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Conceptual and procedural learning of first-degree inequalities in the context of teaching via problem solving

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Abstract: This research aimed to analyze the conceptual and procedural learning of first-degree inequalities by 26 students in a 7th-grade class of elementary school, using a didactic sequence based on the teaching via problem-solving approach. The didactic sequence was structured in four stages: use of the problem as a starting point, concept formation, content definition, and application to new problems, evaluating student performance through a post-test and a postponed post-test (after 108 days). The analysis is based on the stages of representation, planning, execution, and monitoring. The results revealed that procedural knowledge is stable, while conceptual knowledge is fragile, marked by difficulty in understanding inequalities as a set of solutions rather than unique results. It is concluded that, although problem-solving favors the development of conceptual knowledge, it is necessary to reinforce the stages of representation and monitoring to consolidate it.

Keywords: Basic Education. Mathematics Teaching. Concept Formation.

Aprendizaje conceptual y procedimental de desigualdades de primer grado en el contexto de la enseñanza de resolución de problemas

***Resumen**:* Esta investigación tuvo como objetivo analizar el aprendizaje conceptual y procedimental de desigualdades de primer grado por 26 estudiantes en una clase de 7º grado de escuela primaria, utilizando una secuencia didáctica basada en el enfoque de enseñanza de Resolución de Problemas. La secuencia didáctica se estructuró en cuatro etapas: uso del problema como punto de partida, formación de conceptos, definición de contenido y aplicación a nuevos problemas, evaluando el desempeño de los estudiantes a través de un Post-test y un Post-test Pospuesto (después de 108 días). El análisis se basa en las etapas de representación, planificación, ejecución y seguimiento. Los resultados revelaron que el conocimiento procedimental es estable, mientras que el conocimiento conceptual es frágil, marcado por la dificultad de comprender las desigualdades como un conjunto de soluciones en lugar de resultados únicos. Se concluye que, si bien la resolución de problemas favorece el desarrollo del conocimiento conceptual, es necesario reforzar las etapas de representación y seguimiento para consolidarlo.

Palabras clave: Educación Básica. Enseñanza de las Matemáticas. Formación de Conceptos.

Aprendizagem conceitual e procedimental de inequações do 1º grau no contexto do ensino via resolução de problemas

Resumo: Esta pesquisa teve como objetivo analisar a aprendizagem conceitual e procedimental

de inequações de 1º grau de 26 alunos de uma turma do 7º ano do Ensino Fundamental, utilizando uma sequência didática baseada na abordagem de ensino via Resolução de Problemas. A sequência didática foi estruturada em quatro etapas: uso do problema como ponto de partida, formação do conceito, definição do conteúdo e aplicação em novos problemas, avaliando o desempenho dos alunos por meio de um Pós-teste e um Pós-teste Postergado (após 108 dias). A análise baseou-se nas etapas de representação, planejamento, execução e monitoramento. Os resultados revelaram que o conhecimento procedimental é estável, enquanto o conceitual é frágil, marcado pela dificuldade em compreender inequações como um conjunto de soluções em vez de resultados únicos. Conclui-se que, embora a resolução de problemas favoreça o desenvolvimento do conhecimento conceitual, é preciso reforçar as etapas de representação e o monitoramento para consolidá-lo.

Palavras-chave: Educação Básica. Ensino de Matemática. Formação do Conceito.

1 Introduction

The teaching of 1st degree inequalities is fundamental in the transition from arithmetic to algebraic thinking (Brasil, 1998), going beyond algorithmic manipulation when modeling real situations. Unlike equations, inequalities introduce concepts of sets and intervals, requiring abstraction to understand the variability of quantities.

This perspective develops relational thinking and decision-making in contexts such as budgets and physical limits, among others. In line with the curricular guidelines (Brasil, 1998, 2018), the domain of inequalities allows the investigation of phenomena with multiple results, preparing the student for the study of functions. Thus, the contextualization of the theme facilitates learning and fosters the scientific literacy necessary for critical citizen action (Brasil, 2018).

However, despite this pedagogical and social relevance, the practical implementation of this teaching reveals a complex scenario. Learning the concept of inequality is marked by obstacles that persist from Elementary School to Higher Education, as pointed out by several scientific research. We can categorize them into three main fronts: conceptual, representation (semantic) and procedural.

1 – Conceptual difficulties: Students often apply properties of equality in contexts of inequality because they do not distinguish objects (Tsamir & Almog, 2001; Palupi, Sumarto & Purbaningrum, 2022). There is a persistent confusion between unknown and variable, leading to the belief that inequality has a single solution rather than a range of values (Blanco & Garrote, 2007).

2 - Difficulties in representation and semantic knowledge: The transition from natural to algebraic language is a critical point (Conceição Junior, 2011; Frizzarini, 2014). Students fail to translate terms such as "at most" or "at least" into the correct symbols ($\leq$,$\geq$), making representation the biggest barrier to problem-solving (Proença, Maia-Afonso, Mendes & Travassos, 2022). Above all, Queiroz (2021) as well as Travassos (2023) report cases in which students used the symbol of less ($<$) instead of less than or equal to ($\leq$) because they did not understand the scope of the term in the statement.

3 - Procedural difficulties and algebraic manipulation: even when the concept is understood, the execution of the algorithm involved (algebraic treatment) often fails. A persistent flaw is not to reverse the direction of inequality by multiplying or dividing the expression by a negative number. This error is highlighted by El-Khateeb (2016) and Booth, Barbiere, Eyer and Paré-Blagoev (2014). Research also shows students' difficulties with elementary operations, fractions and the manipulation of negative variables as factors that

prevent the arrival of the correct solution (Çiltaş & Tatar, 2011; Travassos, 2023), which occurs even in undergraduate students, in the Mathematics Degree course (Melo, 2007; Souza, 2008; Magalhães, 2013; Bicer, R. Capraro & M. Capraro, 2014; Travassos & Rezende, 2017; Travassos, 2018), suggesting that these misconceptions can be transmitted from teachers to students, perpetuating the cycle of difficulties.

In this panorama of students' difficulties, the role of teaching is important, and the National Common Curricular Base – BNCC (Brasil, 2018) proposes the resolution of problems for the development of skills. Thus, this article (derived from a doctoral thesis) uses teaching via problem solving aiming not only at the technical ability to solve problems, but, primarily, at the construction of mathematical concepts.

In the thesis developed, we identified that most Brazilian research focuses on the identification of errors or traditional approaches, and there are few proposals that use problem solving as a way to form the concept and not only as the application of formulas or validation of learning (Travassos, 2023). Additionally, there is a scarcity of studies that analyze conceptual learning through postponed analysis after a structured pedagogical intervention.

