition, pattern recognition, and algorithms.

Abstraction involves isolating the essential elements of a problem. Decomposition involves dividing the problem into parts to facilitate its resolution. Pattern recognition involves identifying similarities with problems that have already been solved. Algorithms involve creating a sequence of steps to solve the problem (Brackmann, 2017).

Teaching these skills in education has been discussed in several countries and has received special attention in Brazil since the resolution that made computer science compulsory in schools was approved (Brasil, 2022). This resolution complements the learning rights of students in basic education, as defined in the National Common Core Curriculum (BNCC) (Brasil, 2018). While these documents provide examples of applying these skills, they do not discuss how to integrate them with other areas of knowledge. This is a relevant point since problem solving involves multiple areas. Given the complexity of the current reality in which digital technologies are embedded, teaching must consider these interrelationships and avoid fragmented approaches (Morin, 2005).

Considering this, we investigated how the teaching methodology of MM can link the development of CT to mathematics. Based on the work of Burak and Aragão (2012), we incorporated MM into our dialogue with TML because understanding the learning process is essential for developing strategies that promote meaningful learning.

According to David Ausubel, meaningful learning occurs when new information takes on meaning by interacting with the student's existing knowledge, modifying it and expanding the student's cognitive structure (Moreira, 2018). This alteration can occur in three ways: subordination, superordination, and combination (Ausubel, 2003).

Based on this, we conclude that meaningful learning requires skills related to the development of CT. Subordinating new information to prior knowledge involves progressively differentiating meanings. Superordinating involves reorganizing and recombining these meanings (Ausubel, 2003). Both processes require abstracting information, decomposing and composing ideas and meanings, and identifying patterns.

According to Moreira (2018), meaningful learning occurs through the three types of learning proposed by TML: representational (assigning meaning to symbols), conceptual (assigning meaning to symbols representing objects with common properties), and propositional (forming ideas from concepts). These processes require abstraction to assign meanings and pattern recognition to identify common properties.

TML's principles highlight the relationship between meaningful learning and language. Language can favor or hinder meaningful learning, depending on how the learner interprets it (Masini, 2008). Language is essential to learning, and so is abstraction, given their close connection (Abbagnano, 1998).

Meaningful learning involves mental processes aligned with the skills associated with CT, and it is important that students develop these skills to promote this type of learning. In mathematics, these skills can facilitate learning because, according to Navarro and Sousa (2023), they enhance mathematical thinking in problem solving.

It is up to professors to mediate the teaching process by seeking methodologies that mobilize mental processes favorable to meaningful learning. Among the possibilities discussed in the literature, problem-solving-based methodologies stand out because they promote this type of learning by requiring students to reorganize their prior knowledge (Costa, 2008; Assunção, Moreira, & Sahelices, 2018; Moura Junior & Alves, 2023; Puhl, Müller, & Lima, 2020).

Based on this principle, one might believe that CT alone is a sufficient problem-solving strategy to facilitate meaningful learning. However, CT is not characterized as a methodology for intentionally organizing teaching based on epistemological, psychological, and pedagogical assumptions. Furthermore, especially in Mathematics Education, it is necessary to intentionally employ problem-solving methodologies that stimulate the development of mathematical thinking (Navarro & Sousa, 2023, p. 86), which goes beyond the skills associated with CT.

We agree that MM is consistent with the principles of meaningful learning, as argued by Burak and Aragão (2012), Moura Junior and Alves (2023), and Souza (2021), since it focuses on problem solving. Modeling practices have the potential to foster the interpretation, classification, ordering, analysis, and synthesis of data using different languages and abstractions. These are relevant actions when one wishes to develop CT skills (Navarro & Sousa, 2023). Thus, we understand that MM encourages students to use skills to develop CT while solving problems.

At the same time, MM can facilitate meaningful learning of mathematics by enabling the development of content that emerges from problems. This allows students to understand the world through mathematics. The modeling stages favor this approach because they require students to study and research the problem, which may involve knowledge beyond mathematics itself (Burak & Aragão, 2012). Thus, there are convergences between MM and some principles of TML. MM is taken as a guiding axis for pedagogical practices that give students the opportunity to develop CT by exercising their skills in problem situations and mobilizing these skills to develop thinking and learn mathematical concepts in a meaningful way.

From this perspective, MM can be characterized as a two-way street because it enables students to improve their mathematical thinking and enhance their CT development skills in a dialectical process.

In this section, we present CT, MM, and TML together because they were analyzed in conjunction, not in isolation, in the research. Armed with these theoretical reflections, we conducted a field study to investigate which skills emerge from modeling practices developed from the aforementioned perspective, as detailed in the next section.

3 The procedures of *Grounded Theory*: production and analysis

Theoretical analyses using Grounded Theory (GT) methods are based on data collected from observations, interactions, and reflections in the study environment (Charmaz, 2009, p. 19).

Considering that GT is epistemologically aligned with the mode of data production and the tension between the triad that we constitute as the object of study (CT, MM, and TML), we adopted GT in our study. In other words, it is necessary to understand the actions of students participating in the classroom process. Therefore, the data emerges from situations that can occur in classes where modeling practices are developed from the Meaningful Learning perspective, opening up the possibility of theorizing themes that depend not only on theory but also on students' actions.

Field research took place from September to December 2022 with fourth and fifth grade classes from a school in the Municipal Network of Cascavel, Paraná, Brazil. Table 1 shows the organization of the classes and the age profile of the students, who represent all classes in these grade levels at the school. We chose classes from the final years of elementary school because the students demonstrate greater cognitive maturity and literacy skills, which facilitates data production and recording.

Table 1: Classes participating in data production

<table>
<tr><th rowspan="2">Class</th><th rowspan="2">Period</th><th rowspan="2">Number of students</th><th colspan="2">Age group</th></tr>
<tr><th>Age</th><th>Qty.</th></tr>
<tr><td rowspan="4">4th grade</td><td rowspan="4">Regular morning</td><td rowspan="4">26</td><td>9</td><td>12</td></tr>
<tr><td>10</td><td>11</td></tr>
<tr><td>11</td><td>2</td></tr>
<tr><td>13</td><td>1</td></tr>
<tr><td rowspan="3">5th grade</td><td rowspan="3">Regular morning</td><td rowspan="3">28</td><td>10</td><td>20</td></tr>
<tr><td>11</td><td>7</td></tr>
<tr><td>13</td><td>1</td></tr>
<tr><td rowspan="2">4th grade</td><td rowspan="2">Regular afternoon</td><td rowspan="2">27</td><td>9</td><td>22</td></tr>
<tr><td>10</td><td>5</td></tr>
<tr><td rowspan="3">5th grade</td><td rowspan="3">Regular afternoon</td><td rowspan="3">27</td><td>10</td><td>17</td></tr>
<tr><td>11</td><td>9</td></tr>
<tr><td>12</td><td>1</td></tr>
</table>

Source: Research data.

The practices took place during weekly computer classes, during regular school hours. This allowed the field research to be supported by the teaching practice itself within the "habitual" dynamics of the classes. It is important to note that we did not investigate the practices themselves, which were merely vehicles for producing research data in a non-simulated context. To record the data for later GT-based analysis, the classes were video recorded, resulting in 32 hours and 40 minutes of footage, as shown in Table 2.

Table 2: Number of classes and recording time per class

Class	Number of classes	Class duration	Total recording hours
4th Grade – Class 1	12	40min	8h
4th Grade – Class 2	13	40min	8h40min
5th Grade – Class 1	12	40min	8h
5th Grade – Class 2	12	40min	8h
Total	**49**		**32h40min**

Source: Research data.

We imported these videos into the Atlas.ti[2] software to organize the analysis through data coding and categorization without transcribing them, which was unnecessary for our grounded theory construction since transcription occurred throughout the recording and analysis process.

In the next section, we present a brief description of the modeling practices developed.

3.1 The Development of Modeling Practices for Data Production

The practices were developed based on the five steps advocated by Burak and Aragão (2012, p. 89): "1) Choose a theme. 2) Do exploratory research. 3) Identify the problem(s). 4)

[2] The version of the *software used* was duly licensed for the Postgraduate Program in which the research was conducted.

Solve the problems. Develop content within the context of the theme. 5) Critically analyze the solution(s)."

First, we chose a theme based on the students' interests. According to the authors, the professor should assist by providing input that contributes to the group's deliberation. To encourage the learners in this decision-making process, we asked them, "What problems exist in the school environment?"

After this prompt, the students had one week to observe the school environment and record issues they considered problems, that is, difficult or uncomfortable situations. When they brought their records to the next class, we discussed the situations pointed out by each class. Then, we voted on the following themes: 1) "Toilet paper balls on the bathroom ceiling" and 2) "Neglected (ugly) garden." Two classes chose each theme, without interference from the research professor.

Next, we moved on to the exploratory research stage for the selected topics. In both cases, the students searched the internet for information to better understand each topic. They highlighted what they considered important, such as the origin of toilet paper and ways to care for a garden. These issues were then discussed collectively to guide the exploratory research.

The problem identification stage was based on the groups sharing the results of their exploratory research. Each group presented the information they found, and through this dialogue, the problems related to each topic were defined. For Topic 1, the problem was defined as, "How much paper and money is wasted with the balls thrown in the bathroom?" For Topic 2, the problem was defined as, "How is the garden being cared for, and what can be improved?"

