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Software GeoGebra no ensino e aprendizagem de Integrais definidas

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Resumo: Este estudo explora como o GeoGebra pode transformar o ensino de integrais definidas proporcionando novas abordagens metodológicas que favoreçam a sua aprendizagem. A pesquisa de natureza qualitativa foi realizada em uma Instituição de Ensino Superior na cidade de Nampula em Moçambique, onde participaram da pesquisa vinte e dois estudantes do 1º ano. Foram usados como instrumentos para a recolha de dados, atividades exploratório-investigativas e observações. Os resultados da pesquisa mostraram que o uso do software GeoGebra na resolução de integrais definidas, propiciou o rompimento da representação algébrica e conectou os estudantes as representações algébricas e gráficas. Com o estudo concluiu-se que o GeoGebra facilita a compreensão do conteúdo a partir da visualização, manipulação, bem como desperta no estudante a capacidade de conjecturar e tirar conclusões a partir daquilo que observa na tela do computador.

Palavras-chave: Aprendizagem Significativa. Educação Matemática. Integrais Definidas. Software GeoGebra.

GeoGebra software for teaching and learning Definite Integrals

Abstract: This study explores how GeoGebra can transform the teaching of definite integrals, providing new methodological approaches that facilitate its learning. The qualitative research was carried out at a Higher Education Institution in Nampula city - Mozambique, where twenty-two first-year students participated in the research. They were used as instruments for data collection, exploratory-investigative activities and observations. The research results showed that the use of GeoGebra software to solve definite integrals led to the disruption of the algebraic representation and connected students to algebraic and graphical representations. The study concluded that GeoGebra facilitates the understanding of content through visualization and manipulation, as well as awakening in students the ability to conjecture and draw conclusions based on what they observe on the computer screen.

Keywords: Meaningful Learning. Mathematics Education. Definite Integrals. GeoGebra Software.

Software GeoGebra para la enseñanza y aprendizaje de integrales definidas

Resumen: Este estudio explora cómo GeoGebra puede transformar la enseñanza de las integrales definidas al brindar nuevos enfoques metodológicos que favorezcan su aprendizaje. La investigación cualitativa se llevó a cabo en una Institución de Educación Superior de la ciudad de Nampula en Mozambique, donde participaron veintidós estudiantes de 1er año. Se utilizaron actividades exploratorio-investigativas y observaciones como instrumentos para la recolección de datos. Los resultados de la investigación mostraron que el uso del software GeoGebra para resolver integrales definidas condujo a la ruptura de la representación algebraica y conectó a los estudiantes con las representaciones algebraicas y gráficas. El estudio concluyó que GeoGebra facilita la comprensión de contenidos a través de la visualización y manipulación, además de despertar en el estudiante la capacidad de conjeturar y sacar conclusiones de lo que observa en la pantalla del ordenador.

Palabras clave: Aprendizaje Significativo. Educación Matemática. Integrales Definidas. Software GeoGebra.

1 Introduction

The learning of definite integrals represents, for first-year students at the Institution where the research was conducted, the beginning of a new and difficult stage in their studies, since they present difficulties in understanding the concept, which has contributed to poor performance in the subject where this content is taught. As Oliveira and Reis (2017) point out, comprehension difficulties become quite visible because this content presents a high degree of abstraction, resulting in less accessible teaching for many students.

The difficulties that students present in learning definite integrals are related to the traditional approach, which is centered on the memorization of formulas and the mechanical solving of exercises. This idea is corroborated by Mateus (2019) when stating that the teaching of Mathematics in Mozambique follows the traditional model in which the teacher presents the teaching content, the student observes and then solves the exercises proposed by the teacher in an attempt to grasp the content presented. This methodology, criticized for its fragmentation and decontextualization, limits students' conceptual understanding, as they frequently do not associate algebraic techniques with the underlying geometric representations.

According to Fontes (2021), traditional methodologies are not effective and do not stimulate student learning, since they prioritize expository classes followed by the solving and repetition of exercises, with emphasis on the accumulation of information and the reproduction of formulas and concepts. Beyond traditional methodologies, Silvano, Silva, Procópio and David (2022) highlight that another factor related to learning difficulties in Calculus concepts is students' lack of command of basic Mathematics content, which needs to be observed by the teacher so that new strategies can be sought to overcome and reframe these concepts.

This situation falls within what Ausubel (2003) termed mechanical learning, in which, according to Moreira (2022), the student learns new knowledge without any meaning, being able to apply it to provide correct answers in the short term while quickly forgetting it. The search for methodological alternatives that minimize the difficulties faced by students and make the learning of definite integrals meaningful constituted a major motivation for conducting the present study. Thus, the study sought to answer the following research problem: In what way can pedagogical mediation with the aid of GeoGebra favor the meaningful learning of definite integrals?

This problem leads us to rethink teaching practice, where the teacher needs to prepare lessons differently, seeking to use technological resources, contextualize content, and value the prior knowledge that the student possesses, since, as Silva (2017) states, the mathematics teacher needs to address the issue of introducing the conceptual teaching of Calculus so that there is a correct understanding of what is done and why it is done, without confusing it with the mechanical teaching of demonstrations that commonly occur.

It is from this perspective that the study aims to analyze how pedagogical mediation with the aid of GeoGebra can favor the meaningful learning of definite integrals.

2 ICTs in the Teaching and Learning Process of Integral Calculus

The integration of Information and Communication Technologies (ICTs) in the educational landscape dates back to the mid-twentieth century. This process intensified in the 1960s and 1970s, a period marked by pioneering experimental projects introducing the use of computers in schools and universities (Cutrim *et al*., 2025).

Within this context of technological integration, the 1980s were marked by a growing concern among many mathematicians regarding the quality of student learning in Calculus. According to Richit (2010, p.28), this concern led to the "Calculus Reform movement in the United States, proposing the integration of ICTs as a way to make concepts more meaningful for a greater number of students."

Following the proposal of the Calculus teaching reform, research was developed to analyze the use of computers in the teaching and learning of this subject, which we examine below.

According to Escarlate (2008), TALL conducted research in 1986 on the concept of the integral, in which he proposed the use of the computer for a deeper understanding of the fundamental theorem of calculus, using graphical representations enhanced by the machine. He concluded that through the exploration of the Graphic Calculus software, students had the opportunity to develop a meaningful perception of some properties of the definite integral, such as the negative sign in the result of the integral when the function lies below the x-axis.

Barufi (1999) highlights in her research that the use of computers in the classroom can provide a dynamic learning environment, as it enables the creation of discussions and reflections around mathematical knowledge. This idea is supported by Marin (2009), who states that ICTs have become a very important didactic resource in the teaching of Calculus, and their use has been widely recommended by mathematics education researchers, as they allow the teacher to explore various mathematical concepts and algebraic and geometric representations quickly and effectively.