In this context, the teaching of the specific content of 1st degree polynomial inequalities faces challenges that demand pedagogical strategies capable of ensuring conceptual apprehension. To overcome such barriers, teaching via Problem Solving is advocated, which uses the contextualized situation as a starting point for the construction of knowledge. The pedagogical teaching strategy we adopted is the proposal of Proença (2021), organized in four fundamental stages: the use of the problem as a starting point; formation of the concept; definition of content; and, finally, application to new problems. It is a proposal for the *organization of teaching via problem solving*, which emphasizes the work for the students' learning about the conceptual and procedural knowledge of inequalities.

Thus, we aim to analyze the conceptual and procedural learning of polynomial inequalities of the 1st degree of students of a class of the 7th grade of Elementary School, in the context of a didactic sequence based on a proposal of organization of teaching through problem solving. To achieve this objective, we structured the article by presenting the theoretical foundation, then the methodology and the results and discussions and, finally, we weaved the conclusion.

2 Problem Solving: foundations on its theoretical field

The concept of mathematical problem varies across theoretical perspectives, but converges with the idea of a new situation that requires intense mental activity in the absence of an immediate path to the solution (Resnick & Glaser, 1975; Krulik & Rudnik, 1980; Silver, 1985; Echeverría, 1998). Unlike a "recall task" (Sternberg, 2008), the problem is subjective and depends on the student's prior knowledge (Dante, 2009). Thus, the definition by Proença (2018) is adopted. According to the author,

> [...] a Mathematics situation becomes a problem when the person needs to mobilize mathematical concepts, principles and procedures previously learned to arrive at an answer. It is not, therefore, the direct use of a known formula or rule – when this occurs, the situation tends to be configured as an exercise (Proença, 2018, p.17-18).

This conceptualization/definition of problem implies precisely the way that thought goes through when solving a problem. The process of solving mathematical problems requires the mobilization of various cognitive resources and the overcoming of obstacles that do not have an immediate solution path. Authors such as Pólya (1957), Sternberg (2012), Mayer

(1992) and Proença (2018) point out stages or phases that the person goes through when solving a problem, which moves from understanding, presenting a strategy, executing it and monitoring the response and the solution followed, so that this involves mathematical knowledge (conceptual and procedural) and even linguistic knowledge. In addition, factors such as control (executive decisions), beliefs about mathematics, affect (emotions and attitudes) and the social context in which the individual is inserted also influence the student's performance (Lester, 1987).

To support classroom practice, Proença (2018) synthesizes the act of solving problems in four fundamental steps: representation, planning, execution, and monitoring. In the representation stage, the student builds a mental understanding of the problem by mobilizing linguistic knowledge (referring to the terms/words of the mother language that influence comprehension), semantic knowledge (referring to mathematical terms) and schematic knowledge (referring to the type of basic content of the problem, its essence) to interpret the formal structure of the utterance. In planning, the solver uses their strategic knowledge (types of strategies: trial and error, diagram, equation, drawing, etc.) to trace a path or strategy that connects the organized data to the desired solution. The execution stage is the moment when the calculations and mathematical procedures are effectively carried out, revealing the student's procedural knowledge. Finally, monitoring consists of verifying that the answer obtained is consistent with the real context and the conditions imposed by the mathematics situation, as well as verifying that the process followed is coherent.

In this sense, dealing with problem solving is justified, among other points, by changing the dynamics of the classroom, considering: (i) *the student's protagonism*, who leaves passivity to actively build his knowledge; (ii) *development of cognitive skills*, stimulating critical thinking through interaction and practice; and (iii) *dialogue and reflection*, allowing the teacher to act as a mediator who encourages reflection on one's own reasoning (Lester & Cai, 2016).

2.1 Problem-Solving Teaching Approaches

Problem Solving in the teaching of Mathematics has been consolidated as a fundamental field since the 1980s, when the National Council of Teachers of Mathematics [NCTM] (1989) established it as the central focus of school curricula. To systematize the pedagogical practice in this area, Schroeder and Lester (1989) formalized three distinct approaches: teaching *About*, *to* e *via* troubleshooting.

The approach to teaching *about* problem solving focuses on the process and models of solving itself, and is strongly influenced by the work of George Pólya (Pólya, 1957), who divides the act of solving into four phases: understanding the problem, making a plan, executing it, and looking back (or looking back). In this model, the teacher explicitly teaches heuristic strategies, such as pattern finding, data simplification, or backtracking, so that the student learns the mechanisms needed to tackle any mathematical challenge.

Unlike this perspective, teaching *for* problem solving focuses on the application of previously taught content. In this approach, the teacher presents the theory and algorithms first, so that the student is able to transfer this knowledge and apply it in the solution of routine or non-routine problems. As Schroeder and Lester (1989) point out, this view is considered limited by many educators, as the problem ceases to be a point of discovery to become only a tool for validating knowledge already acquired, treating Mathematics as an accumulation of techniques that are applied after formalization.

Finally, the approach of teaching *via* problem solving uses the problem as the starting point for learning new concepts. To structure this practice, Proença (2021) proposes an *organization of teaching via problem-solving* with a focus on conceptual training, organized in four fundamental stages:

- *1st Stage – use of the problem as a starting point*: in this initial stage, a mathematics situation that is configured as a problem is used to introduce the new content or concept. The objective is to allow students to mobilize their previous knowledge and propose their own strategies before any formalization of the content. As a basis, we follow the five actions of the Mathematics Teaching-Learning via Problem Solving approach (MTLvPS) of Proença (2018): choosing the problem, introduction the problem, helping to students during the resolution, discussing students' strategies, and the articulating students' strategies with the content. Thus, the student builds the mathematical idea of the new content through their own strategies to articulate it with its mathematical form, in a process of abstraction;
- *2nd Stage – concept formation*: after the initial contact, students carry out activities to understand the properties and characteristics of the concept, allowing its distinction from other mathematical objects. In this stage, students explore examples and non-examples, present their own definitions (mental constructs) and analyze variations of the concept to broaden understanding;
- *3rd Stage – definition of the content*: the focus of this stage is the formal mathematical definition and the insertion of the student in the symbolic-formal language. Teaching addresses both the public entity (formal definition) and the algorithmic resolution procedures, discussing the structures studied previously;
- *4th Stage – application in new problems*: the final stage aims at the transfer of learning and the application of knowledge built in new contextualized situations. These new problems must involve different spheres (social, political, economic) or other areas of knowledge, and may present incomplete or superfluous information to challenge students' understanding and interpretation. This organization allows teaching to promote a conceptual understanding instead of a merely procedural and mechanical instruction.

Proença (2018, 2021) reinforces that this teaching organization transforms the student's role into an active participant in the creation of knowledge, requiring him to mobilize concepts and principles previously learned to develop his own strategies, above all, the construction by the student of conceptual knowledge, procedural knowledge and the transfer of this knowledge to problem solving.

