

The Global Interpretation of Figural Properties in the Interdisciplinary Study of Functions with Modellus 4.0 Software

Elias Rodrigues de Souza

Secretaria de Educação de Minas Gerais
Londrina, MG — Brasil

✉ mestreers03@gmail.com

ORCID [0009-0002-8007-2356](https://orcid.org/0009-0002-8007-2356)

Josué Antunes de Macêdo

Instituto Federal de Educação Ciência e Tecnologia do Norte de
Minas Gerais

Montes Claros, MG — Brasil

✉ josueama@gmail.com

ORCID [0000-0002-0521-1910](https://orcid.org/0000-0002-0521-1910)



2238-0345 

10.37001/ripem.v16i2.4928 

Recebido • 15/06/2026

Aprovado • 03/08/2026

Publicado • 30/09/2026

Editoria • Edvonete Souza de Alencar 

Veridiana Rezende 

Abstract: This article aims to discuss, in light of the Theory of Registers of Semiotic Representations (TRSR), the process of Global Interpretation of Figural Properties mobilized by students when using the *Modellus* 4.0 software for the interdisciplinary study of Affine Functions through Kinematics problems. This qualitative research was developed by applying a sequence of activities to eighteen students in full-time upper secondary education. The data analysis was based on the principles of the TRSR, with emphasis on the Global Interpretation of Figural Properties. The results indicated that students using the *Modellus* 4.0 software were able to establish connections between different registers of representation, such as the graphical and algebraic, by identifying visual variables and their symbolic correspondences. It is concluded that *Modellus* 4.0 is a tool applicable to the interdisciplinary teaching of Affine Functions with Kinematics, enabling deeper conceptual understanding and moving beyond rote procedures of conversion between the semiotic registers of this object.

Keywords: Polynomial Functions. Interdisciplinary Teaching. *Modellus* 4.0 software. Global Figural Interpretation.

La Interpretación Global de las Propiedades Figurales en el Estudio Interdisciplinario de Funciones con el Software Modellus 4.0

Resumen: Este artículo tiene como objetivo discutir, a la luz de la Teoría de Registros de Representación Semiótica, el proceso de Interpretación Global de Propiedades Figurales movilizadas por estudiantes al utilizar el *software Modellus* 4.0 para el estudio interdisciplinario de Funciones Afines con problemas Cinemáticos. Esta investigación cualitativa se desarrolló mediante la aplicación de una secuencia de actividades a dieciocho estudiantes del programa de Escuela Secundaria de Tiempo Completo. El análisis de datos se basó en los principios de la Teoría de Registros de Representación Semiótica, con énfasis en la Interpretación Global de Propiedades Figurales. Los resultados indicaron que los estudiantes que utilizan el *software Modellus* 4.0 lograron establecer conexiones entre diferentes registros de representación, como el gráfico y el algebraico, mediante la identificación de variables visuales y sus correspondencias simbólicas. Se concluye que *Modellus* 4.0 es una herramienta aplicable a la enseñanza interdisciplinaria de Funciones Afines con Cinemática, que permite la profundización conceptual y la descomposición de los procesos mecanizados de conversión entre registros semióticos de este objeto.

Palabras clave: Funciones Polinómicas. Enseñanza Interdisciplinaria. *Software Modellus* 4.0.

A Interpretação Global de Propriedades Figurais no Estudo Interdisciplinar de Funções com o Software *Modellus 4.0*

Resumo: Este artigo tem por objetivo discutir, à luz da Teoria dos Registros de Representação Semiótica (TRRS), o processo de Interpretação Global de Propriedades Figurais mobilizado pelos estudantes ao utilizarem o *software Modellus 4.0* para o estudo interdisciplinar de Função Afim com problemas da Cinemática. Esta pesquisa qualitativa foi desenvolvida com aplicação de uma sequência de atividades a dezoito estudantes do Ensino Médio Integral. A análise dos dados fundamentou-se nos princípios da TRRS, com destaque para a Interpretação Global de Propriedades Figurais. Os resultados mostraram que os estudantes, utilizando o *software Modellus 4.0*, conseguiram estabelecer conexões entre diferentes registros de representação, como o gráfico e o algébrico, ao identificar as variáveis visuais e suas correspondências simbólicas. Conclui-se que o *Modellus 4.0* é uma ferramenta aplicável ao ensino de Função Afim de forma interdisciplinar com Cinemática, e possibilita o aprofundamento conceitual e o rompimento de processos mecanizados de conversão entre registros semióticos desse objeto.

Palavras-chave: Funções Polinomiais. Ensino Interdisciplinar. *Software Modellus 4.0*. Interpretação Global Figural.

1 Introduction

Within Brazilian Basic Education curricula, Polynomial Functions are part of the thematic unit of algebra. According to the Base Nacional Comum Curricular (BNCC) (Brasil, 2018), their study is included in the curriculum for the final year of elementary and lower secondary education through their numerical, algebraic, and graphical representations; they are to be studied in greater depth in upper secondary education, consolidating, among other skills, the ability to “construct models using 1st- or 2nd-degree polynomial functions to solve problems in various contexts, with or without the support of digital technologies” (Brasil, 2018, p. 536).

According to Fernandes (2014), the difficulties inherent in teaching these and other mathematical objects have led teachers and researchers to seek pedagogical approaches that foster understanding and application of the concepts. In this context, interdisciplinarity, characterized by Japiassu (1976) and Fazenda (2003, 2015) as a teaching method that enables an approach involving two or more disciplines that intertwine, integrate, and collaborate with one another, relating them to solve a single problem, has the potential to provide varied learning situations involving Affine Functions by exploring situations drawn from Kinematics¹.

This perspective of differentiated, engaging, and collaborative teaching is further complemented by the use of digital tools, such as the educational software package *Modellus*². Designed specifically for educational use in Physics and Mathematics teaching, it makes it possible to create simulations/animations of natural phenomena, such as those described in kinematics, through Mathematical Modeling using algebraic expressions, including Affine Functions. Using simple mathematical notation, the software facilitates the development of solutions through data manipulation and analysis of different results. The interactive animations, which require no advanced knowledge of programming languages, allow students

¹ The branch of Physics that studies the motion of inertial bodies governed by Newton’s laws.

² The *Modellus* software, created by Vitor Duarte Teodoro in his 2002 doctoral thesis at the Faculdade de Ciência e Tecnologia da Universidade Nova de Lisboa, Portugal, has undergone updates that have broadened its scope and made its operating system more accessible. In this study, we use *Modellus 4.0*, an updated version adopted by several Brazilian educational institutions, including Instituto Federal de Educação, Ciência e Tecnologia do Rio Grande do Norte.

to explore and visualize phenomena represented abstractly, one of the distinguishing features of this software (Teodoro, 2002).

When addressing the challenges of teaching and learning Affine Functions, Duval (2011) emphasizes that “the underlying reason for these difficulties is not to be found in the mathematical concepts associated with the affine function, but in the lack of knowledge of the rules of semiotic correspondence between the register of the graphical representation and the register of the algebraic expression” (Duval, 2011, p. 97). Thus, when the teaching of functions involves representations in graphical and algebraic registers, the author proposes an approach based on the Global Interpretation of Figural Properties, which entails coordinating the representations in these registers for the process of Conversion between them (Duval, 2011).

Building on this cognitive aspect of Raymond Duval’s Theory of Registers of Semiotic Representations and drawing on the interdisciplinarity between Physics and Mathematics and the use of Digital Technologies (DT) in mathematics teaching, this study aimed to discuss, in light of the Theory of Registers of Semiotic Representations (TRSR), the process of Global Interpretation of Figural Properties mobilized by students when using the *Modellus* 4.0 software for the interdisciplinary study of Affine Functions through Kinematics problems.

Following approval by the Comitê de Ética em Pesquisa, as documented in the consolidated opinion under No. 6.910.921, dated June 26, 2024, and Certificado de Apresentação de Apreciação Ética (CAAE) No. 79857124.1.0000.5146, we conducted this participatory research with students in full-time upper secondary education (EMTI) at a school in the state public school system in the municipality of Lontra, Minas Gerais, Brazil.

This article³ is organized into five sections: Section 1—the present Introduction, which introduces the study and provides the reader with its topic and objective; Section 2—the Theoretical Framework; Section 3—the methodological procedures applied in the study; Section 4—the sequence of activities conducted, the data collected, and discussion of the results; Section 5—the Concluding Remarks of this study; and the final section presents the bibliographic references consulted in this investigation.