However, it is important to note that the incorporation of ICTs in the teaching of Integral Calculus should not be reduced to a merely instrumental use, but understood as a reconfiguration of teaching practice that redefines pedagogical mediation. This reconfiguration implies a shift in the teacher's stance, in which they not only teach through technology, but also develop problem situations that challenge students to think critically and to interpret the results generated by technology.

This perspective converges with the thinking of Villarreal (1999), for whom the computer is considered a 'supplement' when used simply to perform calculations, and as a 'reorganizer' when it is assumed as a tool 'to think with' — that is, when it produces modifications in the organization of content and in the activities developed in the classroom.

We agree with the author in recognizing the need to use the computer as a reorganizer in the classroom, so as to prevent the student from becoming a passive observer concerned only with the graphical aesthetics the computer offers, with the teacher acting as a mediator of knowledge through continuous questioning in order to foster student agency in the learning of content.

Regarding pedagogical dynamics, Cutrim *et al*. (2025) state that the integration of ICTs in higher education proves to be a fundamental strategy for the teaching of Calculus, as it mitigates difficulties in conceptual understanding and broadens student engagement through the direct manipulation of mathematical objects. In this way, technologies transcend the condition of mere supports and begin to act as mediators in the meaningful construction of knowledge.

In this regard, Ballesteros, Lozano and Rodriguez (2020) view technological tools as new ways of visualizing concepts, enabling the dynamic resolution of problems. Mathias (2023) complements this view by highlighting that when learning environments are enriched with technology, they can stimulate students to increase their capacity to explore, reconstruct (or reinvent), and explain mathematical concepts.

In contrast to these technological potentialities, graphical constructions made using pen and paper or on the board are static and, in some cases, limit the understanding of certain concepts.

According to Escher and Miskulin (2019), the technological advances experienced in today's society open up new possibilities for the application of ICTs in the classroom through software that allows the visualization and manipulation of objects, representing a gain for the teaching and learning of mathematics, particularly for Calculus concepts. In this line of thinking, the use of educational software for the teaching and learning of mathematical content is encouraged, not only as a calculation tool, but also as a means of enabling the simulation and modeling of everyday situations.

As Borba and Penteado (2001) point out, the use of software enables experimentation with mathematical concepts while also stimulating students' visual perception. This view is corroborated by Menoncini (2018), who states that the use of computers with mathematical software has broadened the possibilities for the visual transformation of figures, allowing the exploration of mathematical properties and relationships.

From this perspective, Mourarias (2024) highlights the relevance of ICTs in the teaching of Differential and Integral Calculus, emphasizing that these tools enhance learning by enabling interactive approaches. Furthermore, ICTs facilitate the visualization of abstract concepts, making the educational process more dynamic.

Thus, the incorporation of mathematical software in Integral Calculus classes can minimize the issue of algebraic manipulation, enabling the transition between the student's interaction with ICTs and the mathematical representation of a concept. However, for its implementation to be successful, it is essential to ensure "teacher training, adequate access to resources, and the intelligent integration of these tools into the curriculum" (Silva & Mota, 2024), without neglecting the physical and technological infrastructure of educational institutions.

Taking into account the authors' arguments above regarding the importance of computer use in the teaching and learning of Calculus, with emphasis on educational mathematical software, this research chose to use GeoGebra due to the fact that it is a free, open-access software with a user-friendly interface that makes it possible to work with algebraic and graphical representations simultaneously.

3 The use of GeoGebra in the calculation of definite integrals

GeoGebra software offers significant advantages in the teaching and learning of definite integrals, as it allows the visualization of complex graphs, the solving of challenging exercises, and the verification of calculation accuracy (Coelho & Biass, 2024). Its interactive tools enable a dynamic exploration of Integral Calculus concepts (Oliveira & Reis, 2017; Lacerda, Carvalho, Esquincalha & Luz, 2020; Fontes, 2021; Navarrete-Villavicencio, Merino-Córdova, Estupiñán-Cox, Caicedo-Márquez, 2022; Silvano *et al*., 2022), serving as an anchor for the construction of new knowledge through visualization and experimentation.

In this context, the software provides a dynamic perception of the definite integral through the adjustment of parameters such as coefficients and limits of integration, leading the student to understand how such variations impact the calculated areas. In addition to facilitating the construction of graphs and the visualization of integration regions, the software assists in the understanding of complex area calculations, overcoming the limitations of static representations found in textbooks or on whiteboards. Manipulation via sliders acts as an advance organizer, enabling the student to connect the abstract concepts of calculus to their prior knowledge of areas, functions, limits, and others within their cognitive structure.

This anchoring, provided by the manipulation of mathematical objects in GeoGebra, facilitates meaningful learning which, according to Ausubel (2003), only occurs when the new concept relates in a non-arbitrary and substantive (non-literal) way to other concepts already present in the student's cognitive structure. Thus, by establishing a bridge between the student's prior knowledge and the new concept of the definite integral, the software allows the cognitive structure to be modified, giving meaning to the new knowledge.

Studies confirm that GeoGebra is a fundamental didactic resource for carrying out investigative activities and solving problem situations (Lacerda *et al*., 2020; Fontes, 2021). Furthermore, Oliveira and Reis (2017) highlight that the dynamics of using software can motivate students to research, experiment, and seek new solutions related to a problem. Along these lines, Navarrete-Villavicencio *et al*. (2022) suggest the use of GeoGebra software in the learning of Calculus to enable the development of competencies and the construction of new knowledge interactively, contributing to the strengthening of meaningful learning in students.

Ballesteros *et al*. (2020) add that the use of GeoGebra in mathematics learning brings possibilities for deepening the understanding of definite integral concepts. From the same perspective, Mateus (2019) considers GeoGebra as an appropriate tool for the teaching and learning of Mathematics, as it presents graphical and algebraic capabilities that assist in the development of teaching content and make it possible to enhance the meaning of learning content.

Regarding the teaching and learning of the definite integral concept with the aid of GeoGebra, the student can construct graphs and partitions, perform calculations of lower and upper sums, as well as identify the limits and the region of integration. In this context, Silvano *et al.* (2022), when analyzing constructions made with the aid of GeoGebra involving Integral Calculus, found that it was possible to explore the understanding of relevant concepts and meanings, opening up new theoretical, methodological, and practical horizons related to the study of Integral Calculus, overcoming the challenges of traditional teaching.