3 Methodology

The present study adopts a qualitative nature, based on the assumptions of pedagogical research (Lankshear & Knobel, 2008), in which the researcher acts as a teacher-researcher within the classroom environment to understand the educational process. The participants of the research were 26 students from a 7th grade class of Elementary School of a state public school in northern Paraná. To carry out the activities, the students were organized into eight groups, ranging from one to five members each, aiming to favor the exchange of knowledge and discussions. The context of the study was marked by the post-COVID-19 pandemic period, a time when the educational system transitioned from Emergency Remote Learning to face-to-face instruction (Hodges, Moore, Lockee & Bond, 2020), which required an adaptation of pedagogical practices and a careful look at the subjectivities and learning gaps of the group.

In the pedagogical intervention, a didactic sequence was implemented for the teaching of 1st degree polynomial inequalities, whose structure was based on Proença's (2021) proposal of teaching organization via problem solving with a focus on the formation of mathematical concepts, and articulated with the actions of Mathematics Teaching-Learning via Problem Solving (MTLvPS) (Proença, 2018).

The teaching path was organized in four interdependent stages, as detailed below: the

1st Stage used the problem as a starting point to introduce the content of inequalities without prior formalization; the 2nd Stage focused on the formation of the concept through the exploration of examples and non-examples; the 3rd Stage was aimed at defining the content, addressing the symbolic-formal language and the algorithms for solving inequalities; and the 4th Stage consisted of the application of the knowledge built in the resolution of new contextualized problems, thus promoting the validation of learning. Chart 1 below summarizes the pedagogical actions carried out and the opportunities for thought mobilized by the students in each of these stages.

Table 1: Organization of teaching and pedagogical actions

Stages	Pedagogical Description (worked, observed and assisted)	Opportunities for thinking about inequality
1. Use problem as a starting point	Use of two contextualized mathematics situations to introduce the content without prior formalization. The researcher acted as a mediator, encouraging the groups to use previous knowledge (arithmetic and equations).	Mobilization of own strategies; initial perception that in some problems there is not only one correct answer (idea of interval/solution set).
2. Formation of the concept	Use of examples and non-examples to differentiate inequality from equations. Assistance in the interpretation of specific terms, such as 'do not overtake' using visual examples (a run on an Olympic track).	Distinction of the properties of the concept; semantic understanding of specific lexicons indicating inequality ($<, >, \leq, \geq$).
3. Definition of content	Formalization of the definition of inequality as a mathematical sentence of inequality; differentiation of unknown and variable; work focused on the representation of the statements of the problems for the algebraic language and correct use of operative rules and additive/multiplicative principles in the resolution process.	Insertion in symbolic-formal language; understanding that multiplying/dividing by a negative number inverts the sign of inequality.
4. Application to new problems	Transfer of learning for five unpublished problems with social and economic contexts. The researcher helped in the management of information and in overcoming interpretative blocks.	Mobilization of the resolution stages (representation, planning, execution and monitoring); re-signification of the concept in varied contexts and improvement of linguistic, semantic, schematic, strategic and procedural knowledge in solving problems of inequalities.

Source: Prepared by the authors

The organization of teaching in these four stages allowed learning to move from a mere mechanical application of algorithms to focus on the progressive construction of students' conceptual and procedural knowledge with regard to 1st degree inequalities. By using the problem as a starting point and exploring in an organized way the confrontation between examples and non-examples, students were encouraged to act as active participants in the creation of knowledge, developing autonomy and critical reflection on the properties of inequalities. This pedagogical basis was fundamental for the students to be able to reframe the mathematical concept in different contexts, preparing them for the stages of solving new problems, which will be analyzed in the results section of this study. In the pedagogical intervention, data collection was structured in two main moments: the didactic sequence and the evaluation process (Post-test and Post-test Postponed).

The 1st Moment, dedicated to the implementation of the didactic sequence, comprised 12 class hours, distributed over seven days. As for the written records, each group was asked to deliver only one official answer sheet (the result of consensus or collective debate), while the others were used for drafts. To capture the interactions, nine audio recorders were used: eight fixed with the groups and one with the researcher, totaling about 90 hours of recording, thus allowing the full recording of the dialogical exchanges between students and the teaching mediations. As a measure of quality control and reliability, the record sheets were collected at the end of each class, avoiding external changes or consultations when the students went home.

The 2nd Moment covered the evaluation processes, composed of two instruments: (i) Post-test: applied right after the didactic sequence, it aimed to verify the students' learning about their mobilized knowledge. As Creswell (2009) and Fraenkel and Wallen (2011) argue, the post-test is essential to measure whether the treatment (in this case, the teaching approach) resulted in changes in the participants' performance or understanding; (ii) Post-test Postponed: carried out 108 days after the first test, it aimed to verify the stability of knowledge. According to Moreira (2011), postponed analysis is essential to distinguish between mechanical and meaningful learning, and long-term retention is an indicator that there was the integration of new concepts into the student's cognitive structure, that is, if there was conceptual understanding (as well as procedural understanding) of inequalities.

Chart 2 below organizes the moments of the research with the instruments used and the data collection procedures.

Chart 2: Research Moments, instruments and collection procedures

Moment (Stage)	Collection instruments	Collection Procedure
1st Moment: Didactic Sequence (Steps 1 to 4)	Written records and nine audio recorders.	Daily collection of group registration sheets to avoid external changes. Continuous recording of classes to capture reasoning and mediations.
2nd Moment: Post-test	Individual written activities (five mathematics situations and four exercises).	Individual application, without consultation or assistance, focused on the validation of learning after the intervention.
2nd Moment: Post-test Postponed (108 days after Post-test)	Same activities as Post-Test.	Individual application after an interval of 108 days. Use of separate sheets (problems first, and exercises later) to avoid algebraic induction.

Source: Prepared by the authors

In this article, our focus is on the 2nd moment, seeking to reveal conceptual and procedural learning, but based only on mathematics situations (possible problems). In the use of post-tests, the strategy of delivering the activities on separate sheets was adopted. Initially, the students solved the mathematics situations (word problems) and, only after completing these, they received the algebraic exercises. This separation aimed to prevent the previous visualization of symbols in the exercises from inducing students to use algebra concepts to solve problems. In this way, it was possible to observe whether the mobilization of the concept of inequality would occur spontaneously and based on the logical structure of the situation presented.

