Sociedade Brasileira de Educação Matemática

RIPEM

Mathematics Learning in Higher Education: contributions from Solidarity Assimilation to teacher training

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10.37001/ripem.v15i2.4479

Received • 08/02/2025
Approved • 29/04/2024
Published • 28/05/2025

Editor • Gilberto Januario

Abstract: This article discusses contributions from a proposal for didactic-pedagogical intervention based on Solidarity Assimilation (SA) to the understanding of mathematical concepts and training of future mathematics teachers. The theoretical framework was grounded in Lacan's psychoanalytical concepts, which provide a foundation to support SA in the comprehension of mathematical concepts. The study highlights aspects of current educational public policies manifested in teaching and assessment methods adopted by Prevailing Traditional Education (PTE), which emphasizes content memorization and mechanical learning. In conclusion, from this perspective, SA emerges as an important tool for deconstructing PTE, as it fosters more equitable ways to assess students and greater awareness and mastery of learning in mathematics teacher training.

Keywords: Mathematics Education. Initial Teacher Training. Learning. Solidarity Assimilation.

Aprendizaje Matemático en la Educación Superior: aportes de la Asimilación Solidaria a la formación docente

Resumen: Este artículo discute las contribuciones de la propuesta de intervención didáctico-pedagógica Asimilación Solidaria (AS) para la comprensión de conceptos matemáticos y para la formación de futuros profesores de matemáticas. La base teórica se basó en los conceptos del psicoanálisis de Lacan (1992), que sustentan la AS en la comprensión de los conceptos matemáticos. Destacamos aspectos relacionados con las actuales políticas educativas públicas, basadas en los métodos de enseñanza y evaluación adoptados por la Enseñanza Tradicional Vigente (ETV), que priorizan la mecanización y memorización de contenidos. Concluimos, desde esta perspectiva, que la AS puede ser un potente mecanismo de deconstrucción de la ETV al buscar formas más justas de evaluar a los estudiantes y una mayor conciencia y dominio del aprendizaje durante la formación de los futuros profesores de matemáticas.

Palabras clave: Educación Matemática. Formación Inicial del Profesorado. Aprendizaje. Asimilación Solidaria.

Aprendizagem Matemática no Ensino Superior: contribuições da Assimilação Solidária para formação de professores

Resumo: São discutidas neste artigo as contribuições da proposta de intervenção didático-pedagógica Assimilação Solidária (AS) para compreensão de conceitos matemáticos e para formação de futuros professores de matemática. A fundamentação teórica foi ancorada nos conceitos da psicanálise de Lacan que fundamentam a AS na compreensão de conceitos matemáticos. Pontuamos aspectos relacionados às políticas públicas educacionais atuais, consubstanciados em métodos de ensino e de avaliação adotados pelo Ensino Tradicional Vigente (ETV) que privilegiam a mecanização e a memorização de conteúdos. Concluímos, a partir dessa perspectiva, que a AS pode, ser um forte mecanismo de desconstrução do ETV por buscar formas mais justas de avaliar os alunos e mais consciência e domínio do aprendizado durante a formação de futuros professores de matemática.

Palavras-chave: Educação Matemática. Formação Inicial de Professores. Aprendizagem. Assimilação Solidária.

1 Introduction

Mathematics teaching and learning are complex processes, as they involve various factors that may define whether students effectively learn a particular content. In this context, in addition to the didactic aspects related to the classroom, it is important to take other elements into account, such as cultural, social and political factors, which are not usually considered by researchers and teachers/professors within the Mathematics Education community. In this discussion, the literature presents publications by Cabral and Baldino (Baldino, 1997; Baldino and Cabral, 1998, 1999, 2006, 2008, 2010, 2013; Baldino and Carrera, 1999; Cabral, 1998, 2015), which investigate aspects related to mathematics teaching and learning that go beyond the pedagogical components of a mathematics class.

In the same vein, we observed that the Mathematics Education field has been going through the so-called Sociopolitical Turn (Gutiérrez, 2013), a movement that transitions away from the centrality of studies in the field, which are focused on content teaching and learning processes, and sheds light on the cultural, social and political aspects that shape contemporary mathematical practices. Regarding the Sociopolitical Turn, we highlight Alexandre Pais' publications, which explore problems experienced by students and teachers in working with mathematics. These issues cannot be fully understood within the concept of 'learning' that sustains Mathematics Education, requiring a perspective on mathematics and education that considers an institutional structure in which Mathematics Education extends beyond the teaching and learning processes that take place in the classroom (Pais and Valero, 2012; Pais, 2014).

Studies like those aforementioned draw on contemporary theories and philosophies and reveal how mathematics is embedded in the political and social fabric of present-day society, offering possibilities to address the challenges educators face in their practice. These challenges arise in various contexts, especially in Higher Education, in mathematics teaching degree programs. In light of this, the authors (ibid.) propose pedagogical interventions aimed at understanding mathematical concepts, such as the methodological proposal named Solidarity Assimilation (SA), seeking to challenge the criteria imposed by Prevailing Traditional Education (PTE), which are often underlying and rarely discussed with students. In the traditional organization of teaching, students are typically seated in a grid formation, their participation is limited, and the teacher is regarded as the sole holder of knowledge.

From the perspective of SA, according to Cabral and Baldino (2010), students learn by speaking while teachers teach by listening. This proposal for pedagogical intervention has been shaped by various theorists, especially those from Lacanian Psychoanalysis, and is featured in several publications across national and international journals (Cabral and Baldino, 2019, 1998). While reviewing these numerous texts, we observed that several studies did not address mathematics teacher training. Therefore, in this text, we aim to explore contributions from Solidarity Assimilation (SA) to the understanding of mathematical concepts and training of future mathematics teachers. The data used in this study were extracted from a master's paper developed within the Programa de Pós-Graduação em Ensino de Ciências e Matemática (PPGECMat – Postgraduate Program in Science and Mathematics Education, in free translation) at the Federal University of Grande Dourados (UFGD), in Dourados, Mato Grosso do Sul state, in Brazil.