2 The Theory of Registers of Semiotic Representations

The Theory of Registers of Semiotic Representations (TRSR) provides a theoretical framework for analyzing mathematics teaching and learning. It was established and developed by the French professor, philosopher, and psychologist Raymond Duval on the basis of studies in cognitive psychology. These studies began in the 1970s at the Institut de Recherche sur l’Enseignement des Mathématiques (Irem), in Strasbourg, France, and were consolidated as TRSR in 1995.

In Brazil, according to Costa (2021), this theoretical framework was first applied in Mathematics Education research in 1996. Since then, it has gained prominence in Brazilian research, driven by the work of researchers such as Lênio Fernandes Levy, Marisa Rosâni Abreu da Silveira, Mérciles Thadeu Moretti, Célia Finck Brandt, among others, who, in addition to using TRSR in their research, devoted themselves to translating into Portuguese part of the scholarly work produced by Raymond Duval on TRSR.

According to Duval (2009), Mathematics consists of abstract concepts, which are accessed through semiotic representations. The development of a mathematical activity

³ This article is part of the master’s dissertation (Souza, 2025) defended in the Programa de Pós-Graduação em Educação da Universidade Estadual de Montes Claros, organized in *multipaper* format, written by the first author and supervised by the second author.

depends primarily on the cognitive activities of Treatment and Conversion. Both involve transformations of semiotic representations preceded by the activity of recognizing the mathematical rules and characteristics of the object under study—Formation.

Duval (2009) emphasizes that the activity of **Treatment**, characterized by transforming a representation within the same semiotic system, without changing registers, is well established in mathematics teaching. The author notes, however, that **Conversion**—the cognitive activity involving the transformation of a representation in one register into another representation belonging to a different system—receives little attention. Consequently, the “lack of coordination between different registers very frequently creates a deficiency in conceptual learning” (Duval, 2009, p. 63). According to the author, learning occurs when students can move between at least two different registers through Conversion.

Duval (2011) identifies three possible approaches to the teaching and learning of functions. The first approach, which the author calls the *pointwise* approach, consists of the rote substitution of specific values into the algebraic expression, generating reference points to be placed on the Cartesian plane and then connected to form a graph (a straight line, curve, parabola, among others). In the graphical study,

With reference to the two scaled axes, a pair of numbers identifies a point (and, conversely, a point is translated into a pair of numbers). This associative mode is limited to certain values and to the points marked on the reference plane. This approach is useful when one wants to PLOT the graph corresponding to a first-degree equation or the graph of a quadratic equation. It is also useful when one wants to READ the coordinates of a point of interest (because it is a point of intersection with the axes or with a line, because it is a maximum, etc.) (Duval, 2011, p. 98).

The other approach, which is also common, is the *extension-of-the-plot approach*, which considers “an infinite set of potential points, that is, on the homogeneous background of the sheet, in the intervals between marked points” (Duval, 2011, p. 63). This approach favors mechanical Conversion procedures from the register of algebraic representations to the graph; “it is completely ineffective because it draws attention away from the visual variables” (Duval, 2011, p. 99); it therefore hinders the reverse process, namely, deriving the algebraic register from the graphical register. This occurs because the visual variables of the representations under consideration are linked to units of meaning.

Given the limitations of these teaching procedures, particularly when applied over the long term and outside the context studied, Duval (2011) proposes a third approach defined as *Global Interpretation of Figural Properties*. According to the author (2011, p. 99), this approach “is generally overlooked in teaching because it depends on visual and algebraic semiotic analysis.” The next section examines this aspect in greater depth, as it underpins the analysis of the collected data.

2.1 Global Interpretation of Figural Properties

Global Interpretation of Figural Properties extends the *pointwise* approach and the *extension-of-the-plot approach*, which focus on data from the plotted graph but do not attend to the form of the algebraic expression or the visual variables (Duval, 2011). Global Interpretation prioritizes the rules of semiotic correspondence in Treatment and Conversion activities for graphical and algebraic approaches to functions, making it an “effective approach when one wishes, for example, to move from a graphical representation to an algebraic representation” (Costa, 2021, p. 60). This process requires students to identify changes in visual variables in graphical registers and their respective correspondences with units of meaning in algebraic

registers.

According to Brandt and Moretti (2018), each register of representation has its own units of meaning. In the algebraic register of an Affine Function ($f(x) = ax + b$), for example, the units of meaning are $a, b, +, -$ and $=$, since variations in these terms allow immediate associations with visual changes in the graph (upward or downward slope, the slope of the line relative to the axis, its position relative to the origin, and others), as shown in Table 1 below.

Table 1: Visual variables and their respective units of meaning for an Affine Function

Visual variables	Values of visual variables	Corresponding symbolic units	
Direction of slope	Upward	Coefficient $a > 0$	No sign
	Downward	Coefficient $a < 0$	Presence of sign $-$
Angle with the axes	The quadrant traversed is divided symmetrically	Variable coefficient $a = 1$	No coefficient is written
	The angle formed with the horizontal axis is smaller than the angle formed with the vertical axis	Variable coefficient $a < 1$	A coefficient is written
	The angle formed with the horizontal axis is greater than the angle formed with the vertical axis	Variable coefficient $a > 1$	A coefficient is written
Position on the axis	Intersects above	Addition of constant b	Sign $+$
	Intersects below	Subtraction of constant b	Sign $-$
	Intersects at the origin	With no additive shift	No sign

Source: Prepared by the authors – adapted from Duval (2011).

Note that, in this table, the first visual variable refers to the direction of slope of the graph of the function plotted on the Cartesian plane. This corresponds to the sign of coefficient a , that is, the two units of meaning: a negative (the line slopes downward from left to right) or a positive (the line slopes upward from left to right).

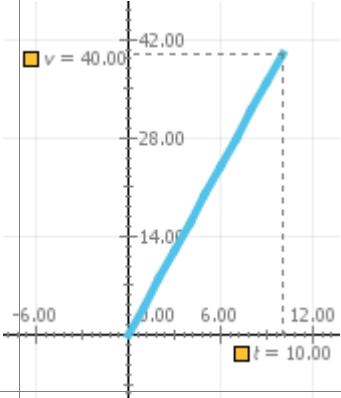
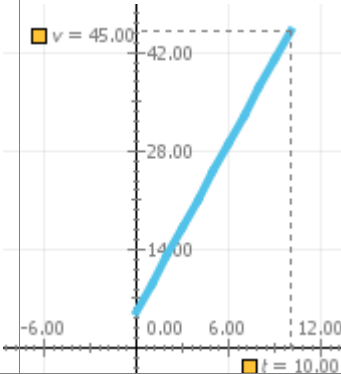
The second visual variable, associated with the angle that the plotted line forms with the axes, corresponds to the value of coefficient a (rate of change), under the following conditions: if $a = 1$, the angles between the plotted line and each axis are symmetric; if $1 < a < -1$ “[...] the angle formed with the horizontal axis is greater than the angle formed with the vertical axis” (Duval, 2011, p. 101); whereas, if $-1 < a < 1$, with $a \neq 0$, “[...] the angle formed with the horizontal axis is smaller than the angle formed with the vertical axis” (Duval, 2011, p. 101).

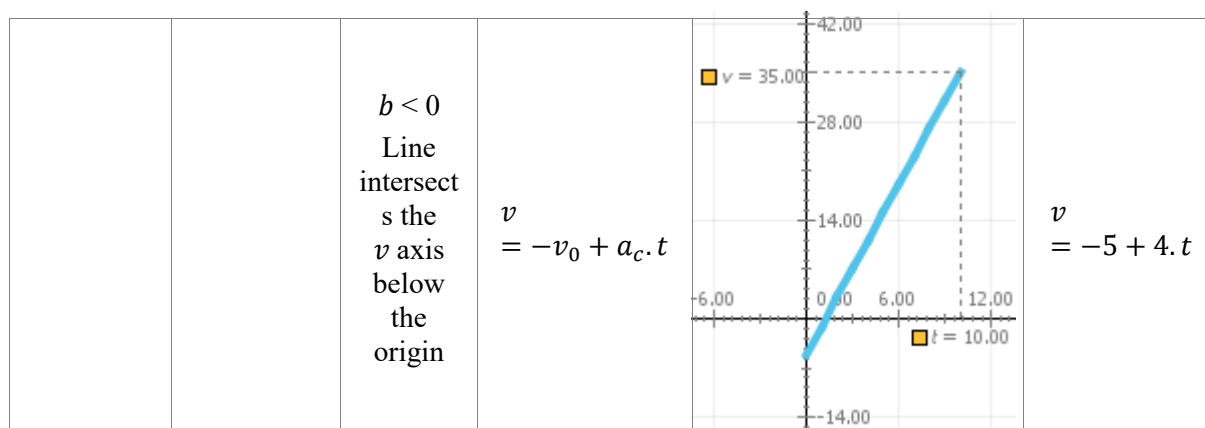
The third visual variable corresponds to the position of the plotted line relative to the origin of the vertical axis; therefore, its unit of meaning is the constant term b . For $b = 0$ (zero constant term), the plotted line passes through the origin of the axes; for $b > 0$ (positive constant term), the plotted line lies above the origin of the vertical axis; consequently, for $b < 0$ (negative constant term), the plotted line intersects the vertical axis below the origin (Duval, 2011).

