

## Communication as a catalyst for the development of mathematical knowledge in the early years

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**Abstract:** This paper presents partial results from doctoral research that discusses communication in mathematics teaching in the early years and its implications for teaching and learning processes. It is classified as a case study with a qualitative approach. The data was produced during an experience with 4th-grade students, recorded by audio and video, and duly transcribed for analysis. The analyses follow the assumptions of Discursive Textual Analysis, based on Discursive Function Theory and other relevant theories. It was concluded that students' understanding of mathematical knowledge is directly related to the teacher's intentionality who, when organizing their teaching actions, seeks to promote communication that privileges dialogue, interaction, collective mobilization, and different registers of semiotic representations. This contributes to discussions in the field of Mathematics Education advancing in the understanding of communication between subjects who share the classroom.

**Keywords:** Dialogue. Mother Tongue. Mathematical Language. Semiotic Representations.

### La comunicación como potenciadora del desarrollo del conocimiento matemático en los primeros años de educación primaria

**Resumen:** Se presentan resultados parciales de una investigación doctoral que discute la comunicación en la enseñanza de las matemáticas en los primeros años de educación primaria y sus implicaciones en los procesos de enseñanza y de aprendizaje. Se clasifica como estudio de caso con enfoque cualitativo. La producción de los datos se realizó mediante una vivencia con estudiantes de 4º año, registrada en audio y video, debidamente transcrita para su análisis. Los análisis siguen los supuestos del Análisis Textual del Discurso, sustentados en la Teoría de las Funciones Discursivas y otras teorías pertinentes. Se concluyó que la comprensión del conocimiento matemático por parte de los estudiantes está directamente relacionada con la intencionalidad del docente, quien, al organizar sus acciones de enseñanza, busca promover una comunicación que privilegie los diálogos, las interacciones, la movilización colectiva y los diferentes registros de representaciones semióticas. Esto contribuye a que las discusiones en el campo de la Educación Matemática avancen en la comprensión de la comunicación entre los sujetos que comparten el aula.

**Palabras clave:** Diálogo. Lengua Materna. Lenguaje Matemático. Representaciones Semióticas.

## A comunicação como potencializadora do desenvolvimento do conhecimento matemático nos anos iniciais

**Resumo:** Apresenta-se resultados parciais de uma pesquisa de doutorado que discute a comunicação no ensino de matemática nos anos iniciais e suas implicações nos processos de ensino e de aprendizagem. É classificada como estudo de caso, com abordagem qualitativa. A produção dos dados ocorreu na realização de uma vivência com alunos do 4º ano, registrada por áudio e vídeo, devidamente transcritos para análise. As análises seguem os pressupostos da Análise Textual Discursiva, alicerçadas na Teoria das Funções Discursivas e demais teorias pertinentes. Concluiu-se que a compreensão do conhecimento matemático pelos alunos está diretamente relacionada à intencionalidade do professor que, ao organizar as suas ações de ensino, busca promover uma comunicação que privilegie os diálogos, as interações, a mobilização coletiva e os diferentes registros de representações semióticas. Contribuindo para que as discussões no campo da Educação Matemática avancem no entendimento da comunicação entre os sujeitos que compartilham a sala de aula.

**Palavras-chave:** Diálogo. Linguagem Materna. Linguagem Matemática. Representações Semióticas.

### 1 Introduction

The early years of elementary school (AIEF) correspond to a significant stage in children's overall development (Moretti & Souza, 2015; Maldaner, 2014). According to the National Common Core Curriculum (BNCC), "During this period of life, children are undergoing important changes in their development process that affect their relationships with themselves, with others, and with the world" (Brasil, 2018, p. 58). Understanding this relevance implies the establishment of ongoing studies that can contribute to the teaching and learning processes at this stage of Basic Education.

Considering mathematics teaching in this context, studies have presented discussions about different teaching activities developed with the aim of promoting students' understanding of concepts, such as ethnomathematics (D'Ambrosio, 2011), problem solving (Onuchic, *et al*, 2014), and mathematical investigations (Ponte, Brocardo & Oliveira, 2020). However, it is necessary to take into account that, regardless of the choices made by the teacher in organizing teaching, other aspects need to be considered in the development of learning, among them, the communication established between the subjects involved in the processes of teaching and learning mathematics.

For Alro and Skovsmose (2010, p. 140), "communication qualities influence learning qualities." Thus, considering that communication is a constituent of human relations in different spaces of society, it is necessary to understand the specificities of this communication when referring to mathematics teaching, especially in AIEFs.

Therefore, this paper aims to present partial results of a doctoral research project that discusses communication in mathematics teaching in AIEFs and its implications for teaching and learning processes, based on the following guiding question: *What attributes of communication established in the classroom, in a collective process of knowledge construction, can qualify teaching actions in AIEFs?* In this production, mathematical knowledge is evidenced through the development of algebraic thinking. In order to clarify the theme addressed in this research, supporting the analyses, discussions, and reflections presented, this study is based on theoretical choices, explained in the following sections.

## 2 Dialogue and social interactions: implications for teaching and learning processes

According to Vigotski (2019), communication is one of the functions of language responsible for the social development of the individual, and it involves relationships established through dialogue and interactions using words, gestures, symbols, and expressions, involving issues related to affectivity, reciprocity, empathy, mutual respect, and other feelings important for good social coexistence. Inseparable from communication is the constitutive function of language (generalizing thinking), responsible for the development of the ability to think.

Understanding the classroom as an environment in which learning develops from collective propositions, it is necessary to consider that both the teaching and learning processes are linked to the interactions produced in this environment. According to Abriliyanto, Saputro, and Riyadi (2017), social interaction enables students to express their thoughts and ideas, encouraging them to reflect.

In mathematics teaching, dialogues and interactions are fundamental for the development of arguments that can reflect and systematize mathematical ideas, which is corroborated by Silveira (2015, p. 90)

From this relationship between the student and others, such as the teacher, classmates, the classroom context, and the subject itself, conditions arise for the movement of action of their mathematical concept. When participating in language games, students use words from the public domain, but loaded with their own meanings that are in accordance with their perceptions, sensations, and experiences.

However, Forero-Sáenz (2008) warns that social interaction does not bring unlimited benefits, as is often thought, and therefore it is necessary to understand the specificities of interactions that can, in fact, lead to the attribution of meanings and the development of mathematical knowledge. Garcia and Nehring (2024) infer that the teacher's intentionality in organizing and conducting their discourse is a determining factor in ensuring that the dialogues and interactions produced in the classroom contribute to effective learning.