2 Solidarity Assimilation and its contributions to mathematics learning

In his psychoanalytical theory, Lacan (1992) developed the idea that, as subjects, we are constituted through language, through speech, through interactions with one another and, from birth, we are subject to alienation[1]. In the formation of a subject, the notion of alienation refers to a forced choice that arises as meaning emerges from the social and cultural field, known as "the Other". In this process, part of a subject's identity fades due to the influence of symbolic elements, such as words and symbols, which veils the presence of subjects themselves.

At the beginning of this process[2], the subject cannot speak, as speech requires connecting at least two words or symbols. It is necessary for the subject to engage with the second symbolic element to overcome this initial limitation and start using language (Lacan, 1973/1988).

Consequently, according to Lacan (1992), a subject is divided into two parts, revealing a division that occurs in their relationship with these symbolic elements. This demonstrates that the subject is not a single entity, but a complex interaction of symbolic elements, emphasizing that language plays a fundamental role in the construction of the subject's identity and in the relationship the subject establishes with the world around them.

This fundamental concept of Lacanian psychoanalysis underpins the aphorism sustained by Solidarity Assimilation, which assumes that students learn by speaking while teachers teach by listening (Cabral; Baldino, 2010). This highlights the importance of dialogue and verbal expression in the construction of knowledge. By speaking, students articulate thoughts, confront their own ideas and often reach a deeper understanding of the content.

When Cabral and Baldino (2010) stated that students learn by speaking, they referred to the specific language of Mathematics, as spoken by students. Through their signifiers, students attribute meaning to the concepts that they study. On the other hand, the idea that teachers teach by listening emphasizes the active role that listening plays in the educator's practice. Attentive listening requires being receptive to students' speech, understanding not only the words they say, but also the potential difficulties they may encounter.

According to the same authors, in Solidarity Assimilation, the directive to "ask questions" during the class is interpreted as a strategic tool designed to lead students toward contradiction. In this context, contradiction is viewed as a critical turning point, aligning with the Lacanian perspective that confronting contradiction is essential to the process of subjective transformation. This is characterized by Lacan (1988) as a form of reverse hypnosis, in which the teacher, in order to lead students toward contradiction, should enter their speech and guide them toward this point.

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[1] According to Lacan (1960/1998), alienation is inherent to the subject; it is not born of language.

[2] Lacan (1988, p. 207, our translation) states that: "We can locate (...) this Vorstellungsrepräsentanz in this first significant pairing that allows us to conceive that the subject appears first in the Other, as the primary signifier, the unary signifier that arises in the field of the Other and how it represents the subject for another signifier, just like another signifier produces the aphanisis of the subject. Hence, the division of the subject – the subject appears in a place as meaning and, in another place, it is manifested as fading. Therefore, if we can say so, there is a matter of life and death between the unary signifier and the subject as a binary signifier, the cause of its fading. Vorstellungsrepräsentanz is the binary signifier".

Entering students' speech is not an easy task because it requires teachers to step out of their own "comfort zone" in order to prompt students to leave theirs. However, for this interaction to happen, pedagogical transference is fundamental, as proposed by Cabral (1998).

Thus, for Baldino and Cabral (2008), SA involves developing clinical guidance within the classroom. This requires the teacher to regulate students' anxiety levels, managing the onset and resolution of their perception of a lack of understanding in Mathematics. By adopting the position of "the Other", teachers acknowledge that they may have limitations in mathematical comprehension, too, thus establishing a more empathetic connection with students. This stance may foster a better relational dynamic between teacher and student.

Therefore, the idea of pedagogical transference was directly adapted from the concept of transference proposed by Lacan (1988)[3], which refers to the process of individuals directing feelings, expectations and desires, usually unconsciously, toward another person. Nevertheless, transference is not only about the patient placing emotions and expectations onto a person whom they perceive as having the answer to their lack, but also about the patient's subjective position regarding this figure.

When teachers adopt a listening stance, students are faced with the ambiguity of having to discern whether the teacher is merely trying to understand what is being said or intentionally providing time for reflection. In this context, SA seeks to assure students that the teacher does not represent an Other capable of fulfilling the lack that would allow mechanical learning to occur and, consequently, earn academic credit. Instead, in SA, the teacher aims to keep the gap in students' comprehension open, encouraging internal questions such as "Do I really understand this? What do I aim to achieve?" (Cabral; Baldino, 2010, p. 631).

In this context, the authors state that anxiety arises[4] when the student assumes that they have understood a concept, making them vulnerable to the possibility that the teacher will challenge this understanding. This vulnerability occurs when the student believes that their lack of knowledge has been resolved and is then faced with the threat of a negative verdict in their next answer. Teachers' selective listening can identify signs of anxiety in students' speech and gestures. In SA, there is a careful approach to teachers' negative verdicts on students' speech, monitoring their potential to trigger unbearable distress. Thus, the strategy adopted by teachers in SA is to replace negative verdicts with new questions. This practice seeks to maintain a learning environment that encourages continuous exploration, avoiding the penalization of students for wrong answers.

This is also a way to support students' lack of knowledge, regulating and accepting their initial position of not wanting to learn something they supposedly already know. In the context of this approach, the student asks "Can I do this?", and the answer depends on the accuracy of the proposal: if it is correct, they can; if it is incorrect, they cannot; let us verify it together[5]. This illustrates how SA enables students to learn by speaking while the teacher teaches by listening (Cabral; Baldino, 2010).