In an interdisciplinary context involving Mathematics and Physics, in a problem involving physics and mathematics related to uniformly accelerated rectilinear motion (constant acceleration), velocity as a physical quantity is, by definition, given by an Affine Function $v(t) = v_0 + a \cdot t$. Thus, we have a velocity-time function.

Table 2 presents the visual variables of the velocity-time function when $a > 1$. It should first be noted that, in the graphical representation of the velocity-time function, time (t) is a scalar quantity and therefore does not take negative values. Table 3 presents the visual variables and units of meaning for $v \times t$ when $a < -1$.

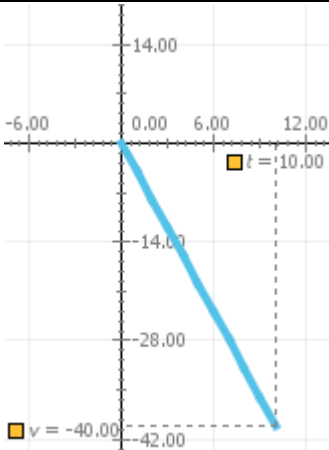
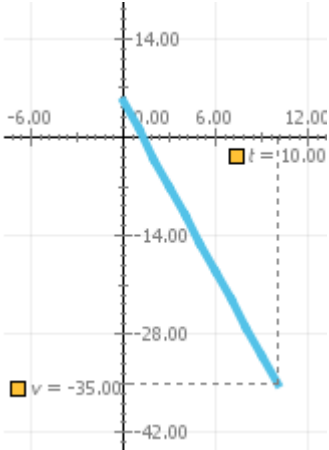
Table 2: Visual variables and symbolic units of the velocity-time function for $a > 1$

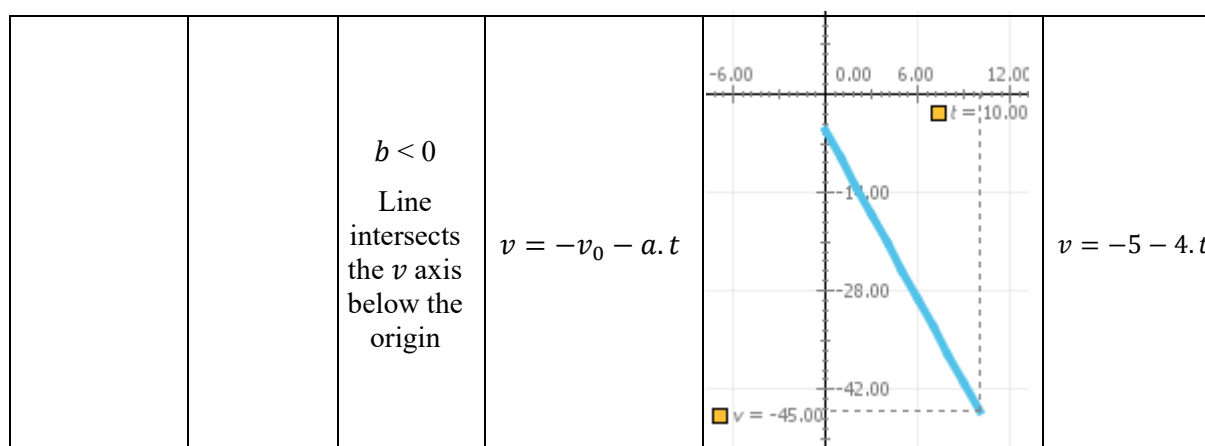
Visual variables			Symbolic Units	Examples	
Direction of slope	Angle with the t-axis	Position of the line relative to the origin	Algebraic expression	Graph	Algebraic
$a > 0$ Upward-sloping line	$a > 1$ The angle formed between the plotted line and the horizontal axis is greater than the angle formed with the vertical axis	$b = 0$ Line passes through the origin	$v = a \cdot t$		$v = 4 \cdot t$
		$b > 0$ Line intersects the v axis above the origin	$v = v_0 + a_c \cdot t$		$v = 5 + 4 \cdot t$



Source: Prepared by the authors – adapted from Amplatz (2020).

Table 3: Visual variables and symbolic units of the velocity-time function for $a < -1$

Visual variables			Symbolic Units	Examples	
Direction of slope	Angle with the t-axis	Position of the line relative to the origin	Algebraic expression	Graph	Algebraic
$a > 0$ Downward-sloping line	$a < -1$ The angle formed between the plotted line and the horizontal axis is greater than the angle formed with the vertical axis	$b = 0$ Line passes through the origin	$v = -a \cdot t$		$v = -4 \cdot t$
		$b > 0$ Line intersects the v axis above the origin	$v = v_0 - a \cdot t$		$v = 5 - 4 \cdot t$



Source: Prepared by the authors – adapted from Amplatz (2020).

If $a = 1$, the graph shows symmetrical partitioning and 45° angles between the plotted line and the axes. The same holds for $a = -1$, except that, in this case, the line slopes downward. Recall that, graphically, if $v_0 = 0$, the plotted line begins at the origin (an object starting from rest), whereas for $v_0 < 0$ it begins below the origin (nonzero initial velocity in the direction opposite to the displacement), and for $v_0 > 0$, it begins above the origin (nonzero initial velocity in the same direction as the displacement). When $a < 0$, this physically means that the object is undergoing decelerating motion.

For the coefficient $a < -1$, the angle formed by the plotted line with the horizontal axis is greater than the angle formed between the plotted line and the vertical axis; the same applies when $a > 1$. Therefore, the greater the absolute value (magnitude) of a , the closer the plotted line is to the vertical axis; physically, this indicates a higher rate of change in velocity. For $0 > a > -1$, the graph behaves similarly to the case in which a takes values in the interval $0 < a < 1$, where the angle formed between the plotted line and the horizontal axis is smaller than the angle formed with the vertical axis; physically, the closer the plotted line is to the horizontal axis, the smaller the change in velocity, since at constant velocity ($a = 0$) the plotted line is parallel to the horizontal axis of the graph $v \times t$.

It is evident that coordination between the visual variables and units of meaning enables a student to move between the graphical and algebraic registers of this function and to understand the connections between Mathematics and the physical phenomenon under study.

Duval (2011) emphasizes that, if this approach is ignored, the teaching and learning of functions are compromised, because “the interpretation of Cartesian graphical representations depends on the precise identification of all values of the relevant visual variables and on qualitative recognition of the units of the corresponding symbolic expression” (p. 111). Because Affine Functions are foundational to the study of Kinematics, gaps in the learning of Polynomial Functions in their graphical and algebraic registers will affect the interpretation and description of physical phenomena in Kinematics, since these processes rely on conversions between graphical and algebraic representations.

According to Moretti and Luiz (2014), technological resources primarily facilitate graphing because graphs “can easily be produced in a computer program, thereby giving students more free time for the more important task of interpretation and thus allowing them to answer the question posed in the activity with greater confidence” (p. 79). Moreover, several theorists advocate the use of technological resources aligned with the principles of theories related to Mathematics teaching.

In an interview with Freitas and Rezende (2013), Duval argues that the use of *software applications* entails three major innovations from a cognitive standpoint: it can promote visualization of content relevant to all areas of knowledge; perform simulation and Modeling functions; and, finally, produce something immediately (Amplatz, 2020, p. 62).