These problems required data collection. For the first theme, the instrument constructed collectively was a table to record the number of "paper balls" thrown in each bathroom of the school over the course of a week. For the second theme, students interviewed the principal, coordinator, and professor responsible for the garden project to determine if someone was assigned to care for the school garden and identify the challenges involved.

In the stage corresponding to problem solving and developing mathematical content within the context of the theme, we estimated the amount of paper wasted and the cost involved for periods beyond collection in the case of the "balls." To accomplish this, we conducted new research and developed graphs, tables, and calculations using a spreadsheet editor. After the groups discussed the different solutions they found for this estimate, they organized a campaign with posters and a collective video for the class. The campaign included guidelines on the consequences of paper waste and how to use this resource consciously.

Regarding Theme 2, the main issue was maintaining garden irrigation. The students suggested building an automatic sprinkler system, which required researching the necessary equipment and developing the appropriate programming. An undergraduate in Computer Science supported the structural and electronic assembly of the equipment in Scientific Initiation, while the groups developed the programming with the researcher's mediation using the MBlock software. Details of this process are beyond the scope of this article but can be found in the thesis (Kaminski, 2023).

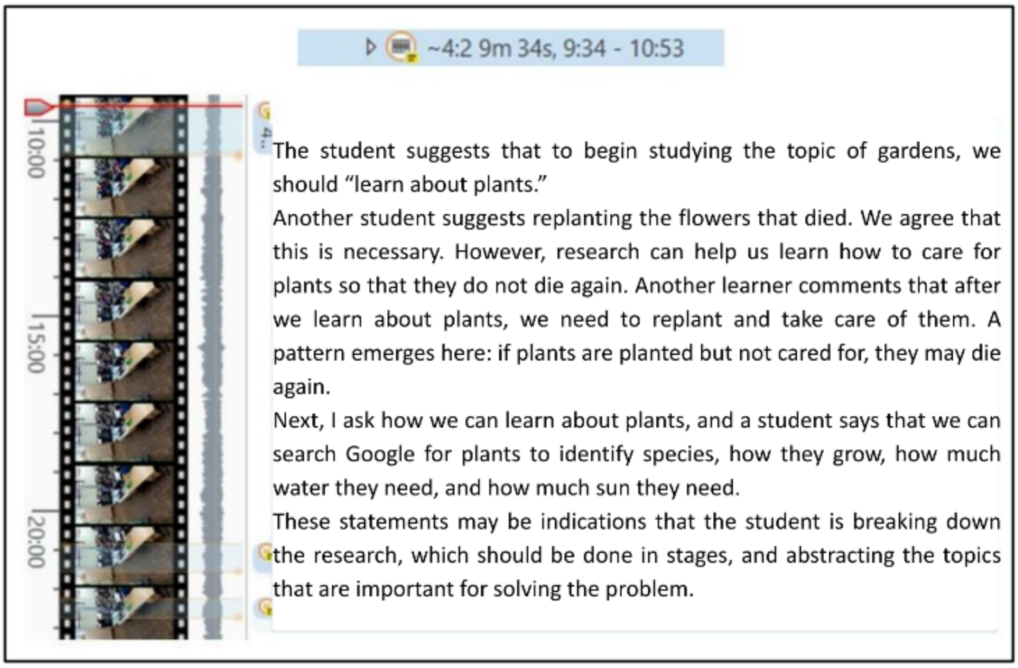
3.2 The process and results of the analyses

In GT, emerging theory is constructed simultaneously with the production and analysis of data (Charmaz, 2009). At the end of each class, the videos were imported into Atlas.ti. Using the software's "memos" tool, a memo was written to record the organization of the class, noteworthy situations (e.g., comments, interactions, and student questions), and the researcher's initial interpretations, insights, and analytical notes regarding the data. According to Charmaz (2009), writing memos is a "fundamental intermediate step between data production and the writing of research reports [...] because it encourages data analysis from the beginning of the investigation" (p. 107).

Next, we watched the videos and highlighted excerpts of episodes (moments during classes) in which situations occurred that revealed the need for or use of skills for developing CT in the students' attitudes during the modeling practice.

We highlighted these excerpts or episodes using the "free quote" tool in Atlas.ti. After selecting an excerpt, we inserted a comment that highlighted the situation and the corresponding CT-related skill. Figure 1 illustrates an example of a free quote created for one of the classes.

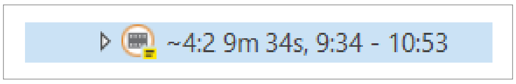
Figure 1: Example of free citations



Source: Research data.

The software generates an identification code for each created citation. Figure 2 shows an example of one of these codes. The initial part (4:2) indicates that this is the second citation from primary document (video) number 4. The rest of the code (9:34–10:53) shows the start and end times of the video clip (primary document) number 4 that was used to create this citation.

Figure 2: Example of free citation coding

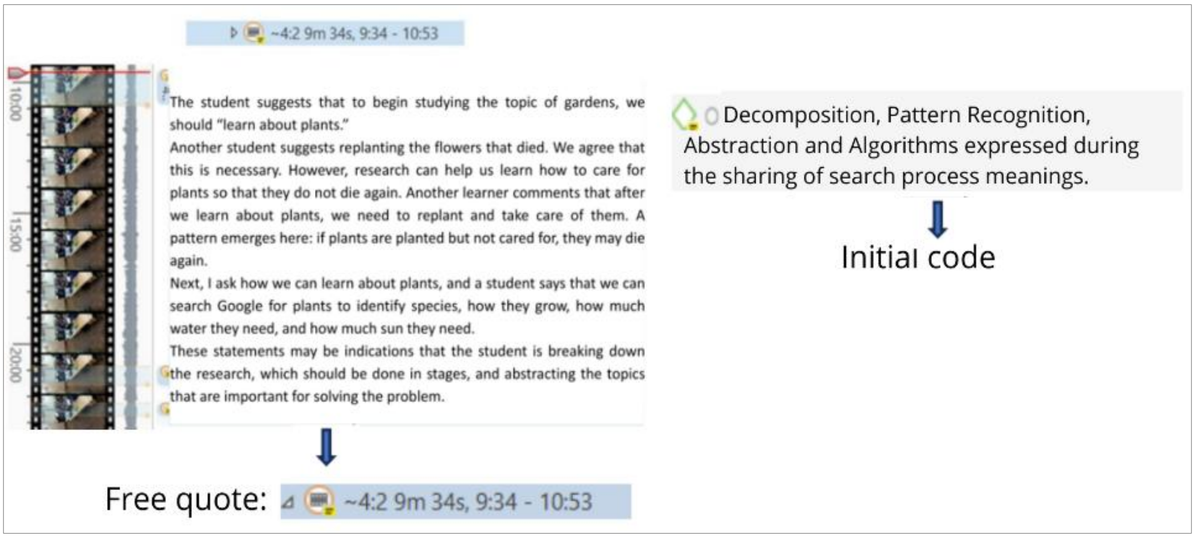


Source: Research data

We arrived at 256 citations following this process. These are 256 episodes in which we could identify the skills for CT development that emerged at different stages and moments of the modeling practice based on the students' statements and actions. Based on these citations, we coded the highlighted episodes, considering the four main processes associated with CT. We assigned a code to "each word, line, or segment of data" (Charmaz, 2009, p. 72). This stage corresponds to the researcher's initial coding, as described by the author.

Our codes were designed to highlight the CT development skills identified in the situations highlighted in the free quotations. We considered the MM stage and the TML principles involved in the process. In the coding stage, we sought to articulate the elements studied in the sense of an emerging theory. We understand that these elements are not independent but occur as related processes in teaching practice. We emphasize that this process required significant analytical effort because the relationship between the elements was not "ready" in the text. This required a dialectical movement between the theoretical triad and the students' actions. Figure 3 exemplifies the code applied to the free quotation in Figure 1.

Figure 3: Example of initial coding created for free citation



Source: Research data.

This code was developed based on an analysis of the students' discourse when suggesting relevant steps and themes for conducting the research. We identified two themes: Decomposition and Abstraction. The students revealed their understanding that knowing the subject is important since replanting flowers without knowledge of the subject could lead to their death. In other words, they recognized that repeating an action can produce a consistent outcome. Additionally, one student demonstrated algorithmic thinking by describing the sequence of steps as research, replant, and care. These skills were expressed in the dialogue between the teacher and students during the exploratory research stage of the modeling practice, when meanings were shared. This code highlights the CT development skills identified in the situation (decomposition, pattern recognition, abstraction, and algorithms), the TML principle involved in the process (sharing meanings), and the stage of the modeling practice in which it occurred (exploratory research).

After applying this methodological analysis procedure to all the highlighted episodes, we arrived at 96 codes, some of which appeared in different free citations. After identifying the predominance of Abstraction in the initial coding, we moved on to axial coding. According to Charmaz (2009), the objectives of axial coding are "to classify, synthesize, and organize large amounts of data and regroup them in new ways after open coding" (p. 91). At this stage, we identified convergent aspects among the 96 codes, grouping those that recorded the same skills for CT development at different stages of the modeling practice and in interaction with different TML principles.