[3] Who, from the subject supposed to know, can feel fully invested? That is not where the question lies. The question is, first, for each subject, where they position themselves in order to address the subject supposed to know. Each time, this function can be, for the subject, embodied in whomever it may be, whether an analyst or not. Based on the definition I have just given you, transference is already established (Lacan, 1988, p. 220, our translation).
[4] In their article *I Love Maths Anxiety* (2008), Baldino and Cabral discuss the importance of mathematical anxiety for student learning. By providing ready explanations, teachers not only hide their own failures, but also students' gaps. This behavior denies the student the opportunity to confront their own uncertainties and contributes to the illusion that understanding has been reached.
[5] The adoption of this attitude by the teacher is emphasized by Baldino and Cabral in several of their publications, such as the text *Mathematics Education Talking to Psychoanalysis* (2010).

3 Classroom organization in Solidarity Assimilation

In the proposal of Solidarity Assimilation, classroom organization is grounded in Lacan's four discourses. SA goes through these discourses to form a logical sequence, aiming to lead students to question themselves and determine whether what they are doing is right or wrong and, thus, learn based on their signifiers.

In common sense, discourse is usually understood as a form of speech directed toward a recipient in order to provoke particular effects. Lacan adopts this popular conception of discourse but transforms it into a specific concept. For Lacan (2008), discourse is a type of stable social bond that has a structure, that is, it follows a particular logic that does not change and is present in various daily relations.

There are various types of social bonds in interpersonal relations. However, in his Seminar 16, Lacan[6] concluded that there are essentially four types of discourses that represent stable forms of social bonds and that can manifest in several situations: the Master's Discourse, the University Discourse, the Hysteric's Discourse and the Analyst's Discourse.

Lacan's discourses are composed of a basic structure defined by four fixed positions: the agent of the discourse, the place of the Other, production (or loss), and truth. This scheme is organized as follows.

Scheme 1: Structure of positions in Lacan's discourses and their elements

$$\frac{Agent}{Truth} \rightarrow \frac{Other}{Production}$$

Source: organized by the authors based on Lacan (1992; 2008; 2009).

The agent is represented by those who propose the discourse. However, the agent never presents themselves as such, and this is why Lacan places truth below the agent. This means that the truth of the agent – that is, what the agent truly is – does not appear: the truth must remain concealed for discourse to function. Therefore, the agent presents themselves with a particular semblant[7] (Lacan, 1992; 2008; 2009).

Next, we have the place of the Other and production. The place of the Other is the one whom the agent of the discourse addresses. Nevertheless, what occupies the place of the Other is not the actual Other, but how they are perceived by the agent of the discourse. In the lower section, there is production, a side effect generated by the discourse. Like truth, production does not appear explicitly (Lacan, 1992; 2008; 2009).

Finally, there is the arrow, which expresses what the agent wants the Other to do. Discourse, therefore, always aims to influence the Other to carry out a particular action. It is important to observe that this objective is never fully achieved. By seeking this direction, discourse introduces an element of lack, contributing to the continuous dynamic of social interactions (Lacan, 1992; 2008; 2009).

The four elements of discourse occupy the four places presented in the structure. As these elements shift positions, discourse is altered, shaping the dynamics of social interactions. These elements are organized as follows.

[6] In his Seminars 16, 17 and 18, Lacan (2008; translation of Seminar 16, from 1968/1969; 1992; translation of Seminar 17, from 1969/1970; 2009; translation of Seminar 18, from 1971) elaborates the designation of the elements of these discourses and the order in which they appear.

[7] Lacan (1998) characterizes semblant as an indication to the Other of how I am. However, this semblant may be deceptive.

Table 1: Elements of Lacan's four discourses

S1: Master signifier.
S2: Knowledge.
\$: Subject.
Object *a*: what language cannot represent/control.

Source: organized by the authors based on Lacan (1992; 2008; 2009).

S1 is the first element, known as the Master Signifier. It is the primary signifier not only chronologically, but also because it is the most important, crucial and fundamental element. The Master Signifier cannot be questioned; it provides the foundation for all other ideas. All other ideas gravitate around this primary signifier, referring to it constantly.

S2 refers to other ideas that reference S1. It is the second signifier, following the Master Signifier (S1), and represents all other ideas[8] that are articulated around S1. The third element in the discourses is the Subject (\$). Lacan represents this element through an S split in half by a bar because, for him, the subject is always alienated[9], lacking completeness, divided by its signifiers.

The fourth element is Object *a*, which represents what is left from the operation of subject constitution. For instance, when we are born, we become part of human society and acquire identity by alienating[10] ourselves to signifiers. However, in this process, something is always left behind. Object *a* symbolizes what remains as a residual element – what is left after the subject has gone through the identity construction process by connecting to its signifiers. Therefore, Object *a* represents what is not fully integrated or represented in the formation of the subject.

After introducing all the elements of the discourse, we will take the Master's Discourse as our starting point, considering the initial arrangement of the scheme. Scheme 2 illustrates the structure of this discourse.

Scheme 2: Structure of the Master's Discourse

$$\frac{S1}{\$} \rightarrow \frac{S2}{Object\ a}$$

Source: organized by the authors based on Lacan (1992; 2008; 2009).

Furthermore, according to Lacan (1992; 2008; 2009), in the Master's Discourse, the agent presents themselves through the semblant of S1, which cannot be questioned and represents a form of authority. Nevertheless, the truth behind this discourse is that the agent is an alienated subject, as needy as any other. Therefore, they are not the complete and unquestionable subject they appear to be; they are full of doubt, like any other.

In the place of the Other, there is S2. In this discourse, the agent perceives the Other as a slave, or as someone who will obey and who comes second. That is exactly what happens when this discourse starts to operate. Yet, the consequence of this bond generates Object *a*,

[8] For instance, we can cite psychoanalysis itself, whose concepts are all derived from the concept of the unconscious. Therefore, the unconscious is the master signifier, and all other ideas of psychoanalysis are S2.

[9] For Lacan (1998), the subject only exists in the realm of language, within a society dominated by the symbolic, which is composed of signifiers, and our subjectivity depends on signifiers. As a signifier does not have meaning in itself and must be articulated with other signifiers, our subjectivity is always situated in the interval between one signifier and another signifier. Therefore, we are alienated subjects.

