



# Perceptions of indigenous mathematics teachers regarding the historical resource of Archimedes' balance

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**Abstract:** This study, aims to analyze the perceptions of seven indigenous Mathematics teachers from the Pataxó ethnic group regarding the production, observation, and manipulation of the historical resource Archimedes' balance for the purpose of constituting mathematical expressions for calculating the spherical surface and volumes of a right cone and a sphere, based on the right equilateral cylinder. The balance and the three solids involved were made of wood (cedar) by a Pataxó artisan at the request of the participants. This is a qualitative research, in the midst of a didactic sequence, organized in two face-to-face meetings that totaled six hours (two in the first and four in the second), in November 2024, with a theoretical basis based on the History of Mathematics from the perspective of Ethnomathematics. The perceptions of the indigenous teachers indicate that they understood the constitution of these expressions, and that they intend to replicate the dynamics in their classes with indigenous students.

Keywords: Indigenous School Education. Mathematical Education. Ethnomathematics. Geometry. History of Mathematics.

# Percepciones de los docentes indígenas de matemáticas respecto al recurso histórico de la balanza de Arquímedes

Resumen: En este trabajo, el objetivo es analizar las percepciones de siete profesores de Matemáticas indígenas de la etnia Pataxó, respecto de la producción, observación y manipulación del recurso histórico balanza de Arquímedes con el propósito de constituir expresiones matemáticas para el cálculo de la superficie esférica y los volúmenes de un cono recto y una esfera, a partir del cilindro recto equilátero. La escala y los tres sólidos involucrados, a pedido de los participantes, fueron elaborados en madera (cedro) por un artesano Pataxó. Se trata de una investigación cualitativa, en medio de una secuencia didáctica, organizada en dos encuentros presenciales que sumaron seis horas (dos en el primero y cuatro en el segundo), en noviembre de 2024, con una base teórica basada en la Historia de la Matemática desde la perspectiva de la Etnomatemática. Las percepciones de los docentes indígenas indican que entendieron la constitución de estas expresiones y que pretenden replicar la dinámica en sus clases con estudiantes indígenas.

Palabras clave: Educación Escolar Indígena. Educación Matemática. Etnomatemáticas.



Geometría. Historia de las Matemáticas.

# Percepções de professores indígenas de Matemática em meio ao recurso histórico da balança de Arquimedes

Resumo: Nesse trabalho, tem-se o objetivo de analisar as percepções de sete professores indígenas de Matemática da etnia Pataxó, sobre a produção, observação e manipulação do recurso histórico balança de Arquimedes com a finalidade de constituição das expressões matemáticas para o cálculo da superfície esférica, e dos volumes de um cone reto e da esfera, a partir do cilindro equilátero reto. A balança e os três sólidos envolvidos, a pedido dos participantes, foram confeccionados em madeira (cedro) por um artesão Pataxó. Trata-se de uma pesquisa qualitativa, em meio a uma sequência didática, organizada em dois encontros presenciais que totalizaram seis horas (duas no primeiro e quatro no segundo), em novembro de 2024, com base teórica pautada na História da Matemática sob a perspectiva da Etnomatemática. As percepções dos professores indígenas apontam que eles compreenderam a constituição das referidas expressões, e pretendem replicar a dinâmica em suas aulas com estudantes indígenas.

**Palavras-chave:** Educação Escolar Indígena. Educação Matemática. Etnomatemática. Geometria. História da Matemática.

#### 1 Introduction

Indigenous teacher training in Brazil takes place through courses in Indigenous Teaching, Intercultural Pedagogy or Indigenous Intercultural Degree, with the aim of enabling these professionals to work in indigenous schools located in their communities. Bicho, Auareke and Miola (2023) emphasize that the structuring of these courses focuses on extolling the diversity and cultural complexity of the peoples involved, as well as seeking to promote dialogue between traditional knowledge and non-indigenous knowledge present in the curricula of each training course. Thus, "[...] indigenous teaching practice has a specificity which is to be knowledgeable about one's own culture and ancestral roots, and it is not enough to know, but to make known in order to provide a liberating indigenous school education" (Mattos & Mattos, 2019, p. 108).

The work of indigenous teachers seeks to meet the needs demanded by public policy for the implementation of indigenous education in Latin American countries, as a result of the struggle of indigenous peoples (Franco & Álvarez, 2023). Zeballosf-Cuathin (2024) presents in her work the struggle for rights achieved by Latin American indigenous peoples, above all, the right to have a heterogeneous education that considers the cultural diversity of each indigenous community in educational dialogue. In Mexico, for example, Intercultural Universities were created in the first decade of this century, which constitute "higher education proposals aimed at serving, primarily, the rural and indigenous population" (Moyo & Pardo, 2024, p. 140). In Brazil, indigenous teachers seek to meet the needs of Indigenous School Education, governed by Intercultural Education and guaranteed by the Federal Constitution of 1988, as a fundamental right of indigenous peoples, regulated by the Law of Guidelines and Bases of Brazilian Education of 1996.

For Candau (2020), Intercultural Education seeks to equalize the cultural confrontations that emerge in mechanisms of radicalization of the affirmation of one culture over another. In it, the educational process is heterogeneous, dynamic, and complex. There are no cultural elements more important than others, given that this model understands that each culture is in permanent construction, disregards the issues that differentiate one culture from another, in



addition to valuing the fight against inequalities that present themselves in our reality. "What I consider important in the intercultural perspective is to stimulate dialogue, mutual respect, and the construction of bridges and common knowledge in the school routine, in the teaching-learning processes developed in the classrooms" (Candau, 2020, p. 42).

According to Moyo and Pardo (2024), Intercultural Universities constitute spaces for cultural and ideological disputes, while the former provides a plural, dynamic and complex educational environment, the latter "can serve to politically position educational projects, but which, in addition, represent an obstacle to understanding the complexity of the phenomena that develop in Intercultural Universities" (Moyo & Pardo, p. 142). Oliveira (2020) highlights that, through Intercultural Education, the training and performance of indigenous teachers require the mediation of conflicts, the appreciation of cultural diversity, and constant dialogue between all those involved (academics, teachers, university, indigenous schools and their communities). "The production and circulation of different knowledge systems in indigenous teacher training courses, at the university and in indigenous schools, develop in a field of many conflicts, of connection and opposition" (Oliveira, 2020, p. 105).