Several recent studies (Macêdo & Martins, 2026; Martins & Macêdo, 2023; Oliveira & Macêdo, 2025, 2026; Ruas, Macêdo & Crisostomo, 2024; Silva & Macêdo, 2025, 2026) have emphasized that teachers' pedagogical practice is shaped by multiple demands and challenges. In this context, according to Santos and Macêdo (2023, 2024), Digital Technologies (DT) seek to support the learning of curricular content and establish an environment for critical reflection. Santos and Macêdo (2023, p. 3) further caution "that the use of Digital Technologies (DT) to teach Mathematics does not guarantee effective learning," because DT are "part of a collective project that envisions democratizing access to technologies developed by society" (Borba & Penteado, 2019, p. 17). Thus, merely incorporating DT into traditional practices for solving mathematical activities will not guarantee learning; the teaching process must be conceived collectively, through knowledge exchange and foundations that ensure the appropriate use of these tools.

In this context, we conducted this study using an interdisciplinary approach to teaching Affine Functions through problems involving physics and mathematics addressed using the *Modellus* 4.0 software, in accordance with the methodological procedures described below.

3 Methodological Procedures

This investigation is a participatory study, qualitative in nature, based on the active participation of the individuals involved (Brandão, 2006). Minayo (2002) emphasizes that the primary purpose of qualitative research is to examine experiences and events in human relationships, based on the search for social meanings and the historical construction of approaches that cannot be expressed in numbers but rather through the researcher's interpretation.

To this end, we used a sequence of activities, the primary data collection instrument in this study, corresponding to a noncompartmentalized instructional unit that followed a cyclical planning process, as Gomes (2011) notes. This sequence comprises two problems involving physics and mathematics constructed from Kinematics concepts, whose interpretation, development, and solution depend on the mathematical concepts of Affine Functions. These problems were explored through an interdisciplinary approach using the *Modellus* 4.0 software with students enrolled in full-time upper secondary education (EMTI) at a state public school in the municipality of Lontra, Minas Gerais, Brazil.

Six students from each upper secondary grade were randomly invited, totaling 18 participants. This number was consistent with the availability of fully functioning computers in the school's computer lab. The choice of the participants' educational level was related to the school curriculum for the Physics and Mathematics concepts addressed. Selecting students from different grades reflected the interest in examining the use of the software across the different years of upper secondary education, as well as the proposal for interdisciplinary teaching of Affine Functions. The institution was selected because it offered upper secondary education, had a fully functioning computer lab available for use, and was connected to the first researcher's educational and professional trajectory. He completed his basic education at this school—the only one in the municipality to provide upper secondary education—and currently holds a position there as a Physics teacher; during the study, however, he was on leave to pursue a master's degree.

We conducted pilot testing of the activity sequence with a randomly selected class in the 1st year of upper secondary education at the same school to verify and guide future adjustments to the activities and to the use of the *Modellus* 4.0 software, since it had not previously been used at the school. In addition to enabling adjustments to the sequence, this practice revealed the need to revise the number of participants—which we set at 18—and also provided an initial introduction to the software.

We emphasize that none of the participants in the pilot testing of the activity sequence took part in the intervention conducted in March 2025. Five sessions were held in the computer lab, totaling 12 class periods. The first two sessions were devoted to an initial immersion in the use of the *Modellus* 4.0 software.

At each session, the pairs received a printed activity with instructions and questions to be completed on paper and on computers with the software installed. All sessions were held in the afternoon because the school operates on a full-time schedule. The students were excused to participate, with their teachers' consent, without adversely affecting their other subjects. Under the guidance and supervision of the researcher, who was primarily responsible for coordinating and assisting with the activities, the students formed pairs to complete the activities in the sequence. Nine pairs were thus formed, each comprising students from the same class. The pairs were interspersed throughout the lab to enable collective dialogue among students from different grade levels. The researcher encouraged participants to engage with the other pairs, especially those from different grades. They could interact regarding the Physics and Mathematics content being explored, computer operations, and interpretation of the results of simulations and animations, thereby enabling the construction of coherent responses about the topic explored. Questions and requests for clarification—about basic computing, commands in the software, organization of the activities, and other matters—were promptly addressed by the first researcher, maintaining a pleasant environment for carrying out the activities.

During the activities, the participants produced physical and digital data, which were collected from copies of the printed materials returned by the participants and from the digital files saved in *Modellus* 4.0. Audio recordings and notes of the participants' statements were also used, as was systematic observation, characterized as “being planned and conducted according to a previously defined objective” (Danna & Matos, 2015, p. 15).

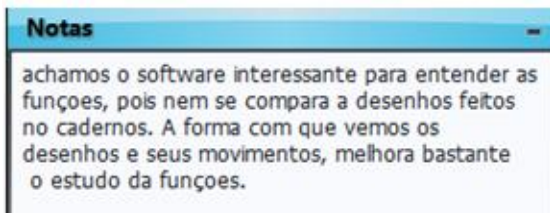
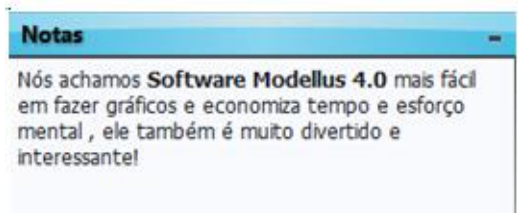
After the sequence of activities had been implemented, data processing began. To facilitate tracking each participant's construction of knowledge, anonymity was maintained. Accordingly, the activities completed by the pairs were identified by random numbers from 1 to 9 and designated D1, D2, D3, ..., D9. Each pair's physical and digital artifacts were organized in an *Excel* spreadsheet and categorized according to the following cognitive activities: Formation, Treatment, Conversion, and, above all, Global Interpretation of Figural Properties. During and after this categorization, all data produced in each activity were analyzed based on the assumptions of the Theory of Registers of Semiotic Representations (TRSR), in line with the research objective, producing the results presented below.

4 Results and Discussion

The implementation of the sequence of activities began with an introductory immersion in the use of the *Modellus* 4.0 software. The students learned about the tool's features and purpose, as well as the functions of its main tabs and commands. Thus, using instructions in the printed material provided, assistance from the first researcher, and exploration of the software environment, the participants completed three simulation examples of Kinematics problems involving physics and mathematics, mathematically described by an Affine Function. At the end of this stage, the pairs recorded their first impressions in the software environment

regarding the use of this technological resource to study the Affine Function through an interdisciplinary approach with physical phenomena. Figure 1 below presents the comments of two pairs.

Figure 1: Responses from pairs D3 and D5, respectively

	
Notes we found the software interesting for understanding functions, because it does not even compare to drawings made in our notebooks. The way we see the drawings and their movements greatly improves the study of functions.	Notes We think the <i>Modellus 4.0</i> software makes it easier to create graphs and saves time and mental effort; it is also very fun and interesting!

Source: Research archive.

These responses show how the *Modellus 4.0* software became an attractive technological resource for studying concepts in two subjects in which difficulties in learning and understanding are frequently reported. The responses refer to the function's graphical register, which is the focus of this study.

Following the immersion activities, in the 3rd session we began administering the data collection instrument. In the first activity of the sequence, participants were to use the *Modellus 4.0* software to create a simulation of the stated problem (Figure 2).

Figure 2: Prompt for Activities 1 and 2 in the sequence

The mathematical expression $S = S_0 + v \cdot t$ is known as the position-time function [$S = f(t)$]. It is used to describe position as a function of time for an object in uniform rectilinear motion (constant velocity). Here, S_0 corresponds to the initial position and v to the velocity. Problem 1 During a track race, one of the participants travels along a segment of the course in uniform rectilinear motion, with her positions described by the position-time function $S = 200 - 5t$, in SI units.

Source: Research archive.

Given the prompt, a series of questions followed in which participants were expected to employ cognitive activities through semiotic representations of the Affine Function to reason about, predict, and visualize the physical behavior of the phenomenon under study (Figure 3).

The students' work clearly shows that the participants mobilized the cognitive activities of Formation and Treatment. We noted a minor mistake by pair D2 in its answer to item *c*, where it entered -2 when the correct value was -5 , corresponding to the function parameter. Another common error in all pairs' responses was the omission of units of measurement from results corresponding to dimensional physical quantities.

Systematic observation showed that all pairs—including those composed of 1st-year upper secondary students who had not studied Kinematics concepts but had already had some exposure to the Affine Function—were able to produce coherent answers. According to Duval (2011), the cognitive activity of Treatment, preceded by Formation, is routinely developed in Mathematics teaching. The participants' work supported this claim and further showed that,

when students discussed functions, they invariably associated them with the use of y and x ; therefore, merely changing the variables to s and t or v and t created difficulty in associating the situation with the concept of function, especially in terms of units of meaning or visual variables.