For Khasawneh, Al-Barakat, and Almahmoud (2023), mathematical learning is directly associated with the teacher's ability to establish constructive and critical interaction in the learning environment, positioning the teacher not as the focal point of interactions, but as the one who conducts them in accordance with conceptual learning objectives. These objectives are defined in the organization of their teaching actions, structuring a participatory learning environment, dominated by collective discussions, acceptance, and respect for others, where students can feel safe in taking risks, without fear of failing to solve mathematical problems, because, according to Alro and Skovsmose (2010, p. 106), “[...] emotional aspects are an essential part of the learning process that provides certain qualities to learning.”

In general, these dialogues and interactions take place in verbal language, in the mother tongue, especially when considering the discourses produced in AIEFs, because, “Before entering school, the construction of knowledge that children carry out with the support of an adult or a more competent peer involves, to a large extent, oral verbal communication: in the beginning, children learn by listening to and talking with adults” (Menezes & Nacarato, 2020, p. 1). According to Duval (2009, p. 106), the mother tongue

translates into all individuals through a discursive spontaneity that serves as an anchor point for all learning linked to teaching, regardless of the fact that this spontaneity

may not respect all the rules of language conformity and that it may be inhibited or favored by the interplay of social interactions.

However, the fact that students can begin to think mathematically through their mother tongue does not mean that mathematical language should be neglected, as each provides individuals with access to different layers of mathematical awareness (Radford & Barwel, 2016). In this sense, it is understood that communication established in the classroom, which effectively contributes to the development of mathematical knowledge, is complex, including different factors that add to the quality of the dialogues and interactions produced, among them the improvement of specialized language, that is, mathematical language, as presented in the following section.

### 3 Communication in mathematics teaching and learning

If we assume that interaction is made possible by communication, language appears as a fundamental aspect in the process of constructing and attributing meaning (Forero-Sáenz, 2008). For Radford and Barwell (2016, p. 1), “[...] language, speech, text, and the production and interpretation of symbols are integral to the creation of learning, teaching, and assessment, particularly in mathematics” (our translation).

Given this, although extremely relevant to teaching and learning processes, it is not possible to sustain mathematics teaching solely through the mother tongue (Duval, 2004). For effective understanding, it is necessary to establish communication that uses different registers of representation, which we call Semiotic Representation Registers (SRR) (Duval, 2009), since mathematical objects are not immediately accessible, and “[...] other semiotic systems such as algebra, geometry, and arithmetic, within the mother tongue, need to be evoked” (Sabel & Moretti, 2021). According to Duval (2009, p. 44),

Semiotic representations are both conscious and external representations. In effect, they allow a “view of the object” through the perception of stimuli (points, lines, characters, sounds...), having the value of a “signifier.” There is a wide variety of possible semiotic representations: figures, diagrams, graphs, symbolic expressions, linguistic expressions, etc.

It is therefore understood that mathematics is a science that has its own language, identified as mathematical language. According to Duval (2004), the language that develops within a science is referred to as specialized language. For this reason, this text assumes the use of mathematical language as specialized language, and the use of the mother tongue as common language.

The dialogues and interactions between the subjects involved in the teaching and learning processes, through the mother tongue, should drive the production of discourses that gradually aggregate the different RRS, developing argumentation skills also through mathematical language, promoting a new perspective of communication for teaching. According to Duval (2004), a combination of words can be considered a discourse when it allows a coherent idea to be expressed, with meaning in the context involved, as an “intentional production by someone” (Duval, 2011, p. 21). According to the author, “All discourse, whether produced orally or in writing, breaks down into units of meaning [...]” and, “The fundamental unit of meaning for any type of discourse is the sentence, whether it is narration, description, explanation, argumentation, etc.” (Duval, 2011, p. 76).

Thus, to communicate something to someone, it is not enough to know a multitude of

words; one must know how to organize them with cohesion and coherence so that the information can be understood. In other words, words are not interpreted individually, but rather based on the meaning produced by the content of the sentence and the context in which it is explained. Therefore, the identification of mathematical learning can be obtained through the discourse produced, when it shows the use of mathematical language, to a greater or lesser extent, according to the stages of development of each individual. Therefore, in AIEF, it is important to consider that the discourses that translate mathematical ideas will be produced predominantly through the use of the mother tongue, as there is still no repertoire for the extensive use of vocabulary related to specialized language. However, its use in the classroom is essential, corroborating Maldaner (2014), who states that Vigotski (2019, p. 31) teaches that

the words that represent concepts that organize scientific knowledge in schools should circulate fully in the school environment. Schoolchildren will use them naturally, even if they do not yet have a fully developed meaning, becoming true signs. These are words that refer to a real context and, as such, make sense.

The production of discourse is linked to the fulfillment of functions that enable a language to function (Duval, 2004). Present in every system of representation and linked to the common use of language are the metadiscursive functions<sup>1</sup>: communication, treatment, and objectification. These enable the development of skills in socialization, information transformation, and awareness of new understandings, respectively. According to Duval (2004), metadiscursive functions are not sufficient for the production of discourse that indicates conceptual learning, which also requires the fulfillment of the following discursive functions<sup>2</sup>: referential function, apophantic function, discursive expansion function, and reflexivity function. In short, these functions indicate the development of skills such as: naming and referencing mathematical objects through signs; categorizing them by their characteristics and describing them; making propositions and elaborating complete statements; expanding inferences by substituting or accumulating information.

Thus, communication capable of qualifying the processes involved in teaching and learning mathematics in AIEFs must consist of the production of discourses that transition through metadiscursive functions (including dialogues and interactions that organize speech) and advance to the discursive functions of language. This allows the establishment of conceptual relationships through language that moves between the mother tongue and specialized language (mathematical language, i.e., language that produces meaningful mathematical content), providing conditions for the improvement of learning through the explanation of coherent mathematical ideas in the form of arguments, hypotheses, and assumptions, which can be presented under different RRS.

For communication to be effective, the teacher's intentionality in organizing and conducting activities is an essential element, since this communication is established based on their own discourse and the mathematical knowledge mobilized. In this sense, the classroom is understood as a privileged space for the development of understanding based on metadiscursive functions, which, although not sufficient for conceptual understanding (which requires discursive functions), are necessary and provide the conditions for learning to develop.

<sup>1</sup> See: Garcia, F. H. & Nehring, C. M. (2024). The importance of teachers' knowledge of content in communication mediated by mathematical language. *Boletim de Conjuntura (BOCA)*, 20(58), 249-275. <https://doi.org/10.5281/zenodo.14397275>

<sup>2</sup> Idem footnote 1.

#### 4 Methodological procedure

The discussions and understandings shared in this paper come from doctoral research, which was approved by the institution's Ethics Committee. In terms of the procedures used, it is classified as a case study (Yin, 2005).