The teaching of Mathematics in indigenous Mathematics teacher training courses is designed

from the perspective of human development and social justice, in addition to the mathematics required for teaching in the final years of elementary school and high school. To consider these aspects, it is necessary to understand the possibilities of establishing relationships between mathematical content and the political, social and cultural dimensions of each sociocultural context. It is about building teaching based on activities that allow for the exercise of investigation, dialogue and criticism (Lima & Lima, 2024, p. 6).

In this scenario, in the field of Mathematics Education, the educational perspective of Ethnomathematics emerges. According to D'Ambrosio (2016), studies in Ethnomathematics seek to intertwine the various ethnomathematics that are developed in each socially identified human community, such as that of indigenous peoples, with the Mathematics itself discussed in schools. For the author, there is no hierarchy between the various ways of mathematizing the daily life of each community. In this context, the mathematical knowledge that is useful for one of these human groups may not be useful for others; it varies in accordance with the cultural environment of each community, just as non-indigenous Ethnomathematics also serves or is important for other societies, such as, for example, the various indigenous ethnic groups (D'Ambrosio, 2016).

The indigenous Mathematics teacher has the role of contextualizing the mathematical knowledge/practice of his/her people with non-indigenous Mathematics, because "[...] access to more intellectual instruments or techniques gives much greater capacity and understanding to deal with new situations and solve problems" (Oliveiras & Gavarrete, 2012, p. 348). For D'Ambrosio (2020), the mathematical knowledge/practice of each indigenous people encompasses the different contextualized ways of practicing "[...] comparing, classifying, quantifying, measuring, explaining, generalizing, inferring and, in some way, evaluating [...] in the search for explanations and ways of dealing with the immediate and remote environment [...] is imbued with the knowledge and practices specific to the culture" (D'Ambrosio, 2020, p. 24). Indigenous mathematical knowledge/practice requires its protagonism in Indigenous School Education, and it is up to the indigenous Mathematics teacher to foster this prominent role.



According to Santos and Lara (2022), when Ethnomathematics works in partnership with the History of Mathematics, other possibilities for discussions about how the production, formation, and propagation of mathematical knowledge develop are opened up. Furthermore, the authors highlight that, through the study and/or research of the History of Mathematics, one can evoke, analyze, and understand the various ways of mathematizing that human societies have developed and/or develop throughout their histories. For Santos and Lara (2022), mathematical knowledge is organized, constituted, and correlated to the different ways of living and developing in each community, "[...] it is the History of Mathematics that enables Ethnomathematics to understand what are the conditions of possibility for the generation, organization, and dissemination of mathematical knowledge/doing [...]" (Santos & Lara, 2022, p. 477).

The authors Silva, Pereira and Batista (2022, p. 3) consider that

The idea of inserting resources from history into the teaching of Mathematics is based on guidelines that seek to mobilize mathematical concepts from some resources available in history, to have a didactic use in the classroom, in order to build knowledge with some elaborate practice (Silva, Pereira & Batista 2022, p. 3).

From this perspective, one of the resources available in the History of Mathematics is Archimedes' abstract balance, "[..] which was supposed to equilibrate equivalent geometric figures. The objective was to defend a method that would allow us to understand certain mathematical realities through mechanics" (Roque, 2015, p. 198). In this case, the aforementioned balance presents the equilibrium between a sphere and a straight circular cone with a straight equilateral cylinder, with base radii equal to the radius of the sphere and a height equal to twice the radius of the base. Throughout the text, they will be referred to only as straight cone and straight cylinder.

In this context, based on the work of Polegatti (2020), a teaching sequence was developed with the objective of analyzing the perceptions of seven indigenous Mathematics teachers from the Pataxó ethnic group, which involves the process of making the historical resource *Archimedes' balance* and the *three geometric solids* (straight cylinder, straight cone and sphere), goes through the observation and manipulation of these materials, and ends with the constitution of mathematical expressions that serve to calculate the spherical surface, the volume of the sphere and the volume of a straight cone. These teachers work in Indigenous School Education in the district of Coroa Vermelha, located in the municipality of Santa Cruz Cabrália, in the South of the State of Bahia. The teaching sequence was implemented in November 2024, at the Federal Institute of Bahia on its campus in the municipality of Porto Seguro.

This is a qualitative study, as it "reads and pays attention to people and their ideas, seeking to make sense of discourses and narratives that would otherwise be silent" (D'Ambrosio, 2019, p. 21). Furthermore, this article is divided as follows: section 2 presents a brief theoretical framework involving the training of indigenous Mathematics teachers in the context of Intercultural Education, from the perspective of Ethnomathematics and with the support of the History of Mathematics; section 3 presents the methodological procedures; section 4 contemplates the development of the didactic sequence intertwined with the participants' perceptions, as well as some specific analyses within the theoretical basis. Finally, the final considerations are presented regarding the results and the possibility of other investigations that this work can foster. For the purpose of identifying the participants' statements (P), they were numbered from P1 to P7, throughout the text, in accordance with the



manifestation of each of the indigenous Mathematics teachers<sup>1</sup>.

#### 2 Theoretical framework

In this work, as a non-indigenous Mathematics teacher, the researcher is faced with a dynamic and complex context, tense due to conflicts and cultural approximations, in the midst of the Intercultural Education environment. According to Candau (2020, p.36), Intercultural Education is based on the principle that the knowledge discussed, at any school level, can and should be questioned, it is neither neutral nor static, "conceiving school dynamics from this perspective implies rethinking its different components and breaking with the homogenizing and standardizing tendency that permeates its practices". For Delmondez and Pulino (2014), intercultural educational spaces are conducive to dialogue with other customs, other ways of reading and interpreting the world, other experiences and thoughts, after all, constant exercise of this dialogue "[...] is desired as a structuring factor for indigenous pedagogical practices. It is about thinking about the role of the one who carries out the mediation, that is, the role of the teacher in the search for valuing cultural differences" (Delmondez & Pulino, 2014, p. 639).