Figure 3: Work produced by pair D2

Comparando a função horária para as posições da atleta, com a expressão $s = s_0 + v \cdot t$, responda:

a) A função que rege as posições do ciclista, mencionada no problema 1, é uma função do 1º ou 2º grau? Justifique

Primeiro, pois a incógnita tem expoente 1.

b) Qual posição inicial, em relação à origem, da corredora para o trajeto em estudo?

200.

c) De acordo com a função, qual a velocidade da corredora neste trecho? Ele varia com o tempo? Qual a relação com o ritmo da atleta para o trecho em estudo?

A velocidade é -2. Não, pois a velocidade é constante. A atleta permanece em um ritmo constante.

d) Substituindo o valor de t na função das posições da corredora, encontre os valores das posições s do ciclista para cada instante de tempo listados abaixo.

	$t = 1,0 \text{ s}$	$t = 3,0 \text{ s}$	$t = 5,0 \text{ s}$	$t = 10,0 \text{ s}$
$s = 200 - 5t$	$s = 200 - 5 \cdot 1$ $s = 195$	$s = 200 - 5 \cdot 3$ $s = 185$	$s = 200 - 5 \cdot 5$ $s = 175$	$s = 200 - 5 \cdot 10$ $s = 100$

e) O que podemos observar em relação aos valores de s à medida que o tempo aumenta? Qual a relação deste fato com o sinal da velocidade?

Quando o tempo aumenta, a posição diminui. A velocidade é negativa, por isso a cada segundo que o tempo passa, a posição diminui.

f) Em que instante a corredora passa pela origem das posições ($s = 0 \text{ m}$)?

*$0 = 200 - 5t$
 $-200 = -5t$
 $t = 40$*

Compare the athlete's position-time function with the expression $S = S_0 + v \cdot t$, and answer:

a) Is the function governing the cyclist's positions, mentioned in Problem 1, a 1st- or 2nd-degree polynomial function? Justify your answer.

Handwritten response: First degree, because the unknown has exponent 1.

b) What is the runner's initial position relative to the origin for the segment under study?

Handwritten response: 200.

c) According to the function, what is the runner's velocity along this segment? Does it vary over time? How does this relate to the athlete's pace along the segment under study?

Handwritten response: The velocity is -2. No, because the velocity is constant. The athlete maintains a constant pace.

d) Substitute the value of t into the runner's position function and find the cyclist's position values s for each time instant listed below.

	$t = 1,0 \text{ s}$	$t = 3,0 \text{ s}$	$t = 5,0 \text{ s}$	$t = 10,0 \text{ s}$
$s = 200 - 5t$	$s = 200 - 5 \cdot 1$ $s = 195$	$s = 200 - 5 \cdot 3$ $s = 185$	$s = 200 - 5 \cdot 5$ $s = 175$	$s = 200 - 5 \cdot 10$ $s = 100$

e) What can we observe about the values of s as time increases? How does this relate to the sign of the velocity?

Handwritten response: As time increases, position decreases. The velocity is negative, so position decreases with each passing second.

f) At what time does the runner pass through the origin of positions ($s = 0 \text{ m}$)?

Handwritten response:

$$0 = 200 - 5t$$

$$-200 = 5t$$

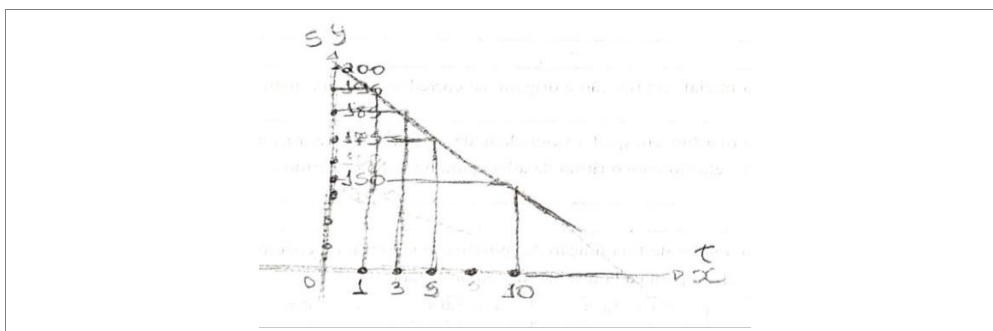
$$t = \frac{200}{5} = 40$$

Source: Research archive.

Duval (2011) further states that Mathematics teaching and learning generally devote little time or space to the cognitive activities of Conversion. Consequently, learning tends to be deficient, since understanding a mathematical concept depends on the student's ability to move between at least two distinct registers, in addition to carrying out Formation and Treatment. Figure 4 shows pair D2's work on the Conversion activity.

Figure 4: Work produced by pair D2

- g) State the graphical characteristics that can be deduced from the available data.
- Handwritten response:* Downward-sloping graph.
- h) Construct the graph according to the pair's understanding.

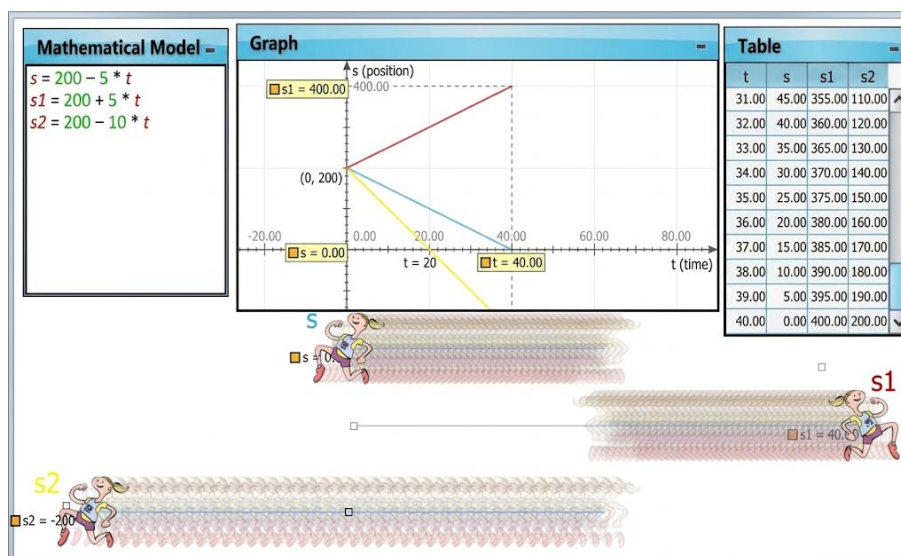


Source: Research archive.

We observed that the pair identifies all the key information about the problem and recognizes that the graph will have a downward-sloping line; however, even after finding the value corresponding to the point at which the object passes through the origin, the pair does not show it on the graph presented in Figure 4. Beyond the difficulty of producing an aesthetically well-defined graphical representation, the pair performs only a partial Conversion to the graphical register because it can plot only the values found in the preceding table. According to Duval (2011), this way of carrying out the activity corresponds to the *pointwise approach*, in which specific values are substituted into the algebraic expression, generating reference points to be entered in the Cartesian plane and joined to form a graph (straight line, curve, parabola, among others). This partial Conversion from the algebraic register was identified in the responses of seven of the nine pairs.

In Activity 2, the participants used the *Modellus 4.0* software to produce simulations of Problem 1. For these activities, they did not need to engage in Mathematical Modeling for the problem because the position-time function was provided in the prompt. The questions in Activity 2 were designed to validate the results obtained by the pairs in Activity 1. To do so, the participants moved among the graphical, tabular, and figural (animation) registers produced in the software, connecting the mathematical interpretations to the physical phenomenon under study—that is, manipulating units of meaning and analyzing visual variables—as shown in Figure 5, which presents the simulation produced by pair D9.