According to its objective, it is categorized as descriptive and interpretive, addressing the data qualitatively (Yin, 2016). The analyses are organized according to the assumptions of Discursive Textual Analysis (DTA) by Moraes and Galiuzzi (2016) and based on Duval's Theory of Discursive Functions (2004) (Garcia & Nehring, 2024) and other postulates presented in the theoretical framework of this study.

The data was produced during an experience with a class of 4th-grade elementary school students from a public school in the state of Rio Grande do Sul, during regular class hours. The choice of the 4th grade was due to the fact that the teachers of those classes welcomed the research, and the invitation was extended to all AIEF teachers at the school.

The analysis presented is the result of the activity proposed and carried out during the experience, whose objective was to mobilize the resolution of a mathematical problem, of an investigative and collaborative nature (Alro & Skovsmose, 2010), which would allow the establishment of understanding and comprehension through discussions and interactions that enabled the development of mathematical arguments produced during the communicative process, established in the discursive game (Garcia & Nehring, 2024), helping to develop students' algebraic thinking and promote mathematical literacy. Table 1 presents the problem.

**Table 1:** Problem proposed during the experience

I have a faucet in my house that I believe is defective in its shut-off valve because, even when I turn it off, the faucet continues to drip a lot of water. One day, I decided to check the volume of water being wasted to see if the flow was a cause for concern. Using a measuring cup and the stopwatch on my cell phone, I collected water for 1 minute and obtained 12 milliliters (ml). Now, I need your help to assess the extent of the waste and whether there is an urgent need to call a plumber to fix the problem. Do you have any idea how to do this?

**Source:** Garcia; Nehring (2024).

In this production, it is acknowledged that algebraic thinking involves “[...] forming generalizations from experiences with numbers and operations, formalizing these ideas using a meaningful system of symbols, and exploring the concepts of pattern and function” (Van de Walle, 2009, p. 287). Furthermore, mathematical literacy “[...] ensures that students recognize that mathematical knowledge is fundamental to understanding and acting in the world” (Brasil, 2018, p. 266).

The problem was intentionally organized around the theme of *Water Consumption*, enabling the contextualization of mathematics with the children's daily experiences and other areas of knowledge. Eighteen students participated in the research and, in order to comply with ethical precepts, each of them was identified with the letter A, followed by numbering in ascending order (A1, A2, A3...). The researcher is referred to by the letter P. The experience was recorded via audio and video, which were transcribed by the researcher, organized into episodes, and subsequently analyzed. Based on the analysis procedure, the research *corpus* was constituted, together with the students' written records.

During the experience, the researcher assumed the role of visiting teacher and was given control of the class. Upon arriving in the classroom, she intentionally asked the students for help in solving a problem at home (a math problem). First, discussions and interactions on the topic of *water* were proposed, organized and conducted so that a bond of trust and affection

could be established between the researcher and the students.

After this period, the researcher presented the math problem that needed to be solved in order to help her. To carry out the activity, the class was divided into four groups, and each student received a sheet with the problem printed on it and blank space for individual notes. The dialogues and discussions, both in the collective and in the groups, gave rise to the fragments selected for the composition of the Episodes.

This fragmentation led to the creation of a unitarization process, which consists of a “[...] deconstruction of the texts in the corpus in order to differentiate and identify constituent unitary elements” (Moraes & Galiuzzi, 2016, p. 80). Thus, the *corpus* was reorganized into units of meaning, based on the identification of recurrences between common ideas and understandings. These recurrences were identified based on what gave meaning to the defined research problem, giving rise to the selection and organization of excerpts, identified by codes. A group of excerpts, organized in chronological order of occurrence, is called an Episode (EP).

When analyzed according to the researcher's interpretations and understandings, the units of meaning gave rise to units of significance, presented through sentences, which, when analyzed in light of the theories underlying the research, generated the categories of analysis, classified as emerging. For Moraes and Galiuzzi (2016, p. 110), “The adoption of the emerging process requires a gradual definition of categories. The clarity and validity of the set of categories are only completed at the end of the analysis. The process is recursive, requiring constant revisits for its qualification.”

This leads to the establishment of relationships between the units of meaning, the categories, and the theories that underpin the study, “[...] giving rise to propositions in the search for answers to the research problem established as a result of data triangulation” (Garcia & Nehring, 2024, p. 261). Table 2 presents the systematization of the analysis.

**Table 2:** Summary of the analysis, according to the assumptions of ATD

| Units of meaning  | Units of meaning  | Category                 | Propositions   |
|---|---|--------------------------|--|
| EP1T(1)P, EP1T(2)P, EP1T(3)AS, EP1T(4)P, EP1T(5)A9, EP1T(6)A5, EP1T(7)A9, EP1T(8)P, EP1T(9)AS, EP1T(10)A18, EP1T(11)A10, EP1T(12)A9, EP1T(13)A8, EP1T(14)A5, EP1T(15)P  | Establishing a dialogic relationship between the researcher and the class.            | Dialogue and interaction | The teacher's intentionality in organizing and conducting their discourse, even in their native language, is a condition for establishing <b>dialogues and interactions</b> that allow students to engage in the appropriation of knowledge as a <b>collective act</b> . |
| EP2T(1)P, EP2T(2)A5, EP2T(3)A9, EP2T(4)P, EP2T(5)A9, EP2T(6)A3, EP2T(7)P, EP2T(8)A9   | Students' prior knowledge: mathematical ideas through the use of their mother tongue. |                          |  |
| EP3T(1)A9, EP3T(2)A9, EP3T(3)P, EP3T(4)A8, EP3T(5)P, EP3T(6)A8, EP3T(7)P, EP3T(8)A9, EP3T(9)P, EP3T(10)G, EP3T(11)P, EP3T(12)A9, EP3T(13)P, EP3T(14)A9, EP3T(15)P, EP3T(16)A8, EP3T(17)P, EP3T(18)A8, EP3T(19)P, EP3T(20)A8, EP3T(21)P, | work in groups: "strangeness" and "confusion."  | production of discourse  | In order to develop students' mathematical knowledge, teachers need to organize and conduct their discourse, <b>articulating the mother tongue and mathematical</b>  |