[10] Being subject to signifiers.

representing what cannot be captured by the symbolic: it is what remains – the dissatisfaction that the Other expresses at being controlled. Finally, in this discourse, the arrow represents the desire to control the Other. Thus, the agent needs to view the Other as a controllable person.

When we compare this discourse with the dynamics of the classroom, we understand that it is reflected in the traditional educational model that still prevails. In this context, teachers position themselves as masters (S1), as holders of all knowledge related to scientific and didactic-pedagogical fields, without showing any fragility or weakness.

Students, in turn, take the position of S2 and agree to merely obey the teacher without questioning. As a result of this discourse, there is a gap for students (Object *a*), as their subjectivity is not taken into consideration, resulting in an incomplete subject that does not content themselves with solely reproducing the information imposed by the teacher. This lack is what encourages subjects oppressed by their masters to revolt; it is this lack that enables the end of dictatorships, for example, because at a particular moment, the oppressed realize what they are undergoing, which makes them rebel against the situation.

With a counterclockwise shift of the elements arranged in Scheme 2, we arrive at the University Discourse (science), represented by Scheme 3.

Scheme 3: Structure of the University Discourse

$$\frac{S2}{S1} \rightarrow \frac{Object\ a}{\$}$$

Source: organized by the authors based on Lacan (1992; 2008; 2009).

In the University Discourse, the agent presents themselves through the semblant of S2 – the set of articulated ideas sitting above S1. It is possible to observe that S1 occupies the place of truth, which means that every S2 always points to an S1 – that is, S2 does not sustain itself. Nevertheless, in the University Discourse, S1 is concealed; it strategically does not appear. This shows that universities, educational institutions and even science – considering those who represent these institutions, such as teachers, professors and scientists – present themselves as individuals that do not explicitly obey any master (Lacan, 1992; 2008; 2009).

On the contrary, still according to Lacan, they present themselves as transmitters of a form of knowledge that does not claim absolute truth and does not serve a specific interest. However, the truth in this discourse is that there is, in fact, an interest in domination and control, represented by S1.

In this discourse, the agent views the other as Object *a*, that which cannot be dominated. This is exactly what the agent desires in this discourse: to capture even what remains of the Other in order to explain everything through their own method. The side effect of this process is the alienated subject, as they are subjected to the agent's signifiers. Thus, the subject loses their essence, as the Other must be represented through these signifiers.

When the educator assumes the role of teaching all content to the students, presuming that their method is sufficient, they end up generating students who are alienated to their signifiers. This confines students to the teacher's particular idea, which is not necessarily the idea the student would have formulated. If the teacher positions themselves as the sole holder of knowledge ready to be transmitted, they view students as those who must learn everything that they, the teacher, have to teach.

Considering the arrangement of the elements in the Master's discourse, if we move them all clockwise simultaneously, we have the Hysteric's Discourse, represented by Scheme 4:

Scheme 4: Structure of the Hysteric's Discourse

$$\frac{\$}{Object\ a} \to \frac{S1}{S2}$$

Source: organized by the authors based on Lacan (1992; 2008; 2009).

In the Hysteric's Discourse, according to Lacan (1992; 2008; 2009), the agent presents themselves through the semblant of the Divided Subject. Since this subject is alienated by language, they believe there will come a moment when they will have a fully defined identity, given that they still have not understood that, due to the structure of language itself, it is impossible to have identity stability – that is, to know precisely who they are. Moreover, in this discourse, in the place of truth, we have Object a, representing what cannot be assimilated, controlled or dominated by language.

The agent of this discourse views the Other as S1, the one who holds the answer they seek, who will fulfill it in such a way that they will feel fully satisfied. Nevertheless, the Other cannot give in to the temptation to believe that they are, in fact, that person, because if they do, the Master's Discourse will be produced. Therefore, this discourse produces knowledge as its effect. The agent compels the one they position as master to work in order to fulfill their demands – that is, to study so as to obtain answers about how to engage with the agent –, thus producing knowledge.

If the teacher presents themselves to students as an alienated and incomplete subject, they will necessarily position students as S1 – those who will provide the final word and present signifiers in response to specific questioning –, and this will lead students to attempt to produce specific knowledge, aiming to fulfill the lack expressed by the teacher.

Finally, if we once again move the elements arranged in the Hysteric's Discourse clockwise, we have the Analyst's Discourse, which is represented by Scheme 5.

Scheme 5: Structure of the Analyst's discourse

$$\frac{Object\ a}{S2} \to \frac{\$}{S1}$$

Source: organized by the authors based on Lacan (1992; 2008; 2009).

In this discourse, the agent presents themselves as the semblant of Object a, representing what within us resists the imposing force of culture and language. This means the agent will present themselves to the Other as an instigator – someone in search of answers, who will always challenge the Other. Nevertheless, the truth is that this provocation is based on theoretical knowledge that the agent holds, that is, an S2 (Lacan, 1992; 2008; 2009).

The agent of this discourse views the Other as a barred/divided Subject, and through the semblant of Object a, they make the Other feel uncomfortable, move, think, work, and associate. The side effect of the agent's instigation is S1, because in the process of withholding answers and instead asking more questions, certain primordial signifiers begin to emerge (Lacan, 1992; 2008; 2009).

When the teacher assumes the position of agent in the Analyst's Discourse, they present themselves as questioners, not to provide answers to students, but to pose more questions. In this process, by relying on their own signifiers, students will be able to reach an understanding of the content, which is S1.