Figure 5: Work produced by pair D9 using the *Modellus 4.0* software



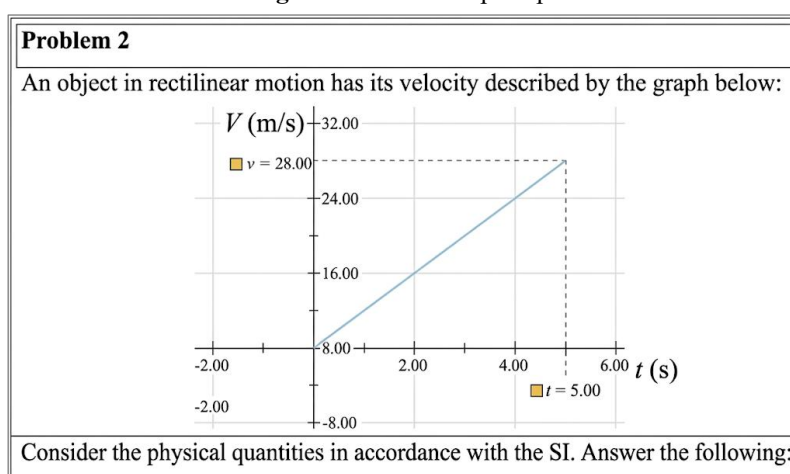
Source: Research archive.

During the discussions held after the simulation was constructed, we recorded the following statements: “The function under study is a first-degree polynomial function because

the exponent of the variable is 1" (D7, 2025); "200 is the initial position and corresponds to the value of b in the function $f(t) = b + at$ " (D8, 2025); "The line does not cross the vertical axis because negative time does not exist" (D4, 2025). In addition, the students developed an understanding of how the sign of the coefficient (velocity) that accompanies the time variable in the function affects the direction of motion, and they connected the value of coefficient a to the path traveled in uniform motion. These responses demonstrate the development of the cognitive aspects that enabled them to relate the visual variables to the units of meaning across the algebraic and graphical registers in this problem involving physics.

The knowledge developed here was important for the next activity in the sequence. In this new activity, the prompt presents a physical problem represented mathematically in the natural-language and graphical registers (Figure 6). The main objective of the second activity was to produce the simulation using the *Modellus 4.0* software. To do so, the participants had to move from the graphical register to the algebraic register via the tabular register.

Figure 6: Problem 2 prompt



Source: Research archive.

Exploring this problem in the software requires use of the algebraic register. For this reason, the participants answered the questions presented (Figure 7) and used their answers to construct the Mathematical Modeling required for the simulation: the equation of the line.

Figure 7: Work produced by pair D6

a) Sendo o eixo vertical, o registro dos valores da velocidade. É correto afirmar que este corpo partiu do repouso? Justifique.

Não. Pois de acordo com a gráfica o corpo partiu da posição 8.

b) Quais os valores da velocidade para

$t = 0,0\text{ s}$	$t = 2,0\text{ s}$	$t = 4,0\text{ s}$	$t = 5,0\text{ s}$
$8 + 5 \cdot 4$	16	24	28

c) Sendo a aceleração dada pela expressão $a = \frac{v_2 - v_1}{t_2 - t_1}$, encontre a aceleração deste corpo.

$\frac{16 - 8}{2 - 0} = 4$ A aceleração desse corpo é 4.

d) Sendo a função horária da velocidade dos corpos em MRUV descrito pela expressão $v = v_0 + a \cdot t$. Escreva a função horária da velocidade do movimento representado no gráfico.

$V = 8 + 4 \cdot t$

a) Given that the vertical axis records velocity values, is it correct to state that this object started from rest? Justify your answer.

Handwritten response: No. According to the graph, the object started at point 8.

b) What are the velocity values for

$t = 0,0\text{ s}$	$t = 2,0\text{ s}$	$t = 4,0\text{ s}$	$t = 5,0\text{ s}$
$8 + 5 \cdot 4$	16	24	28

c) Given that acceleration is defined by the expression $a = \frac{v_2 - v_1}{t_2 - t_1}$, find the acceleration of this object.

Handwritten response:

$\frac{16 - 8}{2 - 0} = 4$ The acceleration of this object is 4.

d) Given that the velocity-time function for objects in uniformly accelerated rectilinear motion (MRUV) is described by the expression $v = v_0 + a \cdot t$. Write the velocity-time function for the motion represented in the graph.

Handwritten response:

$v = 8 + 4 \cdot t$

Source: Research archive.

The responses show that the pair developed knowledge of physics and mathematics by working with the graph's visual variables and the symbolic units of the velocity-time function characteristic of this type of motion, correctly obtaining the velocity-time function represented in the graph.

Duval (2011) states that moving from the graphical register to the algebraic register requires an extension of the *pointwise approach*, that is, mobilizing the Global Interpretation of Figural Properties, through which the characteristics of a graph can be interpreted and connected to the units of meaning in functions. Thus, by mobilizing this cognitive activity and the aspects of Formation and Treatment in an interdisciplinary manner, the participants moved from the graphical register to the tabular register and then to the algebraic register, arriving at the velocity-time function. In this way, even before the simulation, use of the software already promotes the development of cognitive activities.

When asked about the plotted line, pair D7 stated that it was upward sloping because the physical quantity acceleration was positive, indicating accelerating motion. The pair's recognition of symbolic units and visual variables in an interdisciplinary manner is evident, corresponding to a conscious and accurate understanding of the Conversion from the graphical register to the algebraic register.

After finding the velocity-time function, the pairs performed the simulation in the software. For pair D7, the simulation made it possible to verify the consistency between the mathematical model and the graphical representation, as they stated: *"the line started at the same point and slopes upward, just like in the image [...] the values are the same for both time and velocity"* (D7, 2025). By "line," we infer that they were referring to the graph generated, and by "values," to the corresponding table. This work reveals that the digital tool enabled the pair to validate the function they had constructed independently, reinforcing students' agency in the learning process—one of the potential benefits attributed to Digital Technologies, as Romanello (2016) notes.

Pair D6 had the same perception (Figure 8) when answering the question: Is the graph generated in this simulation the same as the graph in the prompt for Activity 3? Justify the similarities or differences. Comment on the table generated by the software: *"Yes. It is identical to the graph presented, with the same starting point, time, and initial velocity, as well as the same angle. The table provided by the software is identical to the one we produced, showing the same results for the specified times"* (2025).

The pair uses the registers present in the simulation to compare them with the results they had produced and concludes that they are similar. In this sense, the digital results were sufficient to support reading, interpretation, and completion of the Conversion among the algebraic, graphical, and tabular registers.

Following the simulation and comparison of the digital representations with the problem statement, the participants answered several questions while simultaneously manipulating the symbolic units and analyzing the visual variables, or vice versa, in order to construct physical interpretations of the problem. Figure 8 presents some of the work produced by pair D2 while manipulating data in the software.

We observed the use of visual information from the graph and animation, as well as the units of meaning in the algebraic register shown in the simulation, to formulate physics-based responses related to the motion variables. Thus, when interpreting the results presented in the representation registers explored through the software, the participants move between different registers and reach appropriate interpretations through the cognitive activity of Global Interpretation of Figural Properties.

Figure 8: Work produced by pair D2

e) Para que o traçado do gráfico fosse decrescente, qual mudança devia acontecer na função horária? Qual o significado físico?

A aceleração deveria ser negativa. Está ocorrendo uma desaceleração.

d) Se o corpo estivesse partindo do repouso, qual seria o valor de v_0 ? E qual a característica gráfica?

$v_0 = 0$. A reta partindo do zero.

e) Quando a aceleração tende a zero, por consequência a variação da velocidade diminui. Faça o valor de a variar até o valor zero e observe e relate as características gráficas ocorridas? Antes na aba "objetos" insira uma partícula, configure a animação na aba "animações" e reproduza

A velocidade foi diminuindo até chegar numa reta constante.

f) Considerando a letra e, o que foi observado na animação e na tabela?

A velocidade da animação e na tabela foi diminuindo a cada variação da aceleração.

e) For the graph to slope downward, what change would have to occur in the velocity-time equation? What is the physical meaning?

Handwritten response: The acceleration should be negative. Deceleration is occurring.

d) If the object were starting from rest, what would the value of v_0 be, and what would the graphical characteristic be?

Handwritten response: $v_0 = 0$. A line starting at zero.

e) As the acceleration approaches zero, the change in velocity consequently decreases. Vary the value of a until it reaches zero, then observe and describe the resulting graphical characteristics. First, on the "objects" tab, insert a particle; configure the animation on the "animations" tab; and run it.

Handwritten response: The velocity decreased until it reached a constant line.