|  |  |  |  |
|--|--|--|--|
| EP3T(22)A8, EP3T(23)P  |  |  | <b>language</b> , so that the  |
| EP4T(1)P, EP4T(2)AS, EP4T(3)P, EP4T(4)A9, EP4T(5)P, EP4T(6)A6, EP4T(7)A7, EP4T(8)P, EP4T(9)AS, EP4T(10)P, EP4T(11)A3, EP4T(12)P, EP4T(13)AS, EP4T(14)P, EP4T(15)AS, EP4T(16)P, EP4T(17)P, EP4T(18)A3, EP4T(19)P, EP4T(20)AS, EP4T(21)P, EP4T(22)P, EP4T(23)A5, EP4T(24)P, EP4T(25)A5, EP4T(26)P, EP4T(27)AS, EP4T(28)P, EP4T(29)A8 | Organization of data and understandings constructed collectively.  |  | <b>use of different semiotic representations</b> can be mobilized, discussed, elaborated, and systematized collectively in the search for <b>conceptual meanings</b> . |
| EP5T(1)P, EP5T(2)A12, EP5T(3)P, EP5T(4)A8, EP5T(5)P, EP5T(6)A9, EP5T(7)P, EP5T(8)A8, EP5T(9)P, EP5T(10)A8, EP5T(11)P, EP5T(12)AS, EP6T(1)P, EP6T(2)AS, EP6T(3)A9, EP6T(4)P, EP6T(5)P, EP6T(6)AS, EP7T(1)P, EP7T(2)P, EP7T(3)AS, EP7T(4)P, EP7T(5)AS, EP7T(6)P, RE(1), RE(2), RE(3)   | Recognition of regularity and algebraic representation: evidence of the development of algebraic thinking. |  |  |
| <b>Legend:</b> EP (Episode); T: Excerpt; A: Student (A1, A2, A3...); AS: All students; G: Collective response from the group; P: Researcher; RE: Written record.   |  |  |  |

**Source:** Prepared by the authors (2025).

For proper visualization in the text, the analyses are presented in the form of Episodes, in which the excerpts are arranged in chronological order of events, in tables. The excerpts in italics correspond to the authors' observations, and those in bold are what they consider noteworthy for the analysis. It is understood that this organization facilitates understanding of the advances evidenced during the experience, considering the development of the children's algebraic thinking, as well as the promotion of mathematical literacy throughout the communicative process, intentionally established by the researcher teacher.

## 5 Analysis and discussion of results

*Water consumption* was chosen because it is a widely discussed topic, both inside and outside school, impacting not only the constitution of school knowledge, but also the critical and civic awareness of each individual. Equally intentionally, we opted for a mathematical problem of an investigative and collaborative nature, in order to awaken in children the need to question and discuss, with the teacher and classmates, different possibilities in the search for solutions.

Next, excerpts produced during dialogues and interactions are presented—as well as understandings inferred through theoretical lenses—as metatexts that are organized according to the study categories, highlighting communication as a constituent in the development of mathematical knowledge.

### 5.1 Dialogue and interaction

In this research, the dialogues and interactions produced in the classroom are characterized as intentional actions, since they are not just any dialogues and interactions, but rather those capable of contributing to the development of mathematical knowledge. They are

linked to the metadiscursive functions of language, that is, they constitute discourses expressed through any language, mainly the mother tongue, which explicates the discourse itself or organizes speech, helping to structure arguments.

To begin working with the class, the researcher organized her actions in order to establish a productive bond with the students, promoting a discussion about daily water consumption. Table 3 presents Episode 1, which characterizes the aforementioned dialogue.

**Table 3:** Episode 1 – Establishing contact

|  |
|--|
| <p>EP1T(1)P: <b>What do you usually use water for?</b> [<i>Students interact to answer the question</i>].</p> <p>EP1T(2)P: Would it be possible to live without water?</p> <p>EP1T(3)AS: <b>No!</b></p> <p>EP1T(4)P: Really? Why?</p> <p>EP1T(5)A9: We would die of thirst, we wouldn't be able to clean ourselves, we wouldn't be able to make food.</p> <p>EP1T(6)A5: There's a "thing" that says 90% of our body is water.</p> <p>EP1T(7)A9: <b>70%...</b></p> <p>EP1T(8)P: Would there be life on the planet if there were no water?</p> <p>EP1T(9)AS: <b>No!</b></p> <p>EP1T(10)A18: First, animals would die, just like us, from thirst. <b>There would be a fruit cycle, but it wouldn't be as abundant...</b></p> <p>EP1T(11)A10: <b>But there wouldn't be any water for planting either...</b></p> <p>EP1T(12)A9: <b>Like cactus water?</b></p> <p>EP1T(13)A8: <b>But if there is no water, how will cacti grow?</b></p> <p>EP1T(14)A5: But cacti live in the desert... [<i>Discussions continue among students...</i>]</p> <p>EP1T(15)P: And at your house or somewhere else, have you ever noticed that the faucet, even when turned off, can keep dripping? Do you have any faucets like that at home? [<i>Several students report some situations.</i>]</p> |
|--|

**Source:** Research data (2023).

Episode 1 shows the extent of the students' involvement, as they all wanted to contribute to the discussion. Considering the topic, the children show awareness of its importance for sustaining life (EP1T(3)AS, EP1T(5)A9, EP1T(9)AS), since, even without being asked to propose solutions for a possible water shortage, they made a point of expressing them verbally. This attitude demonstrates trust in the environment and in the people who were there, as observed in excerpts EP1T(10)A18 to EP1T(14)A5, pointing to evidence for the establishment of an atmosphere capable of encouraging students to explore and deepen their thinking, find alternative ways of thinking, and learn to accept the contributions of others (Abriliyanto, Saputro & Riyadi, 2017).

In addition, when verifying incorrect information given by A5 – when mentioning the percentage of water in the human body (EP1T(6)A5) – A9 immediately corrects it, even if with an approximate value (EP1T(7)A9). This fact highlights the potential of dialogues and interactions in complementing ideas and learning, corroborating Vigotski (2019), who mentions social interactions, mediated by instruments and signs, as structuring the cognitive development of individuals.

Next, Episode 2, shown in Table 4, indicates the continuation of the dialogue that begins to mobilize mathematical knowledge, highlighting the students' prior knowledge. Intuitively and through the common use of language (mother tongue), mathematical ideas are presented

and developed.

**Table 4:** Episode 2 – Prior knowledge

EP2T(1)P: Have you ever stopped to think about how much water is wasted by that tap that drips from time to time? Is it a lot or a little water?  
 EP2T(2)A5: **It depends on how long...**  
 EP2T(3)A9: Because then, maybe we don't see that it's dripping, and **the longer it takes, the more it will drip...**  
 EP2T(4)P: That's right, look how interesting, **the longer it drips...**  
 EP2T(5)A9: **More water goes out.** [*The researcher reads the math problem (Table 1). Murmurs in the class.*]  
 EP2T(6)A3: **Teacher, multiply 24 hours by 60 minutes and then multiply the number of minutes by 12 milliliters to see how many liters there are in a day.** [*Expressions of surprise among the students.*]  
 EP2T(7)P: What a great tip! [*The students discuss, and it is not possible to capture specific audio, with the following exception:*]  
 EP2T(8)A9: **If it uses 12 milliliters in 1 minute, in 2 minutes it uses 24...**

Source: Research data (2023).