Through these discourses and other concepts related to SA, we will analyze the data produced during the development of an elective course called *Tópicos de Matemática* (Mathematics Topics, in free translation), offered to students beginning a Mathematics Teaching degree program at a public institution, with a flexible syllabus that can be adapted to students' needs. The course was organized into 72 class hours, with four classes per week. Transcriptions of audio and video recordings from classes were the material for analysis. Throughout the development of the course, various interesting aspects emerged. However, we limited our discussion to certain points due to time constraints and the volume of data produced. Thus, we selected an episode to fulfill the objective of this text. We emphasize that, to respect participants' privacy, we assigned fictional names to the 13 students who took part in the study: Camila, Luan, Vagner, Ana, André, Vanessa, Pedro, Vitoria, Vivi, Fernando, Fatima, Luiz and Brenda.

4 Solidarity Assimilation as a Proposal for Pedagogical Intervention in a Mathematics Teaching Degree Program.

Some elements are regarded as fundamental for SA to function in the classroom, and one of them is group work. According to Cabral (2015), SA seeks to distribute students into groups of at most four members. This distribution occurs after diagnostic activities are conducted at the beginning of the implementation of the approach. Through these diagnostic activities, the teacher gathers information on students' level of knowledge and individual difficulties with specific content.

In this study, we conducted a diagnostic activity on the first day of class involving operations with number sets, progressions and functions. On the second day, the didactic contract of the course was read, and all students accepted the proposal. This is an important characteristic of the didactic-pedagogical proposal, as the contract is an instrument through which the teacher must outline the entire course organization, assessments methods and any aspects that may be subject to student questioning over time.

Nevertheless, it is important to emphasize that, throughout classes, after the implementation of SA, it is possible to discuss some of the terms initially proposed in the contract. This flexibility allows for adjustments and adaptations based on needs and considerations that may emerge during the process. Therefore, in case a student totally disagrees with the proposed contract and prefers to do the activities in the traditional way, the teacher will not oppose their decision. Baldino (1995, p. 1, our translation) stresses that "no one is forced to participate. Those who do not wish to or cannot attend class and, instead, prefer to engage in other types of work during class time will have their test scores maintained and will not be penalized with a low grade in SA."

Still on the second day, after the diagnostic activity was corrected, the students were divided into groups. As advocated by SA, the teacher aims to form homogeneous groups based on task-related activities, seeking to group students with similar knowledge levels so that discussions and interactions within the group benefit all participants. Yet, regarding each student's ideologies, beliefs, wishes, tastes, expectations, life history, and other aspects, it is preferable for groups to be heterogeneous (Cabral, 2015). Taking all these elements into consideration, SA is grounded in seven basic principles that guide its pedagogical practice and organization, all of which were adhered to in this study. According to Cabral (1992), within this approach, the following aspects are essential:

I. Supremacy of groups over individuals and of the whole class over the groups: everyone should be aware that decisions must be discussed collectively, involving all students. The goal is to foster everybody's active engagement in the decision-making process, considering their opinions and perspectives; thus, the aim is to reach a consensus among all students.

II. Assessing the work process rather than the final product.
III. Measuring the duration of productive work rather than solely the competence achieved.
IV. Increasing the average competence of the group rather than the maximum competence of some individuals.
V. Monitoring the reasoning process, rather than just correcting the result: when it addressing content, it is essential to carefully monitor the logical progression, aiming to foster a broader and deeper understanding instead of an approach relying on rote memorization and solely focused on correction.
VI. Rewards and penalties applied to the whole class and groups rather than to individuals.
VII. Establishing a discussion forum on the role of the school system: this aspect aims to foster critical self-assessment in order to determine whether group work is beneficial and significantly contributes to learning, comparing it with individual achievement and analyzing the results obtained in both scenarios.

Based on these principles, it is essential to stress that this entire structure was discussed with the students prior to the implementation of SA so that everyone was aware of and in agreement with the evaluation process. Formalization took place in the form of a contract outlining the entire structure and all topics – ranging from the execution of activities to the evaluation method –, which were fully detailed and agreed upon by all involved parties. Finally, it is important to emphasize that integrating work assessment into the classroom is a significant contribution, enabling students to be assessed based on their dedication to learning, regardless of limitations imposed by their social background or initial abilities. Moreover, the previously set criteria provided the framework for addressing academic differences, ensuring that all students, including those who have more difficulties, receive the necessary attention during classes. Considering these aspects, the 13-student class was divided into four groups – three groups with three students and one group with four students. Although SA suggests groups of four, the organization into three three-student groups and only one group of 4 students occurred because the classroom where the classes were held featured four round desks, which facilitated this arrangement.

In light of the previously discussed points, we began the activities with worksheets focused on operations within the rational number set. Regarding this aspect, Cabral (2015, p. 230, our translation) explains that "it is customary to adopt a textbook whose text is 'dismantled' and converted into worksheets". In this case, the worksheet content covered operations with rational numbers in the form of fractions, as several students struggled with this type of operation – or did it in a very simplified way –, revealing gaps in their understanding of procedures.

As agreed in the contract, the groups solved the worksheets and were later asked about their solutions – one student per group was selected. It is important to emphasize that this decision plays an important role in SA, as it is made by the educator so as to ensure that all participants within a group reach a shared understanding of the task being performed. In this context, educators cannot be confined to a single form of discourse; they must assume different roles, encouraging students to participate actively and fostering discussion on mathematical problems. In Solidarity Assimilation, according to Cabral and Baldino (2010), the teacher initially adopts the Master's Discourse, transmitting information and guiding students. Then, later, they shift their approach and assume the place of the Other, replacing questions with a fundamental mathematical concept. This change is important, as it encourages students to engage actively in the learning process rather than passively receiving information.

As a result, the students are invited to take on the role of agents (S1), assuming the position of the hysteric. This dynamic is interesting because, in psychoanalysis, the hysteric is viewed as someone who constantly asks questions and searches for answers, challenging established knowledge (Cabral; Baldino, 2010). From this perspective, a discussion session was conducted through an activity led by André and his group during the second class of the course.