The software facilitates progressive differentiation by allowing the student, starting from the general idea of approximating the area using rectangles, to begin discriminating the specificities of Riemann sums — such as the behavior of the limit as the base of the rectangle tends to zero — in order to then consolidate the concept of the Riemann integral. This dynamism also promotes integrative reconciliation by allowing the student to perceive the intrinsic relationship between the algebraic representation of the function and its geometric interpretation, conferring a non-arbitrary and substantive character to the learning of the concept.

Our perception, grounded in the authors' approaches, indicates that GeoGebra's graphical visualization and animation resources have the potential to contribute to improving the teaching and learning process of definite integrals, by enabling the reframing of concepts and a balance between graphical and algebraic processes. The software connects algebraic representations (formulas), numerical representations (calculated values), and geometric representations (graphs), promoting a holistic understanding.

Furthermore, the tool offers instant feedback, allowing students to test hypotheses, identify errors, and reformulate strategies. These aspects stimulate autonomy and mathematical investigation, essential elements for meaningful learning which, according to Moreira (2022), is characterized by the ability to understand, explain, describe, apply, and transfer knowledge to new situations.

In this sense, the use of GeoGebra software in the teaching and learning of definite integrals becomes a preponderant factor for the understanding of concepts, as it acts as a pedagogical mediator that respects the hierarchical structure of knowledge. As it transforms the study of the integral into an investigative process, the software allows the student to visualize and explore mathematical concepts, ensuring that the acquisition of new knowledge occurs in a meaningful way.

Thus, if the concept of the Riemann integral is learned meaningfully, it will be stored in the cognitive structure in a stable and lasting manner, allowing the student to use it as an anchor in the learning of other concepts such as multiple integrals and differential equations.

4 Methodology

In the various areas of knowledge, research is conducted taking into account methodological positions based on both ontological premises of reality and epistemological considerations, among other variables, within a typology divided between quantitative and qualitative paradigms (Sacool, 2009).

With a view to direct contact with the object of study and the description of the ideas and opinions of the study participants, a qualitative, exploratory research approach was chosen, which according to Gil (2019), aims to provide the researcher with greater familiarity with the problem as well as an understanding of the phenomena experienced, in terms of the meanings that the research subjects attribute to them.

It should be noted that, since this is a Mathematics Education research study, the quantitative method could hinder the investigation in terms of deepening discussions regarding the use of GeoGebra in carrying out activities, as well as observing the students' responses throughout this process. As Bicudo (2012) points out, Mathematics Education research that seeks to understand how students construct learning of concepts in specific contexts should be qualitative in nature.

The research was developed from a bibliographic study, in which scientific works were accessed and mapped in databases such as the CAPES Journals Portal, SciELO, and Google Scholar. Articles published between 2017 and 2025 were selected, addressing the use of technologies in the teaching and learning of Differential and Integral Calculus, with greater emphasis on the use of GeoGebra in the teaching of Integral Calculus, the focus of this study.

The following keywords were used as descriptors for the article search: Differential and Integral Calculus, Integral Calculus, Definite Integrals, and GeoGebra software.

In addition to the bibliographic research, two teaching activities were carried out, each composed of three questions. In the first activity, the research participants solved tasks on integral calculus using pen and paper, with the purpose of verifying whether they had meaningfully learned the concepts related to definite integrals as well as their application in the calculation of areas. In the second activity, the participants solved the tasks from the first activity with the aid of GeoGebra software, with a view to providing them with a more interactive learning experience, allowing the visualization of concepts related to definite integrals through dynamic constructions and manipulations.

The activities carried out with GeoGebra were exploratory-investigative in nature, in the view of Miskulin, Escher and Silva (2007), and involved questioning and reflection on the content and graphical representations, with the aim of allowing students to express what they observed on the computer screen through words, since, according to Barufi (1999), the construction of meanings is made possible through language.

Observation was also used with a view to providing greater proximity between the researchers and the phenomenon under investigation, as well as to describe cognitive aspects observed in the learning environment (Marconi & Lakatos, 2017), which helped to understand the students' responses when carrying out the activities on definite integrals with the aid of GeoGebra.

To understand the object of study, an interpretive perspective was adopted, with the purpose of understanding reality as it is seen by the actors who directly engage with it (Ponte, 2006), interpreting the students' opinions obtained through interaction during the resolution of the exploratory-investigative activities.

Twenty-two first-year students from a Higher Education Institution located in Nampula, in northern Mozambique, participated in the research. The participants were instructed to save the work produced in GeoGebra under the code assigned to each of them for identification purposes.

The collected data were analyzed in light of the assumptions of Bardin (2016), comprising the stages of pre-analysis, material exploration, and treatment of results, with the purpose of identifying record units that evidenced the relationship between the use of GeoGebra in carrying out exploratory-investigative activities and the learning of definite integrals.

The pre-analysis consisted of a floating reading of the participants' productions and the selection of material, followed by the transcription of excerpts considered relevant. In the material exploration phase, the data were subjected to a coding process, transforming them into record units which were subsequently grouped into categories of analysis. Finally, in the results treatment phase, the influence of GeoGebra on the attribution of meaning to the teaching and learning process of definite integrals was discussed.

5 Discussion and Analysis of Results

In this study, we present the results of two activities in which the participants first solved tasks using only pen and paper and then solved them with the aid of GeoGebra software. The collected data were organized into two analytical categories: (i) Mobilization of prior knowledge and limitations of static representation; and (ii) Dynamic visualization and reframing of the concept of definite integral.

5.1 Mobilization of prior knowledge and limitations of static representation

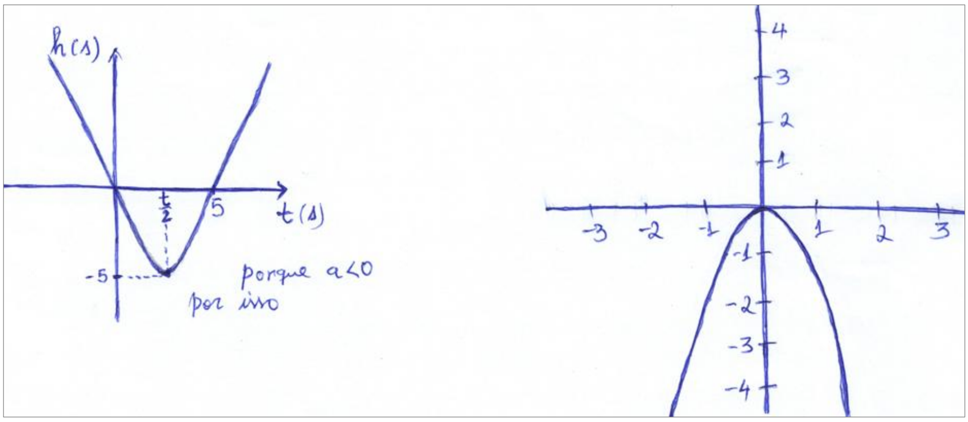
This category diagnoses the difficulties students face in sketching graphs and applying algorithms for the calculation of definite integrals. It also investigates how the cognitive barriers imposed by static constructions can lead to errors in the interpretation of intervals and limits of integration, keeping the concept of the integral confined to an abstract formalism devoid of visual connection.