For example, the code generated for the situation presented in Figure 3 became part of a group of codes identified in other episodes in which the skills of abstraction, decomposition, pattern recognition, and algorithms were also identified. This group was named "Abstraction with Decomposition, Pattern Recognition, and Algorithms" and encompasses three codes referring to different episodes. Figure 4 illustrates the codes that comprise this group. The number in parentheses before a group's name indicates how many initial codes were inserted into it. The number in parentheses before each code indicates how many times it was used to code different episodes. This analysis was performed on all codes to regroup them according to the CT development skills they highlight.

Figure 4: Example of coding grouping

- Abstraction with Decomposition, Pattern Recognition, and Algorithms (3)
  - Decomposition, Abstraction, and Pattern Recognition for Algorithm Production Expressed During Solution Meaning Negotiation {11–0}
  - Decomposition, Abstraction, Pattern Recognition, and Algorithms in the search for solutions expressed during the negotiation of solution meanings {5-0}
  - Decomposition, Pattern Recognition, Abstraction, and Algorithms expressed during the sharing of meanings of the research process {1-0}

Source: Research data.

Figure 5 shows the 18 groups of codes that constitute the initial categories generated from this regrouping in axial coding. Ten of these codes were included in two different groups.

Based on these groups of codes, we performed focused coding, which allowed us to establish relationships between the generated categories, creating a guiding axis for the analysis and interpretation of the data around a recurring central element. We created networks by linking groups of codes related to each other based on a common element.

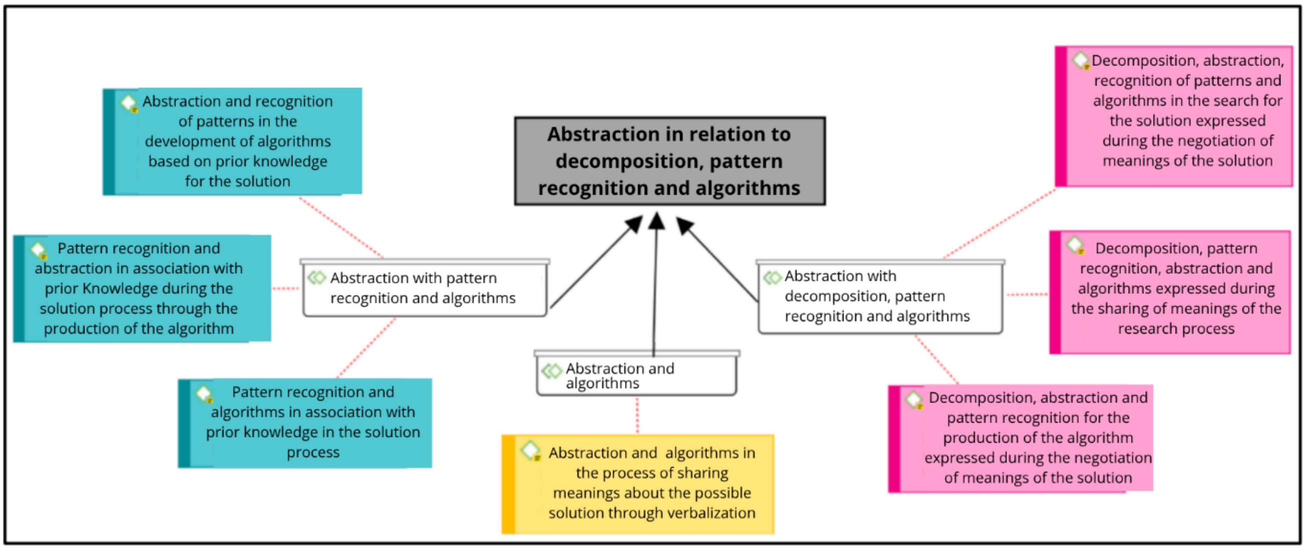
Figure 5: Code groups (initial categories) created during axial coding

- Code Groups
  - Abstraction with Decomposition and Algorithms 〈2〉
  - Abstraction with Decomposition and Pattern Recognition 〈2〉
  - Abstraction with Decomposition and/or Composition 〈15〉
  - Abstraction with Decomposition, Pattern Recognition and Algorithms 〈3〉
  - Abstraction with Pattern Recognition 〈7〉
  - Abstraction with Pattern Recognition and Algorithms 〈3〉
  - Abstraction and Algorithms 〈1〉
  - Abstraction and assignment of meanings to words 〈Semantic Awareness〉 〈8〉
  - Abstraction and Language 〈2〉
  - Abstraction and Mental Models 〈4〉
  - Abstraction and/in research 〈3〉
  - Abstraction and/in the process of analysis and interpretation 〈13〉
  - Abstraction in Decomposition and/or Composition 〈7〉
  - Abstraction in the development of mathematical content 〈9〉
  - Abstraction in Pattern Recognition 〈6〉
  - Difficulties involving abstraction 〈8〉
  - Use of prior knowledge involving abstraction 〈10〉
  - Pattern recognition and attribution of meanings to words 〈semantic awareness〉 〈3〉

Source: Research data.

Figure 6 illustrates one of these networks. Three groups of codes were brought together, and direct and explicit relationships were identified between Abstraction (common to all), Decomposition, Pattern Recognition, and Algorithms. Implicit relationships were also identified with the skill of Algorithms. According to Brackmann (2017), this skill involves all the others. The network formed by the groups indicated by white rectangles is highlighted in gray and connected to the network by black arrows. The codes belonging to each group are connected to it by a dotted red line.

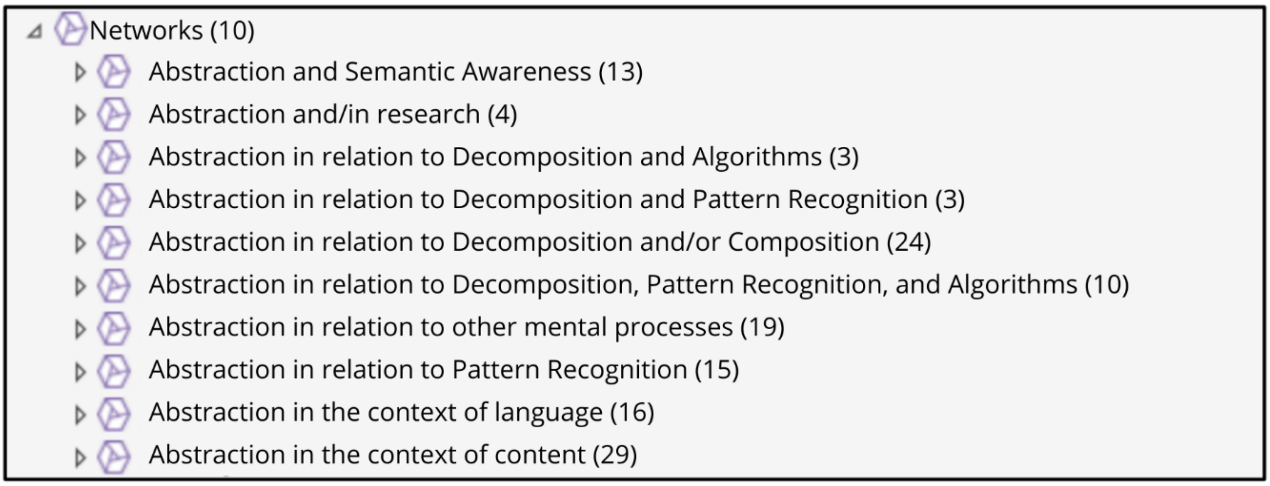
Figure 6: Example of a network formed by code groups



Source: Research data.

Repeating this process, ten networks were formed, bringing together the code groups, as illustrated in Figure 7.

Figure 7: Networks formed from code groups

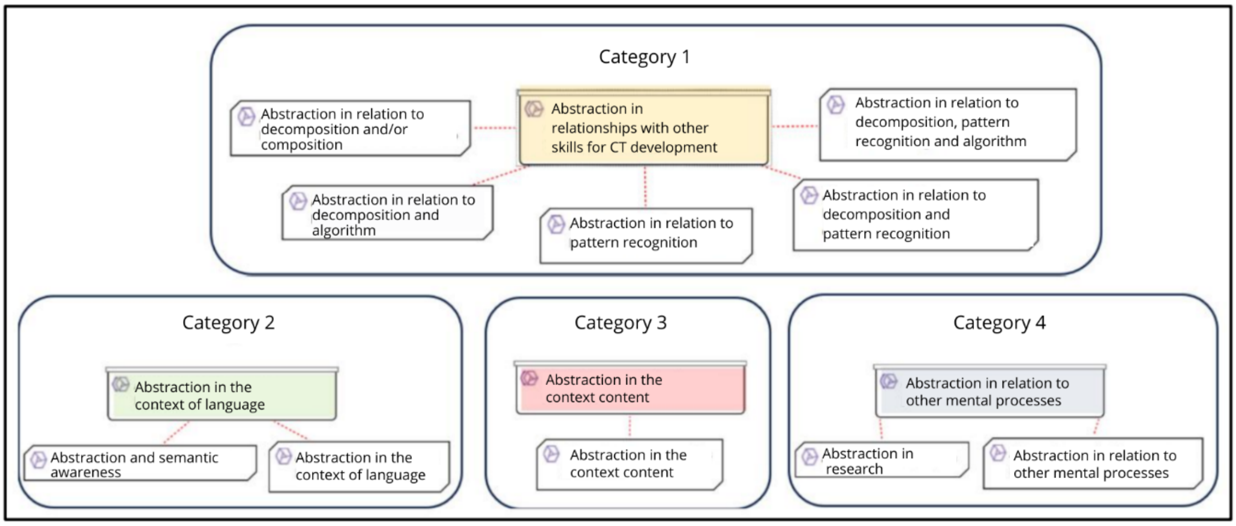


Source: Research data.