The content focused on the representation of division through fractions. André's group, formed by three students, was responsible for creating an example of fraction division and explaining it to the entire class. He appeared to struggle and constantly asked questions to his colleagues during his explanation at the board, as though he was seeking validation of his ideas while presenting the solution. Aware of the representation $\frac{a}{b} \div \frac{c}{d}$, with 'b' and 'd' being different from zero (which was already on the board), André wrote the example $\frac{1}{2} \div \frac{2}{4}$ and argued:

André: in this situation, we should just keep the first fraction and multiply it by the inverse of the second, crosswise: $1 \times 4 = 4$ *and* $2 \times 2 = 4$*. He added: 1 corresponds to "a"; 2 corresponds to "b"; this 2 corresponds to "c"; and 4 corresponds to "d".*

Then, André made the following comment:

André: I don't know why it is inverted. I just learned it this way.

This mechanical learning, according to which students perform calculations without understanding their meaning, is common in Prevailing Traditional Education (PTE). Most students in Basic Education employ mathematical tools without knowing why they are doing that, acting as mere executors of procedures. In a typical PTE-guided class, this approach would suffice for a student to convince the teacher that their answer was correct. However, in Solidarity Assimilation, neither the teacher nor the class is responsible for providing the answer to the student. Instead, the role of the teacher and the class is to guide students in understanding and reaching the solution themselves. Next, the professor asked:

Professor: what's the difference between fraction division and fraction equivalence? You said equality should also be multiplied crosswise. Is it the same as division?

After hearing some comments from the group – such as "I guess it's the same thing" and "I don't think so, because in division you multiply $a \times d$, but as you invert the second fraction, it becomes c" –, André admits feeling confused. Next, one of his classmates, whom we have named Vagner in this text, makes a suggestion:

Vagner: there's also another way: you could place one fraction on top of the other and multiply extreme by extreme and inner by inner. (André solved it this way, too).

Group: but that's the same thing!

Up to that moment, the professor had interfered little and allowed the students to try to help André. However, she reiterated the question she had previously asked.

Professor: you haven't answered whether it's different or not.

André remained silent. The professor then suggested:

Professor: write two fractions of same value – one should be a division and the other one an equality. Check if the results are the same.

André: $\frac{2}{3} = \frac{4}{6}$ *and* $\frac{2}{3} \div \frac{4}{6}$.

Before André solved it, a student commented:

Student: the only difference is that the results will be different.

Then, André solved it the following way:

André: $\frac{2}{3} = \frac{4}{6} = \frac{12}{12}$ *and* $\frac{2}{3} \div \frac{4}{6} = \frac{12}{12}$. *Therefore, it is the same thing.*

At that moment, without the professor's interference, the students tried to help André:

Group: but the operation is different. The first one isn't a division, so you can't write $\frac{12}{12}$.

Guided by his classmates' suggestion, André altered his solution and wrote $\frac{2}{3} = \frac{4}{6} = 12 = 12$. As he followed the steps proposed by the group, it became evident that he did not understand what he was writing; he was merely reproducing what his classmates said. André commented:

André: so, I shouldn't use a fraction here?

Group: no, because that's an equality.

At that moment, the professor asked:

Professor: observe the property in the text. Is that what it says?

André: yes. That was my mistake.

After that, their focus shifted back to division:

Professor: so, can I multiply crosswise in division, too?

Some students said yes, whereas others said no.

André: yes, they're the same. Rather than inverting, you just have to multiply crosswise.

Camila: but the other one was the same.

Vanessa: but he didn't invert the other one, and he was multiplying crosswise.

Professor: well, if you apply this rule he created and multiply crosswise...

Group: [laughing].

Professor: ... then you can't invert it. If you invert it, you can't multiply crosswise.

André continues to insist on his method, arguing that the two operations are equal, but he does not realize that the result is the same because the procedure he is applying is identical. Nevertheless, the operations are different, and he fails to notice that he is performing division simply by multiplying the fractions crosswise $\frac{2}{3} \div \frac{4}{6} = \frac{2x6}{4x3} = \frac{12}{12}$, while the fraction equivalence property leads to $\frac{2}{3} = \frac{4}{6} = 2x6 = 12$, which is equivalent to $3x4 = 12$. Yet, the representations of the results differ: one is $\frac{12}{12}$ and the other is $12 = 12$.

In fact, when André states that fraction division can also be solved by crosswise multiplication, he is not wrong, because performing $\frac{2}{3} \div \frac{4}{6} = \frac{2x6}{4x3} = \frac{12}{12}$ is the same as inverting the second fraction and multiplying it by the first: $\frac{2}{3} x \frac{6}{4} = \frac{2x6}{4x3} = \frac{12}{12}$, leading to the same result. Nevertheless, he fails to notice that a crucial step in this type of operation is being omitted, the invariance of the quotient, in which a quotient remains unchanged when the dividend and the divisor are multiplied by the same number, allowing us to obtain a fraction with a denominator of 1 in fraction division. The professor then offers another suggestion.

Professor: now, represent a division as a standard division. Write this division in fraction form, with a numerator and a denominator, so that we can try to understand why I can multiply the first one by the inverse of the second.

André writes this representation $\frac{\frac{2}{3}}{\frac{6}{3}}$ on the board and asks if it is correct.

Professor: how can I justify that keeping the first fraction and multiplying it by the inverse of the second will give me the correct result?

André: that's a good question. This is something I've never learned. And I don't know why. I just know that I should invert it and solve it. Have you guys learned it? (he asks the group)

The group makes some comments, but they admit that they do not know how to explain it. After this interaction, André seemed to be pleased with his answer, hoping that the professor will clarify why that happens. However, the professor approaches it in a different way:

Professor: you represented this division, in which the numerator is a fraction, and the denominator is also a fraction, and we have seen that we can divide the numerator and the denominator without altering the fractions. If I multiply the numerator by a value, I must also multiply the denominator by the same value, and if I perform a division, I must do it, too. How can that help me in this operation?