In this episode, it is possible to identify that the students already have an intuitive understanding of the relationship between the volume of water wasted by a dripping tap and the time it will remain defective (EP2T(2)A5 and EP2T(3)A9), even though they do not yet use specialized language. In other words, even using their mother tongue, A9 (EP2T(3)A9) is able to express themselves by organizing a sentence, whose unit of meaning consists of a true logical value, making a coherent statement (Duval, 2011) about the facts. The learning potential of children in this period of schooling is evident, often neglected by the prevalence of classes in which repetition and memorization outweigh questioning, interaction, creativity, and critical thinking, which are necessary conditions for conceptual learning.

After reading and explaining the problem to be solved, A3 (EP2T(6)A3) surprises us with the elaboration of a sentence that can be considered a complete statement, referring to the fulfillment of the apophantic function (discursive operation), which has the purpose of "saying something about the objects that are designated, in the form of a declared proposition" (Duval, 2004, p. 88), combining mother tongue and mathematics, whose unit of meaning has logical truth value. However, it should be noted that A3 shows signs of possibly being at an advanced level of cognitive development in relation to his peers, given the expressions of "astonishment" on the faces of the others when they hear their classmate's proposition. In addition, A9 indicates the observation of a proportionality between the values corresponding to time and water volume (EP2T(8)A9).

Thus, if we consider, in Episodes 1 and 2, the prevalence of discourse produced in the mother tongue, without the intention of using specialized language, but intentionally conducted to promote an environment of discovery and rapprochement between the subjects, it can be indicated that, although the metadiscursive functions of language are not sufficient for the identification of conceptual appropriation (Duval, 2004), they can be considered a condition for this, as they assist in the structuring of ideas, even if there are not yet conditions to make conceptual understandings explicit, corroborating Vigotski (2019) when he states that social interactions are inseparable from the development of generalizing thinking.

## 5.2 The production of discourse

In this category, evidence of the development of students' algebraic thinking is made explicit. In general, it is possible to infer strangeness regarding the model of activity proposed in the classroom, difficulties in working in groups, and finding an organized strategy to reach the solution.

Episode 3, shown in Table 5, presents the discussion of one of the groups as they perform the procedure of converting the mathematical problem, whose activity is characterized as “[...] a transformation that moves from one register to another” (Duval, 2009, p. 39), in this case, from the mother tongue to mathematical language. And later, in the procedure of processing information, which, according to Duval (2009, p. 39), “[...] is a transformation that takes place within the same register, the one where the rules of operation are used; processing then mobilizes only one register of representation,” that is, performing calculations, manipulating numbers and mathematical operations in order to find the solution.

It is important to note that the difficulty of working in groups was evident in the fact that, of the four proposed groups, only two produced discussions with an exchange of ideas. The others produced individual solutions, without effective discussions, showing that this is not a frequent practice in classes.

**Table 5:** Episode 3 – Group work

|  |
|--|
| <p>EP3T(1)A9: <b>Let's try 24 times 7...</b> [<i>The group solves the multiplication problem.</i>]</p> <p>EP3T(2)A9: Teacher (T), is this right?</p> <p>EP3T(3)P: What did you calculate?</p> <p>EP3T(4)A8: 24 times 7... 24 for the hours and 7 for the days.</p> <p>EP3T(5)T: So, what does this 168 refer to?</p> <p>EP3T(6)A8: <b>Liters?</b></p> <p>EP3T(7)T: You multiplied hours by days. How can the result be liters?</p> <p>EP3T(8)A9: <b>Would that be minutes?</b></p> <p>EP3T(9)A10: <b>I never thought I would have to think about it, never...</b></p> <p>EP3T(10)P: Oh, every day has...</p> <p>EP3T(11)Group: 24 hours.</p> <p>EP3T(12)P: So, 168, right?</p> <p>EP3T(13)A9: <b>Minutes?</b></p> <p>EP3T(14)P: You used hours and days. Each of those 7 days has 24 hours. So what does this total represent?</p> <p>EP3T(15)A9: <b>Hours?</b></p> <p>EP3T(16)P: Hours of the 7 days. So, in 7 days we have 168 hours.</p> <p>EP3T(17)A8: <b>We need to multiply the number of hours by the milliliters that came out.</b></p> <p>EP3T(18)P: Don't forget that our reference is...</p> <p>EP3T(19)A8: Minutes.</p> <p>EP3T(20)P: So, I can't solve it with hours, I still need to do something else.</p> <p>EP3T(21)A8: What if we multiply by 60? Because 60 is one minute... [<i>referring to the fact that 1 hour has 60 minutes</i>].</p> <p>EP3T(22)P: That's right! [<i>The group performs the calculation.</i>]</p> <p>EP3T(23)A8: Teacher, we each did a calculation and it was wrong, mine came out to 9,680, <b>then milliliters.</b></p> |
|--|

EP3T(24)P: OK, you multiplied **hours by minutes**, so here the result is **minutes, not milliliters yet, right?**

Source: Research data (2023).

According to Duval (2011, p. 76), “[...] the analysis of students’ verbal productions in interpretable units of meaning should be done in terms of the discursive operations that are required in mathematics.” In this sense, it is possible to infer that the dialogues produced in Episode 3 do not show the development of discursive operation, as students have difficulty understanding the articulation between the operations they perform and the meaning of the answers they produce, as explained in excerpts EP3T(6)A8, EP3T(8)A9, EP3T(13)A9, EP3T(17)A8, and EP3T(23)A8.

There is a concern with solving the calculations, even without understanding them, indicating that they do not usually develop activities that require reflection and understanding of the solutions found, clearly evidenced in the statement by A10 (EP3T(9)A10), characterizing the practice as a great challenge at that moment. Realizing the difficulties, P proposes questions with the intention of helping students develop reflections and understanding about the procedures they were proposing. In this regard, Alro and Skovsmose (2010, p. 70) argue that “The teacher can act as a facilitator by asking questions with an investigative attitude, trying to understand how the student interprets the problem.”

Based on the discussions, P organizes a dialogical and collective systematization, mobilizing a new form of representation: the table, which is presented in Episode 4, shown in Table 6. The objective was to help students understand the procedures, solutions, and their meaning in the context established in the proposed mathematical problem, encouraging them to contribute by exposing their strategies and doubts in a collective movement of conceptual elaboration. To this end, P represented a table on the whiteboard, according to the model in Figure 1.

Figure 1: Table model for systematization

| Time (min.) | Volume of water (ml) |
|-------------|----------------------|
| 1           |                      |
| 2           |                      |
| 3           |                      |
| 4           |                      |
| 5           |                      |
| 10          |                      |
| 30          |                      |

Source: Prepared by the authors (2023).

Given the students' unfamiliarity with this representation, P explained its functions and uses. Only after this recognition did the collective filling of the table with the appropriate data begin.