To further refine the categories of analysis and identify new relationships between established networks, we formed groups of networks with converging elements. These groups constituted our categories of analysis, as shown in Figure 8.

Our categories of analysis were thus established around Abstraction, the most recurrent skill, to which the other categories and codes are related. The categories were named as follows: (i) Abstraction in relation to other skills for CT development, (ii) Abstraction in the context of language, (iii) Abstraction in the context of content, and (iv) Abstraction in relation to other mental processes.

Figure 8: Categories generated from network groups



Source: Research data.

Considering that, all categories are centered on Abstraction and are interrelated, in the following section we will present the grounded theory that emerged from the analysis process described here.

4 The theory based on Computational Thinking and Mathematical Education through Mathematical Modeling from the perspective of Meaningful Learning

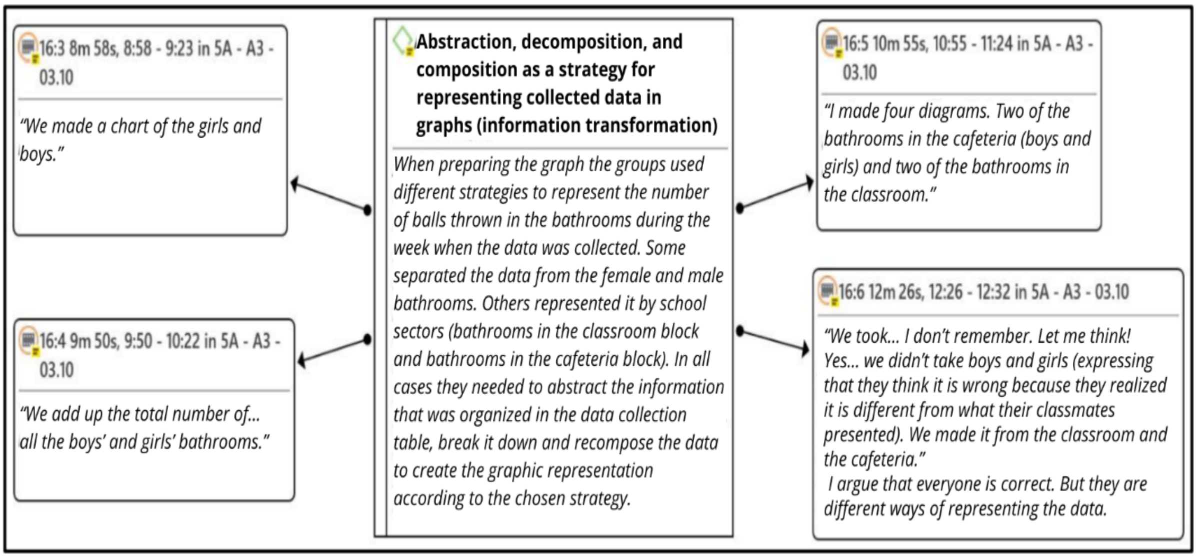
As we have already stated, the process of theorizing about CT in MM practices from the TML perspective is centered on the process of abstraction. Therefore, the unifying principle of this theory is the abstraction that these practices enable.

According to Abbagnano (1998, p. 13), abstraction "is the process of selecting something as an object of perception, attention, observation, consideration, research, or study and isolating it from other things with which it has any relationship." For this author, abstraction is not limited to separating the object from other elements but also involves paying specific attention to the isolated object. In this sense, it becomes clear that "abstraction is inherent in any cognitive procedure and can serve to describe any process of this kind," as Abbagnano (1998, p. 13) points out.

Mora (1994) defends a more comprehensive concept of abstraction, understanding it as objects of thought. He argues that when we abstract something, it is always part of a whole with which it is in a relationship. In this sense, the abstraction that emerged in the modeling practices is not isolated but occurs alongside the other processes involved in constructing knowledge. MM guided these practices, which were developed in interaction with the principles of TML and assumed as a methodology. The data revealed that abstraction is not only present in all processes involved in knowledge construction but also precedes them all. When other skills for CT development emerged in the episodes, they always occurred with Abstraction (Category 1), either implicitly or explicitly.

For example, Figure 9 shows how abstraction emerged alongside Decomposition and Composition during the problem-raising stage of one practice in which the paper problem was raised. After data collection, students were asked to create graphs to represent the problem. During this process, each group used different strategies to represent the data, which were linked to how they abstracted and decomposed and recomposed it until they obtained the desired graph.

Figure 9: Example of episodes in which Abstraction emerges alongside Decomposition and Composition



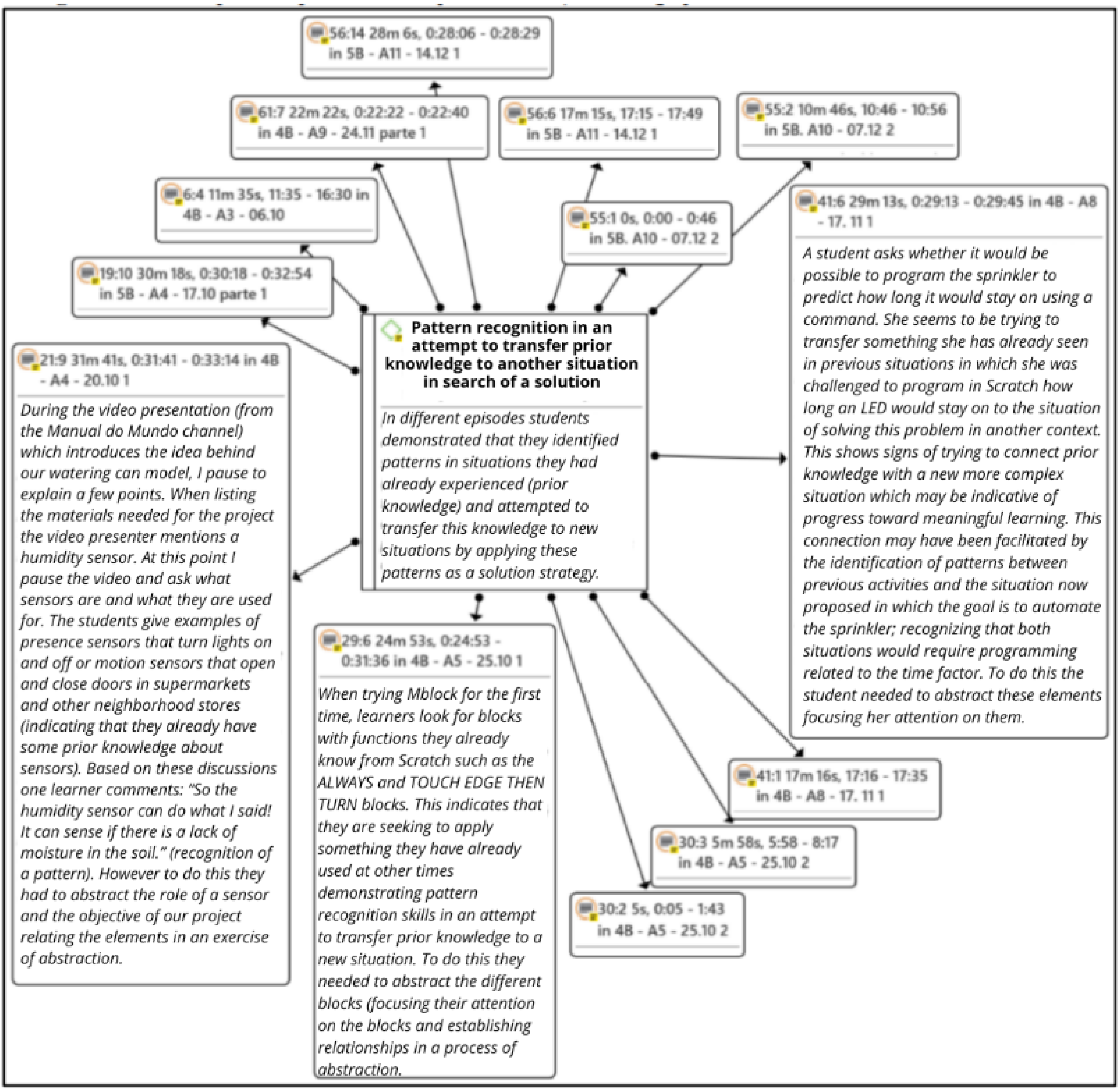
Source: Research data.

The different graphic representations were discussed, and their meanings were interpreted. During this process of negotiating meaning, an exercise in abstraction was performed to interpret and explain the meaning of the interpretation. This interpretation was verbalized to classmates and broken down into steps. Figure 9 shows the codes used in excerpts from the episodes that highlight the students' use of different forms of abstraction.

Situations in which abstraction was always present also occurred with the other CT development skills. Figure 10 shows situations in which abstraction emerges as part of pattern recognition during the problem-solving stage in the garden. At this stage, students began studying how to build the automatic sprinkler and had their first contact with MBlock, the platform chosen to develop the necessary algorithm for the sprinkler to work.