Indigenous school education environments are tense (cultural clash) and challenging for non-indigenous teachers. When it comes to teaching and learning mathematics, the challenge increases, because non-indigenous mathematical knowledge, according to D'Ambrosio (2020), is presented as something from outside the community, originating from the colonizer. In other words, there is, in a way, a cultural gap between the mathematical knowledge/practice of the indigenous community in question and the mathematical knowledge present in the school curriculum of their indigenous schools. In this scenario, Rincón, Osório and Parra (2015) emphasize that the educational and research perspectives of Ethnomathematics emerge to promote the appreciation of the mathematical practices of the indigenous communities involved, that is, the peculiar ways of mathematizing of each ethnic group emerge and are protagonists in the intercultural dialogue with the mathematical knowledge of the non-indigenous. Within the scope of Ethnomathematics, teaching or research methodologies are not delimited; there are dialogues between theories and practices.

Corroborating the debate, according to Mattos and Mattos (2019), it is up to the indigenous Mathematics teacher to build cultural bridges to overcome this abyss, that is, "it is appropriate for indigenous teachers, in their teaching practices, to show the importance of the traditional knowledge and practices of the ethnic group, thus ensuring the presentation of contextualized school mathematical content and working with it interdisciplinarily" (Mattos & Mattos, 2019, p. 10). Thus, the praxis of the indigenous Mathematics teacher needs to occur in educational environments that promote dialogue and cultural appreciation, consider and seek to contextualize the varied ethnomathematical knowledge and practices of those involved, with the mathematical knowledge present in the school curriculum of Indigenous School Education. To this end, according to (D'Ambrosio, 2020), it is necessary for the indigenous Mathematics teacher to recognize that traditional mathematical knowledge, mathematical knowledge/doing, emerges from the indigenous context to be articulated with protagonism in school mathematical knowledge.

Santos (2018) emphasizes that Ethnomathematics acts to establish other truths in the process of teaching and learning Mathematics within indigenous schools. Truths of traditional knowledge meeting or opposing school knowledge. Truths that emerge from the ethnomathematical thinking of indigenous peoples, with emphasis on indigenous Mathematics

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<sup>&</sup>lt;sup>1</sup> The responses obtained were transcribed literally, without undergoing grammatical correction criteria, in order to guarantee a reliable analysis of the phenomena investigated in this research.



teachers. Truths of subversion of the non-indigenous school curriculum, given the consideration of calendars that follow the cultural dynamics of the community, as well as truths produced by the diversity and freedom of pedagogical strategies, adapted and contextualized, to the mathematical knowledge/doing of indigenous daily life.

Thus, the work of indigenous Mathematics teachers, in accordance with the precepts of Ethnomathematics, enables the mathematical knowledge/practice of indigenous students to be deepened amidst the communion of non-indigenous mathematical knowledge with their traditions, stories, myths, legends, beliefs, rituals, interpersonal relationships, and distinct ways of interacting with the environment in which they live and with other indigenous or non-indigenous individuals. In the context of Indigenous School Education, in addition to looking at the surroundings, do it

necessary to look at oneself, human beings, knowledge and actions, because we are part of this environment. This is how indigenous people think and they helped us understand how much we are part of these multispecies that dialogue with nature, with the beings of the waters, the air, the forest (Mattos, 2025, p. 7).

The scenario of Indigenous School Education is conducive to the development of self-research processes, that is, researching one's own practice, or that of other indigenous teachers from the same or other ethnic groups. Or even promoting the intertwining between cultural elements and school knowledge with the help of non-indigenous teachers. In this context, it is recognized that "Ethnomathematics can provide a theoretical basis so that teachers can think, reflect, plan and develop pedagogical practices that mark the place of traditional knowledge in school education" (Oliveira, 2018, p. 179).

According to Polegatti et al. (2024), within the scope of Indigenous School Education, and in line with the principles of Ethnomathematics, the epistemology of the indigenous Mathematics teacher aims to involve with protagonism the individual's own mathematical knowledge/practice in its relationship with the mathematical knowledge of the non-indigenous. From the coexistence with the participants, it is understood that the relationship between the indigenous Mathematics teacher and the indigenous student develops beyond the pedagogical, assumes a degree of kinship, the indigenous teacher gains visibility, becomes representative before his/her indigenous students. In the participants' statements, "the teacher is actually a continuous student. In the community, he is the teacher for all moments, and when he leaves the community, he has the responsibility of representing all his people, of bringing and taking information" (P3) and "the teachers are the relatives themselves, and the students are the nephews, children, cousins, father, mother, and relatives for being indigenous, only those who are indigenous will understand, being relatives without being part of the family" (P5). In this context, it is agreed that "when we see ourselves as human relatives, we will be able to negotiate, exchange and share the environment with other human beings or not in a harmonious way" (Mattos, 2025, p. 8).

According to Zabala (1998, p. 18), the didactic sequence is defined as "A set of ordered, structured and articulated activities for the achievement of certain educational objectives, which have a beginning and an end known by both teachers and students". Each activity in a didactic sequence constitutes a dynamic process that involves students and teachers with the purpose of promoting teaching, learning and school assessment, in a cohesive triad that "has, for example, a dialogued exposition, practical work, observation, study, debate, reading, bibliographic research, note-taking, motivating action, application" (Costa & Gonçalves, 2022, p. 366). For Zabala (1998, p. 20),



The role of teachers and students and, in short, the relationships that occur in class between teacher and students or students and students, affects the degree of communication and the emotional bonds that are established and that give rise to a certain climate of living together.

Thus, in the midst of the proposed didactic sequence, it is understood that the indigenous Mathematics teacher is the one who naturally exalts cultural links in his/her praxis, through the intertwining between the mathematical knowledge/practice of his/her people, with the mathematical knowledge present in the Indigenous School Education curriculum, after all, "it is the indigenous Mathematics teacher who gives voice to traditional mathematical knowledge, or rather, his/her voice in conjunction with his/her indigenous students, family members, elders, among others" (Polegatti *et al.*, 2024, p. 8). In this work, it is emphasized that it is the role of indigenous Mathematics teachers to be autonomous ethnomathematical researchers connected to their cultural realities, that indigenous Mathematics teachers be a voice and exercise the leading role of acting by building bridges and strengthening existing links, between curricular mathematical knowledge and the mathematical knowledge/practice of their people.