The analysis comprises three tasks from the first activity, whose objective was to verify whether students still recalled the procedures and rules for calculating definite integrals previously studied and their relationship with the calculation of areas. To this end, we expected them to graphically represent the integration regions that delimit the intended area, for the subsequent calculation of the integral.

In the first task, participants were required to graphically represent the region bounded by the function, the x-axis, and the lines, and then determine the area of this region by means of the definite integral. In solving this task, 11 (eleven) students sketched the graph of the function and identified the integration region, as well as correctly applying the resolution procedures for the integral until reaching the correct solution; 4 (four) participants had difficulties constructing the graph of the function and consequently were unable to identify the integration region; 7 (seven) students solved the task without sketching the graph, of whom 3 (three) had difficulties finding the correct solution.

In the second task, participants were required to sketch the curve representing the motion of a projectile described by the function, and then determine the area of the region bounded by the curve and the x-axis using the definite integral. In this task, only 10 (ten) students correctly sketched the curve describing the motion of a projectile launch, while the remaining 12 (twelve) had difficulties sketching the curve, as can be seen in Figure 1:

Figure 1: Curves on the motion of a projectile described by the function $h(t) = -t^2 + 5t$[1]



Source: Material produced in the research

It is evident in Figure 1 that the students did not take into account the necessary aspects for sketching the graph of a quadratic function, such as concavity and the zeros of the function, which denotes a lack of prior knowledge about the quadratic function. Regarding the calculation of the area by applying the definite integral, 4 (four) participants stated that it was not possible to determine the area, as the graph presents negative regions and in such cases the area is not defined, while 18 (eighteen) students stated that it was possible to determine the area, of whom 7 (seven) managed to reach the final solution, 5 (five) did not determine it, and 6 (six)

[1] The handwritten Portuguese text inside the figure reads: "porque a<0 por isso", which translates to: "because a<0 therefore."

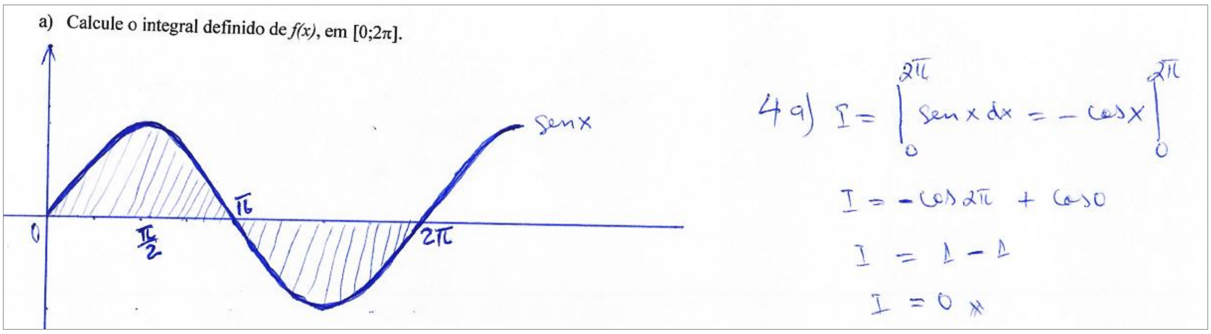
determined it incorrectly, which demonstrated difficulty in working with definite integrals of quadratic functions.

In solving this task, the majority of students presented difficulties in constructing the graph of a quadratic function, which may be related to the type of learning they experienced when studying this content. As Ausubel (2003) points out, when a student does not learn meaningfully, they can hardly retain knowledge in their cognitive structure and consequently are unable to recall what they have learned.

With the third task, we intended for participants to calculate the integral of the function over the given interval. To this end, we expected them to graphically represent the function over the given interval and establish an association between the area of the region bounded by the curve of the function and the x-axis, and the integral of the function over the same interval.

In solving this task, only 3 (three) participants correctly sketched the graph, solved the integral, and calculated the area by partitioning the interval, concluding that the value of the integral equals the value of the area of the region comprised between the function and the x-axis over the given interval; 11 (eleven) students sketched it correctly but did not reach the desired solution, as some of them did not partition the interval, and those who did partition it summed the results without taking into account the sign of the function in the second interval, as shown in Figure 2. The remaining 8 (eight) students did not sketch the graph, stating that they had great difficulties in constructing graphs of trigonometric functions.

Figura 2: Calculation of the integral of the function f(x) in the interval from $[0,2\pi]$



Source: Material produced in the research [2]

In this task, we found evidence that the learning of trigonometric functions and other content related to the definite integral was not meaningful, which is why students have difficulties applying them in other contexts. As Moreira (2022) points out, when students learn concepts mechanically, without meaning, the knowledge is forgotten, as if it were "erased" from the mind. Given that trigonometric functions are introduced in secondary education, the difficulties presented by the participants become even more concerning, as this is fundamental prior knowledge for understanding the definite integral of this type of function and other subsequent content.

In this task, we also noticed that few students are aware of the need to sketch the graph of the function when calculating the integral, in order to visualize the region of integration. This suggests that most students did not understand that calculating the area of the region bounded by the curve of a function and the x-axis over a given interval is the same as calculating the integral of a function over the interval that delimits that region.