With this, we selected three of the five mathematics situations that made up the evaluative instrument. The decision to disregard the others was based on technical validation criteria: one of the situations presented ambiguities in the interpretation by the students, which could compromise the reliability of the results, while the other had a structure involving the concept of equation, which would be outside the objective of this research, which is the conceptual knowledge of inequalities/inequalities. Thus, the data are concentrated on the resolutions of the three situations presented below, called SM1 (Mathematics Situation 1), SM2 (Mathematics Situation 2) and SM3 (Mathematics Situation 3):

> *SM1) With the Black Friday discounts, Moacir, who owns a bookstore, had already sold R$ 850.00 in books, until noon. After noon, in order to "burn the stock", Moacir decided to sell any book for R$ 10.00. At the end of the day, his cash was worth more than R$ 2300.00. How many books did Moacir sell after noon?* (Travassos, 2023, p. 132).

> *SM2) Maria Helena intends to go to a certain shopping center in Maringá to buy some Christmas decorations for her home. Your budget is composed as follows: R$ 50.00 with lights, plus R$ 30.00 with decorative objects and a certain amount of money with the Christmas tree, so that the total does not exceed the amount of R$ 220.00. Determine the price Mary Helena can pay for the Christmas tree.* (Travassos, 2023, p. 132).

> *SM3) Due to the COVID-19 pandemic, one of the sectors that was harmed during this period was the retail sector, such as clothing, footwear, beauty products, decoration items, etc. Nadiny, who sells handmade Chocotones, decided to create a profile on the social network Instagram to sell her products there, with deliveries, since the Lockdown prohibited Nadiny from opening her physical business. Knowing that Nadiny earns R$ 22.00 for each Chocotone sold, how many Chocotones does Nadiny need to sell in order to get at least R$ 770.00 which are the costs of delivery and promotion on Instagram?* (Travassos, 2023, p. 132).

The resolution of these mathematics situations is not limited to the mechanical application of algorithms, it requires the student to move through different levels of cognitive processing. Based on the assumptions of Mayer (1992) and Proença (2018), the effectiveness in solving an inequality depends on the coordinated mobilization of knowledge ranging from the understanding/interpretation of natural language to the final validation of the result.

Thus, each of the proposed situations (SM1, SM2 and SM3) requires the student to activate specific knowledge to understand/interpret the utterance (linguistic and semantic knowledge), structure an action plan (schematic and strategic knowledge) and perform the necessary operations (procedural knowledge). In addition, the nature of inequalities imposes additional rigor in the monitoring stage, since the result must be interpreted not as a fixed value, but as an interval of possibilities. Chart 3 below details the integration of this knowledge in each of the problems analyzed.

Chart 3: Mobilization of knowledge in Mathematics Situations (MH)

Knowledge / Process	SM1 (Moacir - Books)	SM2 (Maria Helena - Tree)	SM3 (Nadiny - Chocotones)
Linguistic	interpret terms such as "had already sold" (fixed amount), "for R$ 10.00" (variable rate) and "value greater than".	Identify fixed values (lights and objects); the value to be discovered for the tree; and to interpret the expression "do not overtake".	Understand the unit gain (R$ 22.00) and the target expression "at least".

Semantic	Relate the unit price to the number of books and understand that the total is the sum of the fixed and the variable, and that the total cannot be less than or equal.	Understand the fixed values and the variable as a mathematical sentence in the budget ceiling relationship: the sum of all expenses must be less than or equal to the limit.	Establish the proportionality relationship between sales and costs to achieve financial viability, working with the concept of greater/equal (≥)
Schematic	Recognize the Open Lower Bound Scheme ($ax + b > c$), where the final value must exceed the target.	Recognize the closed upper limit scheme ($a + b + x \leq c$), focused on spending restrictions.	Recognize the minimum feasibility scheme ($ax \geq c$).
Strategic	Plan the subtraction of the fixed value (2300 - 850) and the subsequent division by the unit value (1450/10).	Plan the Sum of Known Values (50 + 30) and subtract from the total available (220 - 80).	Plan the direct division of the total cost by the unit profit (770 \ 22).
Procedural	Perform arithmetic operations or manipulate inequality: $850 + 10x > 2300$	Perform the addition and subtraction: $x \leq 220 - 80$, resulting in the maximum possible value.	Perform the division of 770 by 22 to find the critical point of the goal.
Monitoring	Validate that the answer is an integer greater than 145 (e.g., 146 books), because 145 would give exactly 2300.	Check if the price found allows you to pay the other bills without exceeding the 220 reais available.	Confirm that the quantity of Chocotones meets the goal of "at least", ensuring the payment of costs.

Source: Prepared by the authors

To analyze the collected data, we followed the assumptions of Content Analysis (CA), by Bardin (2016), which is organized in three phases: pre-analysis; exploration of the material; and treatment of the results, inference and interpretation. In the pre-analysis, it is about the choice of materials, in which we select the three problems and define the quantity as an indicator for the categories. In the material exploration phase, the registration units extracted from the students' resolutions were organized into a system of a *priori* categories, which correspond to the problem-solving stages of Proença (2018). These categories function as the structure of analysis to identify the mobilization of conceptual knowledge and procedural knowledge. The detailed investigation of each category occurs as follows:

i. Representation: In this category, the student's ability to translate from natural language to algebraic or arithmetic language is evaluated. The focus falls on conceptual knowledge, observing whether the student identifies the numerical elements and, above all, the condition of inequality of the problem, structuring a representation that respects the semantics of the utterance.

ii. Planning: The choice of strategy and its logical coherence are analyzed. Here, it is verified whether the student mobilizes schematic knowledge to trace a resolutive path that leads to the solution, opting for algebraic or arithmetic methods that make sense for the nature of the inequality.

iii. Execution: Focuses on mathematical treatment and procedural knowledge. The mastery of the technique is evaluated, such as the application of additive and multiplicative principles, the basic operations and the manipulation of mathematical symbols.

iv. Monitoring: This category verifies the validation of the response. It is the point where conceptual knowledge is most required, as the student must analyze the plausibility of the result in relation to the context, distinguishing, for example, the critical value of the real solution in an interval of inequality.

Thus, in the phase of treatment of the results, inference and interpretation, the results presented in the subsequent section use these criteria to quantify the frequencies of successes, errors and omissions. The analytical tables expose the categories and quantities obtained based on the units of record observed in the resolutions, allowing the inference of the dimension of knowledge (conceptual and procedural) in which the students' difficulties were manifested with greater incidence.

4 Analysis and discussion of the results

The comparative analysis between the performances observed in the Post-Test (PT) and in the Post-Test Delayed (PTP) in the three mathematics situations (SM1, SM2 and SM3) is shown in Chart 4 below, which summarizes the students' successes and errors in each stage of problem solving.