Therefore, in developing this teaching sequence, on the one hand, it is not appropriate to dictate to the participants how *geometric objects* (sphere, straight cone and straight cylinder) and *Archimedes' balance* can emerge from their culture. Instead, a teaching and learning process should be developed, not for them, but with them, based on the cultural appreciation of these indigenous teachers, in the examples they give during the dynamics of the teaching sequence. On the other hand, the researcher plays the role of interlocutor of the mathematical knowledge of the school curriculum (technical academic), with the challenge of bringing the knowledge of the colonizer, without making it transparent, for example, through the use of teaching resources, and, going further, by putting himself in the place of the indigenous teacher, without, however, occupying the space that is his by nature.

Bringing elements from the History of Mathematics to the school debate shows that the mathematical knowledge/practice of each human society is the link, often hidden in the school curriculum, between the mathematical knowledge of the school and the ways people use mathematization in their daily lives. It is understood that sociocultural elements that emerge in investigations, in the face of the History of Mathematics, are important for the understanding that mathematical knowledge is a collective human construction, an intangible and cultural asset of humanity, fundamental for our development as a society, and, therefore, should be within everyone's reach. Using the History of Mathematics is an educational procedure that humanizes Mathematics. "Teachers who have a historical perspective on the evolution of mathematics, as a process of human construction, are able to use the experience and cultural reality of their students to choose motivating and contextual problems" (D'Ambrosio, 2007, p. 401).

Thus, by bringing the History of Mathematics into the development of Mathematics classes, we seek to show indigenous teachers that mathematical knowledge, present in the school curriculum, did not emerge suddenly, but rather, was and continues to be constructed by human beings, in the form of cultural representations that emerge from the art of mathematical knowledge/doing of each socially identified human community. The educational environment provided by Indigenous School Education, combined with the pedagogical perspective of Ethnomathematics, and in partnership with a *praxis* that involves resources from the History of Mathematics, corresponds to the theoretical basis for the constitution, execution and analysis of the proposed didactic sequence.

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## 3 Methodological paths

This work is developed in the context of Indigenous School Education, from the perspective of Ethnomathematics, in line with resources from the History of Mathematics, in the midst of the elaboration and application of a didactic sequence. The objective of this investigation was to analyze the perceptions of seven indigenous Mathematics teachers, from the Pataxó ethnic group, about the process of production, observation and manipulation of the *Archimedes' balance*, with the purpose of constituting the mathematical expressions of the calculation of the spherical surface, and the volumes of the right cone and the sphere, from the right cylinder. The opportunity for research comes from the search, by the management of the Indigenous State School of Coroa Vermelha, located in the municipality of Santa Cruz Cabrália in the State of Bahia, to dialogue with the participating indigenous teachers, about the teaching of Mathematics content, with a focus on Geometry. This research is of a qualitative nature, a type of study characterized, according to Gonzáles Rey (2010, p. 81) as a

permanent process, within which all methodological decisions and options are constantly defined and redefined during the research process itself, [...] involves the immersion of the researcher in the research field, considering this as the social scenario in which the phenomenon studied takes place in the entire set of elements that constitute it, and which, in turn, is constituted by it (Gonzáles Rey, 2010, p. 81).

The researcher does not act neutrally, as there is no way to detach himself from his epistemological lens, which is affected by his readings, experiences, beliefs, reflections, among others. The trajectory of this investigation is not linear; its planning was reviewed and readapted based on the discussions and notes of the participants. Thus, there was no way to establish predetermined regulations for the research, since there is no general model or template for this type of investigation. After all, "the very nature of qualitative research does not allow it to be framed within guidelines" (D'Ambrosio, 2019, p. 22). In this investigation process, both Ethnomathematics and the History of Mathematics grant mathematical knowledge as a human production that develops permeated by the mathematical knowledge/doing present in the cultural contexts of the participants.

According to Zabala (1998, p. 19)

The sequences can indicate the function of each activity in the construction of knowledge or learning of different contents and, therefore, evaluate the relevance or not of each one of them, the lack of others or the emphasis that we should give them (Zabala, 1998, p. 19).

In this scenario, it is worth highlighting that, for Gonzáles Rey (2010), qualitative research seeks to consider all subjects involved in the investigation for the process of constituting the research scenario that "aims to present the research to the possible subjects who will participate in it, and its main function is to involve the subjective sense of those who participate in the research" (Gonzáles Rey, 2010, p. 83). The author emphasizes that the creation of the research scenario, the initial part of this didactic sequence, goes through the essential information about the subject under analysis. In addition, he considers the participants as protagonists in its development, and also emphasizes that it is in the constitution of the research scenario that "people will make the decision to participate in the research, and the researcher will gain confidence and become familiar with the participants and the context in which he will develop the research" (Gonzáles Rey, 2010, p. 83), in agreement with Lima and Lima (2024, p. 7) when they highlight that



The experience of a scenario for investigation in mathematics classes contrasts with a teaching that adopts only lists of exercises that, often, aim only at a mathematical solution based on memorization and the use of formulas, without investigation and critical reflections being requirements for the resolution (Lima & Lima, 2024, p. 7).

Thus, two face-to-face meetings were held with the participants. The first, lasting two hours, was dedicated to the constitution of the research scenario that begins with the reading of the texts by Ávila (2009) and Roque (2015). It went through discussions about basic concepts of Physics (force, lever, weight and specific mass), as well as the definition of the construction of the *abstract balance of Archimedes*' and *geometric solids* (straight cylinder, straight cone and sphere). These elements were used in the second meeting, lasting four hours, with the aim of constructing mathematical expressions of the spherical surface and the volumes of a straight cone and sphere. Thus, "the function of these materials is to arouse interest, promote creativity and ensure argumentation" (Mattos & Mattos, 2022, p. 92).

At the end of the second meeting, participants were asked to respond in writing to three open-ended questions. 1) What did you think of the idea of building and using *Archimedes'* balance? 2) Archimedes' balance and geometric solids were made of wood by an indigenous craftsman. Does this make them familiar to indigenous people? 3) Make a free comment about the activities carried out.