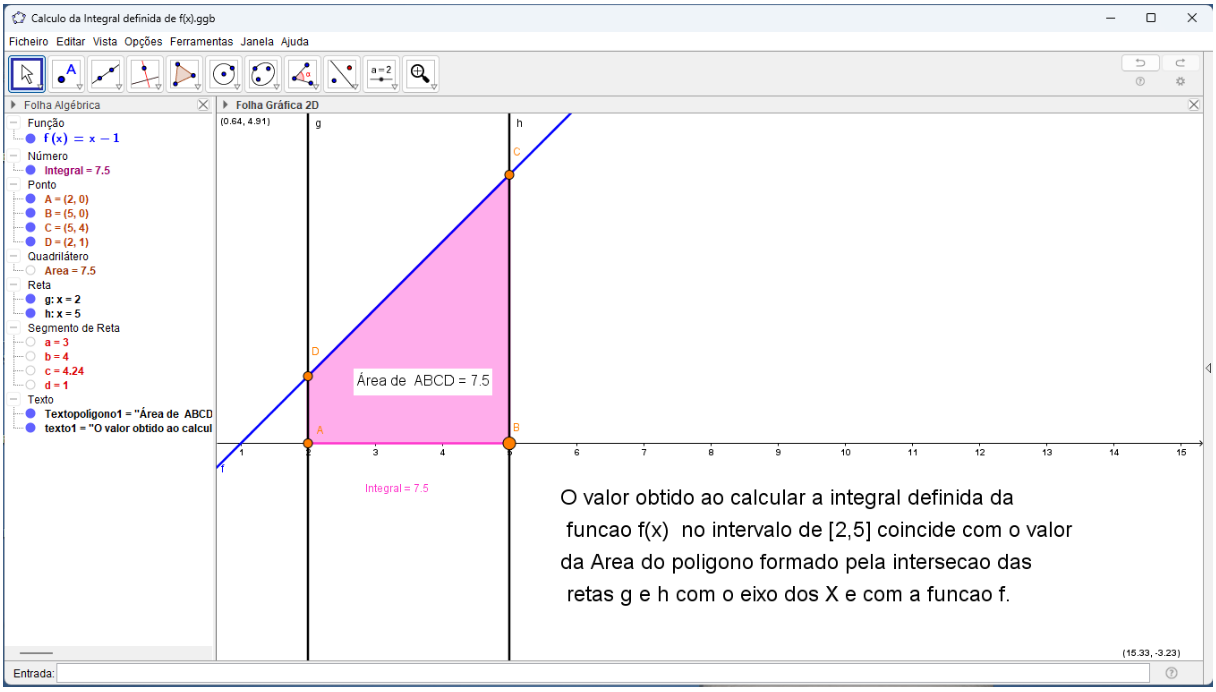
[2] **:** The text in the figure is written in Portuguese. Translation: *"a) Calculate the definite integral of f(x) on [0; 2π]"*

5.2 Dynamic Visualization and Reframing of the Concept of Definite Integral

This category analyzes the transition from solving tasks with paper and pen to the dynamic geometry environment. The focus lies in the mediation exercised by GeoGebra in the learning process, enabling students to assign new meanings to definite integrals. The analysis includes three tasks from the second activity, whose purpose was to provide students with the calculation of definite integrals with the aid of GeoGebra, consolidating the correlation between the concept of the definite integral and the area of the region bounded by the curve of a function and the x-axis over a given interval. At this stage, the dynamic environment enabled the review and correction of errors previously made in manual constructions.

In the first task, all students correctly constructed the graph of the function, bounded by the x-axis and the lines and, and successfully calculated the value of the area and the integral, as shown in Figure 3.

Figure 3: Construction of the graph of the function $f(x) = x - 1$ and calculation of the area under the curve



Source: Material produced in the research[3]

When comparing the results obtained when calculating the area of the polygon formed by the graph of the function f(x) = x − 1, the x-axis and the lines x = 2 and x = 5, and the integral of the function f(x) = x − 1 over the interval [2, 5], the participants were unanimous in stating that:

- *"Despite having used different procedures to solve it, the results obtained are equal in the value of 7.5 u.a.";*
- *"Comparing the results with the calculation of the integral, the value of the definite integral does not differ from the value of the area, since it is the same function and the same interval.".*

[3] The text displayed on the GeoGebra screen is written in Portuguese and reads: *"O valor obtido ao calcular a integral definida da função f(x) no intervalo de [2, 5] coincide com o valor da área do polígono formado pela intersecção das retas g e h com o eixo dos X e com a função f."* — Translation: "The value obtained when calculating the definite integral of the function f(x) over the interval [2, 5] coincides with the value of the area of the polygon formed by the intersection of lines g and h with the x-axis and with the function f."

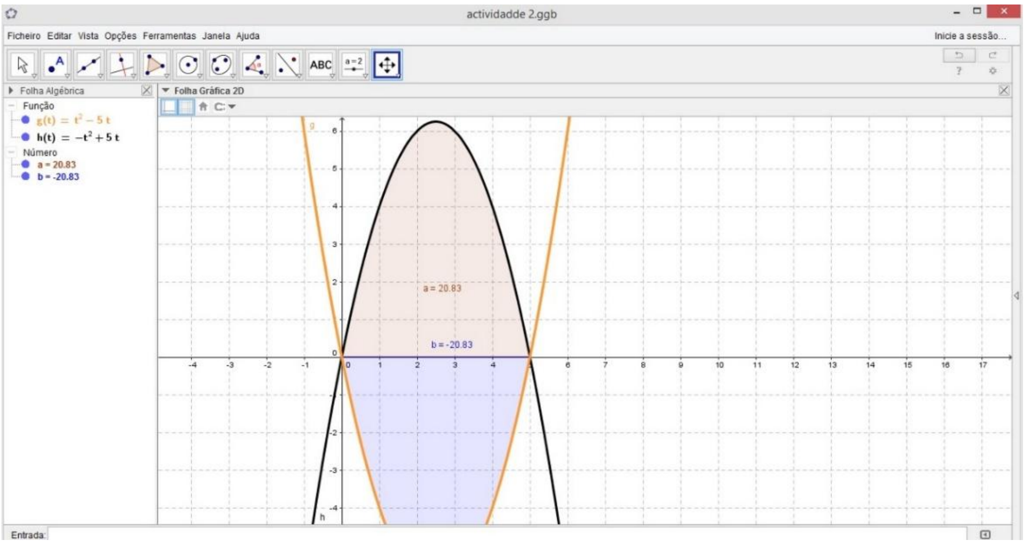
With these comments and based on the construction made in GeoGebra, we note that the students understood that calculating the definite integral means finding the value of the area of the region comprised between the function and the x-axis, over a given interval. In solving this task, the students also observed that GeoGebra facilitates the construction of graphs and the understanding of concepts, as attested by the comments below:

- *"Solving exercises with GeoGebra is relatively easier than normal written resolution. The construction of graphs of functions is simpler compared to written resolution. It is a procedure that inspires simplicity as well as confidence in the activities.";*
- *“"The use of GeoGebra offers a dynamism in terms of the production of figures."*
- *"Solving the activities in GeoGebra is very easy, that is, it allows not only the resolution but also shows in detail cases that we can have and relate to others”*

Comparing the responses presented here with those from the previous activity, where only eleven students sketched the graph and reached the correct solution with the aid of GeoGebra, all students reached the intended solution. This situation shows that the GeoGebra software can contribute to the learning of definite integrals and their relationship to the area under the curve, using graphical visualization that enables the reframing of concepts and provides a balance between the graphical and algebraic process.