Chart 4: Summary of successes and errors of the Post-test and Post-test

Mathematics Situation 1 (n=10)				
Stages	Post-test		Post-test Postponed	
	Hit	Error	Hit	Error
Representation	7	3	1	9
Planning	10	0	5	5
Execution	8	2	8	2
Monitoring	0	10	0	10
Mathematics Situation 2 (n=8)				
Stages	Post-test		Post-test Postponed	
	Hit	Error	Hit	Error
Representation	5	3	1	7
Planning	8	0	8	0
Execution	7	1	8	0
Monitoring	3	5	1	7
Mathematics Situation 3 (n=7)				
Stages	Post-test		Post-test Postponed	
	Hit	Error	Hit	Error
Representation	3	4	1	6
Planning	6	1	6	1
Execution	5	2	4	3
Monitoring	0	7	2	5

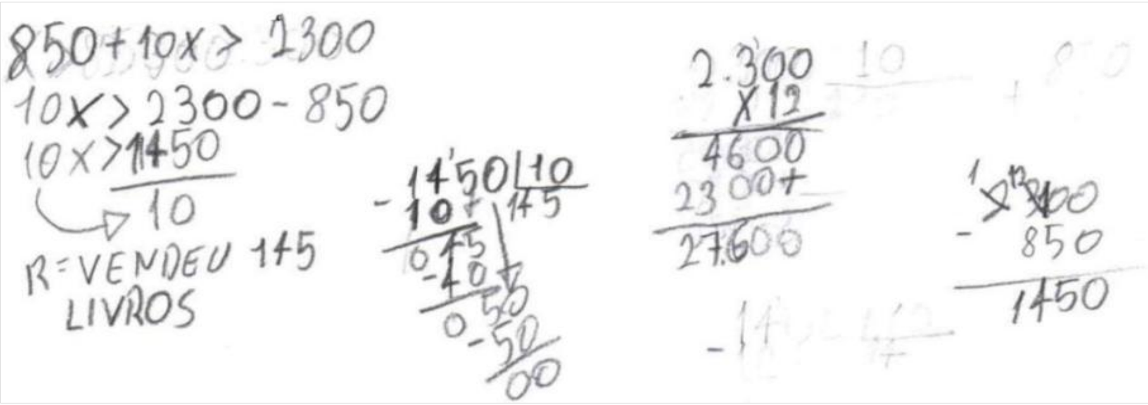
Source: Prepared by the authors

Representation Stage – one of the most significant quantitative data of this investigation is the drop in the use of formal algebraic representation: In SM1, the index fell from 7 correct answers to only 1; in SM2, from 5 to 1, and in SM3, from 3 to 1. This indicates that without immediate intervention, students lost the ability to translate natural language into the algebraic register.

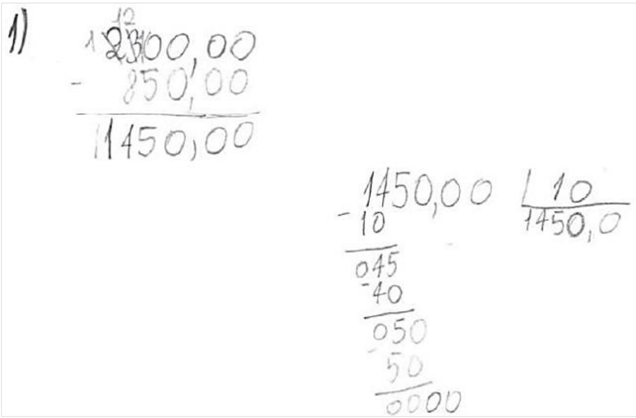
Figure 1 exemplifies the resolution of one of the students when solving the same SM1 in the Post-test and in the Post-Test Posteponed.

Figure 1: Resolution of Mathematics Situation 1 – Post-test and Post-test Postponed

Mathematics Situation 1 applied in the Post-test



Math Situation 1 applied in the Post-Test Postponed



Source: Travassos Collection (2023)

In the Post-Test (PT), the student correctly mobilizes the formal algebraic representation (850 + 10x > 2300). This indicates that, immediately after the didactic sequence, linguistic and semantic knowledge acted efficiently in the process of identifying and interpreting the data, organizing them in a coherent structure (schematic knowledge). In the Post-Delayed Test (PTP), it is observed that the same student, when solving the same problem after the temporal distance of the didactic sequence, no longer uses the formal resources of algebra.

Although the algebraic representation has been replaced by purely arithmetic strategies, it is essential to highlight that the operational logic remains preserved. The student performs the sequence of operations (2300 – 850 and, later, the division by 10) in order to mirror the additive and multiplicative principles that govern the resolution of an inequality. From the perspective of Proença (2018), this phenomenon suggests that strategic knowledge proved to be more conducive to mobilization than specific semantic knowledge of algebra.

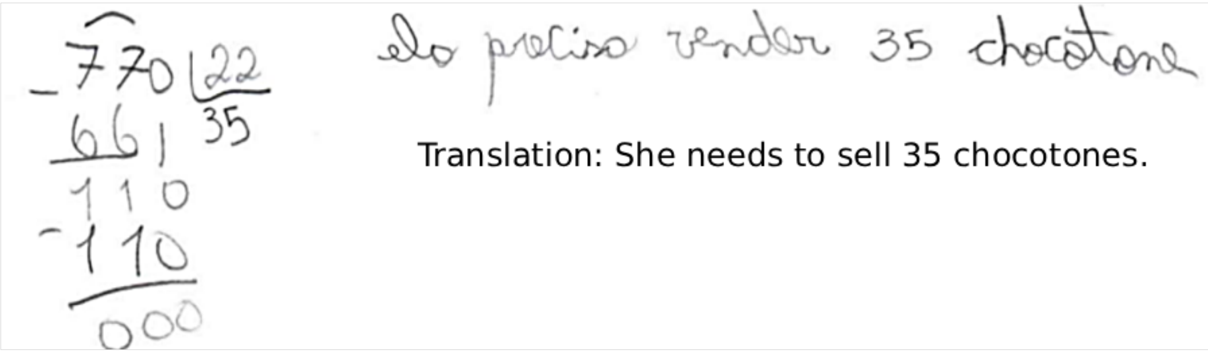
In our understanding, this migration to arithmetic reveals that, despite the erosion of symbolic rigor, the understanding of the relationship of dependence between quantities has been consolidated; The student readjusts his strategy to a numerical field where he feels safer without the immediate mediation of the teacher. However, this strategic simplification brings with it a risk identified in the Monitoring stage, which will be analyzed later.

In general, it is noted that in the absence of mathematical understanding of specific terms of the concept of inequality (semantic knowledge), students choose to work with arithmetic solution strategies. This occurred in 90% of the resolutions of SM1, 87.5% of the resolutions of SM2 and 85.7% of the resolutions of SM3, considering the transition from PT to PTP, which leads us to conjecture that the time factor influenced the reduced mobilization of semantic knowledge to solve problems of inequalities.