# 4 Participants' perceptions during the development of the teaching sequence.

In the first meeting, with the aim of creating the research scenario, the indigenous teachers discussed the texts by Ávila (2009) and Roque (2015), which involve *Archimedes' balance*, the purpose of which was to obtain historical information about the mathematical knowledge developed by this historical resource. Always attentive to consider that the role of the History of Mathematics "beyond the sterile reproduction of anecdotes aimed at motivating the interest of students, it is possible to reinvent the problematic environment in which the concepts were created" (Roque, 2015, p. 32). In the text by Ávila (2009), the participants' attention was drawn to a sketch of *Archimedes' abstract balance* that represents the mechanical equilibrium between a straight cylinder and a sphere with a radius equal to that of the aforementioned cylinder, combined with a straight cone with a height equal to twice the radius of the base of the straight cylinder, both represented in the drawings in Figure 1.

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Figure 1: Drawing of Archimedes' abstract balance

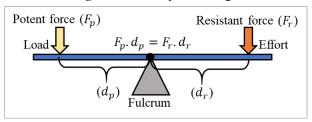
Source: Adapted from Ávila (2009, p. 5)

In order to develop the teaching sequence, prior knowledge of the physical concepts of force, lever, weight force and specific mass is necessary. Within the scope of a teaching sequence, "teaching must help establish as many essential and non-arbitrary links between new content and prior knowledge as the situation allows" (Zabala, 1998, p. 38) and, according to Roque (2015, p. 198) "we know today that some of the results demonstrated geometrically by Archimedes were obtained in a purely mechanical way", thus, the dynamics of the functioning



of the *abstract balance* falls within the scope of Mechanics. It is an lever, in which there is a central fulcrum and two or more forces acting on opposite sides (potent force x resistant force), whose equilibrium, as shown in the drawing in Figure 2, is achieved as a function of the distance *d* between the point of action of each force and the fulcrum of the lever.

Figure 2: Lever system design



Source: Authors' digital archive

In this regard, it was emphasized to the participants that the forces (potent x resistant) acting on the *Archimedes' balance* correspond to the respective weights P of the *geometric solids* involved. "The relationship between the weight and mass of a body is given by the equation P = m.g where m is the mass of the body and g is the magnitude of the free-fall acceleration" (Halliday; Resnick & Walker, 2018, p. 103). In this scenario, the greater or smaller the difference in mass between the bodies, proportionally, the greater or smaller the difference in distance between each body and the fulcrum. With this, it is possible to calculate the relationship between the masses of the objects in the dynamics of promoting equilibrium on the *balance*. For example, if the bodies equilibrium on the lever (*Archimedes' balance*), equidistantly  $(d_p = d_r)$  to the fulcrum, it means that they have the same mass. Therefore, starting from the equilibrium equation present in the drawing in Figure 2, and with the same acceleration g that acts on geometric solids, we have that

$$Pp. dp = Pr. dr \rightarrow m_p. g. d_p = m_r. g. d_r \rightarrow m_p = m_r$$

Considering that "The history of mathematics can perfectly bring out of hiding the problems that constitute the mathematician's field of experience, that is, the concrete side of his work, so that we can better understand the meaning of his concepts" (Roque, 2015, p. 33), it was decided to reconstruct the historical resource of *Archimedes' abstract balance* to use it as teaching material for the didactic sequence.

However, the didactic sequence aims to promote the construction of mathematical expressions of the spherical surface and the volumes of the sphere and the right cone. Therefore, it was necessary to bring the physical concept of specific mass to the discussion. To this end, geometric solids must be made of the same material and be homogeneous, that is, "objects whose specific mass (mass per unit volume), represented by the symbol  $\rho$  (Greek letter rho), is the same for all infinitesimal elements of the object and, therefore, for the object as a whole. In this case, it can be represented:  $\rho = \frac{m}{V}$ " (Halliday; Resnick & Walker, 2018, p. 215-216, our emphasis). Thus, from the equation of specific mass in line with the relationship of the masses of the objects in equilibrium on the balance, the aforementioned mathematical expressions are constituted with the participants in the second meeting.

At the suggestion of the indigenous teachers, the Archimedes' balance and the geometric solids were made of wood (cedar) by an indigenous Pataxó artisan recommended by them. For the participants, "being made of wood gives us the possibility of building the materials ourselves, and we can produce with our indigenous students" (P1); "wood is widely used to make indigenous artifacts, it is part of our culture" (P6); "indigenous peoples use wood a lot



as a means of survival, that is, in moderation so as not to harm the environment" (P4); and "indigenous communities have the custom of using wood or clay, straw, vines, in crafts that can be transformed into geometric shapes" (P3). Figure 3 shows the entrance to the space designated for the Pataxó indigenous trade, located in the district of Coroa Vermelha, as well as five other images of the wooden artifacts in the indigenous artisan's shop (bowls, boats, snack bowls, mugs, flour bowls, flutes, whistles, oars, spears, arrows, bows, among others).

COMERCIO INDIGENA

Figure 3: Wooden artifacts in the artisan's shop at the Pataxó Indigenous Trade

Source: Authors' digital archive

For D'Ambrosio (2020, p. 28, author's emphasis), "human beings act according to their sensory capacity, which responds to the material [artifacts], and their imagination, often called creativity, which responds to the abstract [mindfacts]". According to indigenous teachers, "wood is close to our reality, it is an easily accessible material, and indigenous students like to work with community knowledge, wooden artifacts are familiar to us" (P7); "working with wood is part of our experience and that of our indigenous students, it is related to nature and is a low-cost material for us" (P4); and "using wood to make solids and balance seems familiar to us, for us Pataxó, wood is part of our culture, the solids could also be made of ceramic" (P6). Thus, it is understood that both the balance and the solids, when made with a material familiar to the indigenous teachers, as well as by an indigenous artisan, become artifacts/mindfacts for them, as there is skill and creativity on the part of the artisan in the process of making each piece. Three days later, the objects (artifacts/mindfacts) were ready. Figure 4 shows the images of the balance and the solids photographed in the classroom during the beginning of the second meeting.