As for the second task, the students correctly constructed the graph of the function $h(t) = -t^2 + 5t$ in GeoGebra and were unanimous in stating that the motion could not be represented by $h(t) = t^2 - 5t$, as shown in Figure 4.

Figure 4: Construction on the projectile motion described by the function.



Source: Material produced in the research

Unlike what happened when solving this task with pen and paper, where some students had difficulty sketching the graph as well as verifying that the graph of $h(t) = -t^2 + 5t$ was opposite to $h(t) = t^2 - 5t$, with the help of the software the students were able to visualize the graphs and draw the following conclusions:

- *"This motion cannot be represented by the function $h(t) = t^2 - 5t$, because the functions are symmetric.";*
- *"It is not possible because the functions represent opposite motions".*

As can be noted in the participants' comments, the software helped them in the construction of graphs, and from the visualization on the screen they realized that the two functions did not represent the same motion. For Villarreal (1999), the visualization process plays a fundamental role in learning, since the visual, algebraic, and verbal aspects are complementary in the process of learning mathematics. Corroborating this view, Mathias (2023) adds that through visualization the student establishes connections between graphical representations and formal definitions, which may allow for a better understanding of mathematical concepts.

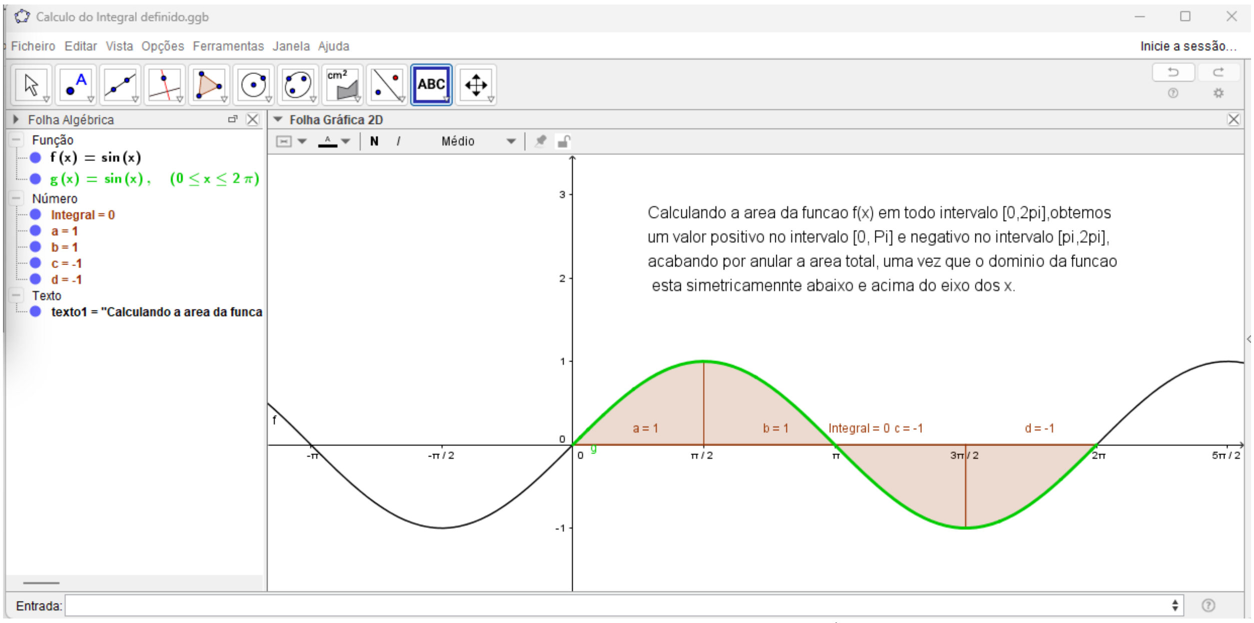
In this regard, the literature shows that visualization through the computer screen makes it possible to develop a set of arguments (conjectures) and also to use them to solve problems, allowing students to construct and relate the various representations of information and build mathematical concepts (Mathias, 2023).

In solving the third task, students were required to construct the graph of the sine function with the aid of GeoGebra, in order to subsequently determine the integral as well as the area over the interval [0; 2π], so as to verify and compare the solutions found without the use of the software.

All students were able to construct the graph of the function f(x) = sin(x), which facilitated the visualization of the region in which the area was intended to be calculated — something that is fundamental to the learning of definite integrals. In the view of Oliveira and Reis (2017), the use of software in Calculus classes can represent a break from the teaching of certain concepts worked almost exclusively through algebraic and symbolic notions, thus hindering the visualization and experimentation of activities.

Since the software does not calculate the area in absolute value when the region of integration lies below the x-axis, some students considered the solution of the integral to be zero and the area to be zero as well, as shown in Figure 5:

Figure 5: Calculation of the area of the function f(x) = sin(x) over the interval [0; 2π] in GeoGebra



Source: Material produced in the research[4]

[4] The text displayed on the GeoGebra screen is written in Portuguese and reads: *"Calculando a área da função f(x) em todo intervalo [0, 2π], obtemos um valor positivo no intervalo [0, π] e negativo no intervalo [π, 2π], acabando por anular a área total, uma vez que o domínio da função está simetricamente abaixo e acima do eixo dos x."* — Translation: "Calculating the area of the function f(x) over the entire interval [0, 2π], we obtain a positive value over the interval [0, π] and a negative value

Thomas (2009) draws attention to this result presented by the software, as well as to some students who did not calculate the absolute value, by stating that:

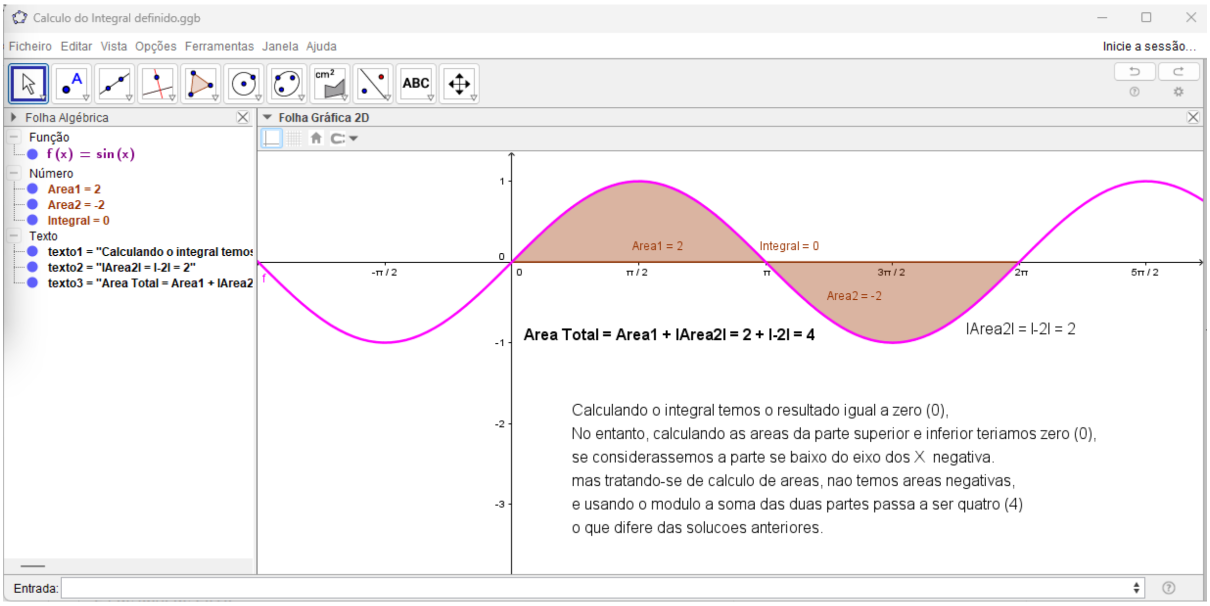
> When calculating the area of the region bounded by the curve of a function and the x-axis over an interval [a, b], special care is needed if the function takes positive and negative values. We need to take care to divide the interval [a, b] into sub-intervals in which the function does not change sign. Otherwise, we may be canceling positive and negative areas between each other, which would lead to an incorrect total" *(Thomas, 2009, p. 394).*

This warning from Thomas (2009) is pertinent to the calculation of definite integrals with or without software, since GeoGebra does not calculate the area in absolute value, and it is up to the teacher to clarify to students what must be done in situations where the function takes negative values.

It is worth noting that in solving the third question, both with and without software, some students made the same type of error by not considering the sign of the graph of the function below the x-axis — which is why Thomas (2009, p. 395) warns that "to determine the area between the graph of y = f(x), over the interval [a, b], one must subdivide [a, b] at the roots of f, integrate f over each sub-interval, and add the absolute values of the integrals."

These ideas from Thomas (2009) were illustrated by some students, who realized the need to calculate the absolute value of the area below the x-axis, as shown in Figure 6:

Figure 6: Calculation of the area of the function sin(x) over the interval [0; 2π] in GeoGebra.



Source: Material produced in the research[5]

over the interval [π, 2π], ultimately canceling out the total area, since the domain of the function is symmetrically below and above the x-axis."

[5] The text displayed on the GeoGebra screen is written in Portuguese and reads: *"Calculando o integral temos o resultado igual a zero (0). No entanto, calculando as áreas da parte superior e inferior teríamos zero (0), se considerássemos a parte se baixo do eixo dos x negativa. Mas tratando-se de cálculo de áreas, não temos áreas negativas, e usando o módulo a soma das duas partes passa a ser quatro (4) o que difere das soluções anteriores."* — Translation: "Calculating the integral, we obtain a result equal to zero (0). However, calculating the areas of the upper and lower parts would give us zero (0) if we considered the part below the x-axis as negative. But since we are dealing with the calculation of areas, we do not have negative areas, and by using the absolute value, the sum of the two parts becomes four (4), which differs from the previous solutions."

Despite the software presenting limitations by displaying negative values of the area, the attitude of these students is commendable, as they realized the need to evaluate the behavior of the function over the given interval. This led them to calculate the absolute value where the function takes negative values in order to find the correct solution. This stance leads us to agree with Mateus (2019), who states that GeoGebra stimulates students' curiosity in mathematical exploration activities. In this context, it becomes evident that the integration of ICTs in Mathematics Education is not merely a trend, but a necessity for developing critical and creative individuals.

6 Final Remarks

When considering the use of GeoGebra software for the teaching and learning of the definite integral, the aim is, in a certain way, to minimize students' difficulties in learning this content, as well as to encourage them to be capable of constructing concepts, investigating, and making sense of numerical solutions from their visual field — preventing them from remaining passive in the knowledge construction process and distancing themselves from mechanical learning.

When analyzing students' performance in solving the proposed tasks, it was found that the difficulties initially observed decreased significantly. The majority of participants presented more detailed responses with the aid of GeoGebra compared to when they solved the tasks using pen and paper. These results corroborate the conclusions of Barufi (1999), who highlights the computer as a facilitating instrument for learning that opens space for the negotiation of meanings, allowing students to validate their own conjectures.

The use of the software enabled the construction of graphs, the visualization of the geometric behavior of the curve, and the exploration of mathematical concepts by connecting algebraic and graphical representations — which is fundamental for the construction of meaning. Furthermore, the research data indicate that GeoGebra is a didactic resource that facilitates the understanding of content through the manipulation of mathematical objects, awakening in the student the ability to conjecture and draw conclusions from what they observe on the computer screen.

The graphical visualization of areas under curves and the comparison between numerical and analytical methods fostered a deeper understanding of definite integrals, making learning more dynamic and accessible. However, the adoption of this tool for mathematics classes requires investment in teacher training, revision of pedagogical practices, and a commitment to technological inclusion so that it can promote student autonomy.

Although the results suggest that the construction of meaning developed during the activities with GeoGebra can positively reflect on the learning of subsequent content, it is important to note that the fact that the research was conducted at a single institution, with only 22 participants, and focused exclusively on definite integrals, limits the possibility of generalization to other contexts. Therefore, it is recommended that future studies explore other Calculus concepts and expand the number of participants and educational institutions.

References

Ausubel, D. P. (2003). *Aquisição e retenção de conhecimentos: uma perspectiva cognitiva.* Lisboa: Plátano.

Ballesteros, B. V. A.; Lozano, S. F. & Rodriguez, O. I. C. (2020). Noção de aproximação da área sob a curva usando o aplicativo Calculadora Gráfica de GeoGebra. *Prax. Saber*, 11(26), 1-16. https://doi.org/10.19053/22160159.v11.n26.2020.9989

Bardin, L. (2016). *Análise de conteúdo.* (3. ed.). São Paulo, SP: Edições 70.

Barufi, M. C. B. (1999). *A construção / negociação de significados no curso universitário inicial de Cálculo Diferencial e Integral.* 195f. Tese (Doutorado em Educação). Universidade de São Paulo. São Paulo, SP.

Bicudo, M. A. V. (2012). A pesquisa em Educação Matemática: a prevalência da pesquisa qualitativa. *Revista Brasileira de Ensino de Ciência e Tecnologia*, .5(2), p. 15-26. DOI: 10.3895/S1982-873X2012000200002

Borba, M. C. & Penteado, M. G. (2001). *Informática e Educação Matemática.* Belo Horizonte, MG: Autêntica.