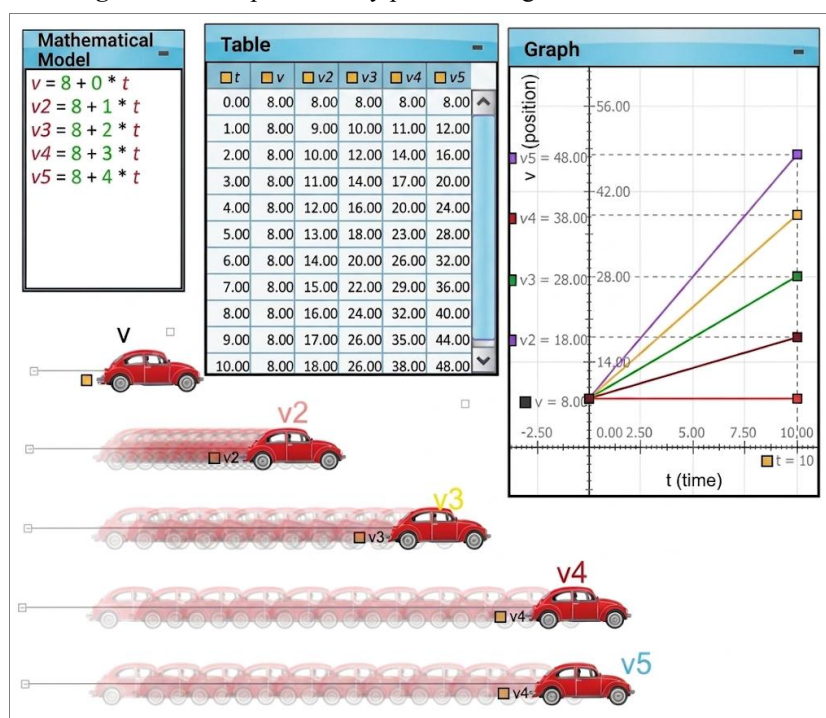
f) With reference to item e, what was observed in the animation and the table?

Handwritten response: The velocity in the animation and the table decreased with each change in acceleration.

Source: Research archive.

When analyzing the animations produced by the pairs, we observed the use of different models of "particles" associated with everyday objects that commonly appear in physics problems. Figure 9 shows the complete simulation produced by pair D2 for Activity 2.

Figure 9: Work produced by pair D2 using Modellus 4.0 software



Source: Research archive.

This pair used the *car* particle and manipulated the function according to the problem questions, producing a simulation that represents the physical phenomenon of the rectilinear motion of an object subject to constant acceleration. The ability to plot lines in different colors on the same Cartesian plane allowed them to observe how the slope of the plotted line varied as the units of meaning were changed. Combining these four registers for the five situations developed in a single window enabled them to relate the approach to the Affine Function to the

physical phenomenon under study. Interpretation is facilitated by visualizing these registers and the influence of each parameter/unit of meaning in the function on the description of the phenomenon.

At this level of simulation, it is clear that the participants quickly achieved meaningful proficiency with the technological tool and were able to visualize, within a single physical situation, the consequences for the car's motion as the acceleration changed. Because the simulation began with a mathematical object and then arrived at an illustration of a physics problem, it enabled them to understand the problem mathematically. Moreover, changes in the unit of meaning represented by the constant term in the time-dependent function enabled the pair to perceive the graphical, tabular, and figural behavior of the problem—a dynamic different from the practices used in the classroom by most Basic Education Physics and Mathematics teachers. Pair D1 recorded what the simulation enabled them to understand (Figure 10).

Figure 10: Work produced by pair D1

d) O que mais, sobre esta função, foi possível compreender e aplicar nestes problemas físicos? <i>Compreendemos que quando a aceleração ou a velocidade for positiva, o gráfico será crescente e quando negativa, será decrescente.</i> e) Como a dupla se auto avalia em relação a interpretação dos gráficos destas atividades? Conseguem chegar na função partindo do gráfico? Compreende os significados físicos descritos no gráfico? Qual a principal dificuldade? <i>Melhoramos na interpretação a cada atividade desenvolvida. Conseguimos compreender os significados e a chegar na função partindo do gráfico.</i>	d) What else about this function could be understood and applied in these physics problems? <i>Handwritten response:</i> We understood that when the acceleration or velocity is positive, the graph is increasing, and when it is negative, the graph is decreasing. e) How do you assess yourselves in terms of interpreting the graphs in these activities? Can you derive the function from the graph? Do you understand the physical meanings described by the graph? What is your main difficulty? <i>Handwritten response:</i> Our interpretation improved with each activity we completed. We were able to understand the meanings and derive the function from the graph.
--	--

Source: Research archive.

These responses provide important evidence of the development of the cognitive activity of Global Interpretation of Figural Properties, because the pair correctly relates the units of meaning in the algebraic expression to the visual variables in the graphical register. In addition to demonstrating the ability to construct the algebraic register from the graphical register, this task represents, according to Duval (2011), one of the major difficulties in learning functions.

The work analyzed shows that the use of the software helped students better understand the physical phenomena under study as represented through functions. The animation and manipulation capabilities offered by the digital tool allowed the pairs to visualize the physical phenomenon dynamically, as the following statements reveal: “we found out where it moved and when it moved” (D7, 2025) and “we can visualize the problem, which made it easier to understand” (D9, 2025). These statements show that the students were able to establish meaningful associations between the physical quantities involved and their corresponding units, as illustrated by the work of pair D7 shown in Figure 11.

Figure 11: Work produced by pair D7

e) Qual o tipo de função aplicada nas atividades 1, 2 e 3? Cite as funções aplicadas no software? Qual a variável independente? Para cada uma delas definam o valor associado $f(x)$, a e b, considerando $f(x) = ax + b$. <i>Nas atividades 1 e 2 nos utilizamos a função $S = s_0 + v \cdot t$ e na atividade 3 utilizamos a função $V = v_0 + a \cdot t$. Foram aplicadas no software a função $20 - 2t$ nas atividades 1 e 2 e a função $V = 8 + 4t$ na atividade 3.</i> <i>$f(x) = a = -2x, b = 20$</i> <i>$f(x) = a = -2x, b = 20$</i> <i>$f(x) = a = 4x, b = 8$</i>	e) What type of function was used in Activities 1, 2, and 3? State the functions entered into the software. What is the independent variable? For each function, identify the corresponding values of $f(x)$, a and b, given $f(x) = ax + b$. <i>Handwritten response:</i> In Activities 1 and 2, we used the function $s = s_0 + v \cdot t$, and in Activity 3, we used the function $v = v_0 + a \cdot t$. The function $20 - 2t$ was entered into the software in Activities 1 and 2, and the function $v = 8 + 4t$ in Activity 3. <i>$f(x) = a = -2x, b = 20$</i> <i>$f(x) = a = -2x, b = 20$</i> <i>$f(x) = a = 4x, b = 8$</i>
--	--

Source: Research archive.

This response shows that the pair can distinguish each unit of meaning in the function and relate it to the corresponding time-dependent function. Although they had some difficulty operating the software and moving between different representation registers in an interdisciplinary manner, we infer that most participants were able to mobilize Global Interpretation of Figural Properties by developing the simulation using *Modellus* 4.0 software.

5 Concluding Remarks

This study arose from a question about the potential of using educational software—a Digital Technology—to support the teaching and learning of the mathematical object Affine Function. Our objective was to discuss, in light of the Theory of Registers of Semiotic Representations, the process of Global Interpretation of Figural Properties mobilized by students when using *Modellus* 4.0 software for the interdisciplinary study of Affine Function through Kinematics problems. To this end, we conducted a qualitative participatory study using a sequence of activities designed around an interdisciplinary approach to Kinematics and Affine Function. These activities were carried out by 18 upper secondary students using *Modellus* 4.0 software in the computer lab of a school in the state public school system.

The data analysis enabled us to answer our research question by allowing us to observe the process of Global Interpretation of Figural Properties developed through the use of the software. Beyond this activity, the participants used Formation and Treatment and, above all, Conversion, as evidenced by their movement among graphical, algebraic, tabular, natural-language, and figural registers. We observed the development of the cognitive activity described by Duval (2009, 2011, 2018), which moves beyond the rote process of conversion between algebraic and graphical registers frequently adopted when teaching the mathematical object under study: Global Interpretation of Figural Properties.

From the students' work, we inferred their autonomy in drawing on prior knowledge of content already covered in Mathematics and Physics, as well as knowledge acquired while working through the sequence, beginning with the simulations produced in the software. The students predominantly used Treatment, demonstrating greater ease and accuracy when performing this cognitive activity.