Table 6: Episode 4 – Organizing thoughts

EP4T(1)P: In one minute, the tap was wasting 12 ml. In 2 minutes, how much water will be wasted, in milliliters?  
 EP4T(2)A9: **24.**  
 EP4T(3)P: When 3 minutes have passed, what happens?  
 EP4T(4)A6: **36.**  
 EP4T(5)A7: **36 milliliters.**  
 EP4T(6)P: What about after 4 minutes?

EP4T(7)AS: **48 milliliters.** [*The teacher recorded the answers in the table.*]  
 EP4T(8)P: What if it stayed open for 60 minutes?  
 EP4T(9)A3: 720.  
 EP4T(10)P: How long is 60 minutes in hours?  
 EP4T(11)AS: **One hour.**  
 EP4T(12)P: So, in one hour she spent 720...  
 EP4T(13)AS: Milliliters.  
 EP4T(14)P: **Is that more or less than 1 liter?** [*Part of the class answered "more" and the other part "less," all still unsure.*]  
 EP4T(15)P: How many milliliters are in a liter?  
 EP4T(16)A3: One thousand milliliters.  
 EP4T(17)P: So, is 720 more or less than 1 liter? [*Using a measuring cylinder to illustrate.*]  
 EP4T(18)AS: **Less.**  
 EP4T(19)P: So, in one hour, this faucet is wasting 720 milliliters of water. And in a whole day? [*Students are thoughtful*] - [*Discussions resume in groups to determine the number of minutes in a day.*]  
 EP4T(20)P: So, in one day, we have 1,440 minutes. If each minute uses 12 ml, how much will be used in a whole day?  
 EP4T(21)A5: 17,280 milliliters.  
 EP4T(22)P: That's right! If I think about this amount in liters, how many of these (*points to the 1000 ml measuring cup*) would we need?  
 EP4T(23)A5: 17 and a little more.  
 EP4T(24)P: So, is that a little or a lot?  
 EP4T(25)AS: **A lot!**  
 EP4T(26)P: That's 17 of these (measuring cups) worth of wasted water in just one day... Wow, I'm starting to worry about how much water I've been wasting at home! Don't you think?  
 EP4T(27)A8: **One thing is for sure, teacher, the water bill won't be cheap.** [*Discussions continued to determine water consumption over 30 days.*]

Source: Research data (2023).

The teacher (T) begins by returning to the problem. When she asks the first question, only A9 responds timidly (EP4T(2)A9). In the second question, two students join in (EP4T(4)A6 and EP4T(5)A7). In the third question, it is possible to identify the participation of the entire class (EP4T(7)AS), indicating that the necessary treatment to be established in this representation record was being mobilized, showing signs of understanding. Now, the students felt confident to socialize their proposals, because "Participating in a dialogue is something that should not be imposed on anyone. In the classroom, this means that the teacher can invite students to an investigative dialogue, but they must accept the invitation for the dialogue to take place" (Alro & Skovsmose, 2010, p. 132). In this case, the collective response indicates acceptance of the invitation and the beginning of the mobilization of discursive functions in mathematical language.

During the episode, P organizes the dialogue so that, in addition to completing the data in the table, students can identify relationships and equivalencies between different units of measurement of time and capacity, as, for example, in excerpts EP4T(10)P, EP4T(11)AS, EP4T(15)P, EP4T(16)A3, EP4T(17)P, and EP4T(18)AS. The comparisons established with the aid of the measuring cylinder allowed students to get an idea of the amounts of water used, considering each time interval.

The movement subsequently contributed to the assessment of water waste, the initial objective of the activity, as indicated in excerpts EP4T(24)P and EP4T(25)AS, improving critical and social awareness of this important issue and enabling the development of civic consciousness, since “[...] the promotion of ingenuity for critical thinking does not happen automatically [...]” (Freire, 2023, p. 33). The excerpt EP4T(27)A8 highlights this awareness, expanding it to establish the relationship of dependency between the volume of water used and the value of the water bill at the end of each month, thus recognizing the mathematics that permeates everyday actions. This fact indicates the potential of collective discussions as a viable path for the development of mathematical literacy, since a literate individual is “[...] one who learns not only certain techniques for reading, writing, and counting, but also how to use them consciously in different contexts and social practices” (Moretti & Souza, 2015, p. 27).

Episode 4 highlights the importance of the teacher's intentionality in organizing their discourse, since “[...] organizing teaching for the development of scientific concepts in children is an important commitment of their pedagogical practice.” (Moretti & Souza, 2015, p. 28). From this perspective, in addition to dialogues in the mother tongue, interactions were articulated with different RRS, which evoked mathematical language through tables, numerical data, operations, and concrete material (test tubes), establishing a new pattern of communication between the subjects, which enabled the development of a mathematical awareness of the facts, whose understandings will be explained in the following episodes, where the discourses begin to move from the mother tongue to mathematical language, indicating the fulfillment of some discursive functions of language (Duval, 2004).

According to Duval (2004), the fulfillment of the discursive functions required in mathematics, expressed in the discourses produced by the students, indicates a conceptual understanding. Thus, Episodes 5, 6, and 7, presented respectively in Tables 7, 8, and 9, expose the dialogues and interactions, intentionally organized and conducted by P, with the purpose of deepening the initial mathematical ideas, towards the establishment of mathematical knowledge, characterized by the identification of signs of algebraic thinking development.

**Table 7:** Episode 5 – Recognizing regularities

|   |
|---|
| <p>EP5T(1)P: What do these values we are working with correspond to?</p> <p>EP5T(2)A12: Milliliters.</p> <p>EP5T(3)P: We had a value that corresponded to milliliters, which was...</p> <p>EP5T(4)A8: <b>Volume.</b></p> <p>EP5T(5)P: And what was the other value?</p> <p>EP5T(6)A9: <b>Time.</b></p> <p>EP5T(7)P: So, we are working with time, which is the time the tap stays open, and with the volume of water it wastes, right? Considering the data in the table, do you notice anything that is regular between them?</p> <p>EP5T(8)A8: Teacher, one thing I noticed is that the table is more or less <b>the 12 times table.</b></p> <p>EP5T(9)P: Very good! So, every time we wanted to know the final volume, what did you do? How did you calculate it?</p> <p>EP5T(10)A8: <b>We calculated the time spent times 12.</b></p> <p>EP5T(11)P: That's right! So, what operation did you use?</p> <p>EP5T(12)AS: <b>Multiplication.</b></p> |
|---|

**Source:** Research data (2023).

In this episode, it is possible to identify, even if intuitively and through common language use, the designation of the variables *volume* and *time* in excerpts EP5T(4)A8 and

EP5T(6)A9, respectively. According to Duval (2004, p. 95, our translation), pure designation is one of the discursive operations of the referential function, which “consists of identifying an object.” However, it is clear that the fulfillment of this discursive function does not occur spontaneously by the students, but rather through the dialogue established with P during the questioning (EP5T(1)P, EP5T(3)P, and EP5T(5)P), through the complementation of ideas.