In these episodes, students demonstrated their ability to establish relationships between prior knowledge and identify patterns to apply this knowledge to solving the current problem (new situation). These attempts may indicate a move towards meaningful learning (Moreira, 2011). However, they needed to abstract information and focus their attention on aspects that would help them identify patterns. In other words, they had to abstract how this pattern could be used in the new situation by analogy.

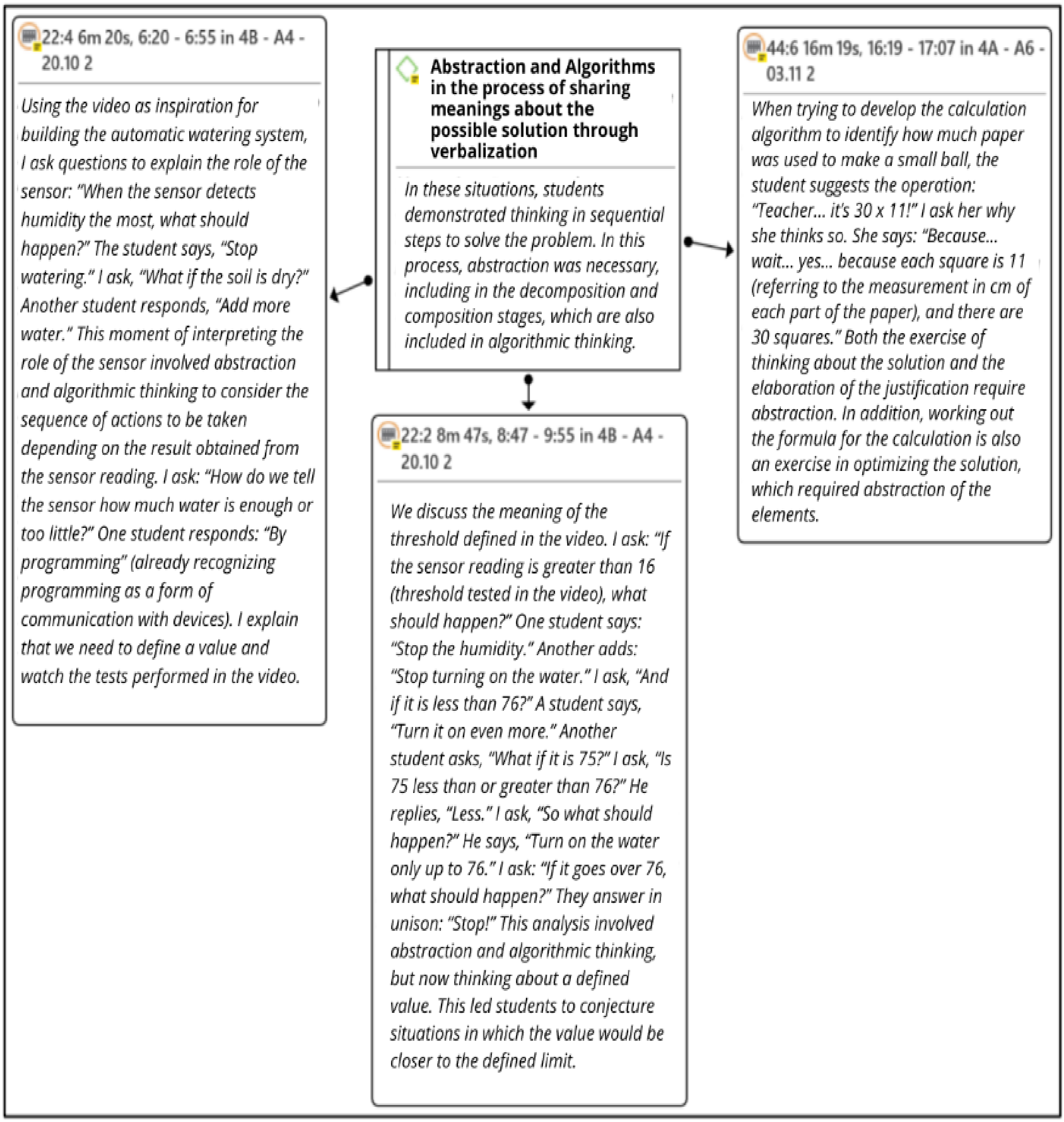
Figure 10: Example of episodes in which Abstraction emerges alongside Pattern Recognition



Source: Research data.

Abstraction emerged alongside the Algorithms skill, as shown in the examples in Figure 11. In these episodes, students abstracted different situations to think about the sequence of steps in the algorithm.

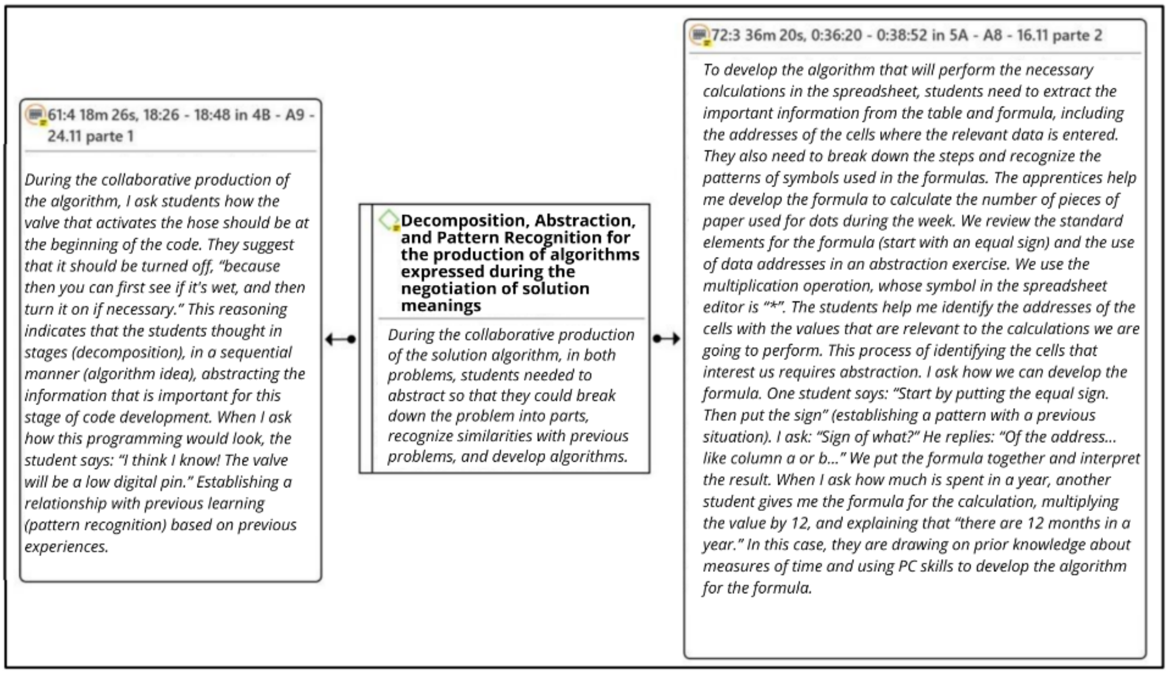
Figure 11: Example of episodes in which Abstraction emerges alongside Algorithms



Source: Research data.

In addition to highlighting the pairs formed by the skill of Abstraction with each of the other skills for CT development, as exemplified in Figures 9, 10, and 11, in various stages of the Modeling practice, the skills emerged together, as shown in Figure 12, which presents moments when Abstraction emerges with Decomposition, Pattern Recognition, and Algorithms.

Figure 12: Example of episodes in which Abstraction emerges with Decomposition, Pattern Recognition, and Algorithms



Source: Research data.

Our data corroborate and clarify Wing's (2008) assertion that "abstraction is the essence of computational thinking" (p. 3717). However, we note that current CT practices have placed a strong emphasis on algorithmic skills. Our analysis is based on empirical data that contribute to our understanding of the movement of abstraction throughout students' actions.