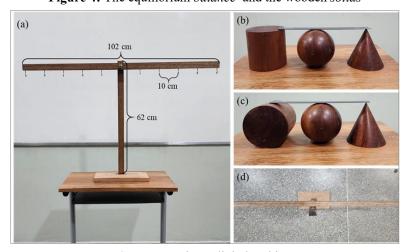


Figure 4: The equilibrium balance and the wooden solids

Source: Authors' digital archive

The radii of the *solids* in Figures 4b and 4c are 6.5 centimeters. The indigenous artisan



explained that he first made the sphere, since it was more difficult to make. The radius was this measurement, which served as a parameter for the other radii of the straight cylinder and the straight cone. In Figures 4b and 4c, a ruler was placed that illustrates the *solids*, with the same height, which is equivalent to twice the radius of their bases, that is, 13 centimeters. As for the measurement of the *balance* lever, as shown in Figure 4a, it was decided to be 102 cm, and every 10 cm, a metal hook was fixed, starting from its central screw, totaling five hooks on each side. The height of the *balance* is 62 cm. It should be noted that the screw, as shown in Figure 4d, allows the free movement of the lever. From the relationships between these artifacts/mindfacts, idealized based on Archimedes, as pointed out by Ávila (2009) and Roque (2015), each dynamic of algebraic manipulation emerges for the construction of the mathematical expressions under study, during the discussions of the second meeting.

For D'Ambrosio (2007, p. 402), "the study of the history of mathematics surprises many students when they realize how geometry establishes the foundation of what we know as algebra today". The mathematical expressions of the spherical surface and the volumes of the right cone and the sphere emerge from the measurement parameters of *geometric solids*. The artifact/mindfacts fact *Archimedes' balance* promotes the encounter of historical aspects arising from the development of Geometry, with mathematical knowledge being enhanced through the enterprise of Algebra, "algebra as a geometric process and the importance of geometry in the mathematical foundation" (D'Ambrosio, 2007, p. 400).

The discussions involve the dynamics of the *balance* with the *solids*, in front of the indigenous teachers, in line with projections, at the back of the *balance* with the *geometric solids*, in a slide by slide format, that is, the image of each projection is constructed step by step, and, with each new slide, there are interspersed dialogues with the indigenous teachers. Thus, at the end of each discussion, the projection shows the result of the algebraic process, which was transposed equation by equation, based on what the indigenous teachers contemplate, discuss and handle to equilibrium the *solids* on the *balance*. It was the participants who requested the projections and that the file be made available to them. Thus, after the initial discussions, they placed the *solids* on the *balance* as indicated by Ávila (2009). In this case, the indigenous teachers chose the second hook on each side, as shown in Figure 5.

Força potente  $(F_p)$  (a)

Força resistante  $(F_p)$  (b)  $(F_p)_{r_p} d_p = F_{r_p} d_r$   $(F_p)_{$ 

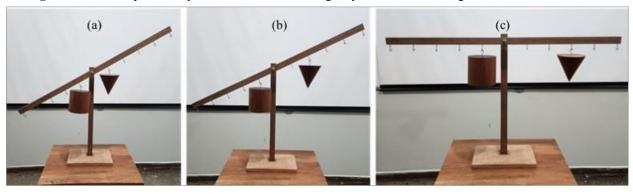
Figure 5: The equilibrium between the cylinder and the union of the cone with the sphere

Source: Authors' digital archive



When placing the cylinder on the second hook on the left side, that is, 20 centimeters from the fulcrum of the balance lever, and the sphere with the straight cone on the second hook on the right side, there was a slight imbalance of the lever, since the set of the sphere with the straight cone has two more metal hooks in relation to the straight cylinder. Thus, to equilibrate the situation, two more hooks were placed fixed to the cylinder, as shown in the image in Figure 5. Then, step by step (slide by slide) in dialogues with the indigenous teachers in the intervals between each slide, the following was projected: a) the design of the lever (discussed in the first meeting, as shown in the Figure; 2) comparisons with Archimedes' balance acting in equilibrium with solids; b) the equation of force equilibrium (resistant x potent) as a function of the distances that separate these forces from the fulcrum of the balance, alternating dialogues with the indigenous teachers, slide by slide, that is, equation by equation, until the conclusion that the mass of the straight cylinder corresponds to the mass of the sphere and straight cone set  $(m_{ci} = m_{es} + m_{co})$ ; c) taking as a premise the fact that the specific mass of the three solids is the same because they are homogeneous bodies and made of the same material (cedar), in the dynamics, slide by slide, equation by equation was presented, with discussions between each new projection. Then, it was concluded that the volume of the straight cylinder is equal to the sum of the volume of the sphere and the volume of the straight cone  $(V_{ci} = V_{es} + V_{co})$ . In the conjuncture, the union between the sphere and the straight cone is considered as a single, homogeneous body. Then, as shown in Figure 6, the equilibrium between the straight cylinder and the straight cone was promoted in communion with the participants.

Figure 6: The manipulation dynamics between the straight cylinder and the straight cone on the balance



Source: Authors' digital archive

During the activity, the indigenous teachers indicated the position of each solid: "put the cylinder on the first hook and the cone on the first hook as well" (P3), as shown in Figure 6a; "no, you can see that the cone is smaller than the cylinder, so they won't be in equilibrium if they're placed on the same hook" (P5); "that's right, we just saw that it would be in equilibrium on the same hook with the sphere and now it's alone" (P3); "leave the cylinder on the first hook and put the cone on the second hook" (P2), Figure 6b; "it still won't be equilibrium, we'll have to put the cylinder even further away" (P2); "let's put the cone on the third hook", Figure 6c; "it worked, we managed to equilibrate it, it was easy" (P1). Thus, equilibrium was achieved with the straight cylinder placed on the first hook on the left side of the balance lever, that is, 10 centimeters from its fulcrum, and the straight cone positioned 30 centimeters from the fulcrum, on the third hook on the right side of the balance lever. Figure 7 shows the dynamics of constructing the mathematical expression for calculating the volume of the straight cone, resulting from the equilibrium with the straight cylinder.



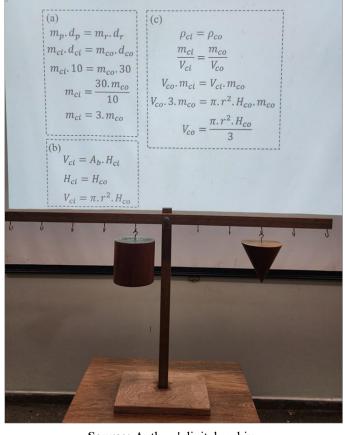


Figure 7: The volume of the right cone from the volume of the right cylinder.