The recognition of a regularity is also an important finding observed in excerpt EP5T(8)A8, and contributes significantly to the development of algebraic thinking. In this case, when expressing “*more or less the 12 times table*,” A8 recognizes a sequence whose elements are multiples of 12, but shows some uncertainty in his statement, as the numbers presented in the *Time* column of the table are not arranged in sequential units of one.

This recognition suggests evidence of the fulfillment of another operation of the referential function, simple categorization, which “consists of identifying an object based on one of its qualities” (Duval, 2004, p. 95). According to Van de Walle (2009), the ability to pay attention to details and observe regularities present in different activities allows individuals to formulate generalizations and apply them in other situations.

In addition, when asked about the procedure used to find the volume of water used in a given time (EP5T(9)P), A8 responds in their native language (EP5T(10)A8), indicating the identification of a standard procedure for all times that the calculation was performed, which can later be related to the law of function formation. For Van de Walle (2009, p. 296), “Learning to look for patterns and how to describe, translate, and expand them is part of doing mathematics and thinking algebraically.”

In this case, EP5T(10)A8 points toward the fulfillment of the apophantic function, through the operation of the illocutionary act, in which, “[...] through the production of the statement, it gives this statement a social value of the act that commits the speaker or the recipient (Duval, 2004, p. 107, our translation), as a response was formulated in response to the question asked by P.

Next, Episode 6 presents excerpts from the collective discussion, intentionally organized so that mathematical language would circulate more emphatically, producing new meanings for the established understandings.

**Table 8:** Episode 6 – Introduction of mathematical language

|  |
|--|
| <p>EP6T(1)P: The longer the tap drips, will the volume of water wasted be greater or less?<br/>         EP6T(2)AS: <b>Greater.</b><br/>         EP6T(3)A9: That's cool!<br/>         EP6T(4)P: Does the volume of water wasted <b>depend</b> on how long the tap was dripping? Or not? [<i>The answers are divided into "yes" and "no."</i>]<br/>         EP6T(5)P: Does the amount of water wasted depend on how long the faucet dripped?<br/>         EP6T(6)AS: <b>Yes!</b></p> |
|--|

**Source:** Survey data (2023).

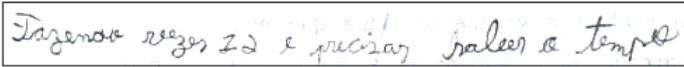
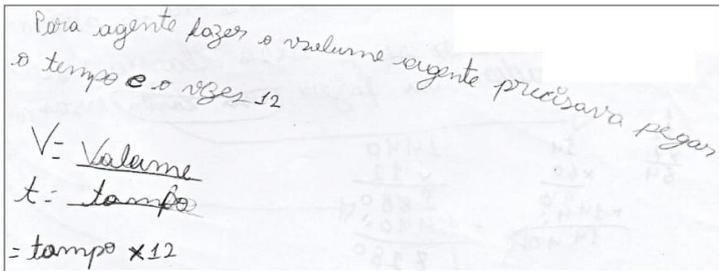
There was significant participation from the class in this episode. When asked, implicitly and through common language, about the relationship between time and volume of water (EP6T(1)P), all students answered correctly and with conviction (EP6T(2)AS). A9 is excited (EP6T(3)A9) when he realizes that the entire process that has been established and developed now makes sense, indicating a possible objectification, which “[...] corresponds to the subject’s own discovery of what he himself had not previously supposed [...]” (Duval, 2009, p. 41).

However, when P introduces the word *depends* in his new question, in order to make explicit the relationship of dependence between the variables, the students respond with a confusion of *yes* and *no*, indicating doubt about the established relationship. Then, P reformulated the question (EP6T(5)P), maintaining the cognitive content and varying the proposition, triggering a unanimous *yes* response. According to Curi (2024, p. 7), “[...] it is important for the teacher to be clear about the objective being addressed, the different possibilities for teaching that content, and the importance of examples and counterexamples.” Although there was unanimity, it was noticed that, after the answer, the children remained silent and thoughtful, showing that the language was not familiar and that this still triggered uncertainties in their understanding.

Given this, P formulated some explanations regarding terms such as *variable quantities*, *dependent variables*, and *independent variables*, and continued the dialogue, reinforcing the teacher's commitment to promoting a discourse that allows for the expansion of ideas and understanding through specialized language, even if such words are not yet widely used by students at this stage of basic education. Thus, “Students construct their mathematical concepts by participating in language games and when they work with intersubjective meanings given to objects” (Silveira, 2015, p. 90). Furthermore, “[...] the use of words is an integral part of the development processes, and words retain their directive function in the formation of true concepts, to which these processes lead” (Vygotsky, 2019, p.101).

Although the aforementioned episode does not present the development of any discursive function, it characterizes an important stage in the path established by the discursive game, mobilized by metadiscursive functions, until the systematization of the activity, expressed in Episode 7, below.

**Table 9:** Episode 7 – The relationship of dependence between time and volume

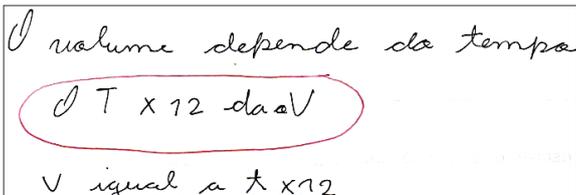
|  |   |
|--|---|
| <p>EP7T(1)P: If we have one <b>variable that is dependent</b> on another, it is because we have a dependency <b>relationship</b> between them. Could we write this relationship down in some way? That one depends on the other? How would you write it? [<i>Silence, attentive eyes and doubts</i>]</p> <p>EP7T(2)P: How do we obtain the volume? Every time we calculate the volume, what do we need to know?</p> <p>EP7T(3)AS: <b>The time...</b></p> <p>EP7T(4)P: And?</p> <p>EP7T(5)AS: <b>Multiply by 12.</b></p> <p>EP7T(6)P: So, try to write down this relationship. [<i>Time for notes.</i>]</p> | <p>RE(1): </p> <p>[Translation of the manuscript: Multiplying by 12 and needing to know the time.]</p> <p>RE(2): </p> |
|--|---|

[Translation of the manuscript: To calculate the volume, we need to take the time and multiply it by 12.

V = volume

T = time

= time x 12]

RE(3): 

[Translation of the manuscript: The volume depends on the time

T x 12 gives V

V equals t x 12]

Source: Research data (2023).