These results are corroborated by the research of Traldi Junior (2002), Blanco and Garrote (2007), Frizzarini (2014) and Queiroz (2021), who also report the resistance of students to use the algebraic representation of an inequality to solve problems, often opting for arithmetic methods or the misuse of the concept of equation. In our case, this resistance was more evident in the PTP.

Planning stage – within the framework of Proença (2018, 2021), it is the moment when the student establishes a strategy to solve the problem, mobilizing previous knowledge to trace a path to obtain the answer. By analyzing the data from situations SM1, SM2 and SM3 in Chart 4, we can observe a significant transition in the strategic behavior of the students over the 108 days. In the PT, the students' planning was mostly algebraic (given the type of representation), reflecting the direct application of the procedures taught. In the PTP, there is a migration to arithmetic resolution plans. Although it may seem like a step backwards, this change demonstrates the strength of linguistic and schematic knowledge, as exemplified in Figure 2 below.

Figure 2: Resolution of Math Situation 3 – Post-test



Source: Travassos Collection (2023)

As observed in Figure 2, even though the student did not represent SM3 algebraically, his planning remained coherent, because he recognized the structure of the situation, that is, he mobilized linguistic (interpretation) and schematic (structuring of the mathematics situation) knowledge.

In SM1, the plan consisted of subtracting the fixed value and dividing it by the unit; in SM2, in adding the expenses and subtracting from the total, and in SM3, planning the direct division of the total cost by the profit. This shows that possibly the logical scheme of the SM was internalized (linguistic and schematic knowledge), since the success rate of the PT for the PTP decreased by 50% in SM1, 0% in SM2 and 0% in SM3 in the planning stage, that is, the majority of those who correctly carried out some type of resolution scheme in the PT, performed correctly also in the PTP.

Execution stage – the execution stage in the problem-solving process is the moment when the student performs the mathematical treatment itself, applying rules and algorithms to arrive at a numerical result. This phase is primarily supported by procedural knowledge, which involves the mastery of mathematical calculations, encompassing the correct use of formulas and the manipulation of algebraic symbols.

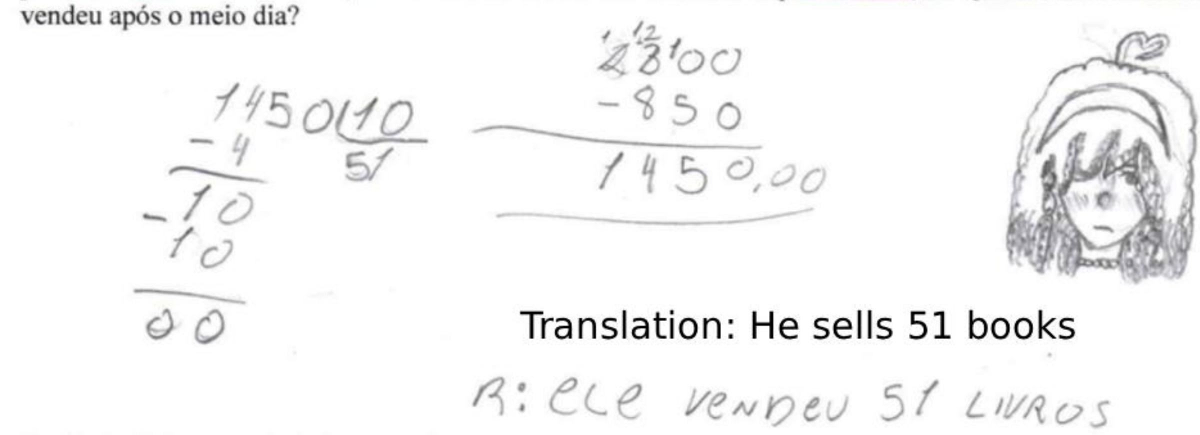
According to the data in Chart 4, execution proved to be the stage with the greatest cognitive stability among students, with high rates of correct answers both in the Post-test and in the Postponed Post-test. In SM1, for example, 80% of the students correctly performed the calculations in both moments, in SM2, 87.5% of correct answers in the PT and 100% in the PTP, and in SM3, 71.4% of correct answers in the PT and 57.1% in the PTP, which suggests that, considering such sampling, procedural knowledge is more resistant to the time factor than conceptual knowledge.

However, when failures occur in this step, they are usually linked to mistakes in basic mathematical operations (addition/subtraction/division/multiplication) or to the lack of rigor with the properties of inequalities (in the case of working with algebraic representation). The following is an example of how an execution error can manifest itself.

Figure 3: Error in the execution step of Math Situation 1 – Post-test

Translation of problem statement: 1) Thanks to Black Friday discounts, Moacir, a bookstore owner, had already sold R$ 850.00 worth of books by noon. After that time, in order to clear his stock, Moacir decided to sell any book for R$ 10.00. By the end of the day, his total sales exceeded R$ 2,300.00. How many books did Moacir sell after noon?

1) Com os descontos da Black Friday, Moacir, que é dono de uma livraria, já havia vendido R$ 850,00 em livros até o meio dia. Após o meio dia, a fim de "queimar o estoque", Moacir decidiu vender qualquer livro por R$ 10,00. No final do dia, seu caixa estava com um valor maior que R$ 2300,00. Quantos livros Moacir vendeu após o meio dia?



Source: Travassos Collection (2023)

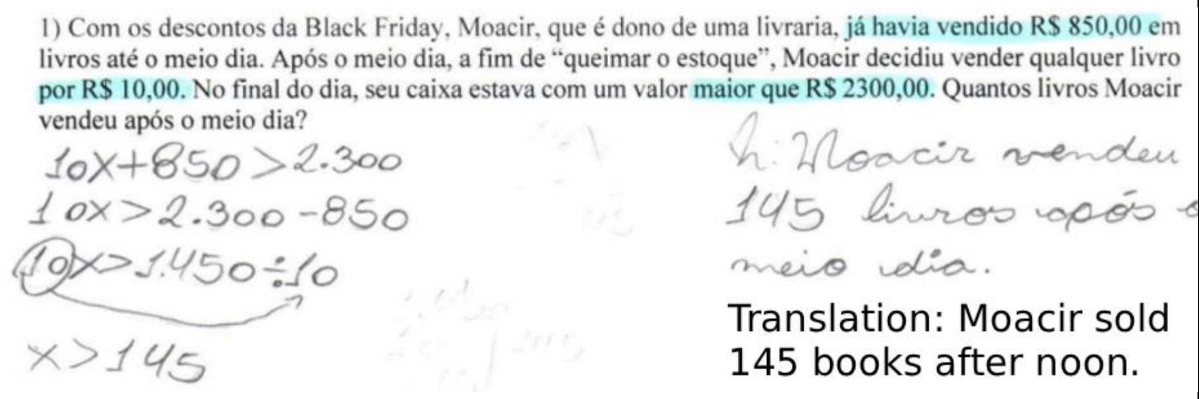
Although the student has demonstrated schematic knowledge by understanding the structure of the problem (subtracting the fixed value to work with the variation), he makes a mistake in the final treatment of the data. The student correctly performs the subtraction 2300 - 850 = 1450. However, when performing the inverse operation of multiplication (dividing the balance by the unit value of each book), it presents the result 51. This error indicates a flaw in the manipulation of the division, which comes from procedural knowledge. The student knows what he should do (divide), but fails in how to do it.