Source: Authors' digital archive

Then, slide by slide was projected, amid discussions with the indigenous teachers in the intervals between each slide: a) we started with the equation of force equlibrium in terms of their distances from the fulcrum, going through the concept of force weight and substituting the distances of the cylinder  $(d_{ci} = 10 \ cm)$  and the cone  $(d_{co} = 30 \ cm)$ , after making the simplifications, we concluded that the mass of the cylinder is three times the mass of the cone,  $(m_{ci} = 3. m_{co})$ ; b) equation by equation (slide by slide), we highlighted the mathematical expression for calculating the volume of a straight cylinder  $(V_{ci})$  through the product of its base area  $(A_b)$  and its height  $(H_{ci})$ ; c) considering that the specific mass of the solids is the same, we worked, slide by slide, intertwining the final equation of (a) with the formula of (b), and we finished the algebraic manipulation with the mathematical expression for calculating the volume of a straight cone.

Indigenous teachers' perceptions: "I had never really thought about it, but looking closely, the cone and the cylinder are very similar, it makes sense that the volume of the cone comes from the cylinder" (P3); "this way, I was able to understand how I can calculate the volume of the cone, and I can do it with my students" (P7); "seeing in practice the balance with the objects on the board, mathematics goes from the concrete to the abstract, I understood that the formula helps in the calculation, now I know where it came from" (P5); "it awakened in me the possibility of working with wooden or other materials with my students" (P2).

In the second moment, the aim was initially to achieve equilibrium between the straight cylinder and the sphere, as shown in Figure 8.



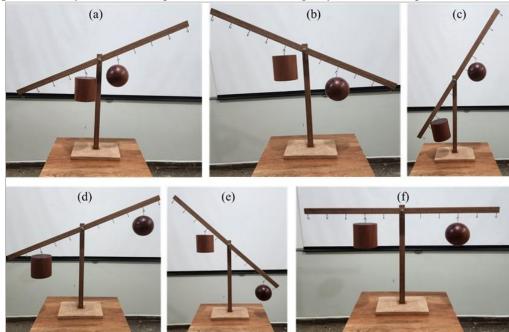
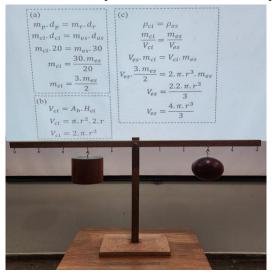


Figure 8: The dynamics of manipulation between the straight cylinder and the sphere on the balance

Source: Authors' digital archive

Participants manipulated the position of each solid "put them both on the first hook", as shown in Figure 8a, "really, it didn't work, the sphere is smaller than the cylinder, put the sphere on the second hook" (P2), Figure 8b; "you'll have to move the cylinder, put it on the second hook, don't put it on the third, if you put it on the second it will be the same as putting them both on the first" (P4), Figure 8c; "now move the sphere, put it on the fourth hook" (P3), Figure 8d; "wow, it still hasn't balanced, try putting the cylinder back on the second hook" (P3), Figure 8e; "I don't think it will equilibrate, the sphere is round and the cylinder looks very different from it" (P5); "put the sphere back on the third hook and leave the cylinder on the second" (P6), Figure 8f; "finally, it balanced" (P2); "I thought there was no way to equilibrate them, they are very different, the cone is similar to the cylinder, it has the same circle, the sphere is rounder, it is different from the cylinder" (P1). After the discussions, the construction of the mathematical expression of the volume of the sphere began, as shown in the projection shown in Figure 9.

Figure 9: The construction of the mathematical expression of the volume of the sphere from the straight cylinder

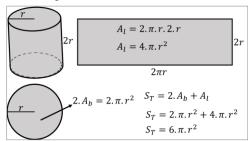


Source: Authors' digital archive



Then, slide by slide was projected, with discussions interspersed with the participants on each slide: a) from the equation of force equilibrium as a function of their distances to the fulcrum, we continued with the concept of force weight, substituting the distances of the cylinder  $(d_{ci} = 20 \ cm)$  and the sphere  $(d_{es} = 30 \ cm)$ , after the simplifications, we concluded that the mass of the cylinder corresponds to  $\frac{3}{2}$  of the mass of the sphere  $(m_{ci} = \frac{3.m_{es}}{2})$ ; b) slide by slide, the mathematical expression for calculating the volume of a straight cylinder  $(V_{ci})$  was highlighted, through the product of its base area  $(A_b)$  by its height  $(H_{ci})$ , however, in this process it is considered that the height of this straight cylinder corresponds to twice its base radius  $(V_{ci} = 2.\pi.r^3)$ ; c) knowing that the specific mass of the *solids* is the same, slide by slide, the final equation of (a) was intertwined with the formula of (b), thus, the algebraic manipulation that constitutes the mathematical expression of the volume of the sphere was completed  $(V_{es} = \frac{4.\pi.r^3}{3})$ . Soon after, the construction of the mathematical expression of the calculation of the total surface of this straight cylinder was promoted, as shown in Figure 10.

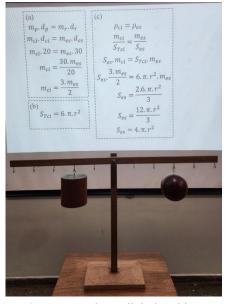
Figure 10: The mathematical expression of the total surface of the right equilateral cylinder



Source: Authors' digital archive

As can be seen, the total area of this straight cylinder corresponds to the sum of its lateral area  $A_l$ , composed of a rectangle of length  $2.\pi.r$  and height 2r ( $A_l = 4.\pi.r^2$ ), with the sum of its two circular areas  $(2.\pi.r^2)$ , which results in the mathematical expression used to calculate the total surface area of a straight equilateral cylinder ( $S_T = 6.\pi.r^2$ ). So, after this presentation, the construction of the mathematical expression of the spherical surface began, with the projection, slide by slide, shown in Figure 11.