This episode presents an attempt to promote the development of an RRS of the dependency relationship between the variables *time* and *volume*. Naturally, the question in excerpt EP7T(1)P did not receive an immediate response. Given this, P revisits the previously developed understandings with the class (EP7T(2)P to EP7T(5)AS), and proposes formalization through a record in the mother tongue, since, “To be aware of the discursive operations specific to mathematical reasoning, it is necessary to go through a written production (Duval, 2011, p. 82).

In general, it can be inferred that the mother tongue stands out in the records, which makes explicit the level of understanding in the AIEF. It is necessary to be aware that, at this stage, it is not up to the student to understand all the RRS that make up mathematical language (Hillesheim, 2022). In any case, students produced statements that show signs of developing algebraic thinking at different levels of understanding.

According to Duval (2016, p. 19), mastering one's native language means “[...] being aware of the operations that allow words to be articulated in nominal phrases to designate objects, propositions, or to make a coherent description.” Thus, RE(1) reflects a lack of mastery of the mother tongue, since the statement does not produce a coherent description and compromises its understanding. However, it is still possible to identify mathematical awareness, as it infers the need to know the value corresponding to time and multiply it by the initial value of the proposition (12 ml) in order to obtain the volume.

RE(2), on the other hand, presents a statement in the mother tongue, followed by a representation that mobilizes words and symbols. From the point of view of the referential function, there are indications of the operation of pure designation (Duval, 2004), since the variables *time* and *volume* were defined. Regarding the apophantic function, there are indications of the fulfillment of the predication operation, which “[...] consists of linking the expression of a property, a relation, or an action with an expression that designates objects”

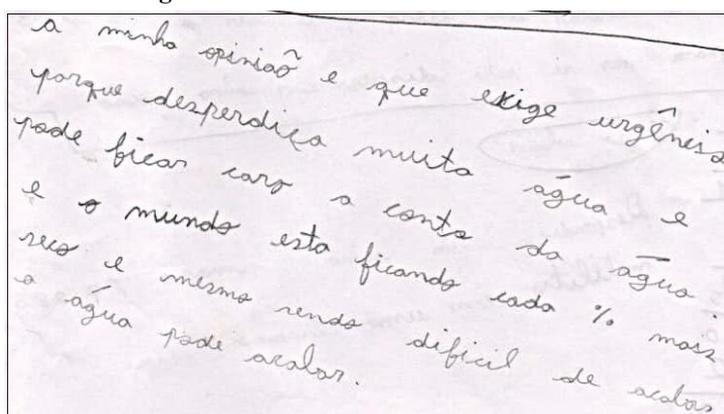
(Duval, 2004, p. 107, our translation), assuming a logical value of truth. The presence of the natural discursive expansion function by substitution can also be inferred, since the proposition was replaced only in the mother tongue with another that includes mathematical symbols, expanding knowledge of mathematical language.

RE(3) shows signs of fulfilling the same discursive functions as RE(2), but with an advance in the declared enunciation, since mathematical language is presented more explicitly. Even mobilizing native language and mathematical symbols, the final proposition “ $V$  equals  $t \times 12$ ” approximates the representation of an algebraic record, very close to fulfilling the formal discursive function, which has as its exclusive resource symbols or the proposition of a functional structure (Duval, 2004). Obviously, the fulfillment of the discursive functions of language is not expected in the case of AIEFs. However, it is necessary to understand that this development is continuous and inherent to the understanding of mathematical knowledge during Basic Education.

Given the above, the records indicate that the discussions produced from the dialogues and interactions between the subjects involved in the teaching and learning processes, constituted by the metadiscursive functions, organized and conducted intentionally by the teacher/researcher, contributed effectively to the students' mathematical understanding. Significant evidence of the development of algebraic thinking can be observed, made explicit during the course of communication, given the perception of regularities, recognition of the variables involved, as well as the existence of a relationship of dependency between them, corroborating Silva and Barbosa (2024, p. 17), when they mention that “[...] it is the communication established in the classroom that ultimately creates the learning environment.”

In addition, the students' inferences show signs of the progressive development of the discursive functions of language, considering the insertion of mathematical language in the propositions and respecting the children's level of development. The promotion of mathematical literacy is evidenced by the record of the response to the question that aimed to reflect on the extent of water waste generated by the faulty tap, as shown in Figure 2.

**Figure 2:** Assessment of the extent of waste



**Source:** Research data (2023).

[Translation of the manuscript: My opinion is that it requires urgency because it wastes a lot of water and can make the water bill expensive. And the world is getting drier by the day, and even though it's difficult to run out of water, it could happen.]

In this case, the student concludes that the tap needs to be repaired urgently, justifying their answer, which relates water waste to the monthly bill and the possibility of water scarcity in the world. This indicates that it was possible to develop civic awareness, which starts from a

local situation and expands to the global level.

## 6 Final Considerations

Taking into account the data and analyses presented, we return to the guiding question of the text and indicate two propositions, with the aim of answering it in whole or in part, considering the theoretical contribution outlined.

The first is related to the category of analysis *of dialogue and interaction* and assumes that the teacher's intentionality in organizing and conducting their discourse, even in their native language, is a condition for establishing dialogues and interactions that allow students to engage in the appropriation of knowledge as a collective act. This proposition reinforces the teacher's responsibility to conduct their teaching activities in such a way that dialogues and interactions are based on mathematical concepts, while at the same time generating affection, respect, and encouragement for active student participation, highlighting the fundamental role of the metadiscursive functions of language in the development of learning.

The second concerns the category *of discourse production* and considers that, in order to develop students' mathematical knowledge, teachers need to organize and conduct their discourse, articulating the mother tongue and mathematics, so that the use of different RRS can be discussed, understood, and systematized collectively in the search for conceptual meanings. To this end, teachers must have a command of this mathematical knowledge so that their intentionality actually leads students to conceptual learning, requiring the mobilization of discursive functions.

Considering the above, it can be inferred that students' understanding of mathematical knowledge is directly related to the teacher's intentionality who, when organizing their teaching actions with a view to addressing mathematical concepts, seeks to promote communication that privileges dialogue, interactions, collective mobilization, and different RRS, so that students' discourse production can indicate the gradual development of the discursive functions of language, highlighting collective and individual learning, expressed through the articulation between the mother tongue and mathematics.

Thus, based on these considerations, it is possible to indicate two limitations of the research, among them, working with only one area of knowledge, considering the AIEF. The other areas, as well as mathematics, are also constituted from a specialized language, making it necessary to conduct studies that relate the communication between the subjects involved in the teaching and learning processes and the different areas that constitute the school curriculum. The second limitation relates to the length of time spent in the classroom, as more time in the classroom context would be essential to deepen understanding of the implications of communication.

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