Difficulties similar to these are highlighted by El-Khateeb (2016) and Booth *et al.* (2014), who identify as a persistent error the fact that the student does not invert the meaning of inequality by multiplying or dividing the expression by a negative number. Studies by Çiltaş and Tatar (2011) also reinforce that gaps in basic mathematical operations prevent the arrival of the correct solution.

Monitoring Stage – in the monitoring stage, the absence of conceptual knowledge prevents the student from identifying the inconsistency between the limit value obtained and the condition of inequality imposed by the statement. The student ends the solving/solving process when reaching the final numerical value, demonstrating that the logic of intervals and solution sets has not been assimilated as an integral part of the concept of inequality. Monitoring requires a metacognitive activity and conduct that remains neglected by most students, both in PT and PTP. Figure 5 below exemplifies the fact.

Figure 5: Error in the monitoring stage of Mathematics Situation 1 – Post-test

Translation of problem statement: 1) Thanks to Black Friday discounts, Moacir, a bookstore owner, had already sold R$ 850.00 worth of books by noon. After that time, in order to clear his stock, Moacir decided to sell any book for R$ 10.00. By the end of the day, his total sales exceeded R$ 2,300.00. How many books did Moacir sell after noon?


1) Com os descontos da Black Friday, Moacir, que é dono de uma livraria, já havia vendido R$ 850,00 em livros até o meio dia. Após o meio dia, a fim de "queimar o estoque", Moacir decidiu vender qualquer livro por R$ 10,00. No final do dia, seu caixa estava com um valor maior que R$ 2300,00. Quantos livros Moacir vendeu após o meio dia?

$10x+850>2.300$

$10x>2.300-850$

$10x>1.450\div 10$

$x>145$

R: Moacir vendeu 145 livros após o meio dia.

Translation: Moacir sold 145 books after noon.

Source: Travassos Collection (2023)

In the case observed in Figure 5, the student was successful in the execution when reaching the algebraic solution x > 145. However, when writing the final answer, he fails to interpret what this interval means in practice, characterizing a monitoring error (and even understanding the concept and semantic interpretation).

The student treats the number 145 as a punctual answer (equation style), ignoring the symbol of inequality (>). By answering '145', he disregards that, in order to achieve Moacir's goal, this value is the limit where the condition has not yet been overcome. According to Proença (2018), monitoring requires the student to 'go back to the problem' to verify that the solution meets the question. By not realizing that the answer should be an integer greater than 145 (such as 146), the student demonstrates that his logical basis is still linked to equational thinking and there is a malformation of the concept of inequality.

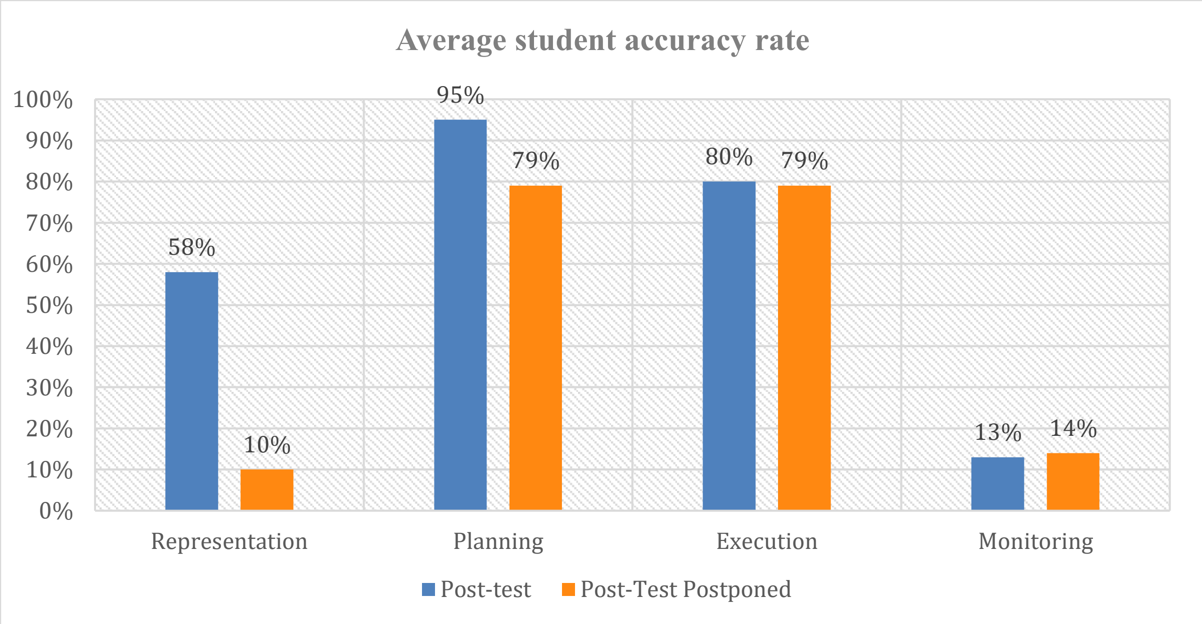
For many students, the greater-than sign (>) is read only as a separator, similar to the equal sign (=). And this moment, above all, is an indication of the lack of conceptual knowledge about inequalities, since the greater-than sign (>), when represented, by itself already evidences the condition of the answer, unlike when using an arithmetic strategy, whose answer is only a number.

This persistence in error, even after the didactic sequence, in which there were only three correct answers among the 25 resolutions analyzed in the PT (10 from SM1, 8 from SM2 and 7 from SM3) indicates that conceptual knowledge was not sufficiently constructed to overcome the epistemological obstacle of the exact result. The analysis of the records allows us to infer that the validation of the response occurred in a merely procedural way, limited to the verification of the algorithm, without reaching the level of reflective analysis necessary to understand the meaning of a solution set.

This misunderstanding of the nature of the solution is an epistemological obstacle verified in the research of Tsamir and Almog (2001) and Palupi *et al*. (2022), which reveal that students often believe that an inequality has a unique answer, confusing the concept of variable with that of unknown. As Blanco and Garrote (2007) point out, for many students the signs of inequality have no semantic value of their own, being used only as a mechanical nexus between the members of the expression.