Figure 11: The construction of the mathematical expression of the spherical surface from the straight cylinder



Source: Authors' digital archive



Thus, slide by slide was projected, with discussions interspersed with the participants on each slide: a) from the force equilibrium equation, according to their distances to the fulcrum, the concept of force weight was continued and the distances of the cylinder ( $d_{ci} = 20 \ cm$ ) and the sphere ( $d_{es} = 30 \ cm$ ) were replaced, after the simplifications, it was concluded that the mass of the cylinder corresponds to  $\frac{3}{2}$  of the mass of the sphere ( $m_{ci} = \frac{3.m_{es}}{2}$ ); b) slide by slide, the mathematical expression of the total surface of the right equilateral cylinder ( $S_{Tci} = 6.\pi.r^2$ ) was highlighted, as previously discussed; c) knowing that the specific mass of the solids is the same, slide by slide, the final equation of (a) was intertwined with the mathematical expression of (b), completing the algebraic manipulation that constitutes the mathematical expression of the spherical surface ( $S_{es} = 4.\pi.r^3$ ).

According to the participants' perceptions, "the idea of the balance with the wooden solids took it out of the abstract, this difference made me understand the subject in a simpler way" (P5); "I had never imagined that this formula had any origin, I use it in geometry class, but I took it from the book and passed it on to my students, now I can explain it to them better" (P7); "I really liked it, seeing the objects, picking them up, moving them on the balance, it took it out of the abstract and then showed where this abstract came from, and the value of  $\pi$  is still 3.14?" (P3). In this case, the indigenous teachers discussed that  $\pi$  is an irrational number, and that, within the scope of Indigenous School Education, it can assume the approximate value of 3.14.

#### 5 Final considerations

This research aimed to analyze the perceptions of seven indigenous Mathematics teachers from the Pataxó ethnic group who work in Indigenous School Education, in the context of High School, about the process of production, observation and manipulation of the historical resource *Archimedes' balance*, with the purpose of constituting mathematical expressions for calculating the spherical surface and the volumes of a right cone and a sphere, from the right cylinder. At first, the participants were not motivated, however, in the process of constituting the research scenario and, mainly, with the action of constructing the historical resource *Archimedes' balance*, in line with the three *geometric solids*, their views changed, indicating curiosity and willingness to participate in the didactic sequence. According to Roque (2015), the mathematical content that we teach today has a long historical and cultural development, involving the contribution of several peoples and civilizations. "We can then analyze the moment in which the concepts were created and how the results, which we now consider classic, were demonstrated, counterbalancing the traditional conception of mathematics as operational, technical or abstract knowledge" (Roque, 2015, p. 33).

The interaction of indigenous teachers with the objects (balance and solids) began timidly, but in a short time the teachers were dialoguing and manipulating other possibilities for promoting equilibrium. So much so that, in the end, they spontaneously performed a dynamic not foreseen in the didactic sequence, between the straight cone and the sphere. "Do you think the cone and the sphere are in equilibrium?" (P2); "put the sphere on the second hook and the cone on the first on the other side" (P4); "it didn't work, the sphere is heavier, put the cone on the third hook" (P1); "it's still leaning towards the sphere, but less, put the cone on the fourth hook" (P3); "it worked, it balanced" (P3), as shown in Figure 12a; "so, if you put the sphere on the first and the cone on the second, it should also equilibrate" (P7), as seen in Figure 12b.

And thus, equilibrium was achieved, since as already seen, the mass of this straight



cylinder corresponds to three times the mass of this straight cone and  $\frac{3}{2}$  the mass of the sphere. Then, the concept of an lever was revisited, and it was manipulated algebraically, according to the projection in Figure 12, and it was concluded that for these *solids* the mass of the sphere is equivalent to twice the mass of the straight cone.

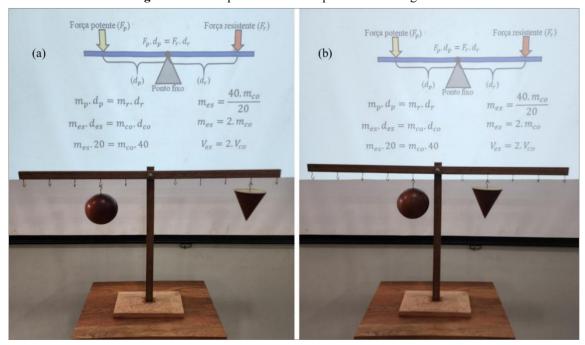


Figure 12: The equilibrium of the sphere with the right cone

Source: Authors' digital archive

In this regard, it was pointed out to the participants that the sphere, the right cone and the right cylinder obey the conditions imposed by Archimedes, that is, both have a base radius equal to the radius of the sphere and a height equal to twice the radius of the base. The indigenous teachers reported the experience of making, observing and manipulating the solids on the screen, using the Archimedes' balance, according to their perceptions: "it was a very interesting activity, the balance and the wooden solids made me think of other possibilities of working with my students, practicality combined with non-indigenous mathematics" (P4); "this way it was even fun to study Math, the class went by quickly and I managed to understand" (P7); "we were able to combine theory with practice, which is a necessity for us indigenous teachers, it is very difficult to teach Math, but this way it was easier, and with a material that I can get from my community, the wooden solids turned out beautiful" (P6); "I intend to build an Archimedes' balance and other wooden solids to use in my Geometry classes. I believe that, like me, my students will like it" (P1); "I can't do it, but I know an indigenous artisan who works with wood. I don't know if he has cedar, but it could be made of another wood. As the teacher said, all the solids have to be made of the same wood. It doesn't have to be just cedar" (P5); "For this class to be better, it needs to be practical in our village. We could take more measurements, in the fields and when fishing, combining the two mathematical knowledges, ours with that of the non-indigenous" (P2); "When I was a child, my father used a balance similar to Archimedes'. He used it to measure the weight of food he bought, such as fish, coffee, and sugar. However, this one has another purpose. However, in terms of comparison, our ancestors already used this tool" (P3).

Thus, topics on the History of Mathematics are promising for the development of teaching and learning processes for Mathematics content, in Indigenous School Education, and



beyond. The historical resource *Archimedes' balance* can be used in the teaching of Geometry as support for algebraic manipulations, as its use favors interaction between students and the possible construction of mathematical knowledge, in Indigenous School Education, in non-indigenous Basic Education and in initial or continuing training processes for Mathematics teachers, indigenous or non-indigenous.

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