Given the importance of the Affine Function, the challenges associated with teaching and, consequently, learning it may affect the learning of concepts in other disciplines, because “there is no *noesis* without *semiosis*; it is *semiosis* that determines the conditions under which *noesis* is possible” (Duval, 2009, p. 9). We view the pursuit of diverse teaching and learning strategies as a means of fostering this knowledge in dynamic, engaging, and productive lessons.

From this perspective, *Modellus* 4.0 software, used in the interdisciplinary teaching of Polynomial Functions and Kinematics, proved effective and useful because it enabled simulations of problems involving physics and mathematics that required students to mobilize the principal cognitive activities. By producing the simulation and animation without needing advanced computing knowledge, the students accessed different graphical registers by manipulating the algebraic register, thereby exploring the mathematical concepts in the problems and relating them to the animation depicting the physical behavior of the situation under study.

The discussions presented in this study reinforce the usefulness of the software for an interdisciplinary approach to Polynomial Functions through Kinematics problems. They also demonstrate the potential of this technological tool and reinforce the need for collaborative work between the two disciplines. Moreover, the usefulness of TRSR for advancing teaching and analyzing Mathematics learning is clear, thereby enabling new approaches to other

mathematical objects that can be addressed interdisciplinarily using *Modellus* 4.0 software.

Conflicts of Interest

- The authors declare that they have no conflicts of interest that could influence the results of the research presented in this article.

Data Availability Statement

- The data generated or collected and analyzed in this article will be made available upon request to the authors.

References

- Amplatz, L. C. (2020). *O estudo da função afim a partir da interpretação global de propriedades figurais: Uma investigação com estudantes do ensino médio* [Dissertação de mestrado, Universidade Estadual do Oeste do Paraná].
- Borba, M. C. & Penteado, M. G. (2019). *Informática e educação matemática* (6ª ed.). Autêntica Editora.
- Brandão, C. R. (2006). A pesquisa participante e a participação da pesquisa: Um olhar entre tempos e espaços a partir da América Latina. In C. R. Brandão & D. Streck (Orgs.), *Pesquisa participante: O saber da partilha* (pp. 21–54). Ideias e Letras.
- Brandt, C. F. & Moretti, M. T. (2018). Aprendizagem da álgebra segundo Raymond Duval. *Revista Brasileira de Educação em Ciências e Educação Matemática*, 2(1), 1–26. <https://doi.org/10.33238/ReBECCEM.2018.v.2.n.1.19419>
- Brasil. Ministério da Educação. Secretaria de Educação Básica. (2018). *Base Nacional Comum Curricular*.
- Costa, C. (2021). *Teoria dos registros de representação semiótica: Estado do conhecimento em dissertações e teses (1996–2019)* [Dissertação de mestrado, Universidade Federal de Santa Catarina].
- Danna, M. F. & Matos, M. A. (2015). *Aprendendo a observar* (3ª ed.). Edicon.
- Duval, R. (2009). *Semiósis e pensamento humano: Registros semióticos e aprendizagens intelectuais* (Fascículo I; L. F. Levy & M. R. A. Silveira, Trans.). Livraria da Física.
- Duval, R. (2011). Gráficos e equações: A articulação de dois registros (M. T. Moretti, Trad.). *Revista Eletrônica de Educação Matemática*, 6(2), 96–112. <https://doi.org/10.5007/1981-1322.2011v6n2p96>
- Duval, R. (2018). Como analisar a questão crucial da compreensão em matemática? (M. T. Moretti, Trad.). *Revista Eletrônica de Educação Matemática*, 13(2), 1–27. <https://doi.org/10.5007/1981-1322.2018v13n2p1>
- Fazenda, I. C. A. (2003). *Interdisciplinaridade: História, teoria e pesquisa* (11ª ed.). Papirus.
- Fazenda, I. C. A. (2015). Interdisciplinaridade: Didática e prática de ensino. *Revista Interdisciplinaridade*, 6(6), 9–17. <https://revistas.pucsp.br/index.php/interdisciplinaridade/article/view/22623>
- Fernandes, M. B. S. (2014). *Funções lineares no ensino médio: Contextualizações e representações* [Tese de doutorado, Universidade Federal da Paraíba].
- Gomes, S. C. (2011). *Elaboração e aplicação de uma sequência de atividades para o ensino*

- da trigonometria numa abordagem histórica* [Dissertação de mestrado, Universidade Federal do Rio Grande do Norte].
- Japiassu, H. (1976). *Interdisciplinaridade e patologia do saber*. Imago.
- Macêdo, J. A. & Martins, C. F. R. (2026). Mathematics teachers' pedagogical practices in emergency remote teaching in a Brazilian municipality. *Revista Colombiana de Educación*, (100), e21580. <https://doi.org/10.17227/rce.num100-21580>
- Martins, C. F. R. & Macêdo, J. A. (2023). Ferramentas digitais: Uma possibilidade educacional em tempos de pandemia. *Revista Internacional de Pesquisa em Educação Matemática*, 13(1), 1–17. <https://doi.org/10.37001/ripem.v13i1.3326>
- Minayo, M. C. S. (2002). Ciência, técnica e arte: O desafio da pesquisa social. In M. C. S. Minayo (Org.), *Pesquisa social: Teoria, método e criatividade* (pp. 9–29). Vozes.
- Moretti, M. T. & Luiz, L. S. (2014). O procedimento informático de interpretação global no esboço de curvas no ensino universitário. In C. F. Brandt & M. T. Moretti (Orgs.), *As contribuições da teoria das representações semióticas para o ensino e pesquisa da educação matemática* (pp. 67–88). Unijuí.
- Oliveira, A. M. R. & Macêdo, J. A. (2025). Desafios de acesso e inovação pedagógica com tecnologias digitais no ensino de matemática: Contribuições do modelo TPACK. *Revista Internacional de Pesquisa em Educação Matemática*, 15(2), 1–21. <https://doi.org/10.37001/ripem.v15i2.4526>
- Oliveira, A. M. R. & Macêdo, J. A. (2026). Percepções de professores da educação básica sobre tecnologias digitais e TPACK no ensino de matemática em escolas públicas do norte de Minas Gerais, Brasil. *RECIE. Revista Caribeña de Investigación Educativa*, 10, e10970. <https://doi.org/10.32541/recie.v10.970>
- Romanello, L. A. (2016). *Potencialidades do uso do celular na sala de aula: Atividades investigativas para o ensino de função* [Dissertação de mestrado, Universidade Estadual Paulista].
- Ruas, V. L. O. F., Macêdo, J. A. & Crisostomo, E. (2024). Decodificando por meio de narrativas o desenvolvimento do TPACK dos docentes de matemática. *REXE – Revista de Estudios y Experiencias en Educación*, 23(51), 153–175. <https://doi.org/10.21703/rexe.v23i51.2210>
- Santos, R. P. & Macêdo, J. A. (2023). Ensino de matemática com uso de tecnologias digitais: Enfoque sociopolítico no pós-pandemia da COVID-19. *Revista Cocar*, 19(37), 1–21.
- Santos, R. P. & Macêdo, J. A. (2024). As possibilidades didático-pedagógicas do uso de softwares matemáticos no ensino de matemática durante a pandemia da COVID-19. *Revista Internacional de Pesquisa em Educação Matemática*, 14(1), 1–14. <https://doi.org/10.37001/ripem.v14i1.3650>
- Silva, M. P. & Macêdo, J. A. (2025). Conhecimentos didático-matemáticos mobilizados por professores durante a aula de matemática. *Revista de Estudios y Experiencias en Educación*, 24(54), 157–182. <https://doi.org/10.21703/rexe.v24i54.2888>
- Silva, M. P. & Macêdo, J. A. (2026). Planejamento de aula e as categorias do conhecimento didático-matemático dos professores dos anos iniciais do ensino fundamental. *Paradigma*, 47(1), e2026006. <https://doi.org/10.37618/PARADIGMA.1011-2251.2026.e2026006.id1752>
- Souza, E. R. (2025). *O uso do software Modellus 4.0 como ferramenta interdisciplinar: Um*

estudo sobre fenômenos físico-matemáticos na perspectiva da teoria dos registros de representação semiótica [Dissertação de mestrado, Universidade Estadual de Montes Claros].

Teodoro, V. D. (2002). *Modellus: Learning physics with mathematical modelling* [Tese de doutorado, Universidade Nova de Lisboa].