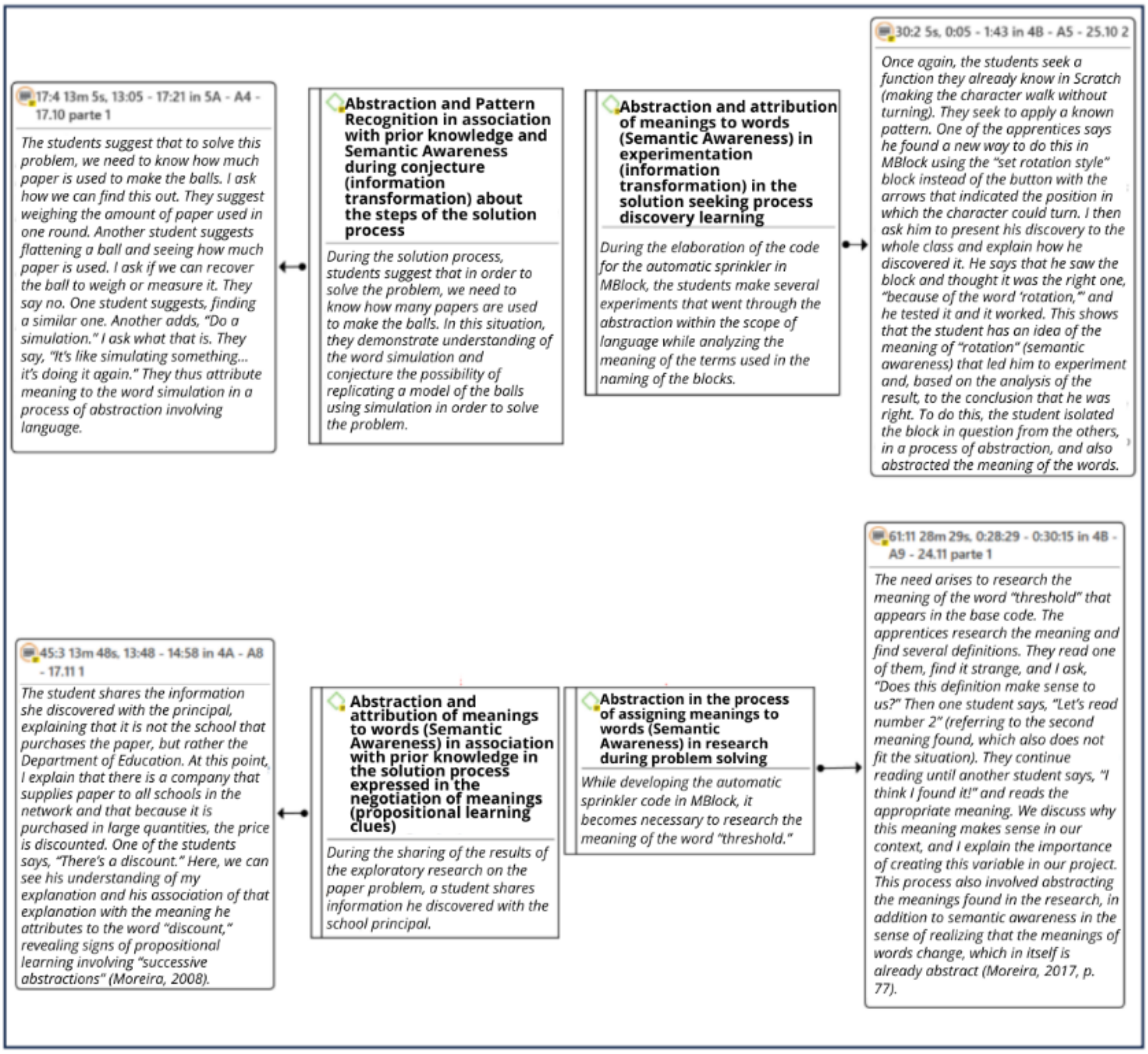
The literature's emphasis on algorithms initially led us to believe that they would be the central skill in our categories. However, this hypothesis was not supported; Abstraction emerged as a prominent skill, setting a precedent and connecting with the others. Furthermore, we highlight the different meanings of abstraction in the context of CT systematically and with the emerging theory.

Regarding decomposition, René Descartes (1596-1650), a leading figure in modern philosophy, highlighted its importance in the production of knowledge (Meneghetti & Bicudo, 2003). However, we must abstract the aspects of a problem to break it down into smaller, more manageable parts. Abstraction makes it possible to visualize the parts of a problem separately and break it down. According to Abbagnano (1998, p. 14), "The limitations of our minds prevent us from understanding composite things except by considering them in their parts and contemplating the various faces they present. This is generally called knowing by abstraction." Thus, decomposition is encompassed by abstraction; the latter is a necessary process for the former.

Regarding decomposition, we emphasize that although it contributes to solving some problems, the process of composing solutions for the parts is equally important, since they are complementary processes. Thus, it is important to help students decompose problems when appropriate but also to highlight the importance of composition for the solution so that the big picture is not lost, since we live in a world in which problems are embedded within a complex whole (Morin, 2005).

Abstraction is involved in Pattern Recognition, since to identify a pattern, it is important to isolate each specific feature of a given situation and focus on it. Abstraction is also necessary for the development of Algorithms (Wing, 2008; Brackmann, 2017). From the data produced, it was possible to conclude that, more than covering Decomposition and Pattern Recognition, the development of an Algorithm depends on these processes, which are supported by Abstraction. Thus, in addition to the step-by-step construction of the Algorithm itself being a process of Abstraction, it is present from the Decomposition of the problem and Pattern Recognition to the production of the Algorithm.

Figure 13: Examples of episodes in which abstraction emerges in processes involving language



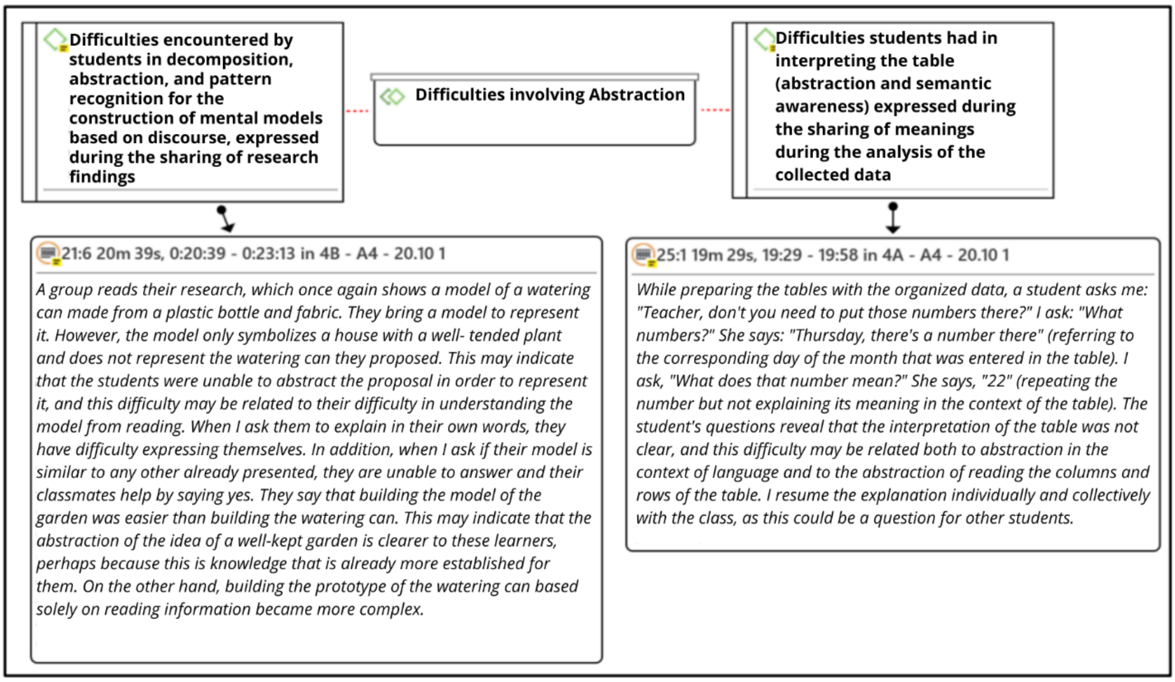
Source: Research data.

Abstraction, which is not unique to CT and emerged during practice as part of other processes, including in the context of language (Category 2, as shown in Figure 8), is essential for any teaching and learning practice, especially meaningful learning (Moreira, 2011). Language supported the development of modeling practices and was present, above all, in the negotiation of meanings, as recommended by TML (Moreira, 2011). In this process, abstraction emerged as fundamental since it is closely associated with language (Abbagnano, 1998). Figure 13 presents examples of excerpts in which abstraction is integrated with language-related semantic awareness.

In the context of language, abstraction is central to meaningful learning because students' understanding of subject matter is directly influenced by their meanings of words and situations. Thus, it is essential for teachers to promote the negotiation of meanings to capture learners' developing meanings (Masini, 2008). This negotiation helps teachers mediate the process.

As illustrated in Figure 14, students' language difficulties emerged during the practices when we perceived, through the negotiation of meanings, the limitations of their understanding of what was proposed and made possible by the dialogue resulting from the modeling methodology.

Figure 14: Examples of episodes involving difficulties in abstraction in language

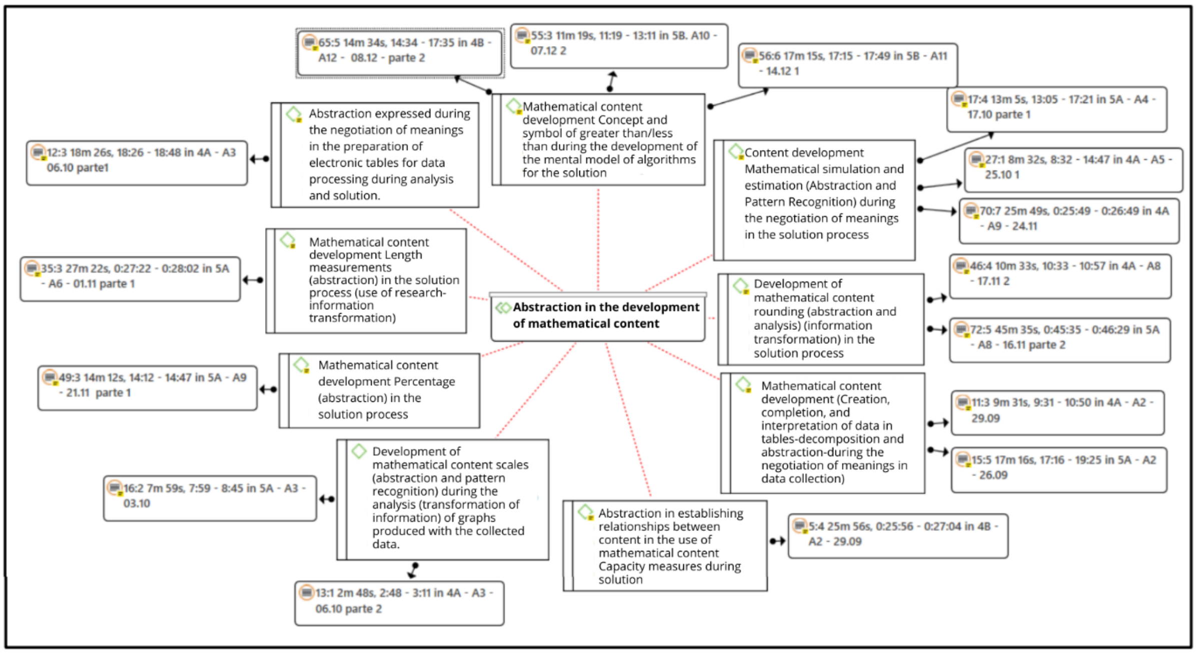


Source: Research data.