At this moment, André remains completely silent, showing that he does not possess the necessary tools to explain what the professor is asking. This is a crucial moment when he has the opportunity to learn something new, because the formula he memorized – "multiply crosswise or preserve the first fraction and multiply it by the inverse of the second" – is insufficient to provide an argument in response to the professor's question. This is the moment when the group can help him develop a new understanding of the subject: the moment when Lacan says the *shutter* is open. Throughout the presentation, the student repeatedly tries to answer the professor's question in his own words, but his response was insufficient to convince her. According to Baldino (1997), when a student lacks the ability to answer a question, the shutter opens (1997). At this moment, the student is open to search for new tools, possibly trying to modify their discourse to solve the question.

Furthermore, it is important to emphasize that the student will always produce a discourse when they stand at the board: they must express what they know about the subject and position themselves, even though their ideas are not fully organized. In this case, the teacher must play a fundamental role in directing them toward a solution to the problem. Thus, based on the previously mentioned properties in the worksheet the group received, the professor continues to ask questions – this time, addressing not only André but the entire group.

Professor: I have a numerator that is a fraction and a denominator that is a fraction. How can I simplify a fraction – generally speaking, since in this fraction, the numerator and the denominator are also fractions – so that only one value remains?

Camila: by transforming it into a decimal?

Vagner: what if I multiply it by 3 in the upper and lower parts?

Pedro: professor, if I divide the fraction's denominator, which is also a fraction, by 2, do I need to divide it by the two numbers of the fraction, that is, the denominator and the numerator, or only the numerator?

Professor: What's that again?

Pedro: for example, let's say I have ½ divided by ½. In the case of the denominator down there – ½ in this case –, do I have to divide it by 2 and by 1 or only by the denominator 2?

Professor: although they look like two separate numbers, the fraction represents a single value.

Camila: professor, there (pointing to the denominator in $\frac{\frac{2}{3}}{\frac{6}{3}}$*), do I have to divide by 3 and by 6? Can I ignore the 3?*

Professor: if you multiply $\frac{6}{3}$ *by 3, how do I represent 3 in fraction form?*

Group: $\frac{3}{1}$.

Professor: so, 6 multiplies 3, and 3 multiplies 1 ($\frac{6}{3} x \frac{3}{1}$*), right?*

Pedro: professor, can't I just simplify $\frac{6}{3}$ *as 2?*

Professor: yes, but how does that help us?

Pedro: I don't know.

In this process, it is natural for students to ask the teacher about their actions, seeking validation, to know whether their answers are correct. The teacher, in response, answers "If it's correct, you may proceed with this particular step; if it's wrong, you cannot; check it", which makes the student turn to their classmates, looking for support to develop their knowledge (Cabral; Baldino, 2010).

The Analyst's Discourse is then established, characterized by the teacher's approach not as someone who provides final answers, but as one who poses questions and encourages students to reach their own conclusions. This method is a response to the hysteric's demand. By assuming this role, the professor prompts students to question the accuracy of their answers and reasoning. Rather than passively accepting information, students are inspired to question their own reactions and thoughts. In this sense, Cabral and Baldino (2010, p. 630, our translation) state that "the teacher's desire is never fulfilled because, once a problem is solved or the logical timing of the solution expires, they intervene, guiding students back through the path they followed, and soon pose a new problem". That is, the teacher continues to reinforce the importance of the learning process rather than focusing solely on the result.

After the group remained silent for some time, the professor continued:

Professor: following this simplification idea, if I consider $\frac{6}{3}$*, I can reach a division really quickly, but I'm considering the numerator as 6 and the denominator as 3; therefore, I can divide both the numerator and the denominator and obtain a single value; in this case, if I divide 6 by 3 and 3 by 3, the result is 2, which equals what?*

Group: $\frac{2}{1}$.

Professor: Now, thinking about it, what can I do in my fraction division ($\frac{\frac{2}{3}}{\frac{6}{3}}$) *so that I finally have a quotient of a single value? What do I need to do with this denominator:* $\frac{6}{3}$ *?*

Camila: simplify it?

Professor: simplify it until the denominator is…?

Camila: 1?

Professor: and how do I make my denominator, which is $\frac{6}{3}$*, equal 1?*

André: divide both the upper and lower parts by 3.

Professor: if I divide by 3, will the result be 1?

Camila: no.

Ana: should it be $\frac{1}{1}$*? Can't I cancel out the 3?*

Professor: cancel it out? How? Can I cancel it out?

Vitória: you can't. It's a whole number, a single value.

Group: no.

André: then you'll have the result you want.

Victor: I can divide by the same value, can't I?

Professor: nothing justifies that here. Whatever you want to do, you need to know why it's done. You can't cancel out that 3 by 3 just because the numbers are the same. There must be a property allowing me to cancel these numbers when they're being divided, which is not the case here. This one here (denominator) must equal 1. What number should $\frac{6}{3}$ *be multiplied by to make it 1? This number will be the same as the one I'll multiply the numerator by. So, we need to know: what number is it?*

Group: it's 6.

Professor: will the result be 1?

Pedro: $\frac{1}{2}$.

Group: $\frac{1}{2}$*?*

Professor: What if I multiply $\frac{6}{3}$ *by* $\frac{1}{2}$*?*

Pedro: multiply by 2, professor.

Professor: if I multiply by 2, it will be $\frac{12}{3}$*. Shouldn't it be 1?*

Luan: then it's $\frac{1}{2}$.

Group: no! YES!

Camila: it'll be 1!

Professor: $\frac{6}{3}$ *multiplied by* $\frac{1}{2}$ *is* $\frac{6}{6}$*, which equals 1. Right. In this case, it worked.*

Camila: but can I do that?

Luan: yes, whatever you do to the upper part, you should also do to the lower part.

Vanessa: but can you multiply by another fraction? Shouldn't it be by an integer?