In order to summarize the results of Chart 4, Graph 1 below presents an average percentage of correct answers in the stages of representation, planning, execution and monitoring of the three mathematics situations in the Post-test and Post-test Postponed.

Graph 1: Average percentage of correct answers of students in the Post-test and Post-test Postponed



Source: Prepared by the authors

The results presented in Graph 1, together with the aforementioned analyses, reveal in a summarized way the formation of conceptual and procedural knowledge of inequalities.

The disparity between the stages of resolution shows that the students' proficiency is fragmented, with a decline in the ability to maintain mathematical formalization over time, in addition to the fact that the formation of procedural knowledge stands out over the formation of conceptual knowledge.

The Representation stage presents the most drastic drop between the two moments of the survey, regressing from an average of 58% in the Post-test to 10% in the Post-Test In the understanding of this investigation, this index (together with the monitoring index) indicates that there was an almost zero identification of inequality as a concept after temporal distancing. This flaw suggests that specific terms and the symbolic language (linguistic and semantic knowledge) and the relational scheme (schematic knowledge) were not properly assimilated, preventing the student from translating the utterance into the algebraic register.

In the Planning stage, it is observed that the average success rates remain high (95% in the PT and 79% in the PTP). This demonstrates that students have a coherent strategic perception, being able to choose between the arithmetic or algebraic route to start the solution. However, this strategic choice arrives at the planning stage (PTP) loaded with difficulties that have not been overcome in representation, since the high rate of success in planning indicates that the student 'knows what he should do', but the lack of conceptual knowledge about the specific points of inequality, such as the use of the variable 'x' and the meaning of inequality, compromises the achievement of the correct answer later.

As for the Execution stage, the failures observed in both the Post-Test (PT) and the Postponed Post-Test (PTP) are based on gaps in procedural knowledge, specifically in the domain and application of algorithms. The investigation identified difficulties that lie in the manipulation of negative variables and in the performance of elementary operations. In this context, the highest incidence of errors/difficulties occurred in the application of the division algorithm, regardless of the approach used (algebraic or arithmetic). Such evidence suggests that, although the operational capacity has demonstrated resilience to time, weaknesses persist in basic calculations that make it impossible to obtain the correct solution. Above all, they presented flaws in the application of the principles of equivalence of inequalities and the improper substitution of the sign of inequality by the sign of equality during the calculation process.

With regard to the Monitoring stage, the data in Graph 1 reveal the most critical scenario of the survey, with success rates ranging from 13% in the PT to 14% in the PTP. This stagnation demonstrates that the ability to validate the solution in the face of the restrictions of the statement has not been consolidated, evidencing a cognitive difficulty, where success in procedural execution does not necessarily translate into a correct answer in the context of the problem.

The analysis of the students' written records indicates that the main barrier in monitoring lies in the persistence in understanding the solution as unique. When reaching the numerical value, the students end the problem-solving process, ignoring the relational nature of the symbol of inequality (when using algebraic representation, according to the majority in the PT) and treating inequality as a search for a unique and exact result. This flaw evidences a gap in conceptual knowledge, as the student does not recognize the solution as an interval or set of possibilities, but rather as an isolated number that often does not even meet the conditions of inequality imposed by the problem.

Thus, it is noted that the monitoring operated in a merely mechanical way, when executed, most of the time limited to the checking of the calculation algorithm, without reaching the level of reflective analysis necessary to understand the meaning of the answer. Thus, in the understanding of this investigation, this difficulty in performing a retrospective view of the problem reinforces that learning remained fragmented: students demonstrate a higher performance in solving inequalities (procedural knowledge), but failed to understand them as a concept (conceptual knowledge), remaining linked to arithmetic and equational concept habits, which make it difficult to perceive the specific properties of inequalities.

Final considerations

The investigation, whose objective was to analyze the conceptual and procedural learning of polynomial inequalities of the 1st degree of students of a class of the 7th grade of Elementary School, in the context of a didactic sequence based on a proposal of organization of teaching through problem solving, allowed to understand the nuances between the construction and mastery of knowledge and conceptual knowledge in students of the 7th grade.

By adopting the organization of teaching via problem solving by Proença (2021), teaching was structured in stages that required the student to represent, plan, execute and, fundamentally, monitor solutions. The results show that this approach contributes to reveal the logical structure of the student's thinking, as well as exposes the intrinsic difficulties in the transition from arithmetic to algebraic thinking, so that it is fundamental not only to lead students to learn to solve problems, but above all to lead them to learn mathematical concepts.

In this research, we conclude that procedural knowledge manifests significant temporal resilience, remaining stable even after the 108-day interval in the Postponed Post-test. However, this stability is strictly operative, in which it was found that the students demonstrate that they know 'how to do' the calculations, but have a profound lack of conceptual knowledge, especially with regard to the nature of the intervals and the interpretation of the symbol of inequality. The drop in the indices of formal algebraic representation, derived from the poor formation of linguistic, semantic and schematic knowledge, and the stagnation in the levels of monitoring indicate that learning, although functional for the resolution of accounts, remains fragmented. The search for a unique and exact result emerges as an epistemological obstacle that prevents the student from validating the answer as a set of possibilities.

The limitations of this study lie, in part, in the difficulty of breaking with ingrained school habits that prioritize the final product to the detriment of the reflective process. It was observed that the intervention time of the didactic sequence, although structured, may have been insufficient to consolidate the rupture with equational thinking (single solution). In addition, the research showed that previous weaknesses in basic operations and in dealing with negative variables overload the student's cognitive load, preventing him from focusing on the conceptual essence of inequalities. In the field of research, a limit identified was the need for instruments that could capture the reasons why monitoring is neglected even when the student has the cognitive resources to perform it.

In the field of scientific investigation, our study contributes by demonstrating that, by following the Proença's (2021) *teaching organization via problem solving* using the two post-tests, we were able to map exactly where the student's cognitive process fails, distinguishing procedural error from conceptual error. The studies already carried out have focused on revealing students' difficulties and errors in the use of inequalities to solve problems, so that our research advances by evidencing the conceptual formation of inequality in problem solving, an aspect not yet addressed in other teaching theories.

Finally, the study reinforces the thesis that solving problems should be the very guiding axis that allows the student to act on the object of knowledge, promoting learning that goes beyond the mere memorization of rules. However, it is suggested the development of research focused on the organization of teaching adopted here for other mathematical contents, with special attention to the stages of comprehension and monitoring, encouraging the student to develop autonomy in Mathematics.

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Conflicts of Interest

The authors declare no conflicts of interest that could influence the results of the research presented in the article.

Data Availability Statement

The data produced and analyzed in the article will be made available upon request to the authors.

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