In Episode 21:6, the necessary abstraction to understand written language through reading, as well as the abstraction to transfer this understanding to verbal language to explain the studied watering can model, proved fragile. This hindered overall understanding. In Episode 25:1, the students' misunderstanding of the table may be linked to their difficulty with language, the abstraction of the table's elements (rows and columns), and the relationship between them. This difficulty also extends to the implicit aspects of thinking necessary to understand tabular organization. In both cases, mediation through dialogue was necessary to clarify the ideas for the students.

Given the mathematical content (Category 3) being worked on, the strong presence of abstraction is not surprising, as Mora (1994) and Abbagnano (1998) have stated; it is part of mathematical knowledge. During the development of the mathematical content necessary to solve the problems, the students required abstraction to develop the solutions. From this, the category "Abstraction in the Context of Content" was formed. These categories were formed from groups of networks. The network "Abstraction in the Context of Content" formed a category with the same name because no other networks were grouped with it. This network consists of groups of distinct codes related to Abstraction and the mathematical content explored in Modeling Practices. These groups of codes are Abstraction in the Development of Mathematical Content, Difficulties Involving Abstraction, and Use of Prior Knowledge Involving Abstraction. Figure 15 shows an example of one of the nodes, highlighting ways in which abstraction emerged in the context of this category.

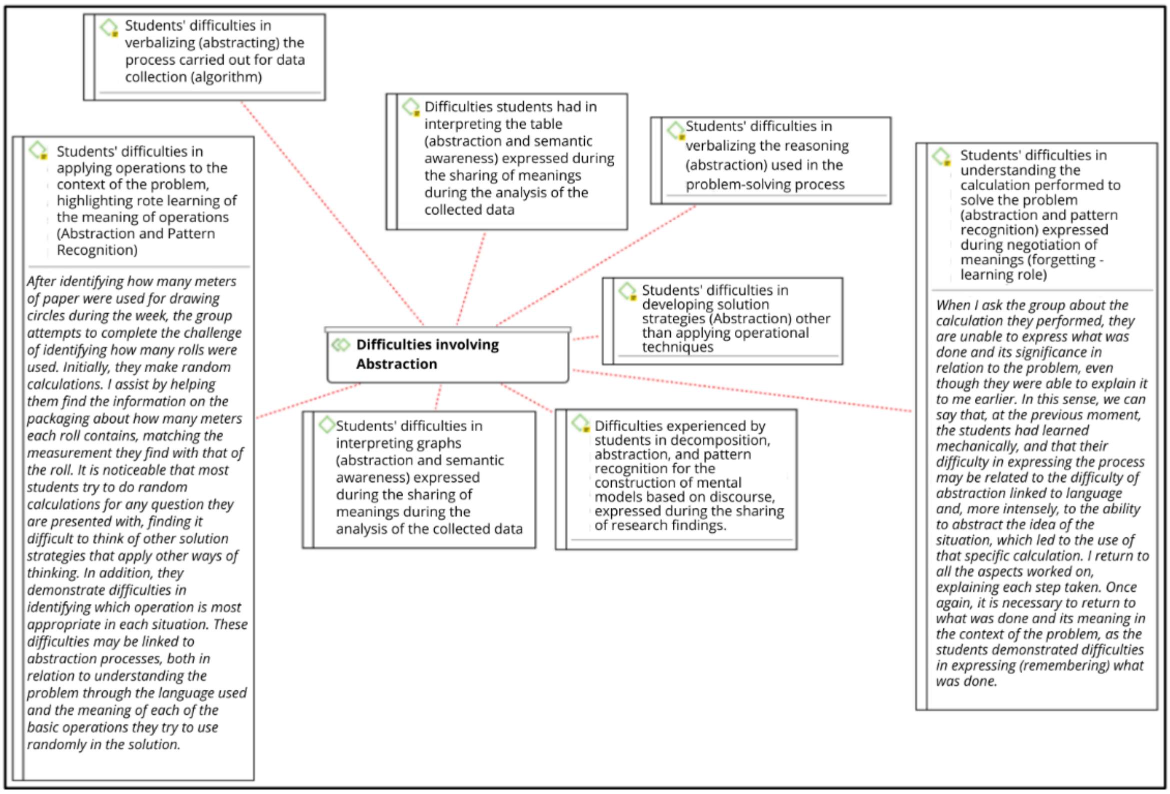
Figure 15: Example of a node in the Abstraction Network in the context of content (Abstraction in the development of mathematical content)



Source: Research data.

Both the episodes in which abstraction emerged during the application of prior knowledge by students in the solution process and the content that they found difficult to develop in the context of the problem, related to abstraction, were part of the nodes of this network, as illustrated in Figure 16.

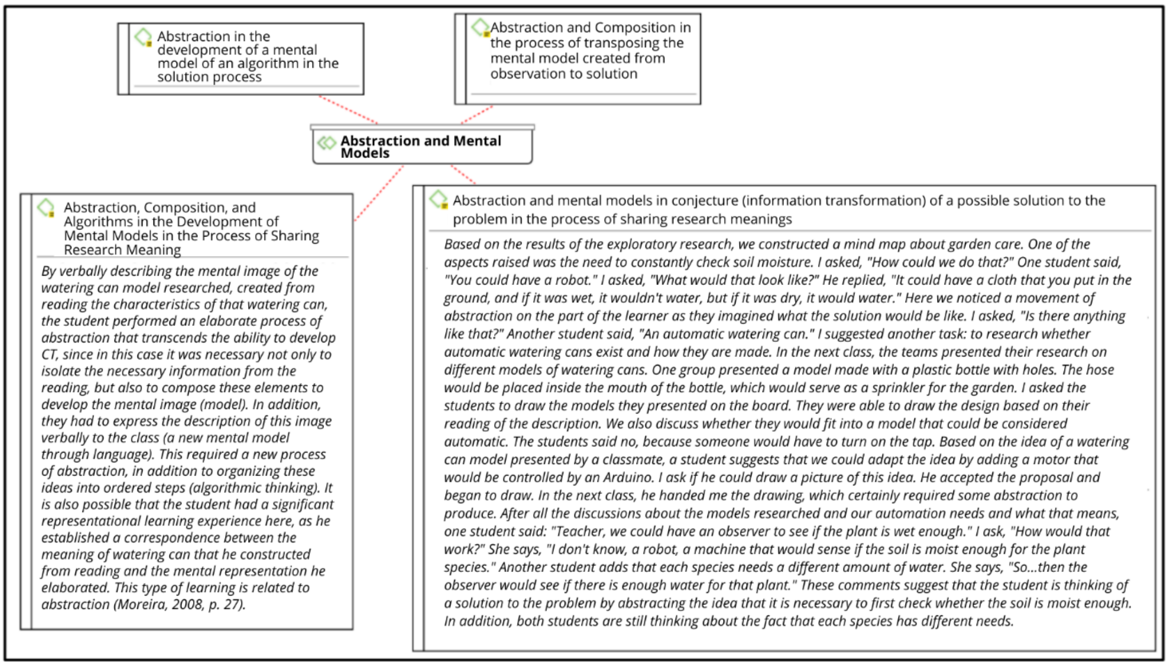
Figure 16: Example of a node in the Abstraction Network in the context of content (Difficulties involving Abstraction)



Source: Research data.

In addition to content, abstraction emerged in relation to other cognitive processes involved in solving problems (Category 4), such as analyzing and interpreting results and developing mental models, as shown in Figure 17. Abstraction is a part of developing mental models, analyzing and interpreting results, and refining them cognitive processes involved in meaningful learning (Costa, 2008).