André: don't we have to find the LCM of 6 and 3?

Group: no!

Professor: ok. In this case, $\frac{1}{2}$ *also works. But what other number besides* $\frac{1}{2}$ *will always result in 1 every time I multiply by it?*

André: $\frac{2}{4}$*?*

Professor: yes, that works, too. But what's happening here? Why are these values correct? What number multiplied by itself always equals 1?

Luan: zero.

Professor: any number times zero will be…

Group: zero.

Professor: if I multiply any fraction by this value, will it always equal 1?

Ana: the fraction itself.

Group: no.

Luan: its inverse.

Group: Oh, that's its inverse!

When we reach the final moments, it is time for the students to adopt the University Discourse. At the professor's request, the students will present the solution steps, preferably by using the board to share their opinions with their classmates. At this stage, the professor plays a fundamental role in pedagogical transference: she questions students, preventing them from imaginarily identifying themselves as subjects who must know everything, imitating their master in the position of S1. Instead, the professor seeks to keep students away from merely reproducing memorized information superficially (Cabral; Baldino, 2010).

Professor: do the calculation, André. Check if it equals 1.

André: $\frac{3}{6} \times \frac{6}{3} = \frac{18}{18} = 1$. *So, that's why it's inverted? Now I get it.*

Professor: and now? Is there anything left to do?

Group: Whatever you do to the denominator you should also do to the numerator.

André: $\frac{\frac{2}{3}}{\frac{6}{3}} = \frac{\frac{2}{3} x \frac{3}{6}}{\frac{6}{3} x \frac{3}{6}} = \frac{\frac{6}{18}}{\frac{18}{18}} = \frac{\frac{6}{18}}{1} = \frac{6}{18}$

Professor: in this case, we've found other fractions.

Luan: because $\frac{6}{3}$ *was simplified.*

Professor: yes.

Vagner: professor, that's exactly it. We study this in basic education.

Fernando: we learn it in 6^{th} *grade, when we start working with fractions.*

Victor: but do we study the justification, too?

Felipe: no. The teacher just tells you to invert it.

This case illustrates how students often enter Higher Education with mechanical conceptions acquired in Basic Education. When confronted with ideas that challenge their usual methods, students may have difficulty in developing and understanding concepts more deeply.

Another significant aspect of this process is that the professor asks questions rather than providing answers – they are hypnotized by the students' answers and ask new questions, leading students' to a contradiction regarding their initial statement, until they understand the irreducible signifier they were defending and show their new comprehension in their discourse –, which reveals what Baldino and Cabral (2010) denominated as "teaching by listening" and "learning by speaking".

5 Conclusion

In order to achieve our objective of discussing contributions from a proposal for didactic-pedagogical intervention based on Solidarity Assimilation (SA) for understanding mathematical concepts and training future mathematics teachers, we used an extract from a situation that occurred during the development of a course in which there was an evident contribution from SA to students' comprehension of the content, leading to significant learning. In this sense, SA was developed under the assumption that comprehension implies learning. While conducting the study, and supported by Baldino and Cabral (2005), we understood that, as educators, we are only responsible for ensuring comprehension, and that learning is the result of one's attitude toward the course, university and life as a whole.

Based on the analyzed episode, we highlight that we identified signs of contributions from SA to mathematics teacher training. The key concept that guided our inquiry was reverse hypnosis, as it encourages students to represent mathematical concepts through signifiers. When André was trying to find examples based solely on memory, he was not able to present enough signifiers to explain the fraction division operation. Nevertheless, when the students looked for other examples – which made them mobilize signifiers that previously existed in the unconscious –, the construction of the 'fraction division' signifier became more comprehensible.

We emphasize the investigated episode was randomly selected, and that many others followed the same format. Our intention was to discuss possible contributions from SA to teacher training, and a wide range of concepts can be employed within it. However, fraction operations are challenging for students across various educational levels, as revealed by research conducted by Miola and Lima (2020), Carolino and Pietropaolo (2019), Miola (2011), and others. Another meaningful contribution from SA to teacher training is leading the participants to understand that educators are unfinished professionals, always able to learn, and that they are represented by signifiers accepted within a particular community. This was precisely demonstrated by Pedro, Wagner, Camila, Vitória, André, Vitor, and others when they represented and positioned themselves through their discourse, gestures, concepts and the authors they studied among their classmates and professors – and this is what they will do in the future, too, within their peer network and the school community.

Considering the discussions and comments drawn from the described episode, we emphasize that SA is a relevant tool to be employed within the assessment system proposed by Prevailing Traditional Education, aiming to assess students in a fairer way by taking into account the work they developed throughout the entire learning process. It is important to highlight that, according to Baldino and Cabral (2010), SA does not seek to replace PTE, but rather to operate within this system.

Another fundamental aspect to emphasize is the context in which SA was created. The initial motivation for developing this pedagogical intervention proposal stemmed from the measures adopted by the military government in the scope of education, especially regarding engineering courses, as the people involved in those programs were financially privileged and had certain advantages, including favorable grading.

It is known that this logic remains embedded in the current Brazilian educational system, perpetuated by the strong structures of capitalism that shape PTE, in which students are viewed as workforce for the job market. These characteristics are especially demonstrated by expository lessons and by how students are assessed (summative tests/exams), supported by methodologies that ignore the global development of individuals and critical thinking. In this regard, it is evident that PTE reinforces capitalism and vice-versa.

Thus, our initial focus was to present a perspective on how mathematics teaching is currently conceived, aiming to challenge certain views and studies in Mathematics Education that portray successful proposals, often masking the actual reality of teaching. Therefore, we centered our efforts on learning and mathematics teacher training, seeking to deepen the understanding of how the educational system operates and of the importance of a more consistent, reflective and in-depth approach to learning that can effectively comprehend reality.

Acknowledgements

I thank the Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES) for funding this research.

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