Coelho, T. G. & Blass, L. (2024). O Impacto das tecnologias no ensino de Cálculo Diferencial e Integral: panorama de pesquisas educacionais no Brasil. *Revista Docência do Ensino Superior*, v. 14, e048471, p. 1-22. https://doi.org/10.35699/2237-5864.2024.48471.

Cutrim, G. E. B.; Ribeiro, F. A. A.; Silva, M. C. A. S.; Barreto, N. S.; Farias, A. S.; Sousa, N. C.; Silva, W. J. B. & Brandão, R. J. B. (2025). GeoGebra no ensino de cálculo avançado com o uso da Inteligência Artificial. *Caderno Pedagógico*, 22(8). https://doi.org/10.54033/cadpedv22n8-205

Escarlate, A. C. (2008). *Uma investigação sobre a aprendizagem de integral.* 154f. Dissertação (Mestrado em Ensino de Matemática). Universidade Federal do Rio de Janeiro. Rio de Janeiro, RJ.

Escher, M. A. & Miskulin, R. G. S. (2019). Dimensões Teórico-Metodológicas do Cálculo Diferencial e Integral: perspectiva histórica e de ensino e aprendizagem. *Revista de Investigação e Divulgação em Educação Matemática*, 3(1), 22–48.

Fontes, L. S. (2021). *As metodologias activas de aprendizagem e sua contribuição para o ensino de Cálculo Diferencial e Integral.* 173f. Tese (Doutorado em Educação) Universidade de Brasília. Brasília, GO.

Gil, A. C. (2019). *Métodos e técnicas de pesquisa social*. (7. ed.). São Paulo, SP: Atlas.

Lacerda, G. K. S.; Carvalho, T. R. S.; Esquincalha, A. C. & Luz, V. C. (2020). A compreensão do Teorema Fundamental do Cálculo em uma atividade exploratória com o uso do GeoGebra. *Revista do Instituto GeoGebra Internacional de São Paulo*, 9(2), 35–51. https://doi.org/10.23925/2237-9657.2020.v9i2p035-051

Marconi, M. A. & Lakatos, E. M. (2017). *Fundamentos de metodologia científica.* (8. ed.). São Paulo, SP: Atlas.

Marin, D. (2009). *Professores de matemática que usam tecnologia de informação e comunicação no ensino superior.* 164f. Dissertação (Mestrado em Educação Matemática). Universidade Estadual Paulista. Rio Claro, SP.

Mateus, P. (2019). Ensino e aprendizagem da matemática: o mérito da integração da tecnologia do GeoGebra nas práticas usuais. *II Fórum Nacional de Educação*. Maputo.

Mathias, C. V. (2023). O potencial do GeoGebra como ferramenta de auxílio as habilidades de visualização. *Revista do Instituto GeoGebra Internacional de São Paulo*, 12(2). https://doi.org/10.23925/2237-9657.2023.v12i2p044-066

Menoncini, L. (2018). *O jogo das operações semióticas na aprendizagem da Integral Definida no cálculo de área*. 274f. Tese (Doutoramento em Educação Científica e Tecnológica). Universidade Federal de Santa Catarina, Florianópolis, SC.

Miskulin, R. G. S.; Escher, M. A. & Silva, C. R. M. (2007). A prática docente do professor de matemática no contexto das TICs: uma experiência com a utilização do MAPLE em cálculo diferencial. *Revista de Educação Matemática*, 10(11), p. 29-37.

Moreira, M. A**. (2022).** Aprendizagem ativa com significado. *Espaço Pedagógico, Passo Fundo*, 29(2), p. 405-416.. https://doi.org/10.5335/rep.v29i2.13887

Mourarias, R. M. S. (2024). *A Integração de Tecnologias Digitais no ensino de Cálculo Integral: Potencialidades, Desafios e Perspectivas – uma Revisão da Literatura*. 92f. Dissertação (Mestrado em Educação). Universidade Estadual de Montes Claros. Montes Claros, MG.

Navarrete-Villavicencio, M. V.; Merino-Córdova, P. A.; Estupiñán-Cox, B. F. & Caicedo-Márquez, J. A. (2022). GeoGebra como ferramenta tecnológico-didática no aprendizado do cálculo integral. *Sapienza: Revista Internacional de Estudos Interdisciplinares*, 3(1), 902-910. https://doi.org/10.51798/sijis.v3i1.271

Oliveira, J. L. & Reis, F. S. (2017). Utilizando o GeoGebra para a construção do Conceito de Integral de Riemann no ensino de análise real. *Vidya*, 37(2), 417- 434.

Ponte, J. P. (2006). Estudos de caso em educação matemática. *Bolema*, 17(25), 105-132.

Richit, A. (2010). *Aspectos conceituais e instrumentais do conhecimento da prática do professor de cálculo diferencial e integral no contexto das tecnologias digitais*. 244f. Dissertação (Mestrado em Educação Matemática). Universidade Estadual Paulista. Rio Claro, SP.

Sacool, A. Z. (2009). Um retorno ao básico: compreendendo os paradigmas de pesquisa e sua aplicação na pesquisa em Administração. *Revista de Administração da UFSM*, 2(2), 250–269. https://doi.org/10.5902/198346591555

Silva, A. P. (2017*). A modalidade EAD Semipresencial e a disciplina de Cálculo Diferencial e Integral.* 227f. Tese (Doutorado em Educação para a Ciência). Universidade Estadual Paulista. Bauru, SP.

Silva, P. A. C. & Mota, J. F. (2024). Dificuldades nos processos de ensino e de aprendizagem de Mudanças de Coordenadas: uma análise no contexto do Cálculo Integral. *Revista Internacional De Pesquisa Em Educação Matemática*, 14(4), 1-16. https://doi.org/10.37001/ripem.v14i4.4367

Silvano, A. M. C.; Silva, J. G.; Procópio, J. W. B. & David, F. F. G. (2022). Uso do software geogebra no desenvolvimento do ensino e aprendizagem de cálculo de integrais. *REAMEC - Rede Amazônica De Educação Em Ciências E Matemática*, 10(3). https://doi.org/10.26571/reamec.v10i3.14273

Thomas, G. B. (2009). *Cálculo I.* (11. ed.). São Paulo, SP: Addison Wesley.

Villarreal, M. E. (1999). *O pensamento matemático de estudantes universitários de Cálculo e Tecnologias Informáticas.* 402f. Tese (Doutorado em Educação Matemática). Universidade Estadual Paulista. Rio Claro, SP.