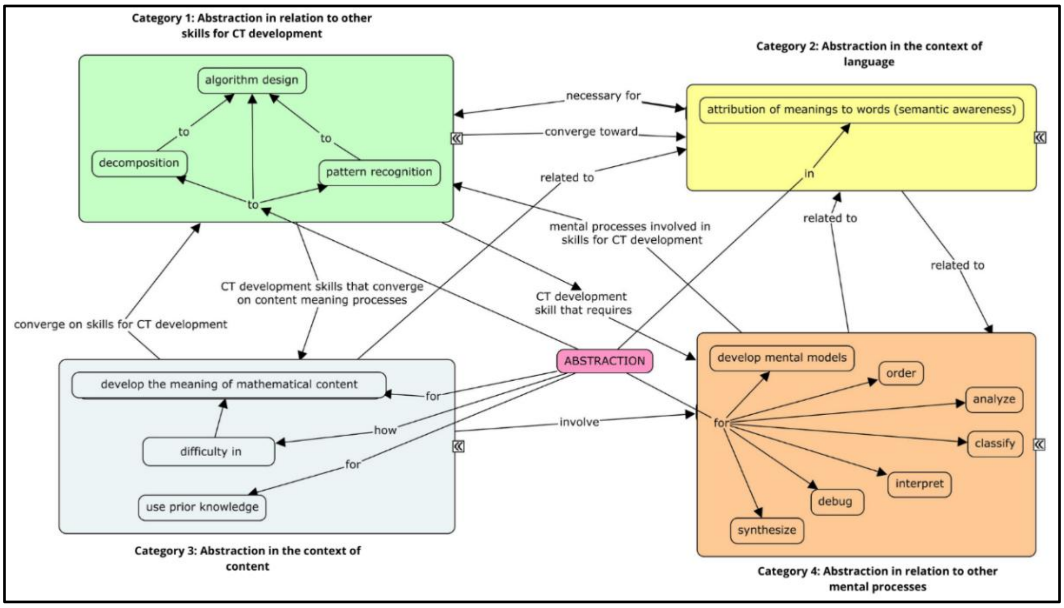
Figure 17: Example of a node belonging to the Abstraction network in relation to other mental processes



Source: Research data.

Although we presented excerpts from each category separately, this was only to facilitate the reader's understanding of the categories' components, which are not independent or isolated. Note that abstraction was present in all the processes that make up each category and in the relationships between them. Figure 18 summarizes these relationships.

Figure 18: Relationships between categories centered on Abstraction



Source: Elaborated by authors.

All categories are related to Category 2, "Abstraction in the Context of Language," because language is a social function formed and transformed throughout life. Language is involved in the organization of thought and depends on the entire cognitive process, cognitive development, and knowledge acquisition (Moreira, 2011).

Any cognitive process, such as the mobilization of skills for CT development, interpretation, analysis, or other elements mentioned in Category 4, as well as content development, is directly related to language. These processes depend on attributing meanings that are not inherent to words but rather to people (Moreira, 2008). Abstraction, which is inherent to language, is directly linked to all categories.

Category 1, "Abstraction in relation to other skills for CT development," is related to Category 3, "Abstraction in the context of content," because meaningful learning involves processes such as progressive differentiation and integrative reconciliation in different types of learning (representational, propositional, and combinatorial; Ausubel, 2003). These processes converge into skills such as decomposing and composing ideas, identifying patterns, and thinking algorithmically. These skills require abstraction. Therefore, skills for developing CT centered on abstraction are aspects that can facilitate meaningful learning. The processes involved in meaningful learning converge toward skills that enable CT development and can mobilize it. Therefore, the development of CT can contribute to meaningful learning, and meaningful learning can favor the development of CT by highlighting the need to evoke the skills necessary for its development.

Category 1 is related to Category 4 – "Abstraction in relation to other mental processes," given that the skills for the development of CT require other mental processes for problem solving, such as analysis, classification, interpretation, data ordering, synthesis, refinement of ideas, data and results, as well as the development of mental models, all of which are centered on Abstraction. Thus, the categories are interrelated so that the cognitive processes involved in each of them are related.

In all stages of Modeling, Abstraction was required along with other skills for the development of CT that emerged in the processes of Meaningful Learning. Based on these data and our reflections on them, we can conclude that Abstraction is at the core of the development of CT and the processes of Meaningful Learning. Therefore, there is a reciprocal influence between the development of CT and TML in the learning processes mediated by Modeling.

5 Summary of the theory

Considering the characteristics of GT, we will not present final considerations but rather a summary of the theory. This summary will highlight the fundamental aspects that emerged from the analyses and explain the "concept" and its content.

*CT development skills, o*ften referred to as CT skills, are not exclusive to CT. It is worth noting that abstraction and decomposition are inherent to the production of knowledge (Abbagnano, 1998; Meneghetti & Bicudo, 2003) and therefore as old as knowledge itself, predating computing (Fonseca Filho, 2007). Regarding Pattern Recognition, its concepts have long been used by humans, even at the level of common sense. For example, humans have used this skill in everyday situations, such as agriculture. It has also been used in medicine for diagnosis. Additionally, Pattern Recognition is related to Algebraic Thinking (Navarro & Sousa, 2023) and has its roots in mathematics. The concept of algorithms also originates in mathematics and therefore predates computing. One example is the division algorithm proposed by Euclid, which Fonseca Filho (2007) considers "the beginning of the quest for the automation of reasoning and calculation" (p. 42).

These skills, used in the production of knowledge in other areas, have been incorporated by computing into its problem-solving and knowledge-building processes. Logically, this area has enabled significant contributions to scientific development. When it comes to these skills, computing has contributed most notably to the development and execution of algorithms through computing-specific languages, leading to important advances.

For this reason, perhaps, working with algorithms has been emphasized in discussions about developing CT practices in schools. Current discussions on this topic originated from movements proposed by computer science researchers. The process of algorithmization is a significant contribution to CT because it can make problem solving feasible and fast by reducing repetitive and excessive efforts. However, it should be noted that this process depends on other skills; without them, achieving CT is impossible.

Disregarding this historical movement and viewing these skills as exclusive to CT and computing can result in their inclusion in the classroom in an "empirical, technical, and utilitarian" manner (Navarro & Sousa, 2023, p. 106). We understand that these are skills for developing CT, not skills that are exclusive to CT. Like Navarro and Sousa (2023), we believe that these skills should be developed in the classroom to help students solve problems in various areas through various strategies that can lead to the development of algorithms.

Centrality in Abstraction Processes: Among the investigated skills, abstraction emerged as the basis for the other processes involved in CT development. It is necessary for decomposition and composition, pattern recognition (especially thinking about what is not immediately given), and developing algorithms. This process requires the mobilization of all these skills, although it is secondary to the others. Based on the analytical efforts undertaken through GT, we conclude that these skills are interdependent and centered on abstraction. It is pertinent to address them with students from this same perspective rather than in isolation since, in the context of real problems, they are always interconnected. Therefore, promoting the development of CT in the classroom requires didactic-pedagogical actions that enable students to use abstraction to solve problems in different modes, levels, and contexts.

Pedagogical practice supported by TML mobilizes abstraction and other skills for developing CT. With problem solving as one of its guiding principles, pedagogical practice supported by TML highlights the need to evoke skills centered on abstraction and other strategies and ways of thinking in the problem-solving process. Furthermore, when language is used to inform classroom actions, negotiation of meaning becomes a central element. This brings constant dialogue between learners and teachers, aiming to understand the meanings students attribute to the subject and verify if these meanings align with the subject. This allows for mediation that facilitates meaningful learning (Moreira, 2011).

In this process, the abstraction inherent in language is constantly required and mobilized. Considering the principles of TML, practices that engage learners in mental processes such as analysis, synthesis, refining results, and developing mental models are valued. These processes are centered on abstraction. Thus, TML based pedagogical practice has the potential to mobilize abstraction in different modes, levels, and contexts.

CT development skills converge with the processes of meaningful learning: CT development skills align with the cognitive processes necessary for meaningful learning. These processes include progressive differentiation, integrative reconciliation, decomposition, composition, and pattern recognition. Above all, they require the abstraction necessary for understanding content. However, this only occurs when the processes are not approached in a merely expository manner or as thinking techniques.

These skills are involved in the cognitive processes of knowledge production and converge with learning processes, especially meaningful learning. Therefore, promoting pedagogical practices that utilize these skills can facilitate meaningful learning. Similarly, it is important to promote practices that enable learners to develop the ability to abstract, since meaningful learning can be hindered by an absence of background knowledge related to this skill.

MM, from the TML perspective, is configured as a methodology that articulates the development of CT and EM: MM aims to contribute to the critical formation of students. However, practices may not lead to the mobilization of thought processes that lead to this formation. In this sense, the theoretical contribution of TML is a differential; given that the principles of this theory, when understood and put into practice in the classroom, enhance Modeling practices in the sense of mobilizing actions by both teachers and learners, which are in fact the guiding thread for this critical formation. Among these actions, we highlight dialogue, reflection-action, research, analysis, argumentation, creativity, the application of what has been learned in new situations, and the learning of mathematical knowledge in contexts of problem solving that make sense to students.

The development of CT can be favored in this process, since the modeling practices thus developed bring out the need to set in motion abstraction at its different levels, from the simplest to the most complex. At the same time, both abstraction and the other skills for CT development converge toward the cognitive processes of meaningful learning. Simultaneously with these practices, the mobilization of these skills can be another facilitator of meaningful learning in a process that requires students to progress in their learning of mathematical content and the development of CT in a process of mutual contribution.

Therefore, we can conclude that mobilizing abstraction in different modes and contexts is central to facilitating the development of CT and the meaningful learning of content. In this process, MM is a methodology that interacts with the principles of TML and serves as a guiding thread.

We emphasize that the present study was limited to one school, thus reflecting its specific context. Nevertheless, we believe it provides valuable insights that can inspire reflection in other contexts with the necessary adaptations.

The data produced will allow for further analysis of the evidence of meaningful learning among students during MM practices, which will be carried out in future studies.

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